Worked Example

Dimensional Analysis of Oil Flow

Oil of viscosity $\nu$ flows through a long straight pipe of cross-sectional area $A$, driven by a pressure drop $p$ per unit length. If $A$ is doubled without altering any of the other parameters, how will the volume flow rate $Q$ (i.e., the volume passing through the pipe per unit time) be affected?

(The dimensions of viscosity are given to be $[\nu] = ML^{-1}T^{-1}$.)

Noting that $p$ has dimensions of pressure per unit length, we see that

$$[A] = L^2$$
$$[p] = [\text{force}][\text{area}]^{-1}L^{-1} = (MLT^{-2})(L^2)(L^{-1}) = ML^{-2}T^{-2}$$
$$[Q] = L^3T^{-1}$$

So if

$$Q = CA^\alpha p^\beta \nu^\gamma$$

where $C$ is a dimensionless constant, then by comparing dimensions of mass, length and time respectively we obtain

$$\begin{align*}
\beta + \gamma &= 0 \\
2\alpha - 2\beta - \gamma &= 3 \\
-2\beta - \gamma &= -1
\end{align*}$$

$$\implies \quad \alpha = 2, \quad \beta = 1, \quad \gamma = -1$$

and hence

$$Q \propto \frac{pA^2}{\nu}.$$  

So doubling the cross-sectional area of the oil pipe makes it possible to pump 4 times as much oil.