Holography with Large N=4

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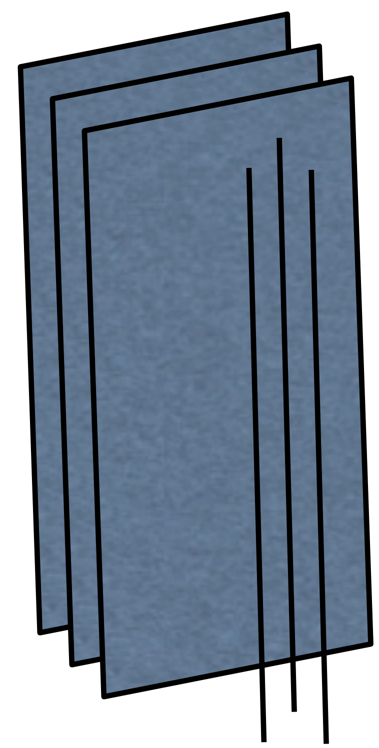
Neve Shalom, April 2014

Holography with *Small* N=4

D1/D5 System



Near horizon limit:

$$AdS_3 \times S^3 \times \mathcal{M}$$

$$\mathcal{M} = T^4 \text{ or } K3$$

Q_5 D5-branes: 012345
Q_1 D1-branes: 05

Holography with *Small* N=4

$$AdS_3 \times S^3 \times \mathcal{M}$$

$$\mathcal{M} = T^4 \text{ or } K3$$

Boundary CFT

The central charge is: $c = 6Q_5Q_1$

The R-symmetry becomes a current algebra

$$SO(4) \cong SU(2)_L \times SU(2)_R$$

Boundary theory has small N=4 superalgebra and is well understood

Holography with *Large* N=4

$$AdS_3 \times \mathbf{S}^3_+ \times \mathbf{S}^3_- \times \mathbf{S}^1$$

Supported by fluxes $Q_5^\pm$ and Q_1

Elitzur, Feinerman, Giveon and Tsabar (1998)
de Boer and Skenderis (1999)
Gukov, Martinec, Moore and Strominger (2004)

Holography with *Large* N=4

The R-symmetry is now:

$$SO(4)^- \times SO(4)^+ \cong SU(2)^-_L \times SU(2)^-_R \times SU(2)^+_L \times SU(2)^+_R$$

- There are two *SU(2)* current algebras.
- There is also a *U(1)* coming from S^1 factor of the geometry.

Boundary theory has large N=4 superalgebra. But what is the theory?!

Elitzur, Feinerman, Giveon and Tsabar (1998)
de Boer and Skenderis (1999)
Gukov, Martinec, Moore and Strominger (2004)

Some Strange Properties of Large N=4

Central Charge

$AdS_3 \times \mathbf{S}^3_+ \times \mathbf{S}^3_- \times \mathbf{S}^1$ supported by fluxes $Q_5^\pm$ and Q_1

$$c = 6Q_1 \frac{Q_5^+ Q_5^-}{Q_5^+ + Q_5^-}$$

Sevrin, Troost, Van Proeyen (1988)

BPS Bound

$AdS_3 \times \mathbf{S}^3_+ \times \mathbf{S}^3_- \times \mathbf{S}^1$ supported by fluxes $Q_5^\pm$ and Q_1

$$h = \frac{Q_5^-}{Q_5^- + Q_5^+} l^+ + \frac{Q_5^+}{Q_5^- + Q_5^+} l^- + \frac{1}{Q_1(Q_5^+ + Q_5^-)} \left[(l^+ - l^-)^2 + u^2 \right]$$

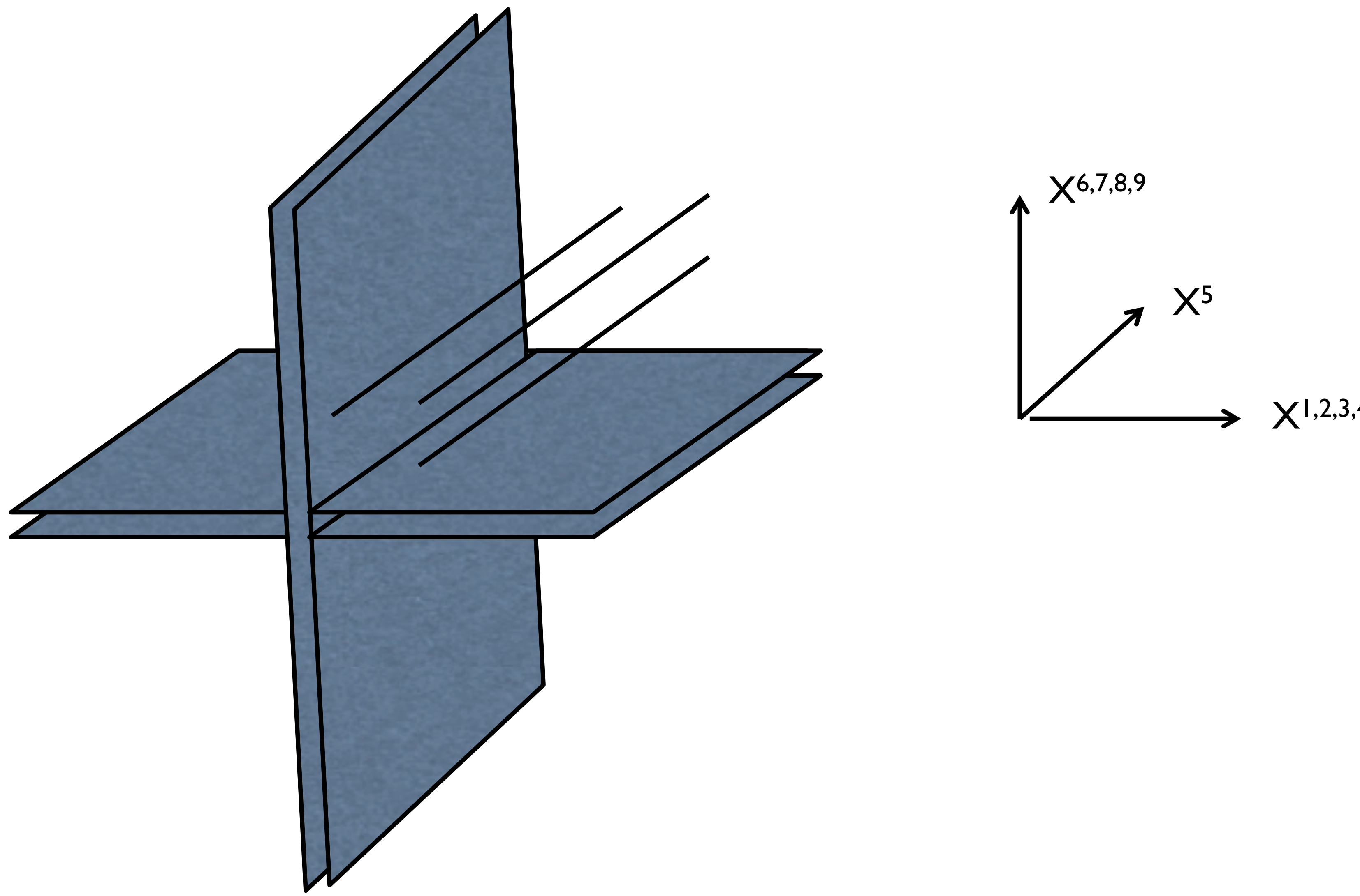
Representation under *SU(2)*$^{+/-}$

Charge under *U(1)*

- The bound is non-linear.
  - There is no chiral ring!
- The non-linearities are "1/N" suppressed.
  - They are not seen in supergravity

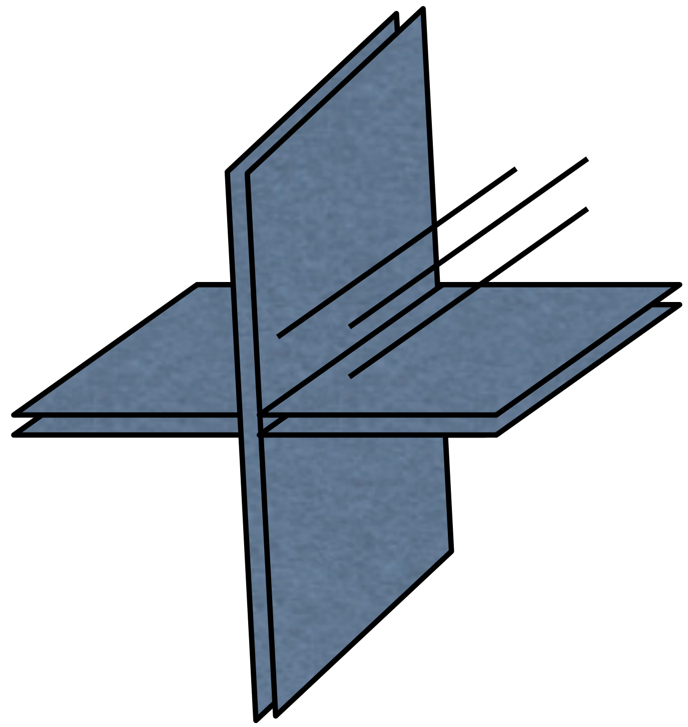
How to Build the Boundary Field Theory

A Tantalising D-Brane Configuration



Q_5^+ D5-branes: 012345
Q_5^- D5$'$-branes: 056789
Q_1 D1-branes: 05

Taking the Near Horizon Limit



Q_5^+ D5-branes: 012345
Q_5^- D5$'$-branes: 056789
Q_1 D1-branes: 05

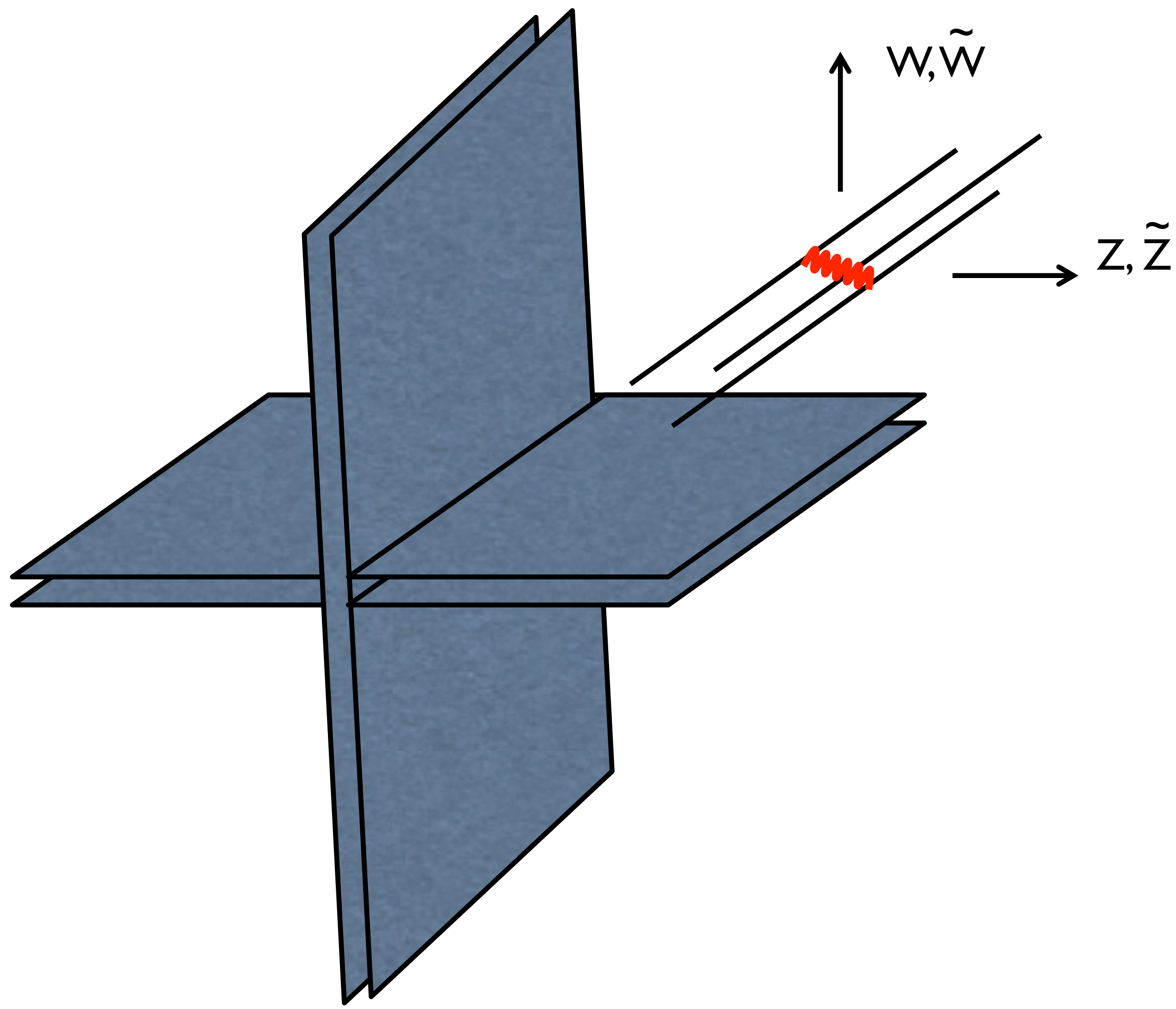
Smear D1-branes along 1234 and 6789. The near horizon limit is:

$$AdS_3 \times \mathbf{S}^3_+ \times \mathbf{S}^3_- \times \mathbf{R}$$

How should we interpret this?!

Basic Idea: Study this D-brane configuration anyway!

The Low-Energy Dynamics of D-Branes

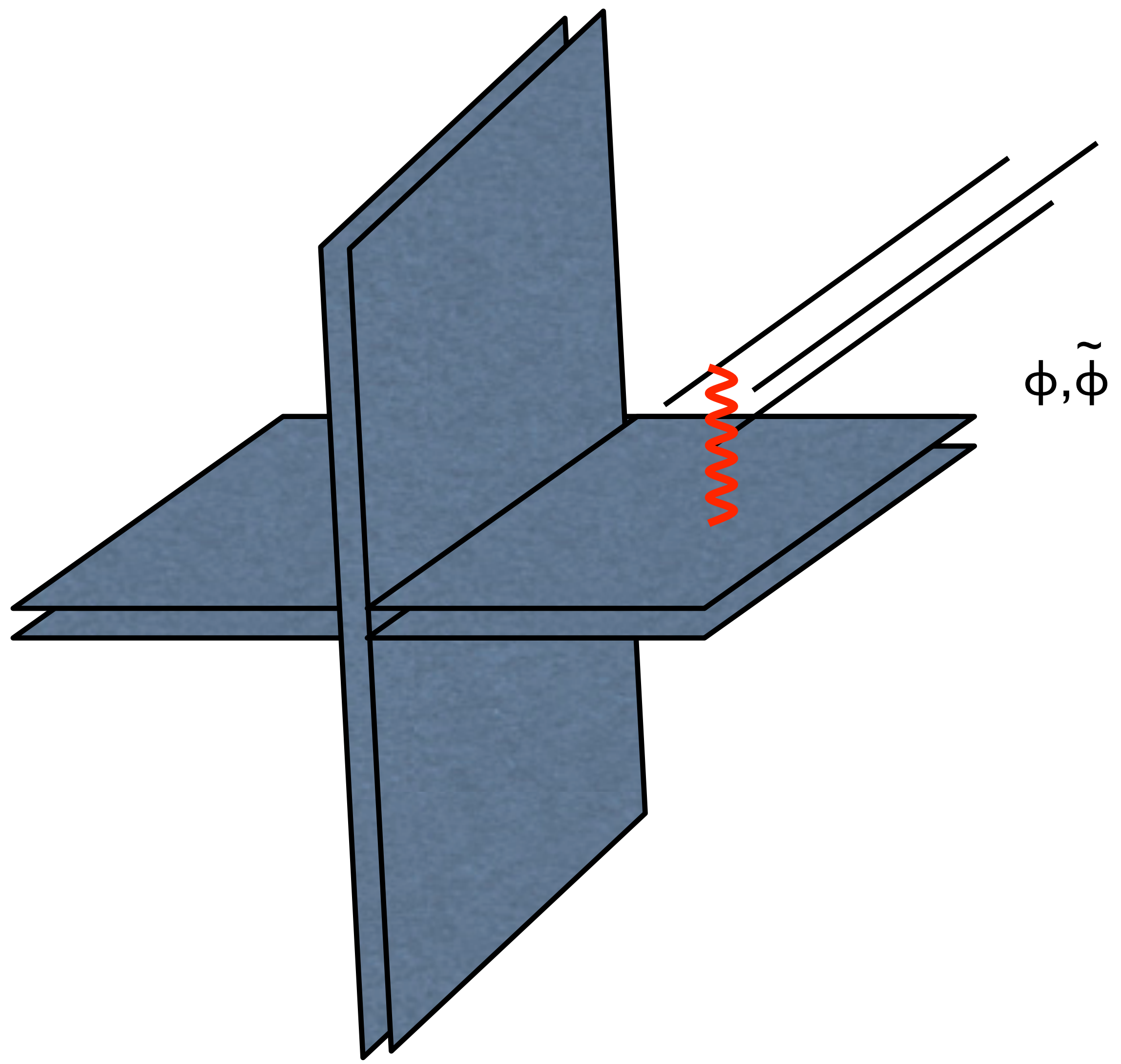


Q_5^+ D5-branes: 012345
Q_5^- D5$^\prime$-branes: 056789
Q_1 D1-branes: 05

D1-D1 strings: $U(Q_1)$ vector multiplet

- *N=(8,8)* supersymmetry
- Gauge field and four complex, adjoint scalars

The Low-Energy Dynamics of D-Branes

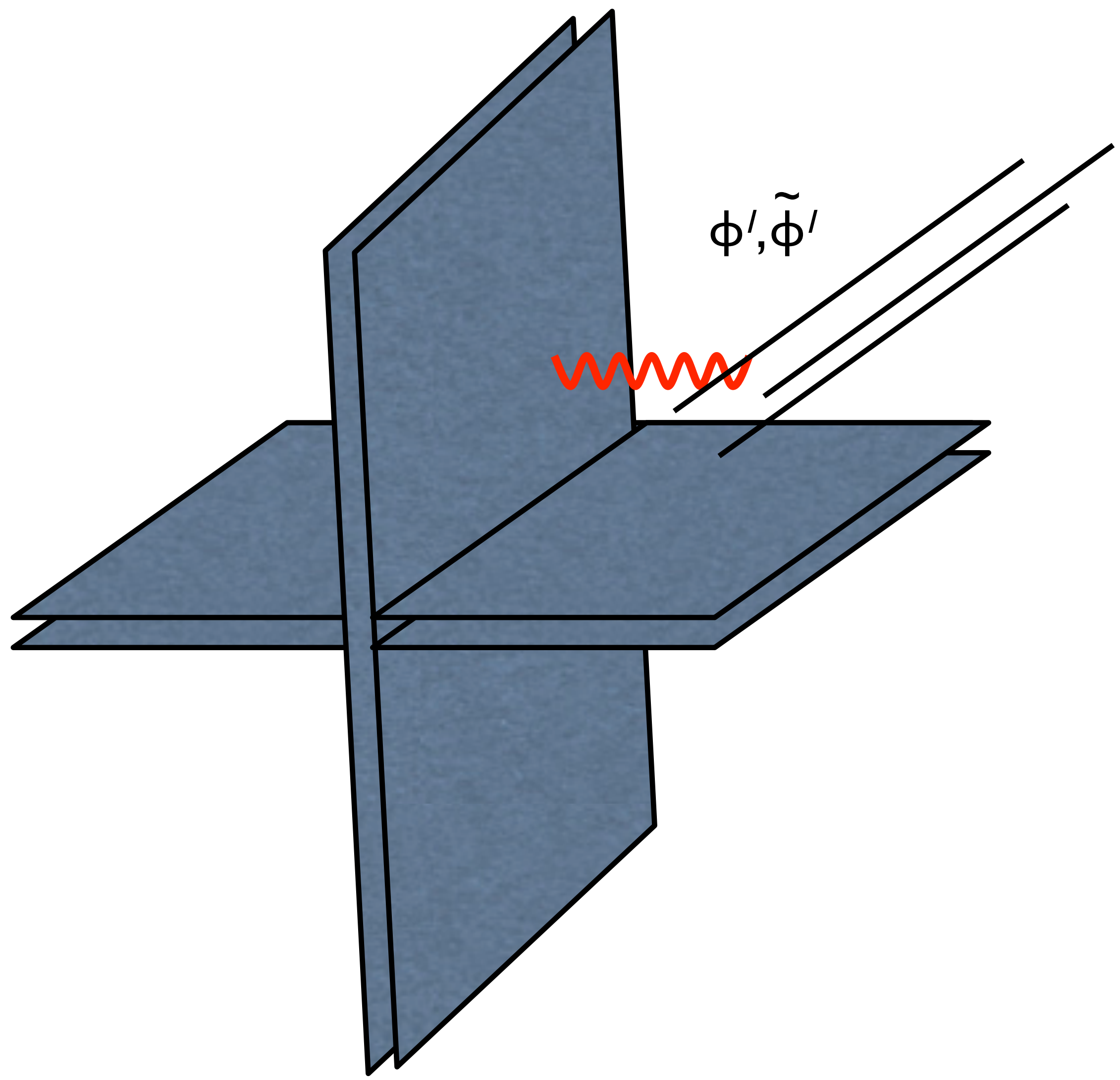


Q_5^+ D5-branes: 012345
Q_5^- D5$^\prime$-branes: 056789
Q_1 D1-branes: 05

D1-D5 strings: Q_5^+ fundamental hypermultiplets

- *N=(4,4)* supersymmetry
- two complex fundamental scalars

The Low-Energy Dynamics of D-Branes

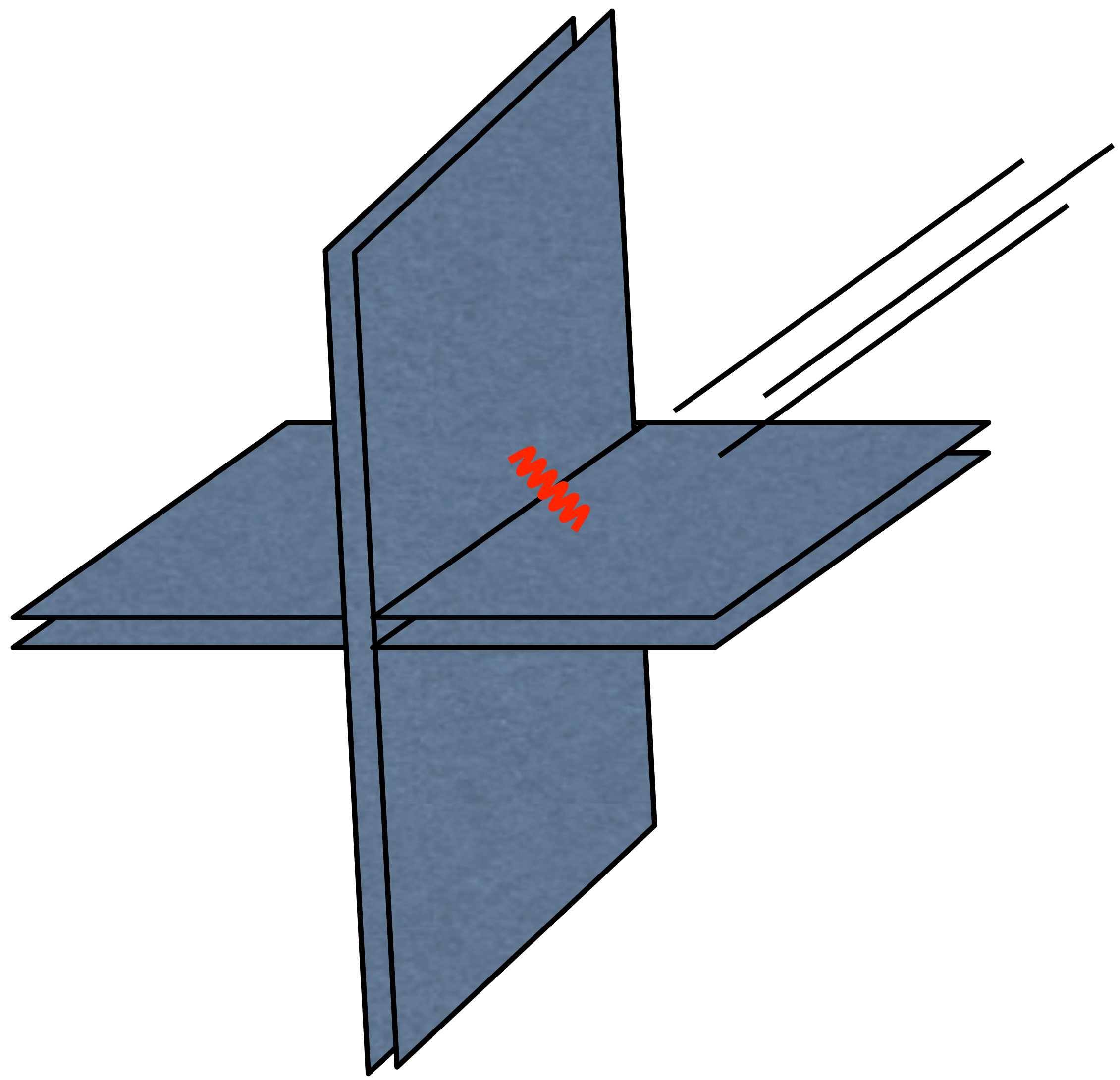


Q_5^+ D5-branes: 012345
Q_5^- D5′-branes: 056789
Q_1 D1-branes: 05

D1-D5′ strings: Q_5^- fundamental (twisted) hypermultiplets

- *N=(4,4)* supersymmetry (but a different one!)
- two complex fundamental scalars

The Low-Energy Dynamics of D-Branes

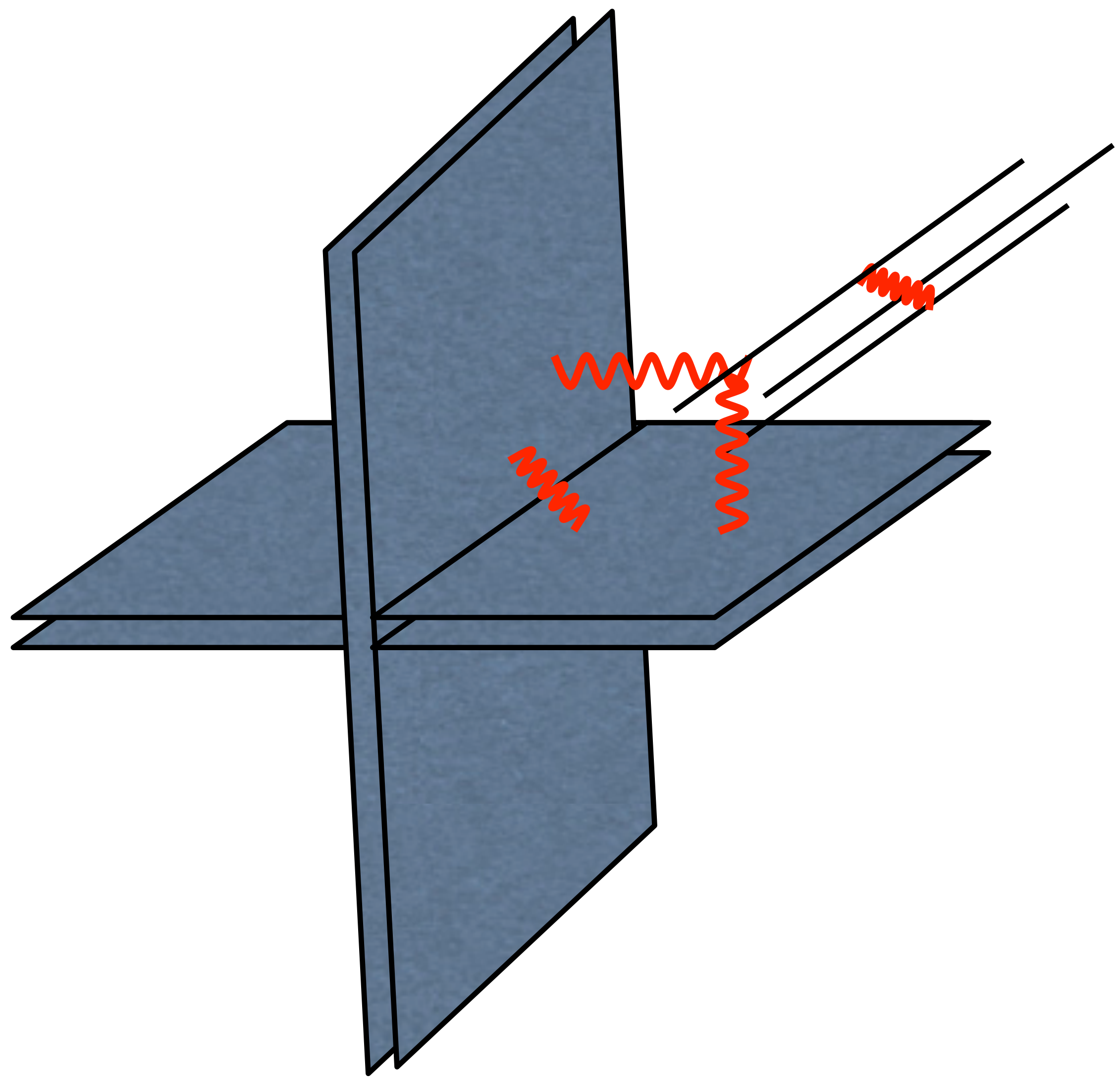


Q_5^+ D5-branes: 012345
Q_5^- D5$'$-branes: 056789
Q_1 D1-branes: 05

D5-D5$'$ strings: $Q_5^+Q_5^-$ Fermi multiplets

- $N=(0,8)$ supersymmetry
- only left-moving fermions

The Low-Energy Dynamics of D-Branes



- All branes together preserve *N=(0,4)* supersymmetry
- The only question: how do chiral fermions couple to the other fields?
    - Surprising answer: the coupling is fixed by supersymmetry

A Brief Explanation of Supersymmetry

Use *N=(0,2)* superfields. Scalar potential terms are built using Fermi multiplets.

$$\Psi = \psi_- - \theta^+ G - i\theta^+\bar{\theta}^+(D_0 + D_1)\psi_- - \bar{\theta}^+ E(\phi_i) + \theta^+\bar{\theta}^+ \frac{\partial E}{\partial \phi^i}\psi_{+i}$$

<u>E-Terms</u>

$$\bar{\mathcal{D}}_+ \Psi_a = E_a(\Phi_i)$$

Holomorphic functions of chiral superfields

<u>J-Terms</u>

$$S_J = \int d^2x \, d\theta^+ \sum_a \Psi_a J^a(\Phi_i) \; + \; \text{h.c.}$$

The scalar potential is: $V = \sum_a |E_a|^2 + |J_a|^2 + D^2$

But there is only *N=(0,2)* supersymmetry if: $E \cdot J \equiv \sum_a E_a J^a = 0.$

The Coupling of the Chiral Fermions

The D5-D5$^{/}$ strings are needed to ensure that *E.J=0*

$$E_\chi = -\frac{1}{2}\tilde{\Phi}\Phi' \qquad E_{\tilde{\chi}} = -\frac{1}{2}\tilde{\Phi}'\Phi$$

$$J_\chi = \frac{1}{2}\tilde{\Phi}'\Phi \qquad J_{\tilde{\chi}} = \frac{1}{2}\tilde{\Phi}\Phi'$$

N=(0,4) *U(Q_1)* Gauge Theory

- Q_5^+ fundamental hypermultiplets
- Q_5^- fundamental twisted hypermultiplets
- $Q_5^+Q_5^-$ neutral Fermi multiplets

Flavour symmetry: $SU(Q_5^+)\times SU(Q_5^-)$

Also: $SO(4)^-\times SO(4)^+\cong SU(2)_L^-\times SU(2)_R^-\times SU(2)_L^+\times SU(2)_R^+$

($Z,\tilde{Z}$) in (2,2)

($Y,\tilde{Y}$) in (2,2)

These are *N=(0,4)* R-symmetries

Finally, the theory has a global flavour symmetry which rotates hypers, twisted hypers and Fermi multiplets

The Scalar Potential

To write it in a way in which the symmetries are manifest, define:

$$\omega = \begin{pmatrix} \phi \\ \tilde{\phi}^\dagger \end{pmatrix} \qquad \omega' = \begin{pmatrix} \phi' \\ \tilde{\phi}'^\dagger \end{pmatrix}$$

and

$$\vec{D}_Z = \vec{\eta}_{ij} Z^i Z^j + \omega^\dagger \vec{\sigma} \omega \qquad \vec{D}_Y = \vec{\eta}_{ij} Y^i Y^j + \omega'^\dagger \vec{\sigma} \omega'$$

In **3** of $SU(2)_R^+$

In **3** of $SU(2)_R^-$

Self-dual
't Hooft matrices

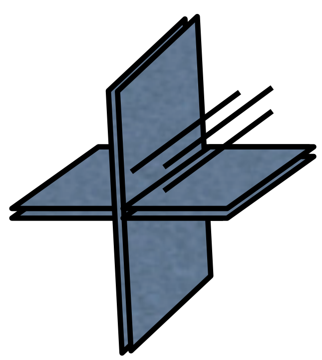
Pauli matrices

$$V = \mathrm{Tr}\,(\vec{D}_Z \cdot \vec{D}_Z + \vec{D}_Y \cdot \vec{D}_Y) + \omega^\dagger Y^i Y^i \omega + \omega'^\dagger Z^i Z^i \omega' + \mathrm{Tr}\,[Y^i, Z^j]^2 + \mathrm{Tr}\,\left(\omega^\dagger \cdot \omega\, \omega'^\dagger \cdot \omega'\right)$$

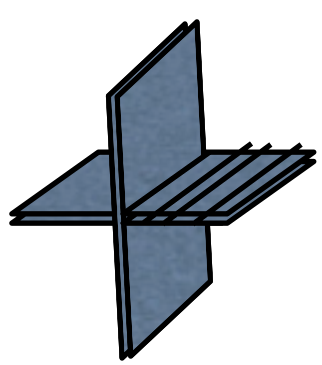
$$\text{last term} = \sum_{a=1}^{Q_5^+} \sum_{b=1}^{Q_5^-} \Big((\phi_a^\dagger \phi_b')(\phi_b'^\dagger \phi_a) + (\tilde{\phi}_a \tilde{\phi}_b'^\dagger)(\tilde{\phi}_b' \tilde{\phi}_a^\dagger) + (\phi_a^\dagger \tilde{\phi}_b'^\dagger)(\tilde{\phi}_b' \phi_a) + (\tilde{\phi}_a \phi_b')(\phi_b'^\dagger \tilde{\phi}_a^\dagger) \Big)$$

Vacuum Moduli Space

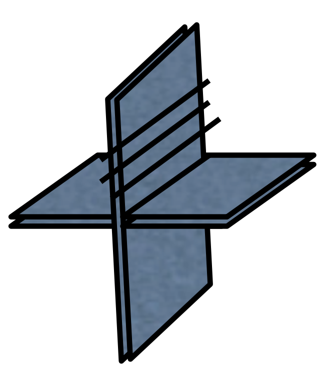
$$V = \mathrm{Tr}\,(\vec{D}_Z \cdot \vec{D}_Z + \vec{D}_Y \cdot \vec{D}_Y) + \omega^\dagger Y^i Y^i \omega + \omega'^\dagger Z^i Z^i \omega' + \mathrm{Tr}\,[Y^i, Z^j]^2 + \mathrm{Tr}\,\left(\omega^\dagger \cdot \omega\, \omega'^\dagger \cdot \omega'\right)$$



- $\omega = \omega' = 0$ with Z^i and Y^i mutually commuting.



- $\vec{D}_Z = 0$ and $Y^i = \omega' = 0$



- $\vec{D}_Y = 0$ and $Z^i = \omega = 0$

But in fact, we'll be interested in modes which appear to localise at the origin…

Flowing to the Infra-Red

Computing the Central Charge of *N=(0,2)* Theories

The OPE of the right-moving R-current includes the term

$$R(x)R(y) \sim \frac{3c_R}{(x^- - y^-)^2} + \ldots$$

But this is the anomaly. Which means that the central charge can be computed in the ultra-violet

$$\boxed{c_R = 3\mathrm{Tr}\, R^2}$$

Sum over right-moving fermions, minus left-moving fermions

Witten (1993)

Computing the Central Charge of *N=(0,2)* Theories

$$c_R = 3\mathrm{Tr}\, R^2$$

There's one small catch: you have to identify the right R-current in the UV. The requirement is:

$$\mathrm{Tr}\, FR = 0$$

Silverstein and Witten (1993)

Any *U(1)* global symmetry *F*

This can be repackaged as "c-extremization"

Adams, Tong and Wecht
Benini and Bobev (2012)

Computing the Central Charge of *N=(0,4)* Theories

Right-moving R-current is: $$SU(2)_R^- \times SU(2)_R^+$$

- No mixing for non-Abelian symmetries
- But *N=(0,2)* R-current is some combination of

$$R^\pm \subset su(2)_R^\pm$$

Both are good *N=(0,2)* R-currents. But there is one combination that is an *N=(0,2)* flavour symmetry

$$U = R^+ - R^-$$

We must have

$$\mathrm{Tr}\, U R = 0$$

$$\mathrm{Tr}\, U R^- = -2 Q_1 Q_5^- \qquad \mathrm{Tr}\, U R^+ = +2 Q_1 Q_5^+ \quad \Longrightarrow \quad R = \frac{Q_5^+}{Q_5^+ + Q_5^-} R^- + \frac{Q_5^-}{Q_5^+ + Q_5^-} R^+$$

Computing the Central Charge of *N=(0,4)* Theories

$$c_R = 3\mathrm{Tr}\, R^2 \quad \Longrightarrow \quad \boxed{c = 6Q_1 \frac{Q_5^+ Q_5^-}{Q_5^+ + Q_5^-}}$$

In agreement with the result from supergravity

Other Anomalies

Each symmetry in the gauge theory has an anomaly. These all agree with the levels of the large *N=4* algebra

$$\mathrm{Tr}\,(R^+)^2 = \mathrm{Tr}\,(L^+)^2 = Q_1 Q_5^+$$ $SU(2)_R^+$ and $SU(2)_L^+$

$$\mathrm{Tr}\,(R^-)^2 = \mathrm{Tr}\,(L^-)^2 = Q_1 Q_5^-$$ $SU(2)_R^-$ and $SU(2)_L^-$

But….

- There is no symmetry corresponding to the S^1 action of the geometry.
- The gauge theory has more degrees of freedom.
  - These show up in the $SU(Q_5^+)$ and $SU(Q_5^-)$ flavour symmetries

$$\mathrm{Tr}\,(F^+)^2 = Q_5^- \qquad \mathrm{Tr}\,(F^-)^2 = Q_5^+$$

and the left-moving central charge

$$c_L = c_R + 2Q_5^+ Q_5^-$$

Proposal

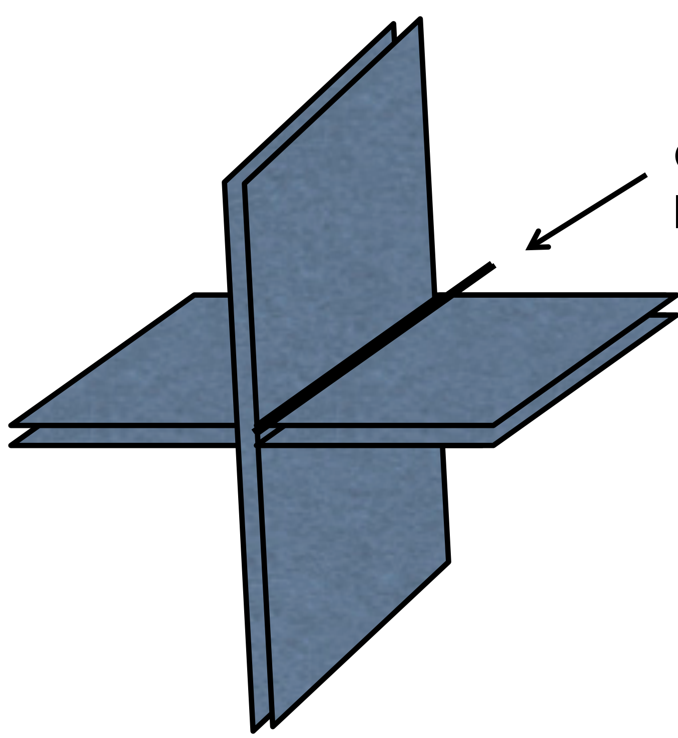
N=(0,4) Gauge Theory ⇨ Large *N=4* CFT dual to $AdS_3 x S^3 x S^3 x S^1$

+ decoupled left-moving fermions

Where does this CFT live?

Right-moving R-symmetries cannot act on scalars in asymptotic (semi-classical) parts of moduli space. But….

$$R[Y] = R[\tilde{Y}] = \frac{Q_5^-}{Q_5^+ + Q_5^-} \qquad R[Z] = R[\tilde{Z}] = \frac{Q_5^+}{Q_5^+ + Q_5^-}$$

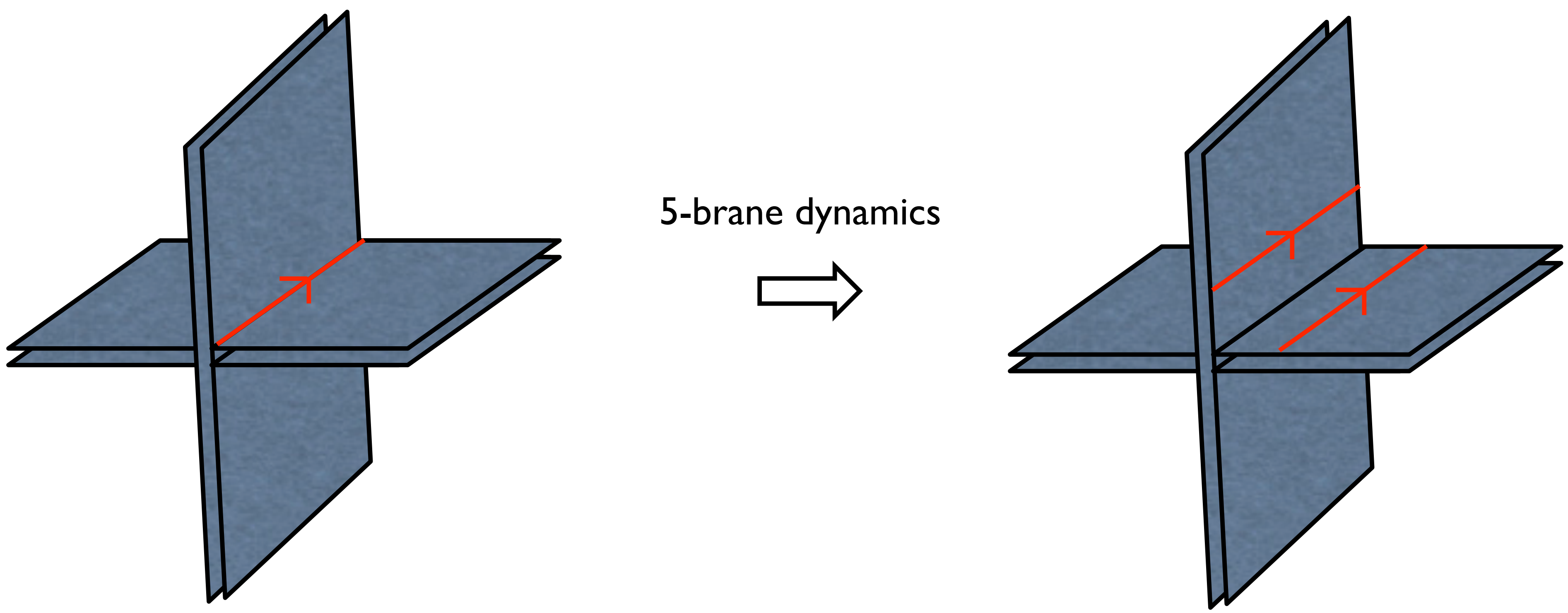


CFT degrees of freedom are localised at the origin.

- This is surprising in d=1+1 (Mermin-Wagner theorem)
- But related things happen in *N=(4,4)*
    - Decoupling of Higgs and Coulomb branches

What happened to the Chiral Modes?

An interesting story was found in the absence of D1-branes. This configuration is called the *I-brane*



- Chiral modes are pushed away from the intersection.
- The intersection has a mass gap
- It also has d=2+1 dimensional symmetry!

Itzhaki, Kutasov and Seiberg (2005)

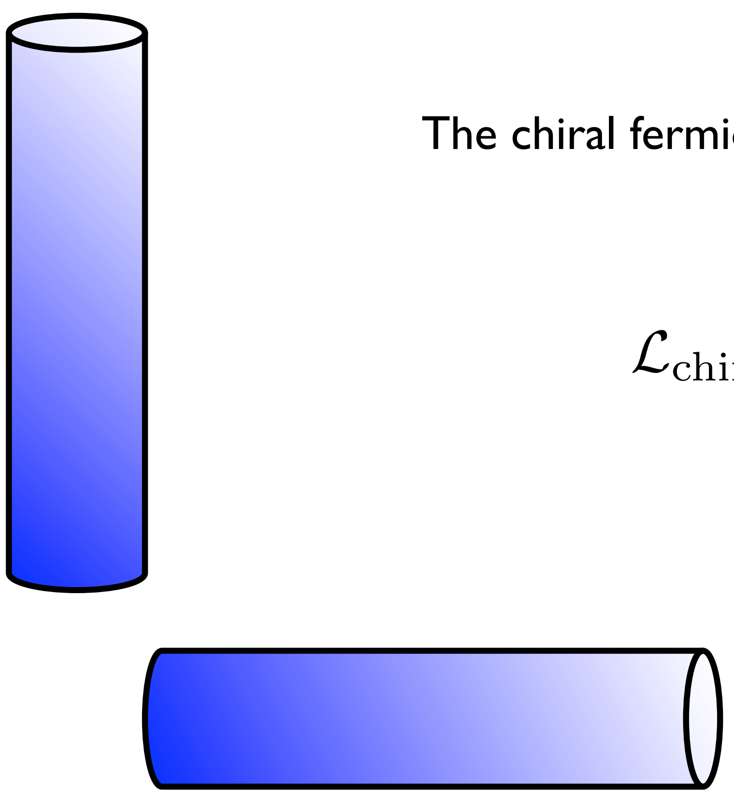
Work in Progress: Decoupling from d=1+1

An (old) idea: Integrate out hyermultiplets. Focus on D1-D1 fields Y and Z

$$
\begin{aligned}
ds^2 &= \left(\frac{1}{g^2}+\frac{Q_5^+}{y^2}\right)\left(dy^2+y^2(d\Omega_3^+)^2\right)+\left(\frac{1}{g^2}+\frac{Q_5^-}{z^2}\right)\left(dz^2+y^2(d\Omega_3^-)^2\right)\\
&\longrightarrow \; Q_5^+\left(\frac{dy^2}{y^2}+(d\Omega_3^+)^2\right)+Q_5^-\left(\frac{dz^2}{z^2}+(d\Omega_3^-)^2\right)
\end{aligned}
$$

There are also torsion terms and background dilaton charge.

Aharony and Berkooz (1999)



The chiral fermions survive this integrating out. They have action

$$
\mathcal{L}_{\mathrm{chiral}} = \left(1+\frac{\log(y^2/z^2)}{y^2-z^2}\right)\bar{\chi}_-\partial_+\chi_- + \dots
$$

$$
\frac{\log(y^2/z^2)}{y^2-z^2} \to \begin{cases} \log y & y\to 0 \\ \frac{1}{y^2} & y=z\to 0 \end{cases}
$$

Work with Kenny Wong

Summary and Open Problems

N=(0,4) Gauge Theory $\Rightarrow$ Large *N=4* CFT dual to $AdS_3 \times S^3 \times S^3 \times S^1$ + decoupled left-moving fermions

- Understand decoupling of chiral fermions from perspective of d=1+1 dimensional gauge theory
- Compare chiral primaries
  - Marginal Operators
  - Any sign of non-linear BPS bound?
- Regimes of parameters where *c* scales linearly.
  - Relationship to higher spin theories?

Gaberdiel and Gopakumar (2013)

$$c = 6Q_1 \frac{Q_5^+ Q_5^-}{Q_5^+ + Q_5^-}$$

Thank you for your attention

The Algebra

$$G^a(z)G^b(w) = \frac{2c}{3}\frac{\delta^{ab}}{(z-w)^3} - \frac{8\gamma\alpha^{+,i}_{ab}A^{+,i}(w) + 8(1-\gamma)\alpha^{-,i}_{ab}A^{-,i}(w)}{(z-w)^2} -$$

$$- \frac{4\gamma\alpha^{+,i}_{ab}\partial A^{+,i}(w) + 4(1-\gamma)\alpha^{-,i}_{ab}\partial A^{-,i}(w)}{z-w} + \frac{2\delta^{ab}L(w)}{z-w} + \ldots,$$

$$A^{\pm,i}(z)A^{\pm,j}(w) = -\frac{k^\pm\delta^{ij}}{2(z-w)^2} + \frac{\epsilon^{ijk}A^{\pm,k}(w)}{z-w} + \ldots,$$

$$Q^a(z)Q^b(w) = -\frac{(k^+ + k^-)\delta^{ab}}{2(z-w)} + \ldots,$$

$$U(z)U(w) = -\frac{k^+ + k^-}{2(z-w)^2} + \ldots,$$

$$A^{\pm,i}(z)G^a(w) = \mp\frac{2k^\pm\alpha^{\pm,i}_{ab}Q^b(w)}{(k^+ + k^-)(z-w)^2} + \frac{\alpha^{\pm,i}_{ab}G^b(w)}{z-w} + \ldots,$$

$$A^{\pm,i}(z)Q^a(w) = \frac{\alpha^{\pm,i}_{ab}Q^b(w)}{z-w} + \ldots,$$

$$Q^a(z)G^b(w) = \frac{2\alpha^{+,i}_{ab}A^{+,i}(w) - 2\alpha^{-,i}_{ab}A^{-,i}(w)}{z-w} + \frac{\delta^{ab}U(w)}{z-w} + \ldots,$$

$$U(z)G^a(w) = \frac{Q^a(w)}{(z-w)^2} + \ldots.$$

$$k^\pm = Q_1 Q_5^\pm$$