Mathematical Tripos Part IA Michaelmas term 2015

Mechanics (non-examinable) Examples sheet 6 Dr P. J. O'Donnell

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On these sheets, no attempt is made to 'model' real-life situations: no trains, cars, cyclists, lifts, governors of steam engines, etc. It is assumed that there are no 'real' forces, such as air-resistance unless they are specifically mentioned. Most questions, but not all, avoid numbers and units, preferring general algebraic formulae with consistent dimensions.

1 A particle is released from rest and falls under the action of gravity. By calculating its speed after falling a distance h, verify that the principle of conservation of energy holds in this situation.

2 A particle of mass m is projected with initial speed u up a line of maximum slope of a rough plane inclined at α to the horizontal. The coefficient of friction is μ (assume the frictional force is $\mu \times$ normal reaction). Show that the acceleration down the plane is $g(\sin\alpha + \mu\cos\alpha)$ and hence calculate the distance s up the plane at which the particle comes to rest. Use the conservation of energy to show that the amount of work done against the frictional force is

$$\frac{1}{2}mu^2\frac{\mu\cos\alpha}{\sin\alpha+\mu\cos\alpha}.$$

Verify this result using work done against frictional force = frictional force $\times$ distance moved.

3 Use the principle of conservation of energy *carefully!* to find the maximum height of a particle projected with speed V at an angle of α to the horizontal, noting that the horizontal velocity is constant.

4 A light inelastic string passes over two small smooth pulleys A and B at the same horizontal level a distance $2a$ apart. Particles of mass m are attached to either end and a particle of mass M (where $M < 2m$) is attached to the midpoint. The system is released from rest with the particle of mass M at the midpoint of AB. Using conservation of energy, show that the system next comes to rest when the particle of mass M has fallen a distance

$$\frac{4amM}{4m^2-M^2}.$$

5 Water is pumped to the surface of the Earth from a depth d and issues from a pipe of cross-sectional area A at a speed of v. The density of the water is ρ. Using energy considerations, find the power of the pump.