Worked Example
Three Balls Colliding in One Dimension

Three solid balls A, B, C, of masses $2m$, m and $5m$ respectively, lie in a row. The coefficient of restitution between any pair of balls is $\frac{1}{2}$. Ball A is projected towards B with speed U: find the speed of ball C after all collisions have taken place.

We first find general formulae governing a collision between two balls with masses m_1 and m_2 travelling at speeds u_1 and u_2 respectively. If their speeds after the collision are v_1 and v_2 respectively then conservation of momentum gives us that

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

while Newton's law of restitution gives us that

$$v_2 - v_1 = \tfrac{1}{2}(u_1 - u_2).$$

These simultaneous equations are easily solved for v_1 and v_2:

$$v_1 = \frac{2m_1 - m_2}{2(m_1 + m_2)} u_1 + \frac{3m_2}{2(m_1 + m_2)} u_2,$$
$$v_2 = \frac{3m_1}{2(m_1 + m_2)} u_1 + \frac{2m_2 - m_1}{2(m_1 + m_2)} u_2.$$

The solution looks simpler if we define $\alpha = m_1/(m_1 + m_2)$, $\beta = m_2/(m_1 + m_2)$:

$$v_1 = (\alpha - \tfrac{1}{2}\beta) u_1 + \tfrac{3}{2}\beta u_2,$$
$$v_2 = \tfrac{3}{2}\alpha u_1 + (\beta - \tfrac{1}{2}\alpha) u_2.$$

We are now ready to examine each collision in turn.

The first collision is between balls A and B, so $\alpha = 2m/(3m) = \frac{2}{3}$, $\beta = m/(3m) = \frac{1}{3}$, $u_1 = U$ and $u_2 = 0$. Hence $v_1 = \frac{1}{2}u_1 + \frac{1}{2}u_2 = \frac{1}{2}U$, $v_2 = u_1 = U$.

The second collision, between B and C, has $\alpha = m/(6m) = \frac{1}{6}$, $\beta = 5m/(6m) = \frac{5}{6}$, $u_1 = U$ and $u_2 = 0$. Hence $v_1 = -\frac{1}{4}u_1 + \frac{5}{4}u_2 = -\frac{1}{4}U$, $v_2 = \frac{1}{4}u_1 + \frac{3}{4}u_2 = \frac{1}{4}U$.

Balls A and B now collide again, with $u_1 = \frac{1}{2}U$, $u_2 = -\frac{1}{4}U$. Hence $v_1 = \frac{1}{2}u_1 + \frac{1}{2}u_2 = \frac{1}{8}U$, $v_2 = u_1 = \frac{1}{2}U$.

Now balls B and C collide again, with $u_1 = \frac{1}{2}U$, $u_2 = \frac{1}{4}U$. Hence $v_1 = -\frac{1}{4}u_1 + \frac{5}{4}u_2 = \frac{3}{16}U$, $v_2 = \frac{1}{4}u_1 + \frac{3}{4}u_2 = \frac{5}{16}U$.

Since the speed of C is now greater than the speed of B which is itself greater than the speed of A, no more collisions will take place. The required speed is therefore $\frac{5}{16}U$.