Supersymmetry: Example Sheet 4

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<u>Note</u>: Questions 2, 3, and 4 below all refer to the global symmetry group of $SU(N_c)$ SQCD, which is

$$G_F = SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$$

Also a warning: for some of the anomaly matching questions, you will save considerable time by expanding the algebraic expressions on Mathematica before comparing.

1. The effective superpotential for $SU(N_c)$ SQCD with $N_f < N_c$ massive flavours is

$$W_{\text{eff}} = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}} + \text{Tr}(mM)$$

where M is the $N_f \times N_f$ matrix of mesons and m is the $N_f \times N_f$ mass matrix. Show that there are N_c supersymmetric ground states at which the meson matrix takes the form

$$M_{j}^{i} = (m^{-1})_{j}^{i} \left(\det m \Lambda^{3N_{c}-N_{f}}\right)^{1/N_{c}}$$

Show that when all flavours can be integrated out, the effective superpotential is

$$W_{\rm eff} = N_c \Lambda_{\rm new}^3$$

where Λ_{new} is an appropriate strong coupling scale that you should identify in terms of Λ and m.

2. When $N_f = N_c$ the vacuum moduli space is deformed to

$$\det M - \tilde{B}B = \Lambda^{2N_c}$$

Write down the unbroken global symmetry $H_F \subset G_F$ and check all 't Hooft anomaly matching conditions at the points

- i) $\tilde{B} = B = 0$ and $M = \Lambda^2 \mathbb{1}_{N_f}$
- ii) M = 0 and $B = \tilde{B} = \Lambda^{N_c}$

3. For $N_f = N_c + 1$, show that the following superpotential is consistent with the symmetries of the theory

$$W = -\frac{1}{\Lambda^{2N_c-1}} \left(\det M - BM\tilde{B} \right)$$

Show that the critical points of this superpotential reproduce the classical constraints

det
$$M(M^{-1})^i{}_j = B^i \tilde{B}_j$$
 and $M_j{}^i B^j = M_j{}^i \tilde{B}_i = 0$

Confirm that the 't Hooft anomalies for G_F are matched by massless baryons and mesons at the origin of moduli space.

4. Write down the non-anomalous symmetries of $SU(N_f - N_c)$ mSQCD. Show that the normalisation of $U(1)_B$ can be chosen so that the charges of the dual baryons $b \sim q^{N_f - N_c}$ and $\tilde{b} \sim \tilde{q}^{N_f - N_c}$ match those of the baryons of $SU(N_c)$ SQCD.

Show that the 't Hooft anomalies of $SU(N_f - N_c)$ mSQCD match those of $SU(N_c)$ SQCD.

5a*. $SO(N_c)$ supersymmetric gauge theory coupled to N_f chiral multiplets Φ^i , with $i = 1, \ldots, N_f$, in the fundamental representation has the following classical symmetries

	$SO(N_c)$	$SU(N_f)$	$U(1)_B$	$U(1)_{R'}$
Φ			1	0

where, as usual, the gluons have R-charge +1.

i) Show that there is a non-anomalous R-symmetry under which the gluinos have charge +1 and

$$R[\Phi] = \frac{N_f - N_c + 2}{N_f}$$

- ii) For what value of N_f does the theory cease to be asymptotically free? Use the relation between R-charge and scaling dimension to show that the meson $\Phi^i \Phi^j$ has dimension 2 at this point.
- iii) For what value of N_f does the meson $\Phi^i \Phi^j$ have the dimension of a free scalar? Hence determine the expected range of the conformal window.

Note: You will need some of the following group theoretic facts about representations of SO(N): dim $(\Box) = N$ and $I(\Box) = 1$; dim $(adj) = \frac{1}{2}N(N-1)$ and I(adj) = N-2.

5b*. It is conjectured that this theory has a dual description given by $SO(N_f - N_c + 4)$ gauge theory coupled to N_f chiral multiplets q_i and a collection singlet chiral multiplets M^{ij} coupled through the superpotential

$$W \sim q_i M^{ij} q_j$$

The classical symmetries of this theory are:

	$SO(N_f - N_c + 4)$	$SU(N_f)$	$U(1)_B$	$U(1)_{R'}$
q			1	0
M	1		-2	+2

where the dual gluons have R-charge +1.

- i) Determine the non-anomalous R-charge of q.
- ii) Determine the R-charge and dimension of the singlets M at the fixed point and show that they coincide with those of the meson in the original $SO(N_c)$ theory.
- iii) Show that the theory ceases to be asymptotically free for a value of N_f that coincides with the lower end of the original conformal window.
- iv) Show that both theories reproduce the following 't Hooft anomalies:

$$SU(N_f)^3 : \mathcal{A} = N_c$$

$$SU(N_f)^2 \cdot U(1)_R : \mathcal{A} = \frac{N_c(2 - N_c)}{N_f}$$

$$U(1)_R : \mathcal{A} = \frac{1}{2}N_c(3 - N_c)$$

$$U(1)_R^3 : \mathcal{A} = \frac{1}{2}N_c(N_c - 1) - \frac{N_c}{N_f^2}(N_c^3 - 6N_c^6 + 12N_c - 8)$$

Note: you will need some of the following group theoretic facts about representations of SU(N):

Irrep		adj	
dim	N	$N^{2} - 1$	$\frac{1}{2}N(N+1)$
I(R)	1	2N	N+2
A(R)	1	0	N+4

and $I(R) = I(\overline{R})$ and $A(R) = -A(\overline{R})$.