Worked Example
A Ping-Pong Ball Bouncing on a Plane

A ping-pong ball is projected with initial speed V_0 at an angle θ_0 to the horizontal, and bounces on a slippery horizontal plane with coefficient of restitution e. For how long does it bounce, and how far does it travel along the plane?

Elementary projectile theory tells us that the time taken for the ball to return to the plane for the first time is

$$t_0 = \frac{2V_0 \sin\theta_0}{g},$$

and the range is

$$l_0 = \frac{2V_0^2 \sin\theta_0 \cos\theta_0}{g}.$$

Its velocity at that time is $(V_0 \cos\theta_0, -V_0 \sin\theta_0)$.

Just after the first impact with the plane, suppose that the ball now has speed V_1 at an angle θ_1 to the horizontal. The effect of the impact is that the ball's horizontal speed is unchanged (because there is no friction on a "slippery plane") but its vertical speed is reversed and multiplied by the factor e.* Hence its total velocity just after the impact is

$$(V_1 \cos\theta_1, V_1 \sin\theta_1) = (V_0 \cos\theta_0, eV_0 \sin\theta_0). \qquad (*)$$

The time taken for the second bounce is

$$t_1 = \frac{2V_1 \sin\theta_1}{g} = \frac{2eV_0 \sin\theta_0}{g} = et_0$$

and the range of the second bounce is

$$l_1 = \frac{2V_1^2 \sin\theta_1 \cos\theta_1}{g} = \frac{2eV_0^2 \sin\theta_0 \cos\theta_0}{g} = el_0$$

using $(*)$. It is clear (formally, by induction) that

$$t_n = e^n t_0, \qquad l_n = e^n l_0.$$

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* In the notation of the notes, the speed of the plane is $u_1 = v_1 = 0$, so $v_2 = -eu_2$.

Hence the total time taken to bounce is

$$T = \sum_{n=0}^{\infty} e^n t_0 = \frac{2V_0 \sin\theta_0}{g(1-e)}$$

(which is finite!) and the distance travelled horizontally is

$$L = \sum_{n=0}^{\infty} e^n l_0 = \frac{2V_0^2 \sin\theta_0 \cos\theta_0}{g(1-e)}.$$

What happens after the ball has finished bouncing? It has no vertical component of velocity left, but it still has horizontal speed $V_0 \cos\theta_0$; so it just slides along the plane at constant speed.