Mathematical Tripos Part IA Michaelmas term 2015

Mechanics (non-examinable) Examples sheet 7 Dr P. J. O'Donnell

Comments and corrections: e-mail to P.J.ODonnell@damtp.cam.ac.uk. *All examples sheets and solutions are available on www.damtp.cam.ac.uk/user/po242/mechanics.html*

On these sheets, no attempt is made to 'model' real-life situations: no trains, cars, cyclists, lifts, governors of steam engines, etc. It is assumed that there are no 'real' forces, such as air-resistance unless they are specifically mentioned. Most questions, but not all, avoid numbers and units, preferring general algebraic formulae with consistent dimensions.

1 Two particles each of mass m collide and coalesce. The velocities of the particles before impact were $(u, 0)$ and $(v\cos\theta, v\sin\theta)$. Use conservation of momentum to find an expression for the speed of the particle after the collision and find also the loss of kinetic energy.

2 Particles of mass m are attached to the ends of a light rigid rod. The rod lies along the x-axis. One particle of the rod is hit by a blow that would cause it, were it not attached to the rod, to move with velocity $(u\cos\theta, u\sin\theta)$. What is the impulse of the blow?

This particle actually moves with initial velocity $(v\cos\phi, v\sin\phi)$. Assuming that total momentum is conserved, show that $\tan\phi = 2\tan\theta$. (Note that the rod is rigid so the other particle's initial velocity is necessarily in the direction of the rod.)

3 Two spheres of masses m_1 and m_2 moving with speeds u_1 and u_2 in the same direction collide head on. The collision is perfectly elastic (e=1). If the speeds after collision are v_1 and v_2, show that

$$v_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2}.$$

Verify in the case $m_1 = m_2$ that kinetic energy in conserved in the collision. What does this result give if one sphere is fixed?

4 A particle of mass m strikes a surface with speed u, its trajectory making an angle θ with the normal to the surface. It rebounds with speed v. Find the coefficient of restitution between the surface and the particle.

Show that the impulse on the surface is $mu(1 + e)\cos\theta$.

[Note: the component of velocity parallel to the surface is unaffected by the collision but the vertical component obeys Newton's experimental law.]