Supersymmetry: Example Sheet 4

David Tong, February 2022

Note: Questions 2, 3, and 4 below all refer to the global symmetry group of $SU(N_c)$ SQCD, which is

$$G_F = SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$$

Also a warning: for some of the anomaly matching questions, you will save considerable time by expanding the algebraic expressions on Mathematica before comparing.

1. The effective superpotential for $SU(N_c)$ SQCD with $N_f < N_c$ massive flavours is

$$W_{\rm eff} = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}} + {\rm Tr}(mM)$$

where M is the $N_f \times N_f$ matrix of mesons and m is the $N_f \times N_f$ mass matrix. Show that there are N_c supersymmetric ground states at which the meson matrix takes the form

$$M_j^{\ i} = (m^{-1})_j^{\ i}\left(\det m\, \Lambda^{3N_c - N_f}\right)^{1/N_c}$$

Show that when all flavours can be integrated out, the effective superpotential is

$$W_{\rm eff} = N_c \Lambda^3_{\rm new}$$

where $\Lambda_{\rm new}$ is an appropriate strong coupling scale that you should identify in terms of Λ and m.

2. When $N_f = N_c$ the vacuum moduli space is deformed to

$$\det M - \tilde{B}B = \Lambda^{2N_c}$$

Write down the unbroken global symmetry $H_F \subset G_F$ and check all 't Hooft anomaly matching conditions at the points

i) $\tilde{B} = B = 0$ and $M = \Lambda^2 \mathbb{1}_{N_f}$

ii) $M = 0$ and $B = \tilde{B} = \Lambda^{N_c}$

3. For $N_f = N_c + 1$, show that the following superpotential is consistent with the symmetries of the theory

$$W = -\frac{1}{\Lambda^{2N_c-1}}\left(\det M - BM\tilde{B}\right)$$

Show that the critical points of this superpotential reproduce the classical constraints

$$\det M (M^{-1})^i{}_j = B^i \tilde{B}_j \quad \text{and} \quad M_j{}^i B^j = M_j{}^i \tilde{B}_i = 0$$

Confirm that the 't Hooft anomalies for G_F are matched by massless baryons and mesons at the origin of moduli space.

4. Write down the non-anomalous symmetries of $SU(N_f - N_c)$ mSQCD. Show that the normalisation of $U(1)_B$ can be chosen so that the charges of the dual baryons $b \sim q^{N_f - N_c}$ and $\tilde{b} \sim \tilde{q}^{N_f - N_c}$ match those of the baryons of $SU(N_c)$ SQCD.

Show that the 't Hooft anomalies of $SU(N_f - N_c)$ mSQCD match those of $SU(N_c)$ SQCD.

5a*. $SO(N_c)$ supersymmetric gauge theory coupled to N_f chiral multiplets Φ^i, with $i = 1, \ldots, N_f$, in the fundamental representation has the following classical symmetries

	$SO(N_c)$	$SU(N_f)$	$U(1)_B$	$U(1)_{R'}$
Φ	$\square$	$\square$	1	0

where, as usual, the gluons have R-charge +1.

i) Show that there is a non-anomalous R-symmetry under which the gluinos have charge +1 and

$$R[\Phi] = \frac{N_f - N_c + 2}{N_f}$$

ii) For what value of N_f does the theory cease to be asymptotically free? Use the relation between R-charge and scaling dimension to show that the meson $\Phi^i\Phi^j$ has dimension 2 at this point.

iii) For what value of N_f does the meson $\Phi^i\Phi^j$ have the dimension of a free scalar? Hence determine the expected range of the conformal window.

Note: You will need some of the following group theoretic facts about representations of $SO(N)$: $\dim(\square) = N$ and $I(\square) = 1$; $\dim(\text{adj}) = \frac{1}{2}N(N-1)$ and $I(\text{adj}) = N - 2$.

5b*. It is conjectured that this theory has a dual description given by $SO(N_f - N_c + 4)$ gauge theory coupled to N_f chiral multiplets q_i and a collection singlet chiral multiplets M^{ij} coupled through the superpotential

$$W \sim q_i M^{ij} q_j$$

The classical symmetries of this theory are:

	$SO(N_f - N_c + 4)$	$SU(N_f)$	$U(1)_B$	$U(1)_{R'}$
q	$\square$	$\overline{\square}$	1	0
M	$\mathbf{1}$	$\square\square$	-2	$+2$

where the dual gluons have R-charge $+1$.

i) Determine the non-anomalous R-charge of q.

ii) Determine the R-charge and dimension of the singlets M at the fixed point and show that they coincide with those of the meson in the original $SO(N_c)$ theory.

iii) Show that the theory ceases to be asymptotically free for a value of N_f that coincides with the lower end of the original conformal window.

iv) Show that both theories reproduce the following 't Hooft anomalies:

$$\begin{aligned}
SU(N_f)^3 \quad &: \quad \mathcal{A} = N_c \\
SU(N_f)^2 \cdot U(1)_R \quad &: \quad \mathcal{A} = \frac{N_c(2 - N_c)}{N_f} \\
U(1)_R \quad &: \quad \mathcal{A} = \frac{1}{2} N_c(3 - N_c) \\
U(1)_R^3 \quad &: \quad \mathcal{A} = \frac{1}{2} N_c(N_c - 1) - \frac{N_c}{N_f^2}(N_c^3 - 6N_c^6 + 12N_c - 8)
\end{aligned}$$

Note: you will need some of the following group theoretic facts about representations of $SU(N)$:

Irrep	$\square$	adj	$\square\square$
dim	N	$N^2 - 1$	$\frac{1}{2}N(N+1)$
$I(R)$	1	$2N$	$N + 2$
$A(R)$	1	0	$N + 4$

and $I(R) = I(\bar{R})$ and $A(R) = -A(\bar{R})$.