Mathematical Tripos Part IA Michaelmas term 2015

Mechanics (non-examinable) Examples sheet 4 Dr P. J. O'Donnell

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On these sheets, no attempt is made to 'model' real-life situations: no trains, cars, cyclists, lifts, etc. It is assumed that there are no 'real' forces, such as air-resistance unless they are specifically mentioned. Most questions, but not all, avoid numbers and units, preferring general algebraic formulae with consistent dimensions.

1 A particle moving in a straight line has initial speed u and accelerates with constant acceleration a, where $a < 0$, until it comes to rest. How long does this take and how far has the particle travelled?

2 A particle moving in a straight line accelerates from rest at constant acceleration b and then decelerates to rest again with constant deceleration $b/2$, moving a total distance d in time t. Draw a graph of velocity against time. Show that $d = \frac{1}{6}bt^2$.

3 The acceleration of a particle moving in a straight line, at time t after starting from rest is $5 + 4t - t^2$ for $0 \leq t \leq 5$ and zero for other values of t. Find:

(i) the maximum acceleration;

(ii) the greatest speed;

(iii) the distance travelled until the particle attains its greatest speed.

4 A particle is projected upwards at initial speed 25 $\mathrm{m\,s^{-1}}$. Taking $g = 10\,\mathrm{ms^{-2}}$ (actually, g is more like $9.8\,\mathrm{ms^{-2}}$), find the length of time for which the height of the above the point of projection exceeds 30 m.

5 A particle, originally at the origin and moving at velocity $\mathbf{u}$, accelerates with constant acceleration $\mathbf{a}$ until, at displacement $\mathbf{x}$ from the origin, it attains velocity $\mathbf{v}$. Show that

$$v^2 - u^2 = 2\mathbf{a}.\mathbf{x},$$

where u and v are the initial and final speeds of the particle.

6 A particle is projected at speed V from ground level at an angle of α above the horizontal. Show that its maximum height is $\frac{1}{2}V^2 \sin^2\alpha/g$ and that the particle lands a distance $V^2 \sin 2\alpha/g$ from the point of projection. Deduce that the maximum range is obtained by firing the particle at $\alpha = \pi/4$.

Question 1: time $= u/(-a)$; distance $= u^2/(-2a)$.

Question 2: the graph looks like a triangle because the velocity increases at a constant rate. The diagram is useful to display the information, but you can also use it for the calculation since the area underneath is the distance moved (check you understand this!).

Question 3: maximum acceleration $= 9$; greatest speed $= 100/3$ (when $t = 5$); distance to greatest speed $= 375/4$.

Question 4: Draw a diagram (distance against time)! It reaches 30 on the way up at $t = 2$ and on the way down at $t = 3$ so time is one second.