Worked Example
Dimensional Analysis of Oil Flow

Oil of viscosity ν flows through a long straight pipe of cross-sectional area A, driven by a pressure drop p per unit length. If A is doubled without altering any of the other parameters, how will the volume flow rate Q (i.e., the volume passing through the pipe per unit time) be affected?

(The dimensions of viscosity are given to be $[\nu] = ML^{-1}T^{-1}$.)

Noting that p has dimensions of pressure *per unit length*, we see that

$$\begin{aligned} [A] &= L^2 \\ [p] &= [\text{force}][\text{area}]^{-1}L^{-1} = (MLT^{-2})(L^2)^{-1}(L^{-1}) = ML^{-2}T^{-2} \\ [Q] &= L^3T^{-1} \end{aligned}$$

So if

$$Q = CA^\alpha p^\beta \nu^\gamma$$

where C is a dimensionless constant, then by comparing dimensions of mass, length and time respectively we obtain

$$\left.\begin{aligned} \beta + \gamma &= 0 \\ 2\alpha - 2\beta - \gamma &= 3 \\ -2\beta - \gamma &= -1 \end{aligned}\right\} \qquad \Longrightarrow \qquad \alpha = 2, \quad \beta = 1, \quad \gamma = -1$$

and hence

$$Q \propto \frac{pA^2}{\nu}.$$

So doubling the cross-sectional area of the oil pipe makes it possible to pump 4 times as much oil.