Mathematical Tripos Part IA

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Dr P. J. O'Donnell

Mechanics (non-examinable) Examples sheet 2

Comments and corrections: e-mail to P.J.ODonnell@damtp.cam.ac.uk. All examples sheets and solutions are available on www.damtp.cam.ac.uk/user/po242/mechanics.html

On these sheets, no attempt is made to 'model' real-life situations: no trains, cars, cyclists, lifts, etc. It is assumed that there are no 'real' forces, such as air-resistance unless they are specifically mentioned. Most questions, but not all, avoid numbers and units, preferring general algebraic formulae with consistent dimensions.

1 A horizontal light rod AB has length l is supported by vertical strings fixed to its endpoints. A mass m is placed on the rod at A and a mass M is placed at the centre of the rod. Find the tension in each of the strings.

2 The foot of a light ladder rests against on a smooth horizontal floor and the other end rests against a smooth vertical wall. The foot of the ladder is held by a horizontal string attached to the wall and the angle between the ladder and the horizontal is α . A mass *m* is placed at the midpoint of the ladder. Show that the normal reaction of the wall on the top of the ladder is $\frac{1}{2}mg \cot \alpha$ and find the tension in the string.

3 A uniform rod AB of mass m rests with end A in contact with a smooth vertical wall and end B attached to a string BC where C is a point on the vertical wall above A (the angle CAB is obtuse). Show that the reaction of the wall on the rod is $\frac{1}{2}mg \tan \theta$, where θ is the acute angle between the wall and the rod.

Show also that

$\tan\theta = 2\tan\phi$

where ϕ is the acute angle between the string and the vertical.

[You can regard the rod as being light with a mass m placed at the midpoint.]

4 The foot of a uniform ladder of weight mg rests on a rough floor and the other end rests against a smooth wall. The ladder makes an angle α to the horizontal, and the coefficient of friction between the ladder and the floor is μ . Show that $\frac{1}{2} \cot \alpha \leq \mu$.

[You can regard the ladder as being light with a mass m placed at the midpoint.]