Mathematical Tripos Part IA Michaelmas term 2015

Mechanics (non-examinable) Examples sheet 8 Dr P. J. O'Donnell

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On these sheets, no attempt is made to 'model' real-life situations: no trains, cars, cyclists, lifts, differential screws, etc. It is assumed that there are no 'real' forces, such as air-resistance unless they are specifically mentioned. Most questions, but not all, avoid numbers and units, preferring general algebraic formulae with consistent dimensions.

1 A particle of mass m is attached to the lower end of a string of natural length (unstretched) l_o suspended from a point A. The modulus of elasticity of the string is λ (so that the restoring force is λ/l_o times the extension of the string from its natural length). Initially, the particle is held vertically below A at a distance of l_o below A. The particle is released. Find the distance below A at which the particle first comes to rest.

[You can do this either by writing down the equation of motion, writing down its solution and applying the initial conditions OR by conservation of energy — the potential energy in the stretched string is $\frac{1}{2}\lambda x^2/l_o$, where x is the extension from its natural length.]

2 The end A of a uniform rod AB of mass m and length $3l$ is hinged on a vertical pole so that it can move in a vertical plane. A light elastic string BC of natural (unstretched) length l is attached to the rod at B. The end C of the string is fixed to the pole a distance $5l$ above A. The rod rests in equilibrium and the angle ABC is a rightangle. Show that the modulus of elasticity of the string is $2mg/15$.

[Again, you need to know that the tension in a stretched string is modulus of elasticity times extension / unstretched length. You can do this by taking moments about a suitably chosen point.]

3 A spring with spring constant k and natural length l lies on a smooth horizontal table and is fixed at its ends to two points a distance l apart. A particle of mass m is attached to its midpoint. The particle is displaced a distance x_0 along the line of the spring and released. Find the period of the subsequent SHM and the position of the particle a time t after it was released.

4 A particle of mass m is suspended from a light spring of natural length l and spring constant k causing an extension d. It rests in equilibrium with the string hanging vertically. A particle of mass M is attached to the particle of mass m. Find the period of the ensuing motion in terms of m, M and k. Also, find the amplitude of the oscillation. Determine the amplitude directly using conservation of energy.