A New 5d Non-Supersymmetric Fixed Point?

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Based on 2001.00023
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SU(2) Super Yang-Mills

$$\mathcal{L}_{\rm YM} = \frac{1}{2g^2}{\rm tr}\left(-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \mathcal{D}_\mu\phi\mathcal{D}^\mu\phi + i\bar{\lambda}\gamma^\mu\mathcal{D}_\mu\lambda + D^iD^i + i\bar{\lambda}[\phi,\lambda]\right)$$

$$[g^2] = -1$$

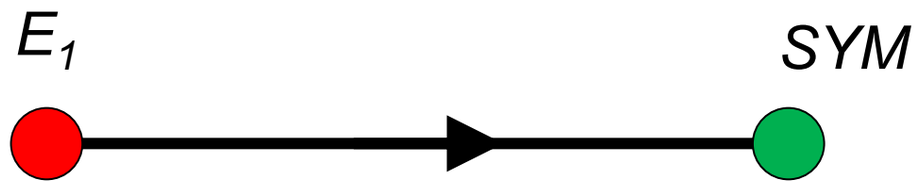
Symmetries: $SU(2)_R \times U(1)_I$

Topological current: $J_I = \frac{1}{8\pi^2} * {\rm tr}\, F \wedge F$

The UV Fixed Point

Seiberg, ’96

The UV fixed point has: $SU(2)_I \times SU(2)_R$



Deform the UV theory by:

$$\delta\mathcal{L} = h^a M^a$$

relevant parameter

$$|h| \sim \frac{1}{g^2}$$

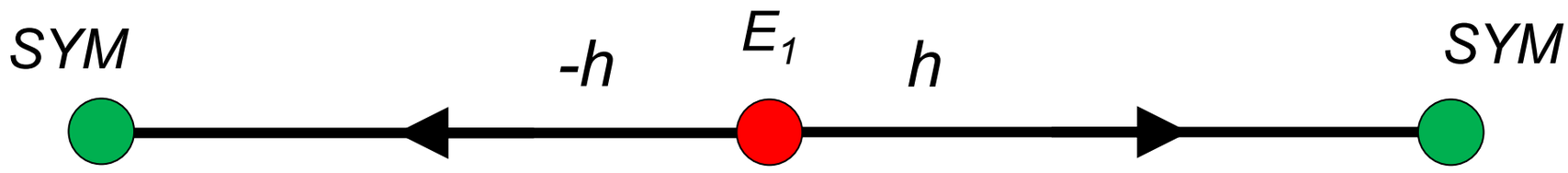
scalar operator in triplet of $SU(2)_I$

$$[M^a] = 4$$

The UV Fixed Point

Seiberg, ’96

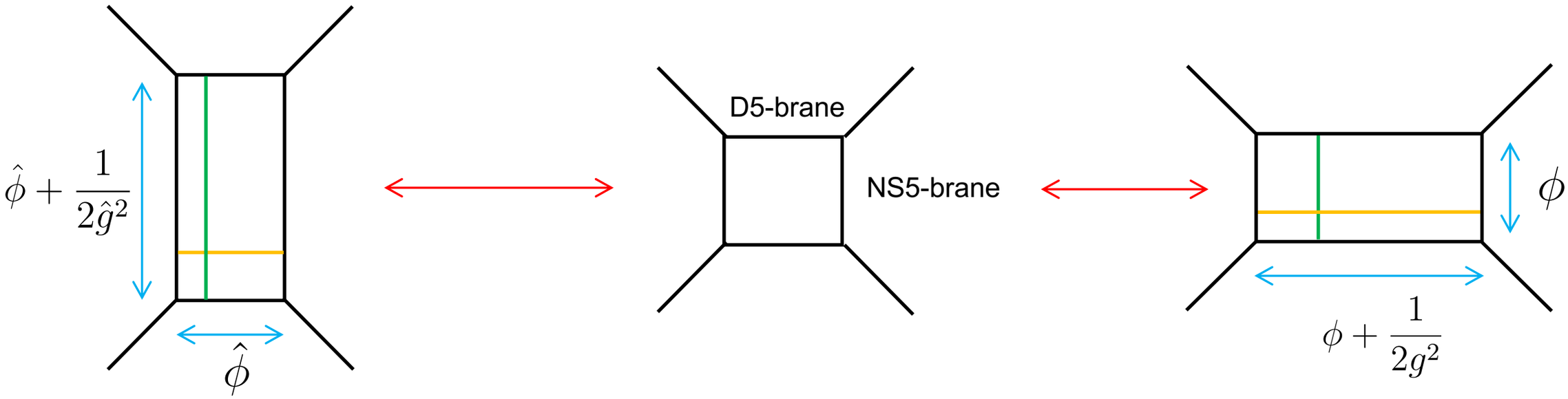
Pick, for example $h^a = (0, 0, h)$



An Aside: (p,q)5-Brane Webs

Aharony, Hanany and Kol, ’97

On the Coulomb branch, we can move smoothly from one SYM to the other.



$$\frac{1}{\hat{g}^2} = -\frac{1}{g^2} \quad \text{and} \quad \hat{\phi} = \phi + \frac{1}{2g^2}$$

Sometimes called “continuation past infinite coupling”, or “UV duality”

Breaking Supersymmetry

The *SU(2)$_I$* current multiplet contains two scalar operators.

superconformal primary:

$$\mu^{ai} \xrightarrow{Q} \psi^{am}_{\alpha} \xrightarrow{Q} M^a, J^a_{\mu}$$

$[\mu] = 3$ $\qquad$ $[M] = 4$

SU(2)$_I$: a=1,2,3
SU(2)$_R$: i=1,2,3

Deform the UV theory by:

$$\delta\mathcal{L} = h^a M^a + d^{ai}\mu^{ai}$$

pick $\quad h^a = h\hat{v}^a \quad \text{and} \quad d^{ai} = \tilde{m}^i\hat{v}^a$

$$\Longrightarrow \qquad SU(2)_R \times SU(2)_I \longrightarrow U(1)_R \times U(1)_I$$

Breaking Supersymmetry

$$\delta\mathcal{L} = h^a M^a + d^{ai}\mu^{ai}$$

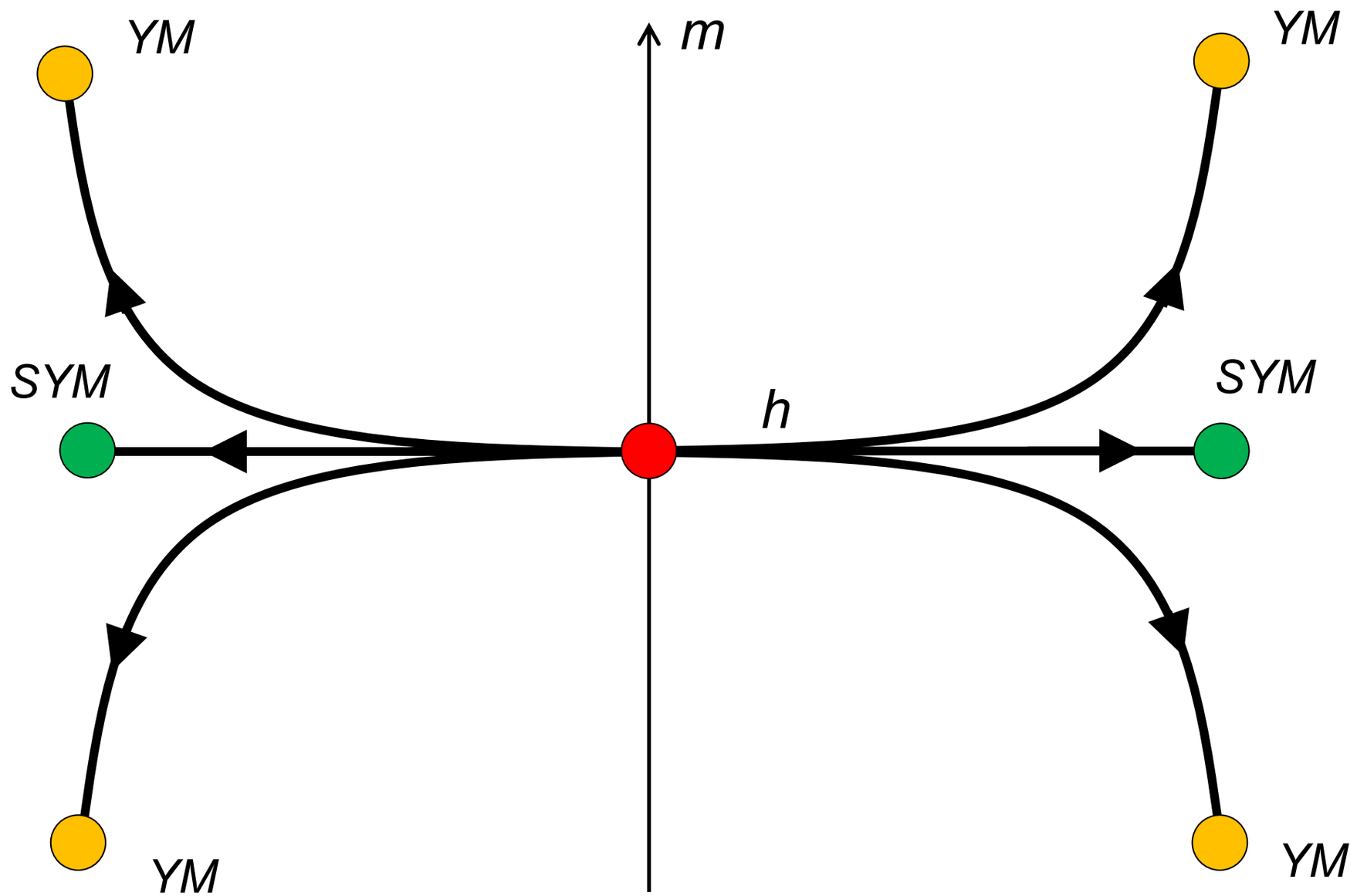
$|\tilde{m}| \ll h^2 \quad \Longrightarrow$ we first flow to SYM, and subsequently break susy.
In this case, the deformation in the IR is

$$\delta\mathcal{L} = m^i \,\mathrm{tr}\left(\frac{i}{4}\,\bar{\lambda}\sigma^i\lambda + \phi D^i\right)$$

i.e. the bosons and fermions are gapped.

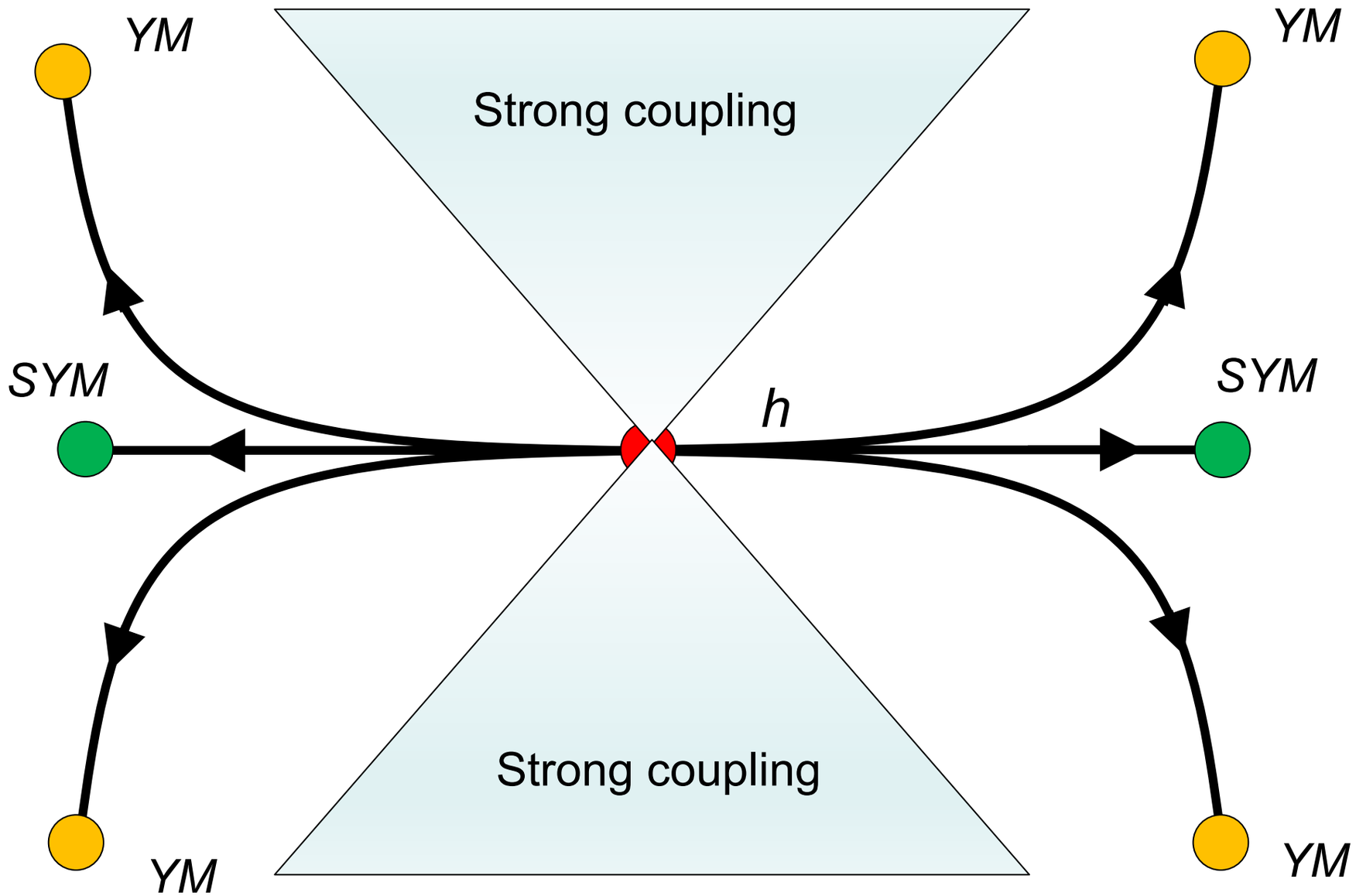
The Phase Diagram

Pick, for example $\tilde{m}^i = (0, 0, \tilde{m})$



The Phase Diagram

What happens in the rest of the phase diagram?



Topological Phases

To make progress: the massive fermions sit in a topological phase.

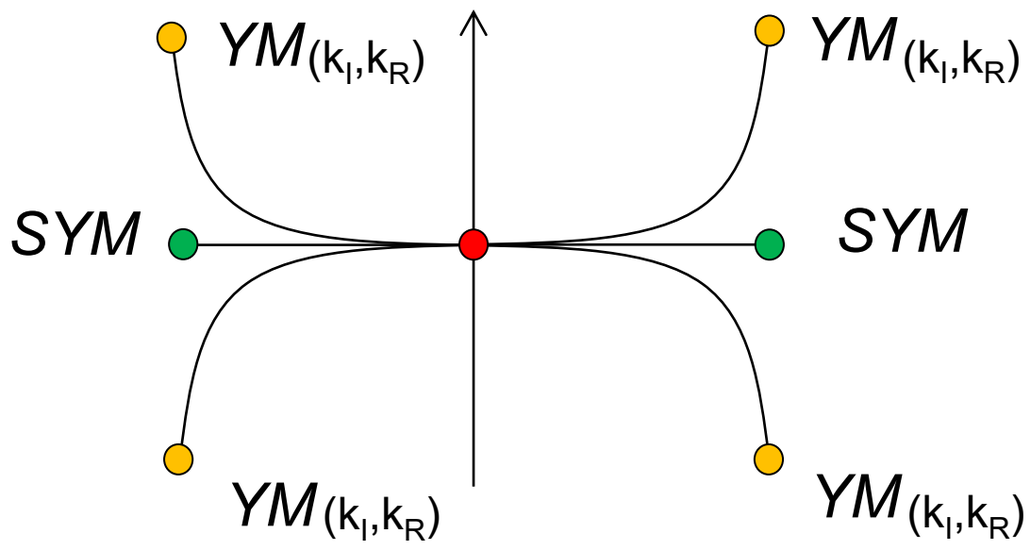
Introduce a background gauge fields: A_R for $U(1)_R \subset SU(2)_R$

A_I for $U(1)_I \subset SU(2)_I$

When the fermions are gapped, there are CS terms for the background gauge fields*

$$S_{CS} = \sum_{a=R,I} \frac{k_a}{24\pi^2} \int A_a \wedge dA_a \wedge dA_a$$

Each *SU(2)* YM theory is dressed with two CS levels.

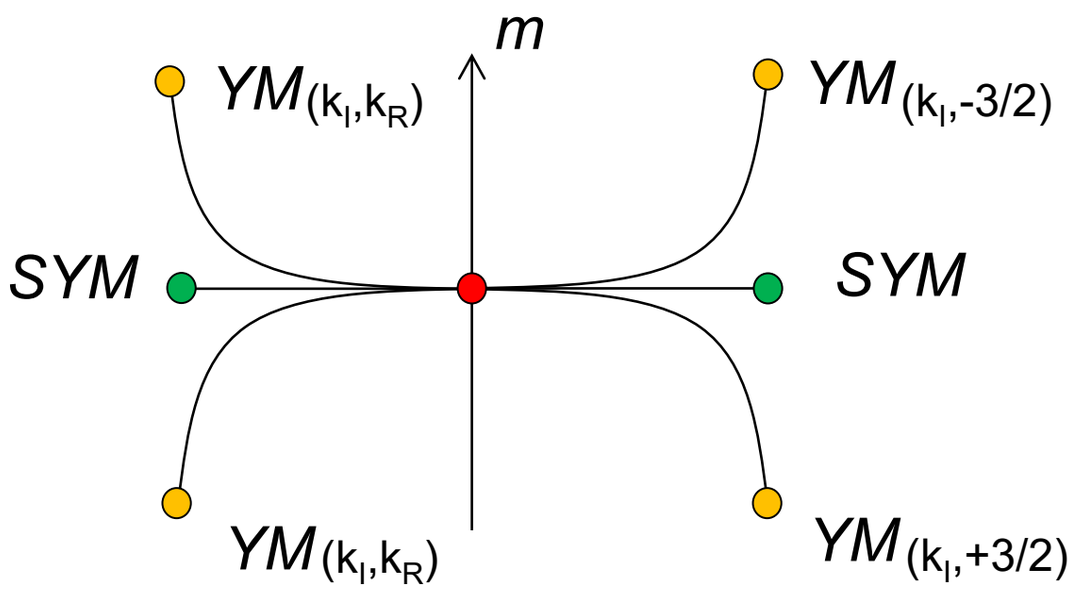


*There are also mixed CS terms

The Chern-Simons Terms

For $SU(2)_R$ the computation of k_R is straightforward.

Integrate out the gauginos to find: $k_R = -\frac{3}{2}\mathrm{sign}(m)$



Note: half-integer related to ‘t Hooft anomaly

The Chern-Simons Terms

For $SU(2)_I$ the computation of k_I involves understanding non-perturbative states

The prepotential is:

$$6\mathcal{F} = 12h\phi^2 + 8\phi^3$$

tree-level one-loop = CS term for dynamical gauge field

But this should be invariant under "UV duality", or $SU(2)_I$ Weyl group

$$h \to -h \quad \text{and} \quad \phi \to \phi + h$$

$$\Longrightarrow \qquad 6\mathcal{F} = 12h\phi^2 + 8\phi^3 - 2h^3$$

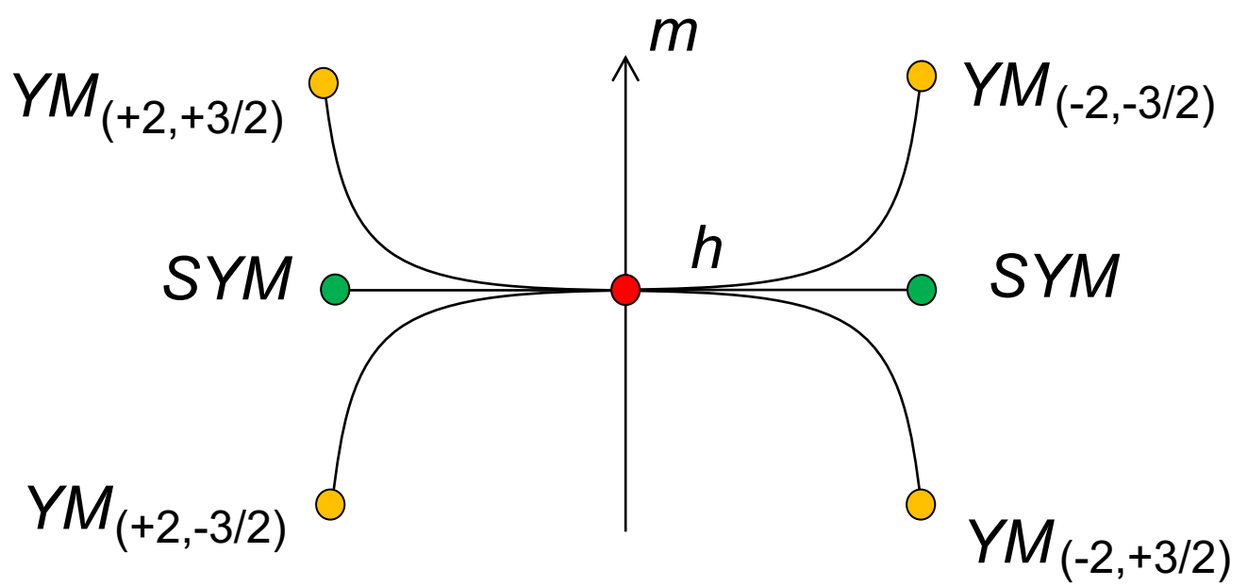
See also Closset, Del Zotto and Saxena, '18
Hayashi, Kim, Lee and Yagi '19

The Chern-Simons Terms

$$6\mathcal{F} = 12h\phi^2 + 8\phi^3 - 2h^3$$

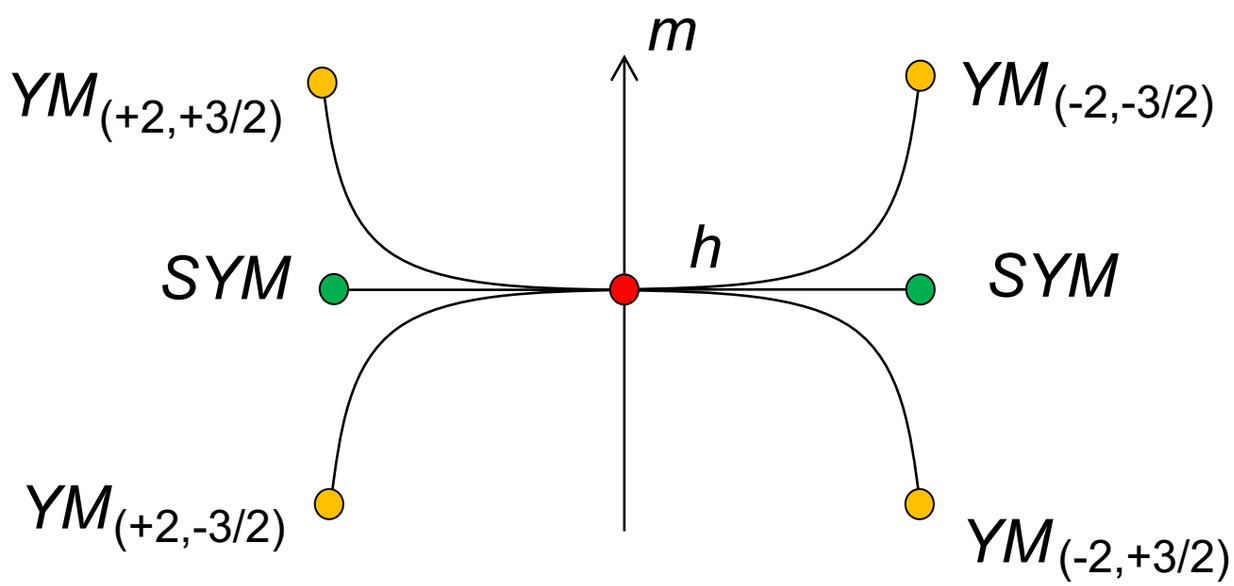
$$\Longrightarrow \qquad k_I = -2\,\mathrm{sign}(h)$$

Given (k_I,k_R) in one quadrant, can use $SU(2)_I$ rotations to compute in others



Topological Phases

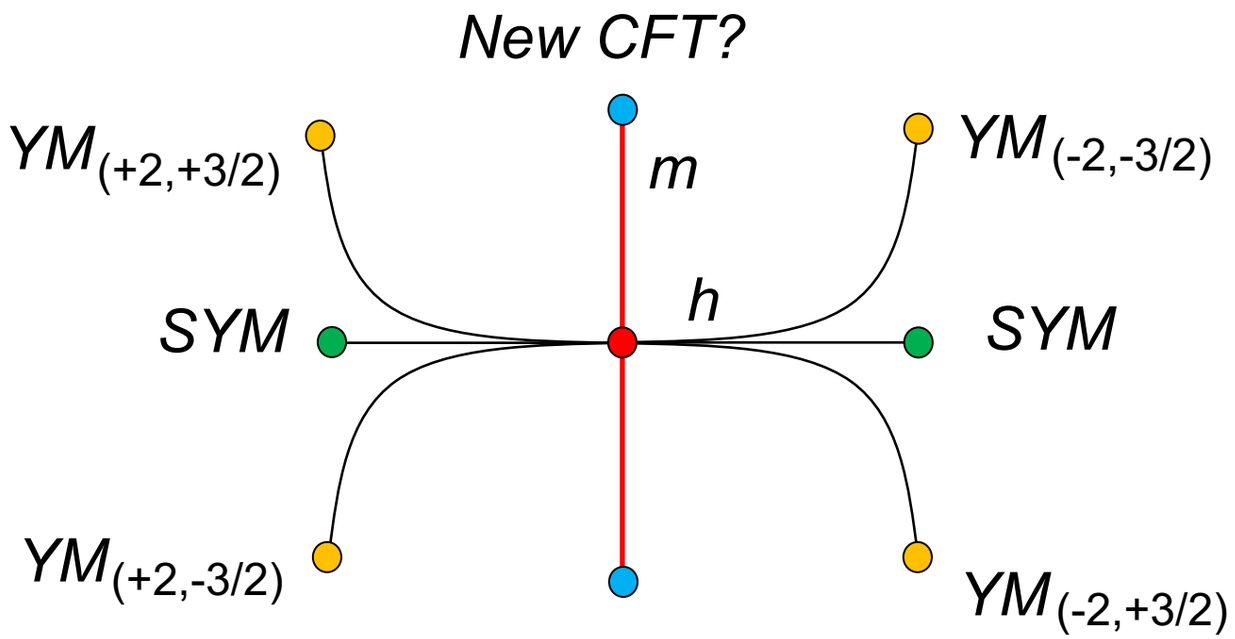
Jump in Chern-Simons term ⇨ something interesting happens



e.g. at $m=0$ line, k_R jumps ⇨ massless fermions

Topological Phases

What happens at *h=0* line?

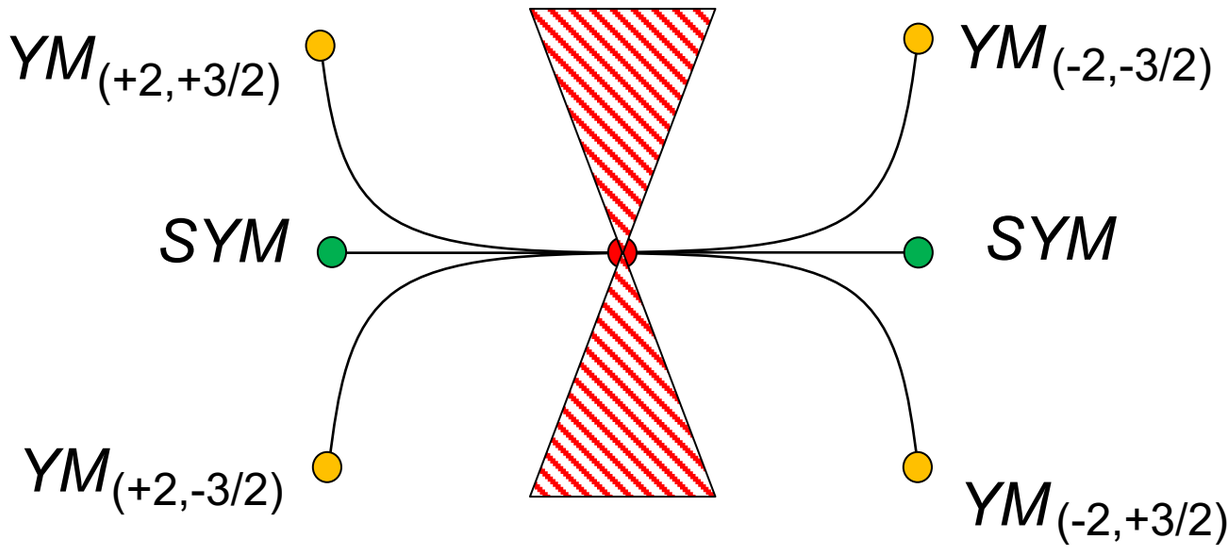


Option 1: Massless non-perturbative states?

- Free or interacting?

Topological Phases

What happens at *h=0* line?

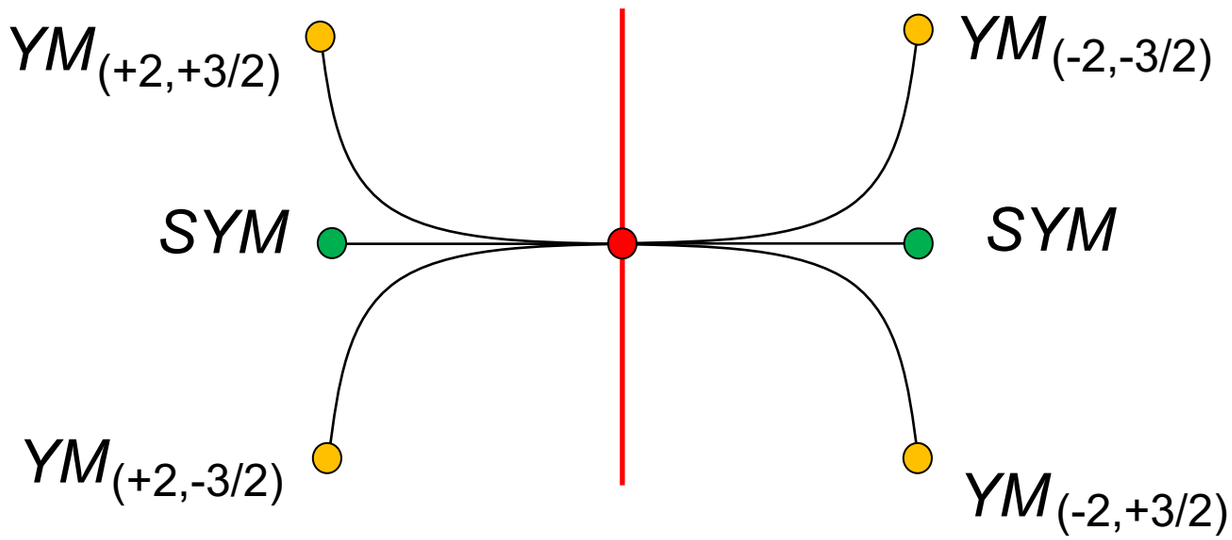


Option 2: Spontaneous breaking of $U(1)_R \times U(1)_I$

- Massless Goldstone modes

Topological Phases

What happens at *h=0* line?



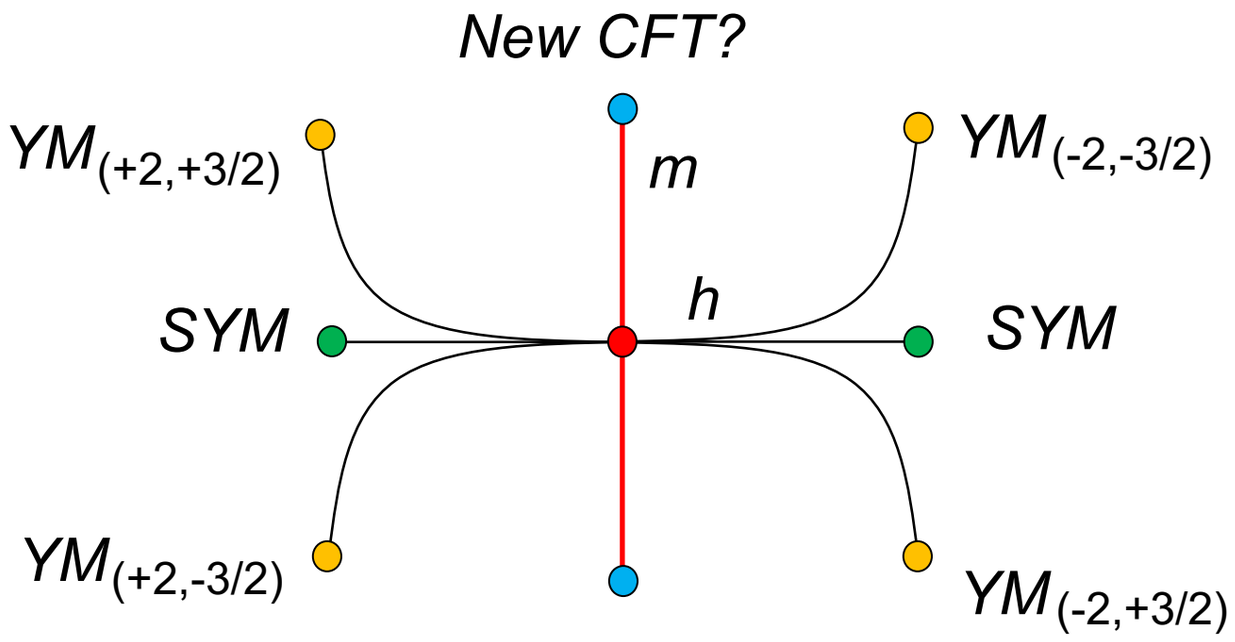
Option 3: First order phase transition

- Spontaneous breaking of $h \rightarrow -h$

Summary

- New non-supersymmetric CFT?
- Or spontaneous symmetry breaking?

How can we tell?



Thank you for your attention