3. Open Strings and D-Branes

In this section we discuss the dynamics of open strings. Clearly their distinguishing feature is the existence of two end points. Our goal is to understand the effect of these end points. The spatial coordinate of the string is parameterized by

$$\sigma \in [0, \pi] \ .$$

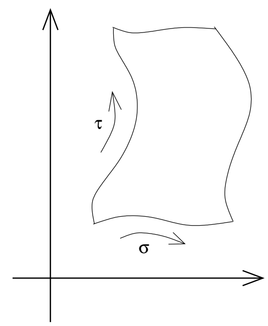

Figure 12:

The dynamics of a generic point on a string is governed by local physics. This means that a generic point has no idea if it is part of a closed string or an open string. The dynamics of an open string must therefore still be described by the Polyakov action. But this must now be supplemented by something else: boundary conditions to tell us how the end points move. To see this, let's look at the Polyakov action in conformal gauge

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \ \partial_\alpha X \cdot \partial^\alpha X \ .$$

As usual, we derive the equations of motion by finding the extrema of the action. This involves an integration by parts. Let's consider the string evolving from some initial configuration at $\tau = \tau_i$ to some final configuration at $\tau = \tau_f$:

$$\begin{aligned} \delta S &= -\frac{1}{2\pi\alpha'} \int_{\tau_i}^{\tau_f} d\tau \int_0^\pi d\sigma \ \ \partial_\alpha X \cdot \partial^\alpha \delta X \\ &= \frac{1}{2\pi\alpha'} \int d^2\sigma \ (\partial^\alpha \partial_\alpha X) \cdot \delta X + \text{total derivative} \end{aligned}$$

For an open string the total derivative picks up the boundary contributions

$$\frac{1}{2\pi\alpha'} \left[\int_0^\pi d\sigma \ \dot{X} \cdot \delta X \right]_{\tau=\tau_i}^{\tau=\tau_f} - \frac{1}{2\pi\alpha'} \left[\int_{\tau_i}^{\tau_f} d\tau \ X' \cdot \delta X \right]_{\sigma=0}^{\sigma=\pi}$$

The first term is the kind that we always get when using the principle of least action. The equations of motion are derived by requiring that $\delta X^\mu = 0$ at $\tau = \tau_i$ and τ_f and so it vanishes. However, the second term is novel. In order for it too to vanish, we require

$$\partial_\sigma X^\mu \, \delta X_\mu = 0 \qquad \text{at } \sigma = 0, \pi$$

There are two different types of boundary conditions that we can impose to satisfy this:

- Neumann boundary conditions.

$$\partial_\sigma X^\mu = 0 \quad \text{at } \sigma = 0, \pi \tag{3.1}$$

Because there is no restriction on δX^μ, this condition allows the end of the string to move freely. To see the consequences of this, it's useful to repeat what we did for the closed string and work in static gauge with $X^0 \equiv t = R\tau$, for some dimensionful constant R. Then, as in equations (1.34), the constraints read

$$\dot{\vec{x}} \cdot \vec{x}' = 0 \quad \text{and} \quad \dot{\vec{x}}^{\,2} + \vec{x}'^{\,2} = R^2$$

But at the end points of the string, $\vec{x}' = 0$. So the second equation tells us that $|d\vec{x}/dt| = 1$. Or, in other words, the end point of the string moves at the speed of light.

- Dirichlet boundary conditions

$$\delta X^\mu = 0 \quad \text{at } \sigma = 0, \pi \tag{3.2}$$

This means that the end points of the string lie at some constant position, $X^\mu = c^\mu$, in space.

At first sight, Dirichlet boundary conditions may seem a little odd. Why on earth would the strings be fixed at some point c^μ? What is special about that point? Historically people were pretty hung up about this and Dirichlet boundary conditions were rarely considered until the mid-1990s. Then everything changed due to an insight of Polchinski...

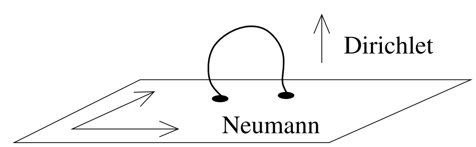

Figure 13:

Let's consider Dirichlet boundary conditions for some coordinates and Neumann for the others. This means that at both end points of the string, we have

$$\begin{aligned} \partial_\sigma X^a &= 0 \qquad \text{for } \; a = 0, \ldots, p \\ X^I &= c^I \qquad \text{for } \; I = p+1, \ldots, D-1 \end{aligned} \tag{3.3}$$

This fixes the end-points of the string to lie in a $(p+1)$-dimensional hypersurface in spacetime such that the $SO(1, D-1)$ Lorentz group is broken to,

$$SO(1, D-1) \to SO(1, p) \times SO(D-p-1) \; .$$

This hypersurface is called a *D-brane* or, when we want to specify its dimension, a Dp-brane. Here D stands for Dirichlet, while p is the number of spatial dimensions of the brane. So, in this language, a D0-brane is a particle; a D1-brane is itself a string; a D2-brane a membrane and so on. The brane sits at specific positions c^I in the transverse space. But what is the interpretation of this hypersurface?

It turns out that the D-brane hypersurface should be thought of as a new, dynamical object in its own right. This is a conceptual leap that is far from obvious. Indeed, it took decades for people to fully appreciate this fact. String theory is not just a theory of strings: it also contains higher dimensional branes. In Section 7.5 we will see how these D-branes develop a life of their own. Some comments:

- We've defined D-branes that are infinite in space. However, we could just as well define finite D-branes by specifying closed surfaces on which the string can end.
- There are many situations where we want to describe strings that have Neumann boundary conditions in all directions, meaning that the string is free to move throughout spacetime. It's best to understand this in terms of a space-filling D-brane. No Dirichlet conditions means D-branes are everywhere!
- The Dp-brane described above always has Neumann boundary conditions in the X^0 direction. What would it mean to have Dirichlet conditions for X^0? Obviously this is a little weird since the object is now localized at a fixed point in time. But there is an interpretation of such an object: it is an *instanton*. This "D-instanton" is usually referred to as a D(-1)-brane. It is related to tunneling effects in the quantum theory.

Mode Expansion

We take the usual mode expansion for the string, with $X^\mu = X^\mu_L(\sigma^+) + X^\mu_R(\sigma^-)$ and

$$\begin{aligned} X^\mu_L(\sigma^+) &= \tfrac{1}{2}x^\mu + \alpha' p^\mu \, \sigma^+ + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \, \tilde{\alpha}^\mu_n \, e^{-in\sigma^+} \;, \\ X^\mu_R(\sigma^-) &= \tfrac{1}{2}x^\mu + \alpha' p^\mu \, \sigma^- + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \, \alpha^\mu_n \, e^{-in\sigma^-} \;. \end{aligned} \tag{3.4}$$

The boundary conditions impose relations on the modes of the string. They are easily checked to be:

- Neumann boundary conditions, $\partial_\sigma X^a = 0$, at the end points require that

$$\alpha^a_n = \tilde{\alpha}^a_n \tag{3.5}$$

- Dirichlet boundary conditions, $X^I = c^I$, at the end points require that

$$x^I = c^I \quad , \quad p^I = 0 \quad , \quad \alpha^I_n = -\tilde{\alpha}^I_n$$

So for both boundary conditions, we only have one set of oscillators, say α_n. The $\tilde{\alpha}_n$ are then determined by the boundary conditions.

It's worth pointing out that there is a factor of 2 difference in the p^μ term between the open string (3.4) and the closed string (1.36). This is to ensure that p^μ for the open string retains the interpretation of the spacetime momentum of the string when $\sigma \in [0, \pi]$. To see this, one needs to check the Noether current associated to translations of X^μ on the worldsheet: it was given in (2.33). The conserved charge is then

$$P^\mu = \int_0^\pi d\sigma \, (P^\tau)^\mu = \frac{1}{2\pi\alpha'} \int_0^\pi d\sigma \, \dot{X}^\mu = p^\mu$$

as advertised. Note that we've needed to use the Neumann conditions (3.5) to ensure that the Fourier modes don't contribute to this integral.

3.1 Quantization

To quantize, we promote the fields x^a and p^a and α_n^μ to operators. The other elements in the mode expansion are fixed by the boundary conditions. An obvious, but important, point is that the position and momentum degrees of freedom, x^a and p^a, have a spacetime index that takes values $a = 0, \ldots, p$. This means that the spatial wavefunctions only depend on the coordinates of the brane not the whole spacetime. Said another, quantizing an open string gives rise to states which are restricted to lie on the brane.

To determine the spectrum, it is again simplest to work in lightcone gauge. The spacetime lightcone coordinate is chosen to lie within the brane,

$$X^\pm = \sqrt{\frac{1}{2}} \, (X^0 \pm X^p)$$

Quantization now proceeds in the same manner as for the closed string until we arrive at the mass formula for states which is a sum over the transverse modes of the string.

$$M^2 = \frac{1}{\alpha'} \left(\sum_{i=1}^{p-1} \sum_{n>0} \alpha_{-n}^i \alpha_n^i + \sum_{i=p+1}^{D-1} \sum_{n>0} \alpha_{-n}^i \alpha_n^i - a \right)$$

The first sum is over modes parallel to the brane, the second over modes perpendicular to the brane. It's worth commenting on the differences with the closed string formula. Firstly, there is an overall factor of 4 difference. This can be traced to the lack of the factor of 1/2 in front of p^μ in the mode expansion that we discussed above. Secondly, there is a sum only over α modes. The $\tilde{\alpha}$ modes are not independent because of the boundary conditions.

Open and Closed

In the mass formula, we have once again left the normal ordering constant a ambiguous. As in the closed string case, requiring the Lorentz symmetry of the quantum theory — this time the reduced symmetry $SO(1, p) \times SO(D - p - 1)$ — forces us to choose

$$D = 26 \quad \text{and} \quad a = 1 \ .$$

These are the same values that we found for the closed string. This reflects an important fact: the open string and closed string are not different theories. They are both different states inside the same theory.

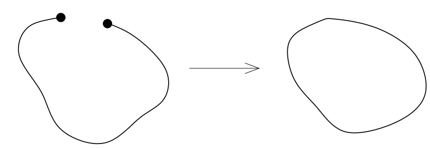

Figure 14:

More precisely, theories of open strings necessarily contain closed strings. This is because, once we consider interactions, an open string can join to form a closed string as shown in the figure. We'll look at interactions in Section 6. The question of whether this works the other way — meaning whether closed string theories require open strings — is a little more involved and is cleanest to state in the context of the superstring. For type II superstrings, the open strings and D-branes are necessary ingredients. For heterotic superstrings, there appear to be no open strings and no D-branes. For the bosonic theory, it seems likely that the open strings are a necessary ingredient although I don't know of a killer argument. But since we're not sure whether the theory exists due to the presence of the tachyon, the point is probably moot. In the remainder of these lectures, we'll view the bosonic string in the same manner as the type II string and assume that the theory includes both closed strings and open strings with their associated D-branes.

3.1.1 The Ground State

The ground state is defined by

$$\alpha_n^i |0; p\rangle = 0 \qquad\qquad n > 0$$

The spatial index now runs over $i = 1, \ldots, p - 1, p + 1, \ldots, D - 1$. The ground state has mass

$$M^2 = -\frac{1}{\alpha'}$$

It is again tachyonic. Its mass is half that of the closed string tachyon. As we commented above, this time the tachyon is confined to the brane. In contrast to the closed string tachyon, the open string tachyon is now fairly well understood and its potential is of the form shown in the figure. The interpretation is that the brane is unstable. It will decay, much like a resonance state in field theory. It does this by dissolving into closed string modes. The end point of this process – corresponding to the minimum at $T > 0$ in the figure – is simply a state with no D-brane. The difference between the value of the potential at the minimum and at $T = 0$ is the tension of the D-brane.

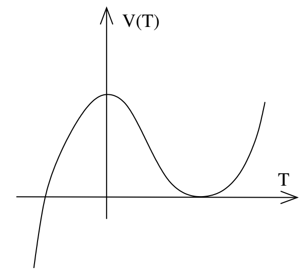

Figure 15:

Notice that although there is a minimum of the potential at $T > 0$, it is not a global minimum. The potential seems to drop off without bound to the left. This is still not well understood. There are suggestions that it is related in some way to the closed string tachyon.

3.1.2 First Excited States: A World of Light

The first excited states are massless. They fall into two classes:

- Oscillators longitudinal to the brane,

$$\alpha_{-1}^{a}|0;p\rangle \qquad a = 1,\ldots,p-1$$

  The spacetime indices a lie within the brane so this state transforms under the $SO(1,p)$ Lorentz group. It is a spin 1 particle on the brane or, in other words, it is a photon. We introduce a gauge field A_a with $a = 0,\ldots,p$ lying on the brane whose quanta are identified with this photon.

- Oscillators transverse to the brane,

$$\alpha_{-1}^{I}|0;p\rangle \qquad I = p+1,\ldots,D-1$$

  These states are scalars under the $SO(1,p)$ Lorentz group of the brane. They can be thought of as arising from scalar fields ϕ^I living on the brane. These scalars have a nice interpretation: they are fluctuations of the brane in the transverse directions. This is our first hint that the D-brane is a dynamical object. Note that although the ϕ^I are scalar fields under the $SO(1,p)$ Lorentz group of the brane, they do transform as a vector under the $SO(D-p-1)$ rotation group transverse to the brane. This appears as a global symmetry on the brane worldvolume.

3.1.3 Higher Excited States and Regge Trajectories

At level N, the mass of the string state is

$$M^2 = \frac{1}{\alpha'}(N-1)$$

The maximal spin of these states arises from the symmetric tensor. It is

$$J_{max} = N = \alpha' M^2 + 1$$

Plotting the spin vs. the mass-squared, we find straight lines. These are usually called *Regge trajectories*. (Or sometimes Chew-Fraschuti trajectories). They are seen in Nature in both the spectrum of mesons and baryons. Some examples involving ρ-mesons are shown in the figure. These stringy Regge trajectories suggest a naive cartoon picture of mesons as two rotating quarks connected by a confining flux tube.

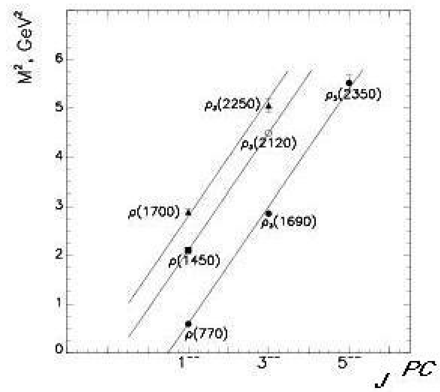

Figure 16:

The value of the string tension required to match the hadron spectrum of QCD is $T \sim 1$ GeV. This relationship between the strong interaction and the open string was one of the original motivations for the development of string theory and it is from here that the parameter α' gets its (admittedly rarely used) name "Regge slope". In these enlightened modern times, the connection between the open string and quarks lives on in the AdS/CFT correspondence.

3.1.4 Another Nod to the Superstring

Just as supersymmetry eliminates the closed string tachyon, so it removes the open string tachyon. Open strings are an ingredient of the type II string theories. The possible D-branes are

- Type IIA string theory has stable Dp-branes with p even.
- Type IIB string theory has stable Dp-branes with p odd.

The most important reason that D-branes are stable in the type II string theories is that they are charged under the Ramond-Ramond fields. (This was actually Polchinski's insight that made people take D-branes seriously). However, type II string theories also contain unstable branes, with p odd in type IIA and p even in type IIB.

The fifth string theory (which was actually the first to be discovered) is called Type I. Unlike the other string theories, it contains both open and closed strings moving in flat ten-dimensional Lorentz-invariant spacetime. It can be thought of as the Type IIB theory with a bunch of space-filling D9-branes, together with something called an orientifold plane. You can read about this in Polchinski.

As we mentioned above, the heterotic string doesn't have (finite energy) D-branes. This is due to an inconsistency in any attempt to reflect left-moving modes into right-moving modes.

3.2 Brane Dynamics: The Dirac Action

We have introduced D-branes as fixed boundary conditions for the open string. However, we've already seen a hint that these objects are dynamical in their own right, since the massless scalar excitations ϕ^I have a natural interpretation as transverse fluctuations of the brane. Indeed, if a theory includes both open strings and closed strings, then the D-branes have to be dynamical because there can be no rigid objects in a theory of gravity. The dynamical nature of D-branes will become clearer as the course progresses.

But any dynamical object should have an action which describes how it moves. Moreover, after our discussion in Section 1, we already know what this is! On grounds of Lorentz invariance and reparameterization invariance alone, the action must be a higher dimensional extension of the Nambu-Goto action. This is

$$S_{Dp} = -T_p \int d^{p+1}\xi \, \sqrt{-\det \gamma} \tag{3.6}$$

where T_p is the tension of the Dp-brane which we will determine later, while ξ^a, $a = 0, \ldots, p$, are the worldvolume coordinates of the brane. γ_{ab} is the pull back of the spacetime metric onto the worldvolume,

$$\gamma_{ab} = \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b} \eta_{\mu\nu} \ .$$

This is called the *Dirac action*. It was first written down by Dirac for a membrane some time before Nambu and Goto rediscovered it in the context of the string.

To make contact with the fields ϕ^I, we can use the reparameterization invariance of the Dirac action to go to static gauge. For an infinite, flat Dp-brane we can choose

$$X^a = \xi^a \qquad a = 0, \ldots, p \ .$$

The dynamical transverse coordinates are then identified with the fluctuations ϕ^I through

$$X^I(\xi) = 2\pi\alpha'\,\phi^I(\xi) \qquad I = p+1, \ldots, D-1$$

However, the Dirac action can't be the whole story. It describes the transverse fluctuations of the D-brane, but has nothing to say about the $U(1)$ gauge field A_μ which lives on the D-brane. There must be some action which describes how this gauge field moves as well. We will return to this in Section 7.

What's Special About Strings?

We could try to quantize the Dirac action (3.6) for a D-brane in the same manner that we quantized the action for the string. Is this possible? The answer, at present, is no. There appear to be both technical and conceptual obstacles . The technical issue is just that it's hard. Weyl invariance was one of our chief weapons in attacking the string, but it doesn't hold for higher dimensional objects.

The conceptual issue is that quantizing a membrane, or higher dimensional object, would not give rise to a discrete spectrum of states which have the interpretation of particles. In this way, they appear to be fundamentally different from the string.

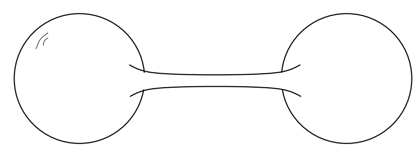

Figure 17:

Let's get some intuition for why this is the case. The energy of a string is proportional to its length. This ensures that strings behave more or less like familiar elastic bands. What about D2-branes? Now the energy is proportional to the area. In the back of your mind, you might be thinking of a rubber-like sheet. But membranes, and higher dimensional objects, governed by the Dirac action don't behave as household rubber sheets. They are more flexible. This is because a membrane can form many different shapes with the same area. For example, a tubular membrane of length L and radius $1/L$ has the same area for all values of L; short and stubby, or long and thin. This means that long thin spikes can develop on a membrane at no extra cost of energy. In particular, objects connected by long thin tubes have the same energy, regardless of their separation. After quantization, this property gives rise to a continuous spectrum of states. A quantum membrane, or higher dimensional object, does not have the single particle interpretation that we saw for the string. The expectation is that the quantum membrane should describe multi-particle states.

3.3 Multiple Branes: A World of Glue

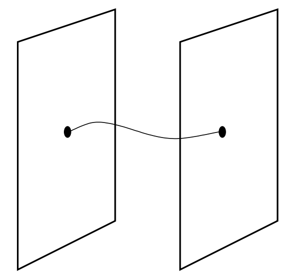

Figure 18:

Consider two parallel Dp-branes. An open string now has options. It could either end on the same brane, or stretch between the two branes. Let's consider the string that stretches between the two. It obeys

$$X^I(0,\tau) = c^I \quad \text{and} \quad X^I(\pi,\tau) = d^I$$

where c^I and d^I are the positions of the two branes. In terms of the mode expansion, this requires

$$X^I = c^I + \frac{(d^I - c^I)\sigma}{\pi} + \text{oscillator modes}$$

The classical constraints then read

$$\partial_+ X \cdot \partial_+ X = \alpha'^2 p^2 + \frac{|\vec{d} - \vec{c}|^2}{4\pi^2} + \text{oscillator modes} = 0$$

which means the classical mass-shell condition is

$$M^2 = \frac{|\vec{d} - \vec{c}|^2}{(2\pi\alpha')^2} + \text{oscillator modes}$$

The extra term has an obvious interpretation: it is the mass of a classical string stretched between the two branes. The quantization of this string proceeds as before. After we include the normal ordering constant, the ground state of this string is only tachyonic if $|\vec{d} - \vec{c}|^2 < 4\pi^2\alpha'$. Or in other words, the ground state is tachyonic if the branes approach to a sub-stringy distance.

There is an obvious generalization of this to the case of N parallel branes. Each end point of the string has N possible places on which to end. We can label each end point with a number $m, n = 1, \ldots, N$ which tell us which brane it ends on. This label is sometimes referred to as a *Chan-Paton factor*.

Consider now the situation where all branes lie at the same position in spacetime. Each end point can lie on one of N different branes, giving N^2 possibilities in total. Each of these strings has the mass spectrum of an open string, meaning that there are now N^2 different particles of each type. It's natural to arrange the associated fields to sit inside $N \times N$ Hermitian matrices. We then have the open string tachyon $T^m_{\ n}$ and the massless fields

$$(\phi^I)^m_{\ n} \quad , \quad (A_a)^m_{\ n} \tag{3.7}$$

Here the components of the matrix tell us which string the field came from. Diagonal components arise from strings which have both ends on the same brane.

The gauge field A_a is particularly interesting. Written in this way, it looks like a $U(N)$ gauge connection. We will later see that this is indeed the case. One can show that as N branes coincide, the $U(1)^N$ gauge symmetry of the branes is enhanced to $U(N)$. The scalar fields ϕ^I transform in the adjoint of this symmetry.