Gapped Chiral Fermions

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Question

What symmetries are broken when fermions get a mass?

Simplest Example

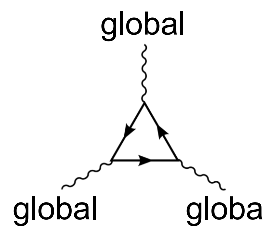
$$\mathcal{L}_{\rm mass} = m\psi_L^\dagger \psi_R$$

Vector symmetry survives, chiral symmetry broken

The Real Obstacle: the 't Hooft Anomaly

A global symmetry *G* has a ‘t Hooft anomaly.

$$\text{Anomaly} = \sum_{\text{fermions}}$$



If the anomaly is non-vanishing then either

- The symmetry *G* is spontaneously broken
- There exist massless fermions to saturate the anomaly

What if the 't Hooft Anomaly Vanishes?

Consider the following examples:

- *G* = *SU(N)* with $\square\!\square$ and *N+4* $\overline{\square}$
- *G* = *SU(N)* with $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ and *N-4* $\overline{\square}$
- *G* = *SU(3)* x *SU(2)* x *U(1)* with 15 fermions carrying the quantum numbers of quarks and leptons in the Standard Model

In each case, can we give a mass to the fermions without breaking *G?*

How to Gap Chiral Fermions

The Rules of the Game

- Start from free massless fermions realising a non-anomalous chiral symmetry G

Add extra degrees of freedom and flow to the IR. The goal is to gap everything while preserving G.

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- Scalars.
  - These can be charged under G (but you better make sure that they don't condense)
- Fermions.
  - If these are charged under *G,* they must come in left/right conjugate pairs.
- Gauge Fields.
  - These gauge a different symmetry *H* providing
    - $[H,G] = 0$
    - There are no mixed anomalies with G.
    - There are scalars that allow a phase in which *H* is Higgsed.

The Basic Idea

Find *H* such that:

Gauge dynamics of *H* with global symmetry *G*

↓ RG

Confinement *without* chiral symmetry breaking

Example 1

G = $SU(N)$ with $\square\!\square$ and $N+4$ $\overline{\square}$

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Gauge $H = SU(N+4)$. Must also add:

- Additional fermion in $\overline{\begin{array}{c}\square\\[-4pt]\square\end{array}}$ of H.
- Scalars that can Higgs H.

- Scalars condense $\Rightarrow$ auxiliary fields heavy and decouple
- Scalars heavy $\Rightarrow$ have to understand dynamics of strongly coupled H gauge theory

Example 1

$G = SU(N)$ with $\Box\Box$ and $N+4$ $\overline{\Box}$

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λ ↓ $\quad \psi$ ↓ $\quad \tilde{\chi}$ ↓

Under G x H, we have $(\Box\Box, \mathbf{1}) + (\overline{\Box}, \Box) + (\mathbf{1}, \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}})$

These two charged under $H = SU(N+4)$

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$$\begin{array}{ccccc} & \lambda & & \psi & & \tilde{\chi} \\ & \downarrow & & \downarrow & & \downarrow \end{array}$$

Under G x H, we have $(\Box\Box, \mathbf{1}) + (\overline{\Box}, \Box) + (\mathbf{1}, \overline{\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}})$

H is expected to confine *without* breaking chiral symmetry G.
The low-energy spectrum us believed to be a massless composite fermion

$$\tilde{\lambda} = \psi \tilde{\chi} \psi \quad \text{in } (\overline{\Box\Box}, \mathbf{1})$$

Georgi '79; Dimopoulos, Raby and Susskind '80; Eichten, Peccei, Preskill and Zeppenfeld, '85

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$$\lambda \qquad\qquad \psi \qquad\qquad \tilde{\chi}$$

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H is expected to confine *without* breaking chiral symmetry *G.*
The low-energy spectrum us believed to be a massless composite fermion

$$\tilde{\lambda} = \psi\tilde{\chi}\psi \quad \text{in } (\overline{\square\!\square}, \mathbf{1})$$

Add in the UV

$$\mathcal{L}_{UV} \sim \lambda\psi\tilde{\chi}\psi \xrightarrow{\text{RG}} \mathcal{L}_{IR} \sim \lambda\tilde{\lambda}$$

This gaps the fermions, preserving *G.*

Example 2

G = *SU(N)* with $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ and *N-4* $\overline{\square}$

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G = *SU(N)* with $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ and *N-4* $\overline{\Box}$

Gauge *H* = *SU(N-4)*. Must also add:

- Additional fermion in $\overline{\Box\Box}$ of *H.*
- Scalars that can Higgs *H.*

Example 2

$G = SU(N)$ with $\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$ and $N\text{-}4$ $\overline{\Box}$

Gauge $H = SU(N\text{-}4)$. Must also add:

- Additional fermion in $\overline{\Box\Box}$ of H.
- Scalars that can Higgs H.

$\chi \downarrow$ $\quad \psi \downarrow$ $\quad \tilde{\lambda} \downarrow$

Under G x H, we have $\left(\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}, \mathbf{1}\right) + \left(\overline{\Box}, \Box\right) + \left(\mathbf{1}, \overline{\Box\Box}\right)$

These two charged under $H = SU(N\text{-}4)$

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- Additional fermion in $\overline{\Box\Box}$ of H.
- Scalars that can Higgs H.

$\chi \downarrow$ $\quad \psi \downarrow$ $\quad \tilde{\lambda} \downarrow$

Under $G \times H$, we have $\left(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}, \mathbf{1}\right) + \left(\overline{\Box}, \Box\right) + \left(\mathbf{1}, \overline{\Box\Box}\right)$

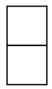
What now happens to H? A simple guess is that it again confines without breaking chiral symmetry, with a massless composite fermion:

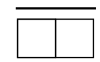
$$\tilde{\chi} = \psi\tilde{\lambda}\psi \quad \text{in } \left(\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}, \mathbf{1}\right)$$

Very likely at large N. But….not true for H = SU(2)

Eichten, Peccei, Preskill and Zeppenfeld, ‘85

The Case of *H* = *SU(2)*

G = *SU(6)* with  and *2* 

Gauge *H* = *SU(2)*. It has 6 doublets and an adjoint = 

ψ (6 doublets) $\quad$ $\tilde{\lambda}$ (adjoint)

Now *H* is a vector-like gauge theory. And this changes things.

Possibilities:

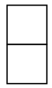
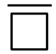
- *H* confines without breaking chiral symmetry *G* with massless

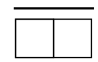
$$\tilde{\chi} = \psi\tilde{\lambda}\psi$$

- Fermion bilinears $\pi = \psi\psi$ condense, breaking *G*

The Case of *H* = *SU(2)*

Weingarten '83;
Aharony, Sonnenschein, Peskin, Yankielowicz, '95

G = *SU(6)* with  and 2 

Gauge *H* = *SU(2)*. It has 6 doublets and an adjoint = 

ψ (doublets), $\tilde{\lambda}$ (adjoint)

Use Weingarten inequalities. Look at the propagator for $\tilde{\chi} = \psi\tilde{\lambda}\psi$

$$\langle \tilde{\chi}(0)\tilde{\chi}^\dagger(R)\rangle \sim \int d\mu \; S_\psi S_\lambda S_\psi \sim e^{-m_{\tilde{\chi}}R}$$

Integral over gauge field with positive definite measure

Propagators for quarks in background gauge field

The Case of *H* = *SU(2)*

Weingarten '83;
Aharony, Sonnenschein, Peskin, Yankielowicz, '95

G = *SU(6)* with $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ and 2 $\overline{\square}$

Gauge *H* = *SU(2)*. It has 6 doublets and an adjoint = $\overline{\square\square}$

ψ (the 6 doublets), $\tilde{\lambda}$ (the adjoint)

Use Weingarten inequalities. Look at the propagator for $\tilde{\chi} = \psi\tilde{\lambda}\psi$

$$\langle\tilde{\chi}(0)\tilde{\chi}^\dagger(R)\rangle \sim \int d\mu\; S_\psi S_\lambda S_\psi \sim e^{-m_{\tilde{\chi}}R}$$

$$\leq \int d\mu\; |S_\psi|^2\left(|S_\lambda|^2\right)^{1/2}$$

$$\leq \int d\mu\;\left(|S_\psi|^2\right)^{1/2}\int d\mu\;\left(|S_\lambda|^2|S_\psi|^2\right)^{1/2} \quad \Longrightarrow \quad m_\pi \leq m_{\tilde{\chi}}$$

$\left(|S_\psi|^2\right)^{1/2}$ term: $\sim e^{-m_\pi R}$

$\left(|S_\lambda|^2|S_\psi|^2\right)^{1/2}$ term: at most a constant

Supersymmetry to the Rescue

H = *SU(2)* with 6 doublets and an adjoint Weyl fermion

- Non-supersymmetric $\Rightarrow$ likely to break *G* = *SU(6)*
- Supersymmetric theory $\Rightarrow$ confinement without chiral symmetry breaking

Seiberg '94

Note: presence of scalars means the measure is *not* positive definite.

Many other examples of supersymmetric theories known

Csaki, Schmaltz and Skiba '96

Example 3: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

(left-handed)^c		right-handed		
<u>leptons</u>	<u>quarks</u>	<u>electron</u>	<u>up quark</u>	<u>down quark</u>
$(\mathbf{1}, \mathbf{2})_{-3}$	$(\bar{\mathbf{3}}, \mathbf{2})_{+1}$	$(\mathbf{1}, \mathbf{1})_{+6}$	$(\mathbf{3}, \mathbf{1})_{-4}$	$(\mathbf{3}, \mathbf{1})_{+2}$

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$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_0$
$(\mathbf{1},\mathbf{2})_{+3}$				$(\bar{\mathbf{3}},\mathbf{1})_{-2}$	

- Add three further pairs of fermions

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$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
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- Add three further pairs of fermions
- Gauge the H = SU(2) symmetry

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- Add three further pairs of fermions
- Gauge the H = SU(2) symmetry
- Supersymmetrize.
  - Add scalar superpartners for all fermions, and a H = SU(2) gaugino

Example 3: The Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

L	Q	E	U	D	N
leptons	quarks	electron	up quark	down quark	neutrino
$(\mathbf{1},\mathbf{2})_{-3}$	$(\bar{\mathbf{3}},\mathbf{2})_{+1}$	$(\mathbf{1},\mathbf{1})_{+6}$	$(\mathbf{3},\mathbf{1})_{-4}$	$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$L' \longrightarrow (\mathbf{1},\mathbf{2})_{+3}$				$D' \longrightarrow (\bar{\mathbf{3}},\mathbf{1})_{-2}$	

- The *H* = SU(2) gauge theory is coupled to six doublets.
- This confines *without* breaking the global symmetry.

Seiberg '94

- The low-energy physics consists of 15 free mesons:

$$\epsilon_{ab}L^aL^b \qquad \epsilon_{ijk}D^iD^j \qquad L^aD^i \qquad L^aN \qquad D^iN$$

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	$(\mathbf{1},\mathbf{2})_{-3}$				$(\mathbf{3},\mathbf{1})_{+2}$	$(\mathbf{1},\mathbf{1})_{0}$
$L' \longrightarrow$	$(\mathbf{1},\mathbf{2})_{+3}$			$D' \longrightarrow$	$(\bar{\mathbf{3}},\mathbf{1})_{-2}$	

If we add the superpotential

$$\mathcal{W}_{UV} = \epsilon_{ab} L^a L^b E + \epsilon_{ijk} D^i D^j U^k + \epsilon_{ab} L^a D^i Q^b_i + \epsilon_{ab} L^a N L'^b + D^i N D'_i$$

But, in the infra-red, this becomes

$$\mathcal{W}_{IR} = \widetilde{E} E + \widetilde{U}_k U^k + \widetilde{Q}^i_b Q^b_i + \widetilde{L}^b L'^b + \widetilde{D}_i D'_i$$

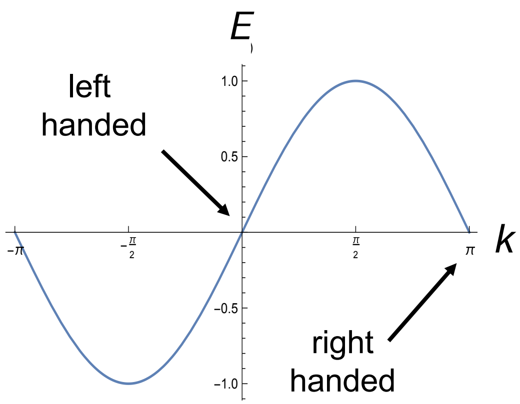
Comments on Domain Wall Fermions

(in the continuum and on the lattice)

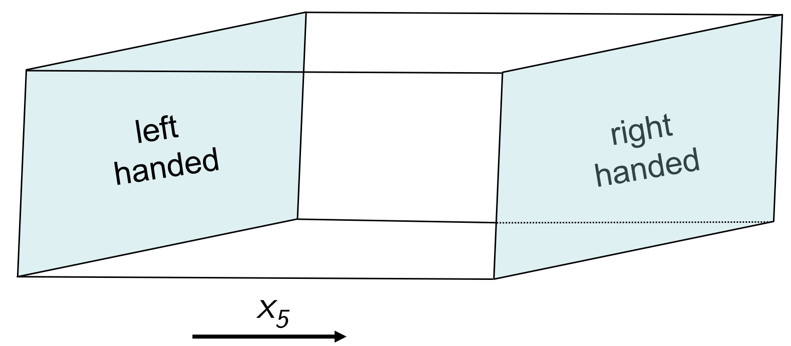
Lattice Fermions

Naïve attempts to put chiral fermions on the lattice result in doublers

Either separated in momentum space…



…or in an extra spatial dimension



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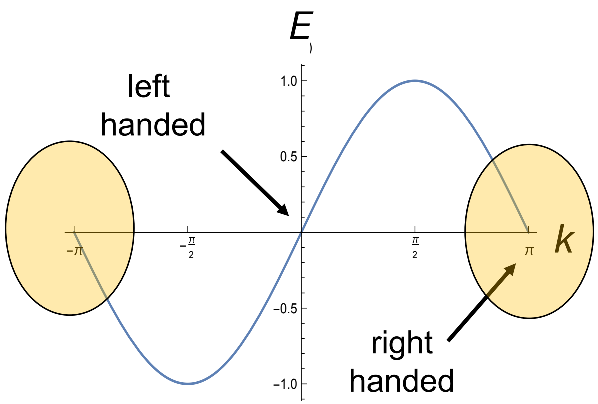

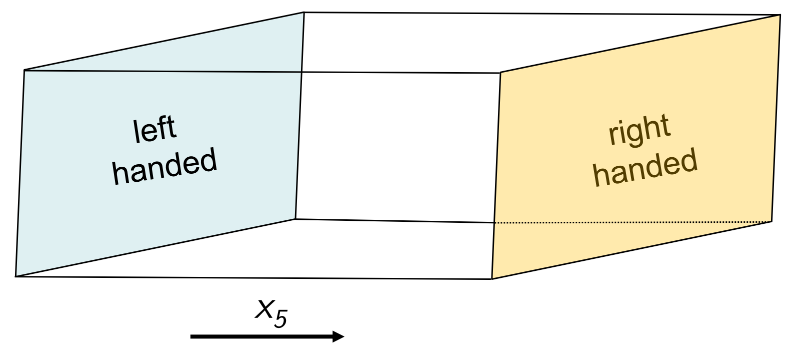

An Old Idea: Find a way to gap the doublers, leaving the original fermions untouched

Challenges:

- Ensure that only the mirror fermions experience the interactions
- Find interactions that gap chiral fermions

Eichten and Preskill ’86

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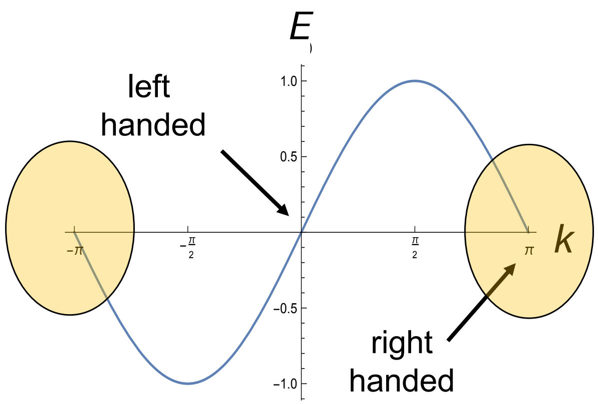
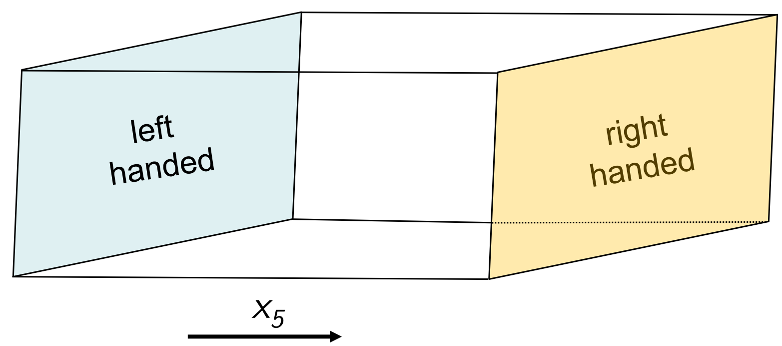




Most attempts work with irrelevant multi-fermion operators, cranked up to the lattice scale

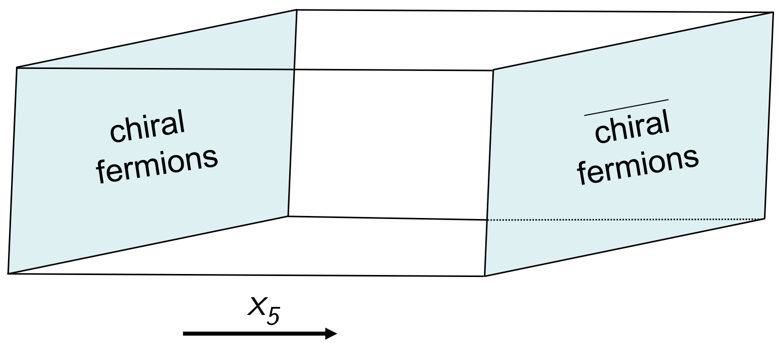
$$\mathcal{L}_{4-\mathrm{fermi}} \sim \psi\psi\psi\psi$$

Sadly, so far, to no avail.

Eichten and Preskill '86, Golterman, Petcher and Rivas '93; Creutz, Rebbi, Tytgat, Xue '96;
Poppitz and Shang '10; Chen, Giedt and Poppitz '12; Wen '13; Wang and Wen '13; Kikukawa '17'; Wang and Wen '18

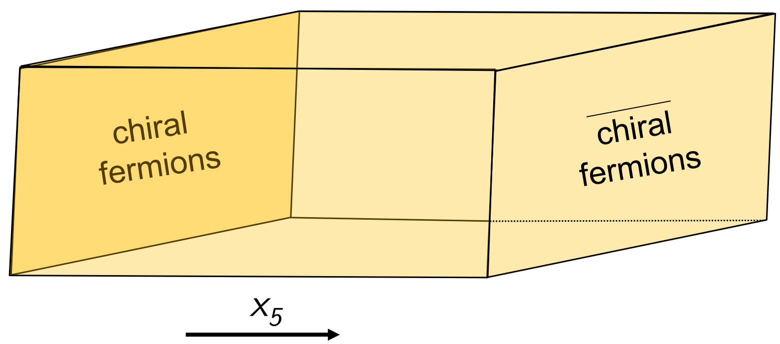
Can we use continuum gapping mechanisms to help us?

Gapping Domain Wall Fermions



- Put gauge field that you care about everywhere in the fifth dimension.
  - e.g. G= *SU(3) x SU(2) x U(1)*
  - It couples to chiral fermions + their conjugates in a vector-like manner

Gapping Domain Wall Fermions



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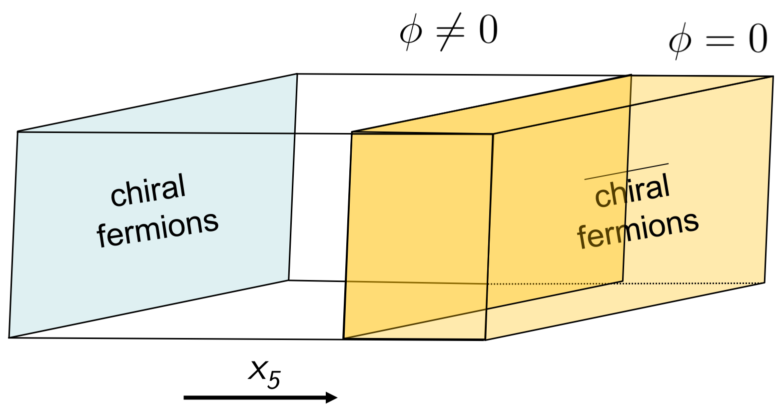
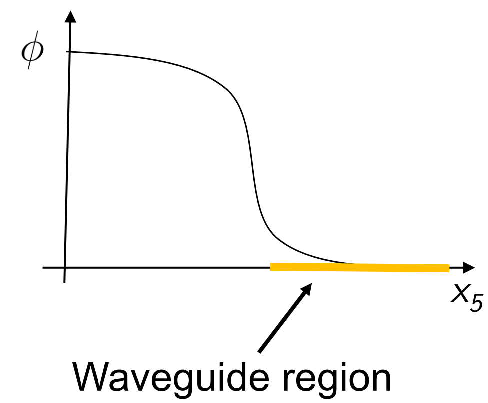
- Put the auxiliary gauge field everywhere.
  - e.g. *H = SU(2)*
  - It too couples to chiral fermions and their conjugates

Gapping Domain Wall Fermions





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- Put the auxiliary gauge field everywhere.
  - e.g. *H* = *SU(2)*
  - It too couples to chiral fermions and their conjugates

- Add Higgs fields for *H* with a profile in the fifth dimension.
  - Add extra fermions coupled to H

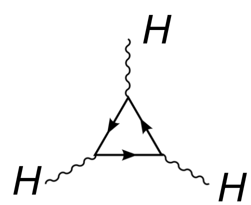
Gapping Domain Wall Fermions

<u>Pitfall 1:</u>

- What if chiral fermion are in anomalous representation of *G?*
  - Then *H* dynamics can't gap them!

- What if chiral fermions are in anomalous representation of *H*?

$$\sum_{\text{5d Fermions}}$$

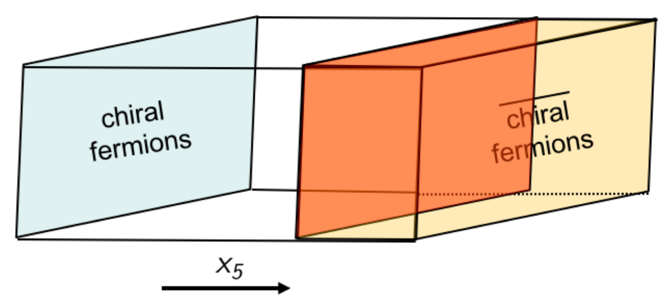


$$= \frac{k}{24\pi^2} \mathrm{tr} A \wedge F \wedge F + \dots$$

The phase of the Higgs field then fails to decouple on the interface.
We get a Wess-Zumino term living at the interface.

$$\mathcal{L}_{\text{eff}} \sim k \, \mathrm{tr} \, \Omega F \wedge F$$
$$\sim k \bar{\psi} \gamma^5 \partial_5 \Omega \psi \qquad \Omega \in H$$

by anomaly

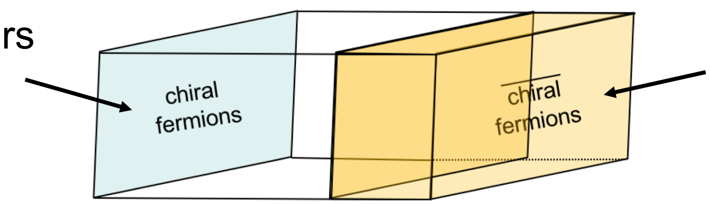


Gapping Domain Wall Fermions

Pitfall 2:

To make *H* anomaly free, we typically need to add extra fermions (singlets or vector-like under G).

Now unwanted doublers show up here



Extra *H* fermions needed here

The examples we've seen fall into two categories:

- *H* is a chiral gauge theory, with new additional chiral fermions.
  - This feels like a vicious circle!
- *H* is a vector-like gauge theory (e.g. supersymmetric)
  - Can gap doublers with Majorana mass without breaking *H*

In the latter case, we have a *continuum* description of chiral gauge theory *G*.

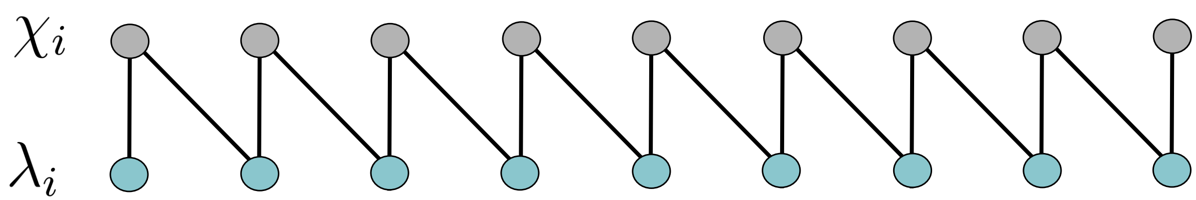
No obstacle to discretization (albeit with a sign problem and significant fine tuning.)

Lattice Domain Wall Fermions

A 5d Dirac fermion $\Psi = \begin{pmatrix} \chi \\ \lambda \end{pmatrix}$. We discretize it in the 5$^{\text{th}}$ dimension with Wilson parameter *r = 1*

4d kinetic terms — 5d hopping terms — masses

$$S = \int d^4x \sum_{i=1}^{N} a \left\{ \overbrace{i\chi_i^\dagger \bar{\sigma}^\mu \partial_\mu \chi_i - i\lambda_i^\dagger \sigma^\mu \partial_\mu \lambda_i}^{\text{4d kinetic terms}} + \frac{1}{a}\left[\overbrace{\chi_i^\dagger(\lambda_i - \lambda_{i-1}) + \lambda_i^\dagger(\chi_i - \chi_{i+1})}^{\text{5d hopping terms}} - \overbrace{ma(\chi_i^\dagger \lambda_i + \lambda_i^\dagger \chi_i)}^{\text{masses}} \right] \right\}$$



$ma > 0$

Left-handed zero mode localized here

Right-handed zero mode localized here

Kaplan '92; Shamir '93

Lattice Domain Wall Fermions

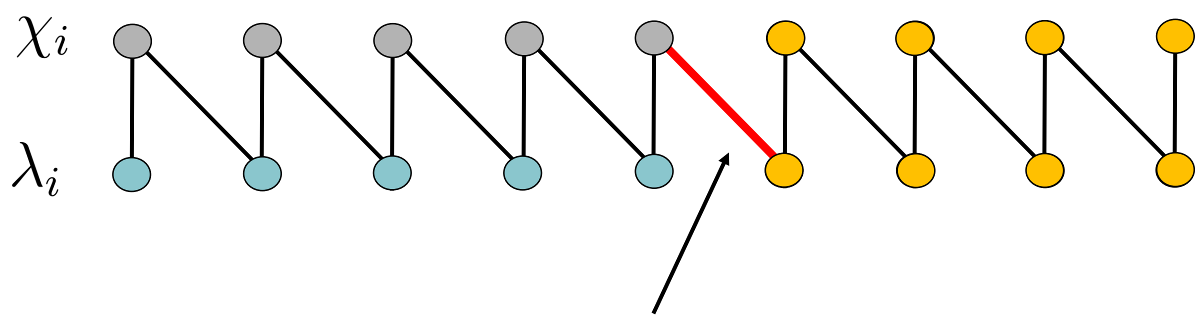
Add a 5d gauge field in waveguide region. At low-energies, only 4d gauge field survives

gauged

$$S = \int d^4x \; a \sum_{i \notin \mathrm{WG}} \left[i\chi_i^\dagger \bar{\sigma}^\mu \partial_\mu \chi_i - i\lambda_i^\dagger \sigma^\mu \partial_\mu \lambda_i \right] + a \sum_{i \in \mathrm{WG}} \left[i\chi_i^\dagger \bar{\sigma}^\mu \mathcal{D}_\mu \chi_i - i\lambda_i^\dagger \sigma^\mu \mathcal{D}_\mu \lambda_i \right]$$

$$+ {\sum_i}' \frac{1}{a} \left[\chi_i^\dagger (\lambda_i - \lambda_{i-1}) + \lambda_i^\dagger (\chi_i - \chi_{i+1}) - ma(\chi_i^\dagger \lambda_i + \lambda_i^\dagger \chi_i) \right]$$

$$+ \; y \left(\frac{1}{a} \chi_\star^\dagger \Omega \lambda_{\star-1} - \chi_{\star-1}^\dagger \Omega^\dagger \lambda_\star \right)$$

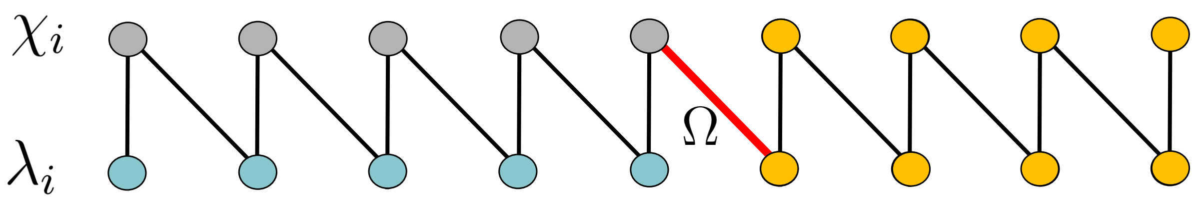


New dynamical field needed here: $\Omega \in H$. This is the Wess-Zumino term on the interface

Golterman, Jansen, Petcher, Vink '93; Golterman and Shamir '94

Lattice Domain Wall Fermions

$$S_{\rm important} = \int d^4x \ \ \text{kinetic terms} + y\left(\frac{1}{a}\chi^\dagger_\star \Omega \lambda_{\star-1} - \chi^\dagger_{\star-1}\Omega^\dagger \lambda_\star\right)$$



Result: Neither Ω nor the two neighbouring fermions are gapped.

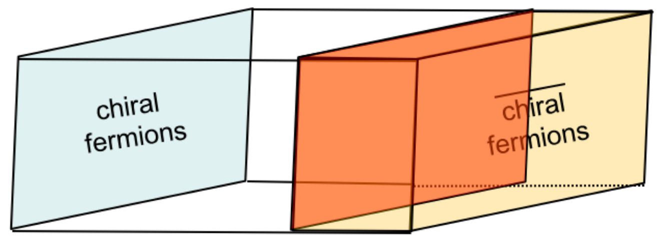
- Seen at small *y*, large *y*, and in *quenched* simulations.

A practical necessity due to sign problem!

Golterman, Jansen, Petcher, Vink ’93; Golterman and Shamir ‘94

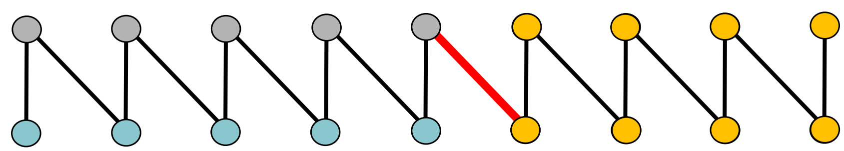
Comparing Domain Wall Fermions

In the continuum:



Nothing fishy at the interface provided that *H* gauge theory is anomaly free.
But the gauge theory will necessarily have a sign problem.

On the lattice:



Challenging to check what happens because the theory has a sign problem!!

Thank you for your attention