Mathematical Tripos Part IA — Michaelmas term 2015

Mechanics (non-examinable) Examples sheet 5 — Dr P. J. O'Donnell

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On these sheets, no attempt is made to 'model' real-life situations: no trains, cars, cyclists, lifts, etc. It is assumed that there are no 'real' forces, such as air-resistance unless they are specifically mentioned. Most questions, but not all, avoid numbers and units, preferring general algebraic formulae with consistent dimensions.

1 A car is connected to a caravan of mass M by a light rod. The two vehicles are moving with acceleration a. Find the tension in the rod.

How would your calculation differ if the rod has mass μ? What would then be the tensions in the rod at its ends?

2 A particle of mass M is placed on a smooth horizontal table and connected by a light inelastic string passing over a small smooth pulley at the edge to a mass m hanging freely below the edge of the table. Find the acceleration of the particles and the tension in the string. Show that the magnitude of the force on the pulley due to the tension in the string is

$$\frac{\sqrt{2}Mmg}{M+m}.$$

3 A particle slides down a rough plane inclined at angle α to the horizontal. The coefficient of (sliding — assume the usual $F = \mu R$ relation) friction is μ. Show that the acceleration of the particle is $g(\sin\alpha - \mu\cos\alpha)$.

If instead the particle were projected up the plane, find its deceleration.

4 A particle of mass m slides down the smooth face of a wedge of mass M and slope α, which is free to slide on a smooth table. Let R be the normal reaction of the wedge on the particle and let S be the normal reaction of the table on the wedge. Give two diagrams, one showing the forces on the particle and the other showing the forces on the wedge.

Let f be the acceleration of the particle down the wedge (relative to the wedge) and let F be the acceleration of the wedge. Show that

$$S = R\cos\alpha + Mg \qquad (1)$$

$$R\sin\alpha = m(f\cos\alpha - F) \qquad (2)$$

and derive two more equations of motion.

Hence (no need to use equation (1)) show that $F = mg\dfrac{\sin\alpha\cos\alpha}{M + m\sin^2\alpha}$.