Concepts in Theoretical Physics

Lecture 5: Quantum Mechanics

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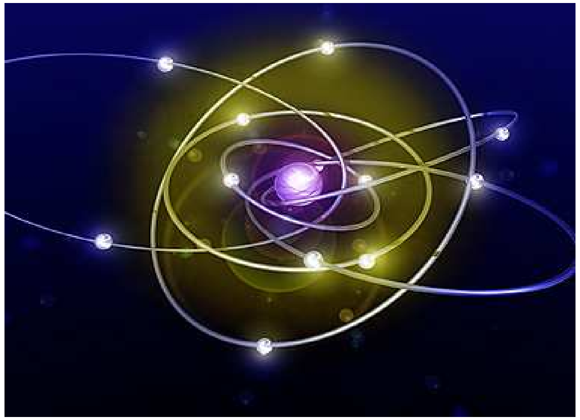
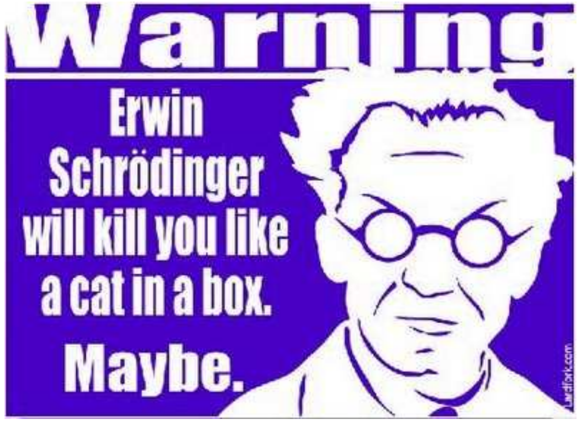
Slogans of Quantum Mechanics

- Wave Particle Duality
- Discrete “Quantum” Energy
- Heisenberg’s Uncertainty Principle
- Schrodinger’s Cat
- Feynman’s Sum over Histories

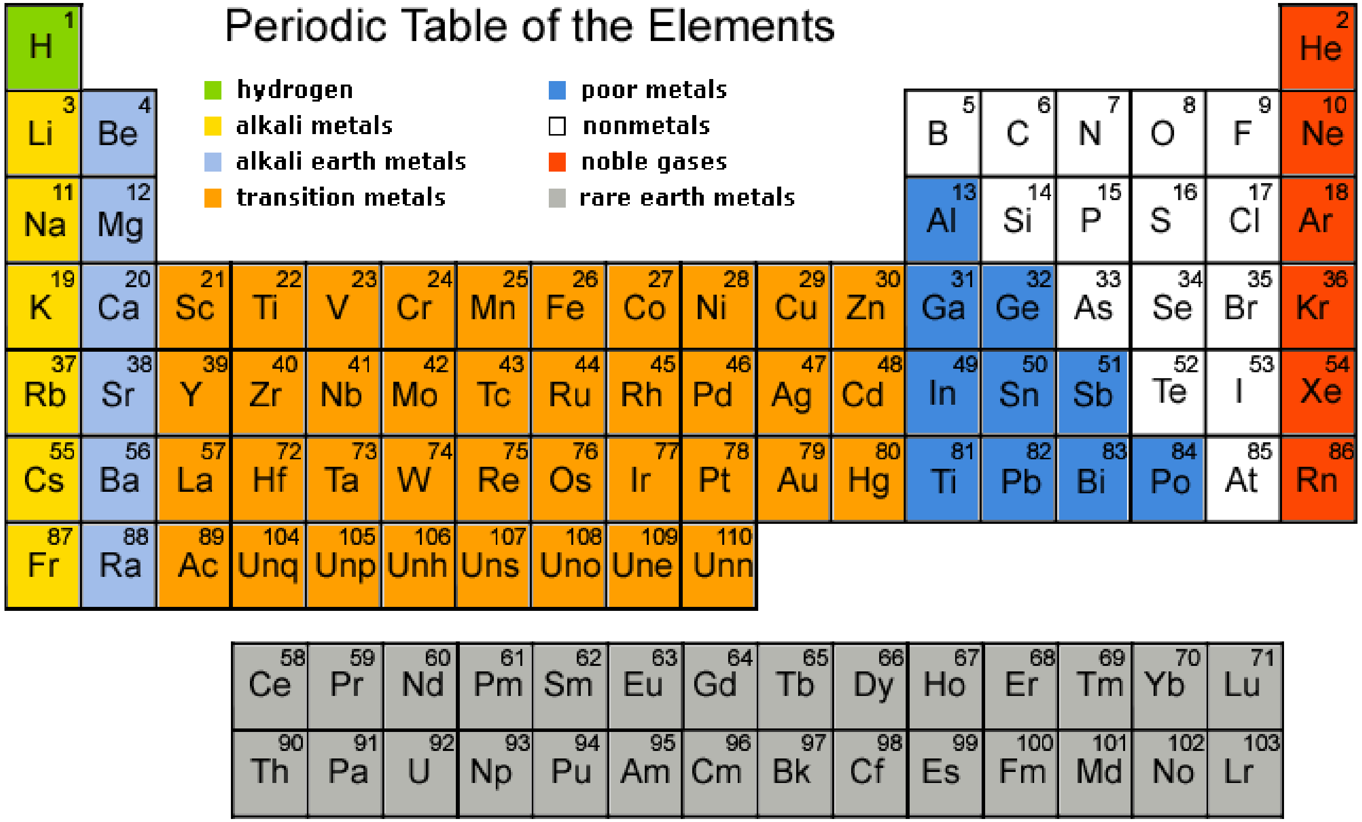




What is it Good For?

- It is the way the universe works at the deepest level.
- Technological developments.
- Philosophical questions.
- New developments in pure mathematics.

Atomic Physics



Periodic Table of the Elements

- 🟩 hydrogen
- 🟨 alkali metals
- ▧ alkali earth metals
- 🟧 transition metals
- 🟦 poor metals
- ☐ nonmetals
- 🟥 noble gases
- ▨ rare earth metals

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Ti	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Unq	105 Unp	106 Unh	107 Uns	108 Uno	109 Une	110 Unn								
			58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
			90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

Cold Physics

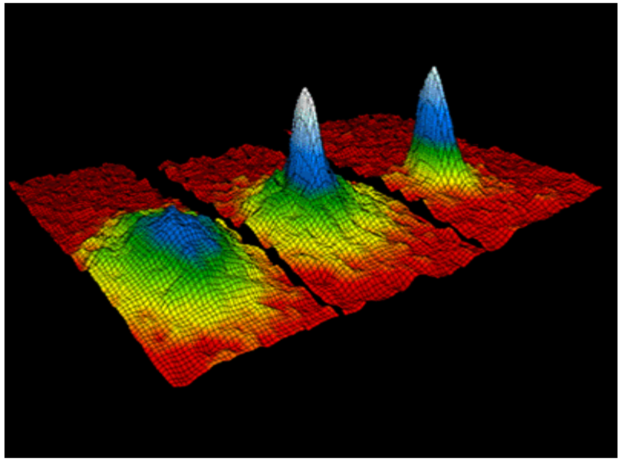
- Quantum Mechanics is the way the universe works. But often its effects are washed out unless we look at very small scales

- Or very cold temperatures

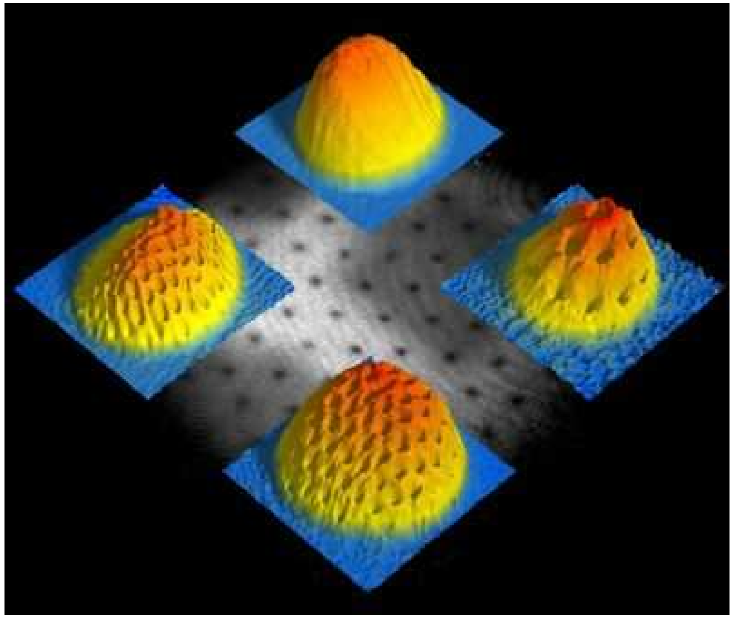
- Much recent progress has been in understanding how quantum mechanics affects macroscopic numbers of atoms

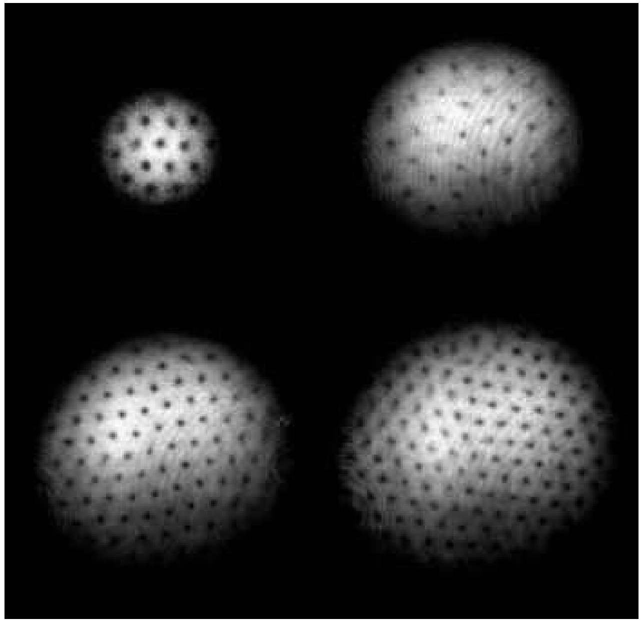
Macroscopic Quantum Effects

- Superconductivity (discovered 1911, understood 1957)
  - High Temperature Superconductivity (discovered 1986, still to be understood)
- Superfluidity (discovered 1937, understood 1950's and 1960's)
- Bose-Einstein Condensation (understood 1925, discovered 1995)



Rotating Bose-Einstein Condensates





- Quantized rotation: number of vortices = spin

Dividing the Indivisible

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News and Views

Nature **452**, 822-823 (17 April 2008) | doi:10.1038/452823a; Published online 1 April 2008

Quantum physics: Debut of the quarter electron

Eduardo Fradkin[1]

A particle-like object with a quarter of an electron's charge is the latest find in a hotbed of quantum-physical experimentation, the fractional quantum Hall fluid. Its significance is more than esoteric.

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- Electrons are elementary particles. They have no constituent parts
- Yet we frequently perform experiments where they split up!
- This is an article from 2 weeks ago

The Framework of Quantum Mechanics

- Quantum Mechanics is a “framework” rather than a “theory”

The Framework of Quantum Mechanics

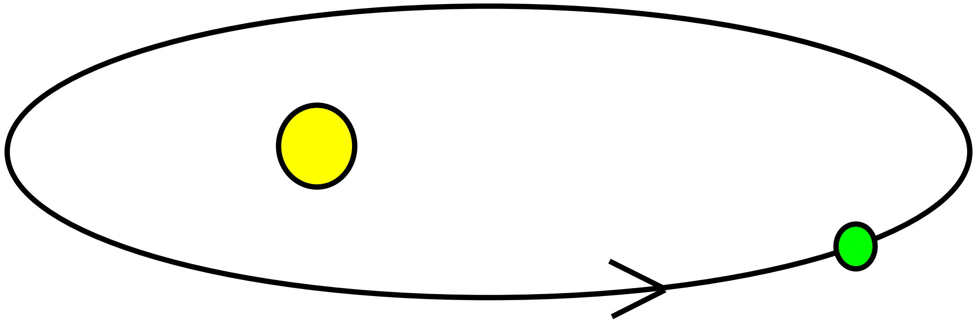
- For example, suppose you’re given Coulomb’s law, describing how electric charges experience a force

$$F = \frac{Q_1 Q_2}{4\pi r^2}$$

- You can either choose to think in the classical framework, which means that we plug this into F=ma.
- Or you could choose to think in the quantum framework, which means that you plug this into Schrodinger’s equation.
- It’s like running a programme on different operating systems...and the operating system of the universe is quantum mechanics

Classical Orbit of an Electron

- Consider the electron orbiting the proton.
- The classical problem (i.e. F=ma) is exactly the same as a planet orbiting the sun.
- The orbits are ellipses, with the sun (or proton) at one focus.

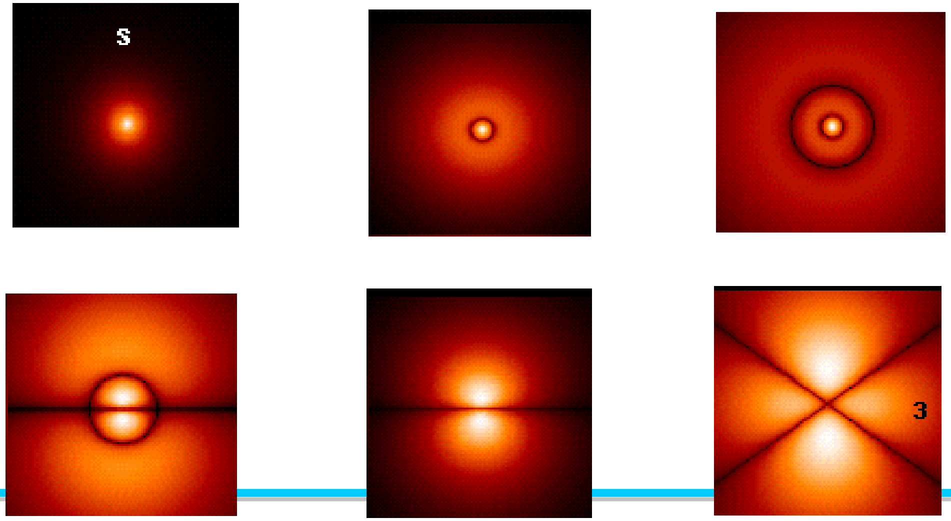


- But there's no restriction on the size or eccentricity of the orbit….that depends only on initial conditions of the problem

The question of the distances between the planets used to be the biggest open problem in science. But we now know that it's a complicated question that isn't answered by the fundamental theory.

Quantum Orbits of an Electron

- In quantum mechanics the answer is very different
- The electron can only sit in very particular orbits.
- Yet, in each of these orbits, its position is undetermined. It is smeared out, a wave of probability.



The Mathematical Framework

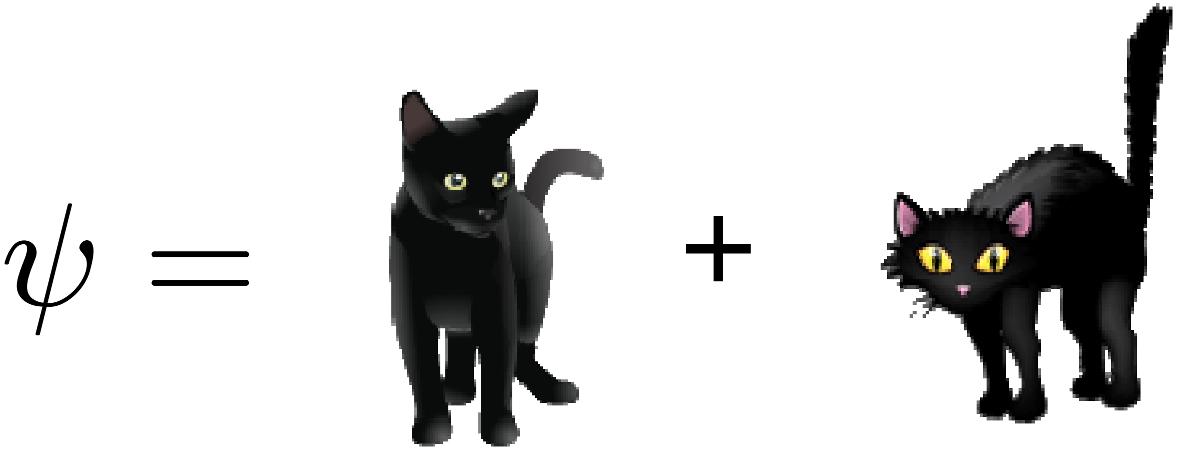
- The mathematical framework of quantum mechanics was covered in “Vectors and Matrices”, with more in next year’s “Linear Algebra”.
- However, this may not be apparent when taking your first “Quantum Mechanics” course next year, where “Differential Equations” will appear more important.

The Principle of Superposition

- The *state* of a system consists of all the information that's required to determine the state of the system at all times in the future
- In Classical Mechanics, the state of a system is by given the positions and momenta of all the particles.

The Principle of Superposition

- In quantum mechanics, the state lives in a vector space. This means that we are allowed to add and subtract states…something which makes no sense in classical mechanics.



- The vector ψ is called the *wavefunction.* It is typically a vector in an infinite dimensional vector space, known as a *Hilbert space.*

Eigenvectors and Eigenvalues

- While the state of the system is a vector, the measurements that we do on a system are matrices. We have a different matrix for each type of measurement: e.g. position, momentum, energy…
- The possible outcomes of a measurement are the eigenvalues of the matrix.

$$H\psi = E\psi$$

matrix

eigenvalue

- For example: if H is the matrix representing a measurement of energy, then the eigenvalues E are the possible outcomes of that measurement
- When H is energy, this is known as the *Schrodinger Equation.*

Probability

- What happens if our state ψ is not an eigenvector of the matrix we are measuring?
- Then the measurement could give any one of the eigenvalues E_i . Each occurs with some probability
- We expand our state in a basis of eigenvectors of H.

actual state

$$\psi = \sum_i c_i \psi_i$$

eigenvectors

- Then the probability of the measurement giving E_i is

$$Prob(E_i) = \frac{|c_i|^2}{\sum_j |c_j|^2}$$

The Necessity of Uncertainty

- Most matrices have different eigenvectors. This means that if the state is in an eigenvector of one matrix, it is unlikely to be in an eigenvector of a different matrix.

- So if one type of measurement is certain, another type becomes uncertain.

- This is *Heisenberg's Uncertainty Principle.*
  If we know, say, the position of the particle
  then it's momentum becomes uncertain.
  And vice versa.

Entanglement

- To end, let’s look at one of the more bewildering aspects of quantum mechanics. It is the fact that strange correlations can exist between experiments. This subject is usually called *entanglement*.

Entanglement

- This subject has a venerable history
  - Einstein, Podolsky and Rosen, 1935, tried to use it to disprove quantum mechanics. The argument was roughly: “entanglement” is so ridiculous that it can’t possibly be right.
  - Bell, 1964 showed that one could test through experiment whether entanglement actually occurs.
  - Aspect et. al. 1982 did the experiment.
- Punchline: the universe is a much stranger place than you imagined!
  - I’ll tell you about a result from 1990 by Greenberger, Horne and Zeilinger,* known as GHZ correlations.

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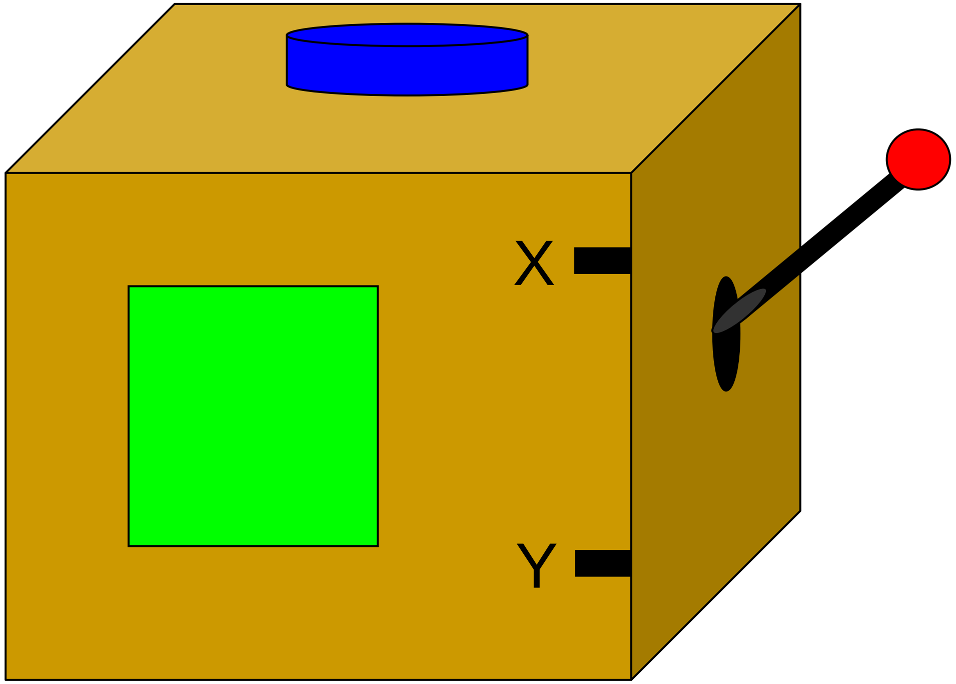
* Via Mermin and Coleman

An Experiment

- Three scientists are each sitting in a lab, separated in spacetime.

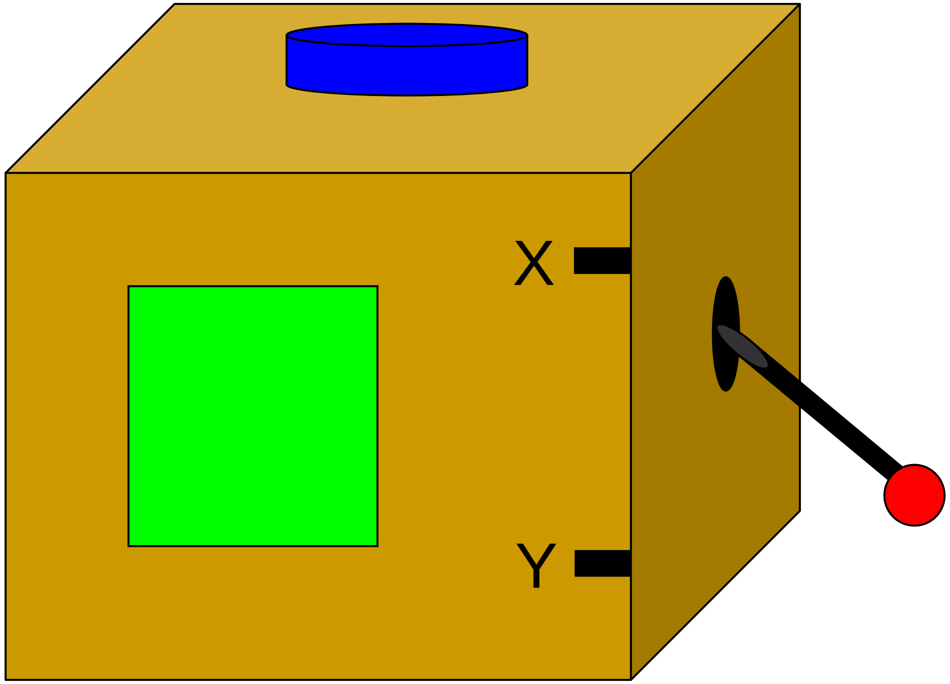
- Every minute, they receive a package sent from a mysterious central station. They are told what they have to do…

An Experiment



- Chose the setting for switch
- Place the sample in the machine
- Press the button, and record whether the result is +1 or -1

An Experiment



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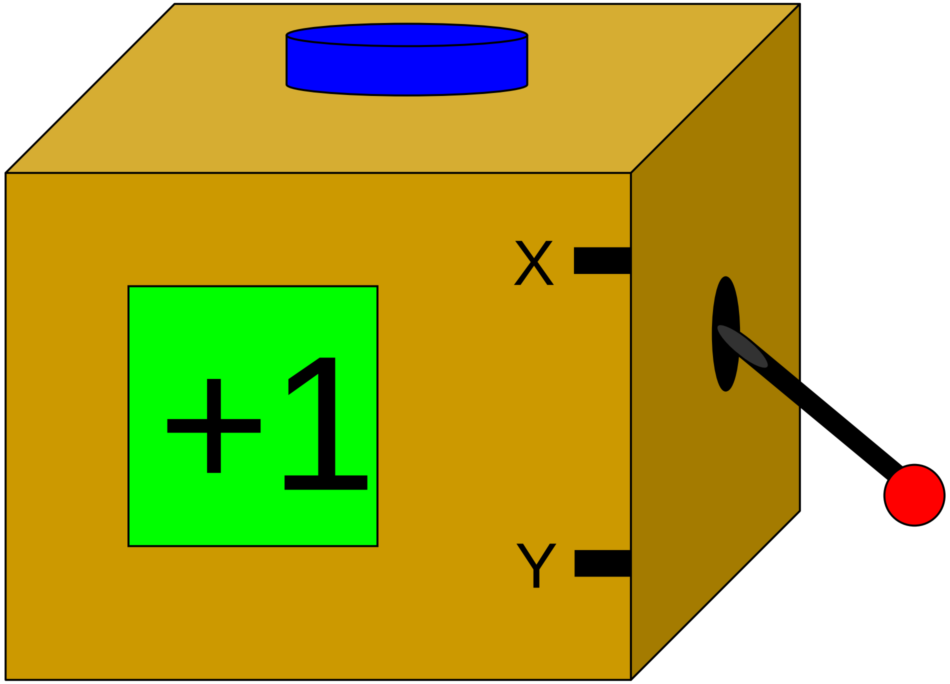

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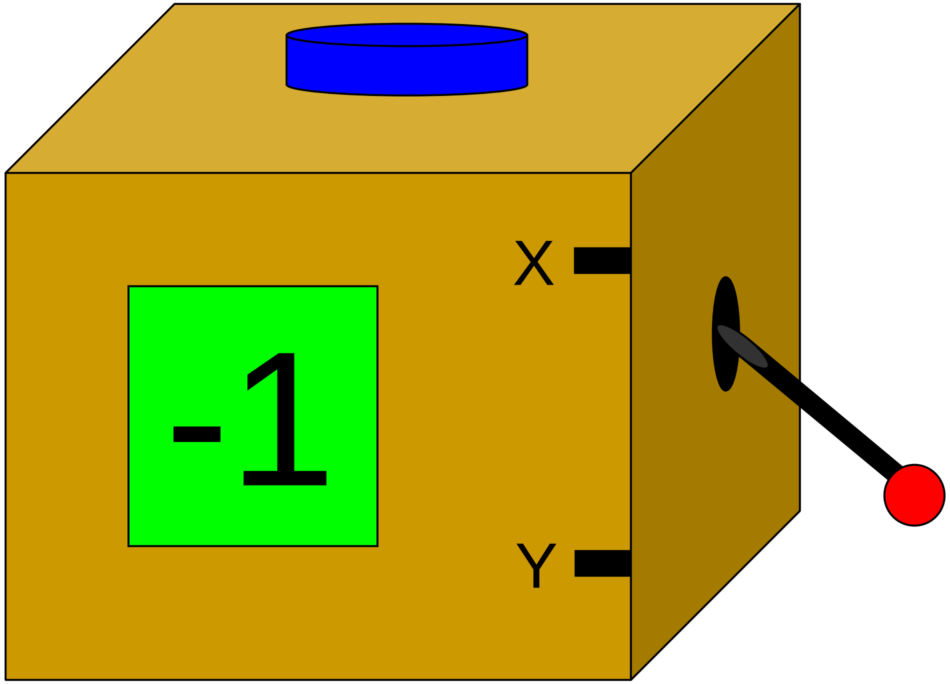

- Chose the setting for switch
- Place the sample in the machine
- Press the button, and record whether the result is +1 or -1

An Experiment

- The scientists are not told what’s in the packages
  - They could be blood samples, with the machine testing for high/low glucose when the switch is on X, and high/low cholesterol when the switch is on Y.
  - They could be elementary particles
  - Or the whole thing could just be a hoax with the machine flashing up +1/-1 at random
- Each measurement is recorded until each scientist has a list that looks like this but with a bazillion entries

X	X	Y	X	Y	Y	X	Y	Y	Y	Y	X	X	Y	X
+1	-1	+1	+1	-1	-1	-1	+1	+1	-1	+1	-1	+1	+1	+1

Looking for Correlations

- Now the scientists get together and start looking for correlations in the measurements. They notice the following.

- Whenever one person measured X, and other two measured Y, the results *always* multiply to +1

$$X_1Y_2Y_3 = Y_1X_2Y_3 = Y_1Y_2X_3 = +1$$

This means the first person measured X, while the second and third people measured Y

Looking for Correlations

- Maybe this occurred because all three got the result +1; or perhaps one got +1 and the other two got -1. There are 8 ways that this could have happened

$$\begin{pmatrix} X_1 = + & Y_1 = + \\ X_2 = + & Y_2 = + \\ X_3 = + & Y_3 = + \end{pmatrix} \quad \begin{pmatrix} X_1 = - & Y_1 = - \\ X_2 = - & Y_2 = - \\ X_3 = + & Y_3 = + \end{pmatrix} \quad \begin{pmatrix} X_1 = - & Y_1 = + \\ X_2 = - & Y_2 = + \\ X_3 = + & Y_3 = - \end{pmatrix}$$

$$\begin{pmatrix} X_1 = - & Y_1 = - \\ X_2 = + & Y_2 = + \\ X_3 = - & Y_3 = - \end{pmatrix} \quad \begin{pmatrix} X_1 = - & Y_1 = + \\ X_2 = + & Y_2 = - \\ X_3 = - & Y_3 = + \end{pmatrix} \quad \begin{pmatrix} X_1 = + & Y_1 = - \\ X_2 = + & Y_2 = - \\ X_3 = + & Y_3 = - \end{pmatrix}$$

$$\begin{pmatrix} X_1 = + & Y_1 = + \\ X_2 = - & Y_2 = - \\ X_3 = - & Y_3 = - \end{pmatrix} \quad \begin{pmatrix} X_1 = + & Y_1 = - \\ X_2 = - & Y_2 = + \\ X_3 = - & Y_3 = + \end{pmatrix}$$

The Prediction

- But this gives a prediction….whenever all three scientists measured X, the results multiplied together *must* give +1

$$\begin{aligned}(X_1Y_2Y_3)(Y_1X_2Y_3)(Y_1Y_2X_3) &= X_1X_2X_3(Y_1Y_2Y_3)^2 \\ &= X_1X_2X_3 \\ &= +1\end{aligned}$$

- This is so simple, it couldn’t even be called a law of physics. It follows from our most basic ideas about how the universe works.

The Astonishing Truth!

- This experiment has been done:*
- The things measured were the polarization of photons. (Spins of elementary particles would work just as well).
- The results are

$$X_1Y_2Y_3 = Y_1X_2Y_3 = Y_1Y_2X_3 = +1$$

$$X_1X_2X_3 = -1$$

- The very basic (classical) intuition for how the universe works is *wrong!*

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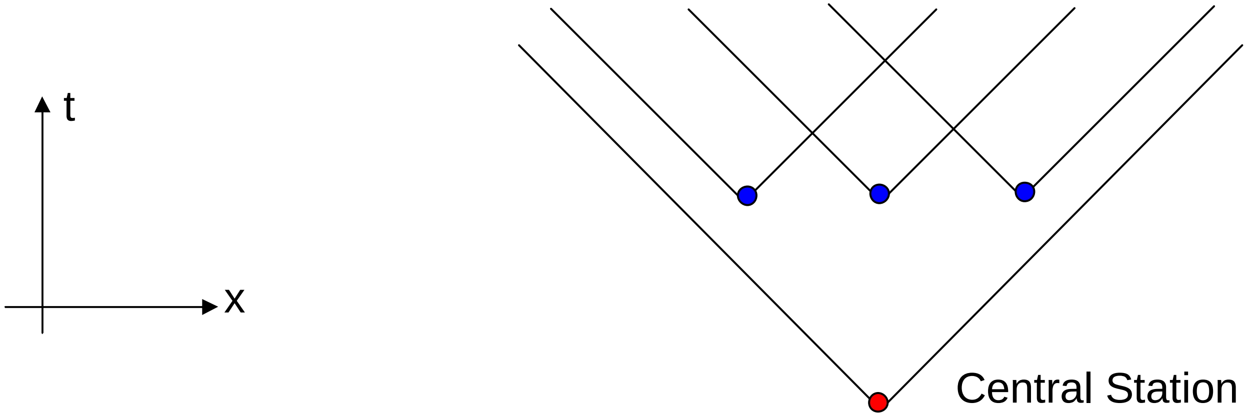
*Pan et. al. Nature 2000, Feb 3;403(6769): 515-519

Violations of Locality?

- An implicit assumption is that the measurements are performed independently, so that experiment 2 has no way of knowing whether the switch on experiment 1 is set to X or Y.
- But we can guarantee that this is true, by placing the scientists at space-like separated points.



- It appears that these correlations require information to be transmitted faster than light!

The Quantum Resolution

- The resolution to the paradox is that we assumed the packages leaving the central station had definite assignments, e.g.

$$\begin{pmatrix} X_1 = - & Y_1 = + \\ X_2 = + & Y_2 = - \\ X_3 = - & Y_3 = + \end{pmatrix}$$

- But in the quantum world, we cannot give definite assignments to all possible measurements. The package that arrived didn't have both X and Y assigned at the same time.

- The GHZ correlations are *almost* nonlocal. In a classical world, the only way you could get such correlations is by non-locality, which implies transmission of information faster than the speed of light.

- But our world is quantum. And such correlations are allowed without faster-than-light communication.

What were they measuring?

- They were measuring spins of particles. The measurement matrices are

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

- You can check that these have eigenvalues +1 and -1, corresponding to the measurements
  - But X and Y do not have the same eigenvectors

- The state that the central station was sending is neither an eigenvector of X nor Y. It is

$$\Psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

What were they measuring?

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- This is an eigenvector of XYY and YXY and YYX.
- And, importantly, it is an eigenvector of XXX.
- Exercise: Check that this gives rise to the observed correlations.