Geometry Through the Eyes of Physics

Professor David Tong

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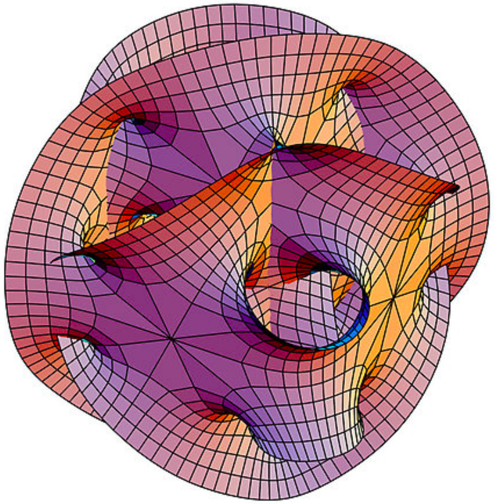
UNIVERSITY OF CAMBRIDGE

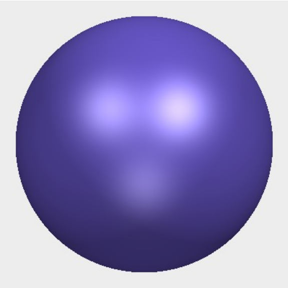
What is Geometry?

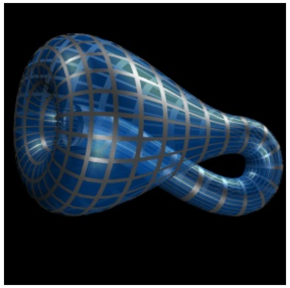
The study of shape and space.



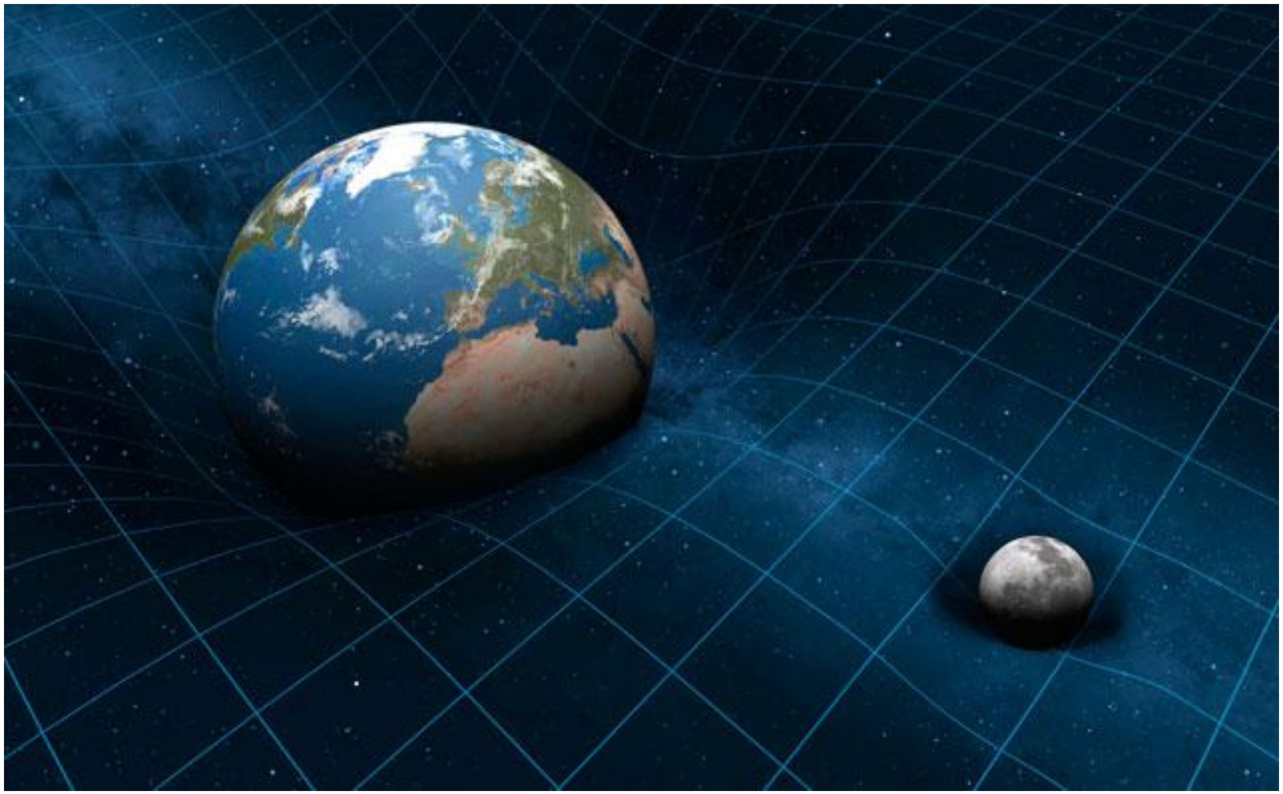
Your word for the day: *manifold*

Some Two Dimensional Manifolds





Geometry in Physics

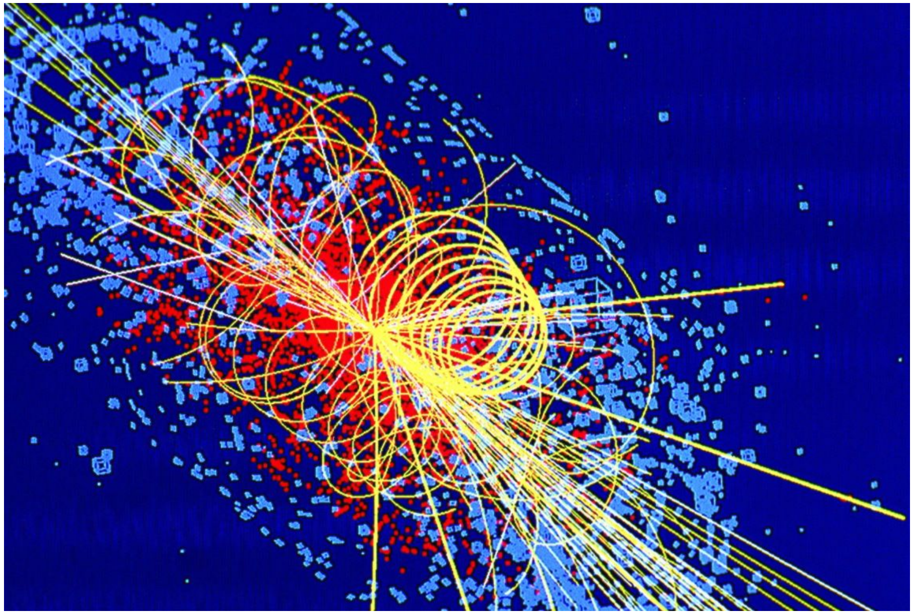


General Relativity: Gravity = Geometry

What About Physics in Geometry?

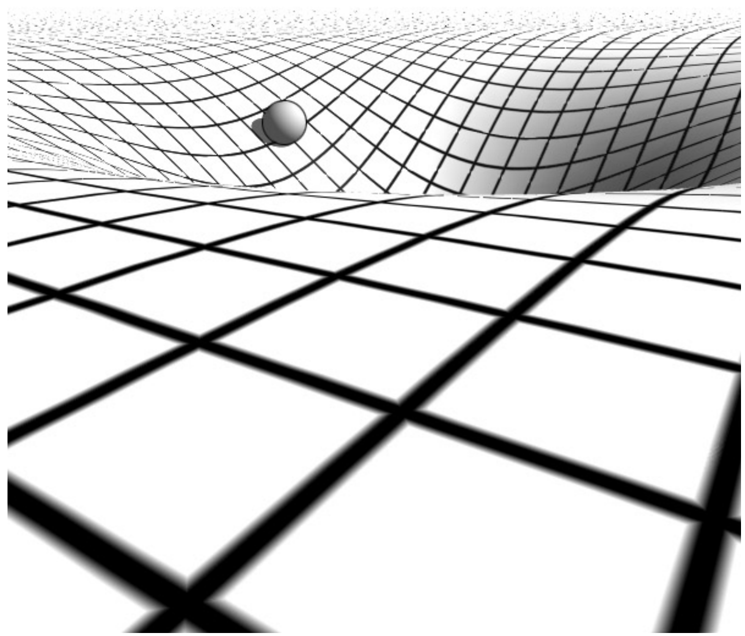
Thinking like a Physicist

This is what we care about…



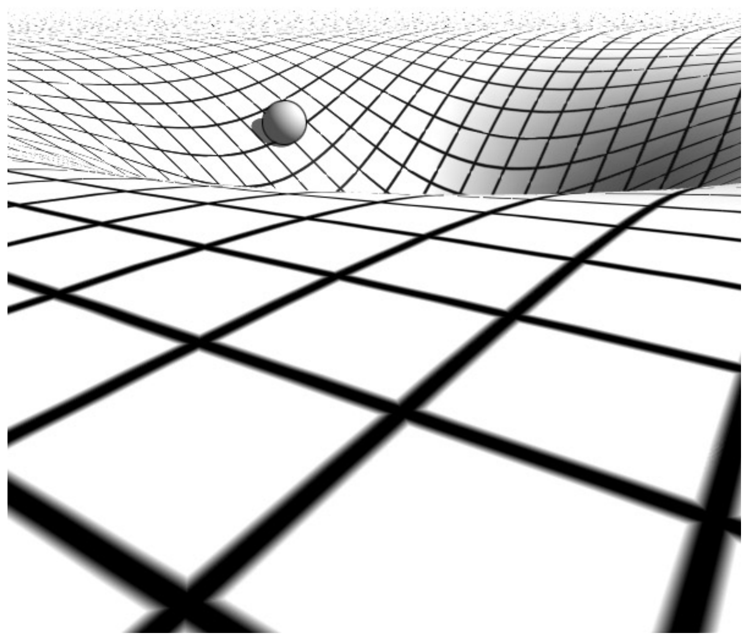
…particles.

Particles Moving on Manifolds



Classical particles just roll around…

Particles Moving on Manifolds

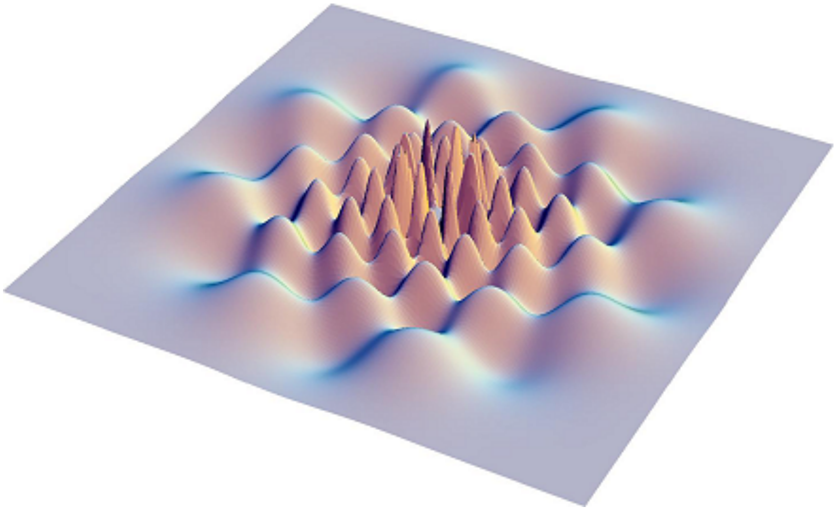


Classical particles just roll around…

…but quantum particles are more interesting

Quantum Mechanics

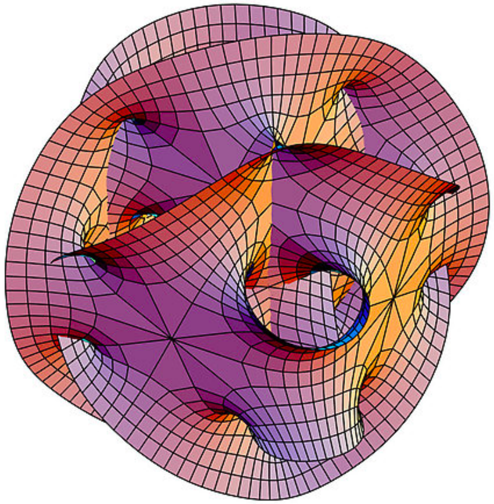
Particles are described by a *wavefunction*



$\psi(\vec{x})$ is a function that tells us the probability that a particle is at position $\vec{x}$

Quantum Particles on Manifolds

Quantum Particles are more interesting because they're everywhere! They feel the whole space at once.



The Schrodinger Equation

$$\nabla^2 \psi = -E^2 \psi$$

The *Laplacian*. This depends on the manifold and its shape.

e.g. in three-dimensional flat space $\psi(x, y, z)$

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$$

The Schrodinger Equation

$$\nabla^2 \psi = -E^2 \psi$$

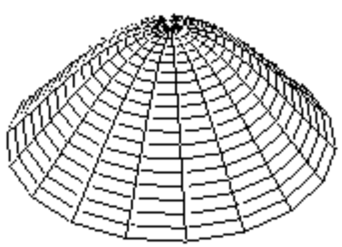
Main Idea: for a given manifold, this equation only has solutions for certain values of *E.*

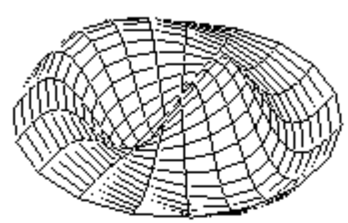
To a physicist, *E* is the energy of the particle.

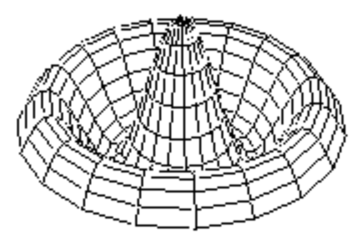
An Analogy

$$\nabla^2 \psi = -E^2 \psi$$

This same equation describes the vibrations of a drum. The different values of *E* are the notes that the drum will play.







A New View on Geomety

$$\nabla^2 \psi = -E^2 \psi$$

Instead of describing the shape of the manifold, I simply give you a list of all the values of *E* which solve this equation!

This is called the *spectrum* of the manifold.

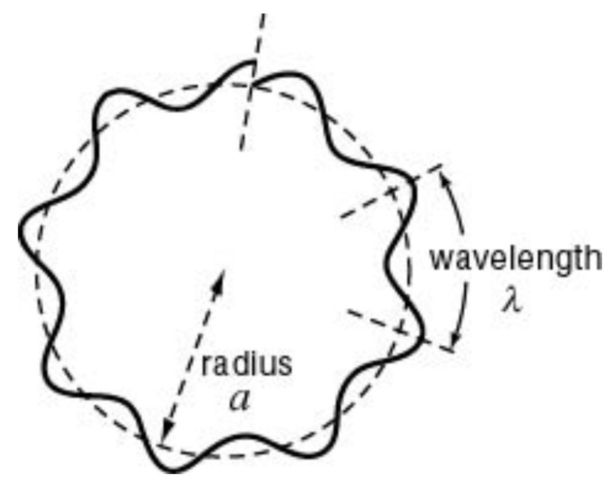
A Simple Example: the Circle

$$\frac{d^2\psi}{dx^2} = -E^2\psi \qquad \text{with} \qquad x \equiv x + 2\pi R$$

Solution is: $\psi(x) = \cos(nx/R)$ $\qquad n = 0, 1, 2, 3, \ldots$

Spectrum of circle of radius *R* is:

$$E = \frac{n}{R}$$



Something New

String Theory

The Circle Again

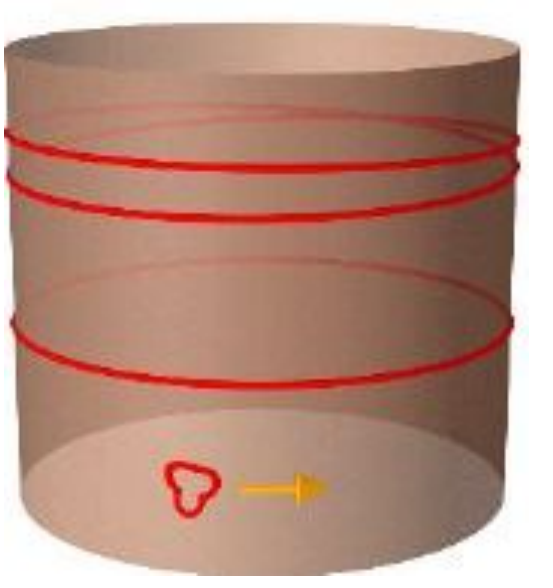
What is the spectrum (i.e. allowed energies) of a string on a circle?

- Small loops of string moving around the circle

$$E = \frac{n}{R} \qquad n = 0, 1, 2, 3, \ldots$$

- Big loops of string winding around the circle

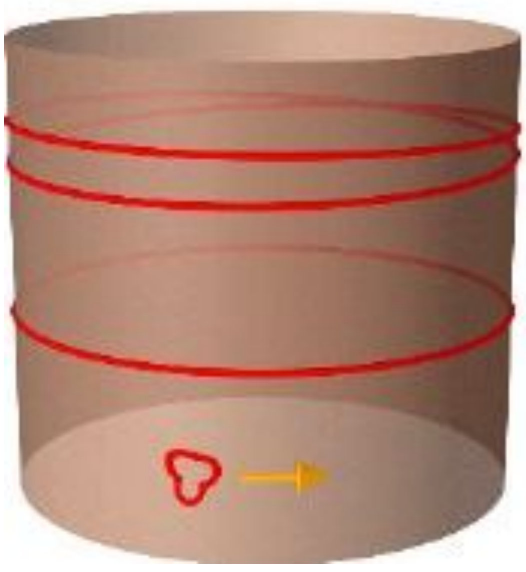
$$E = mR \qquad m = 0, 1, 2, 3, \ldots$$



Strings on the Circle

The spectrum of strings is:

$$E = \frac{n}{R} + mR \qquad n, m = 0, 1, 2, 3, \ldots$$



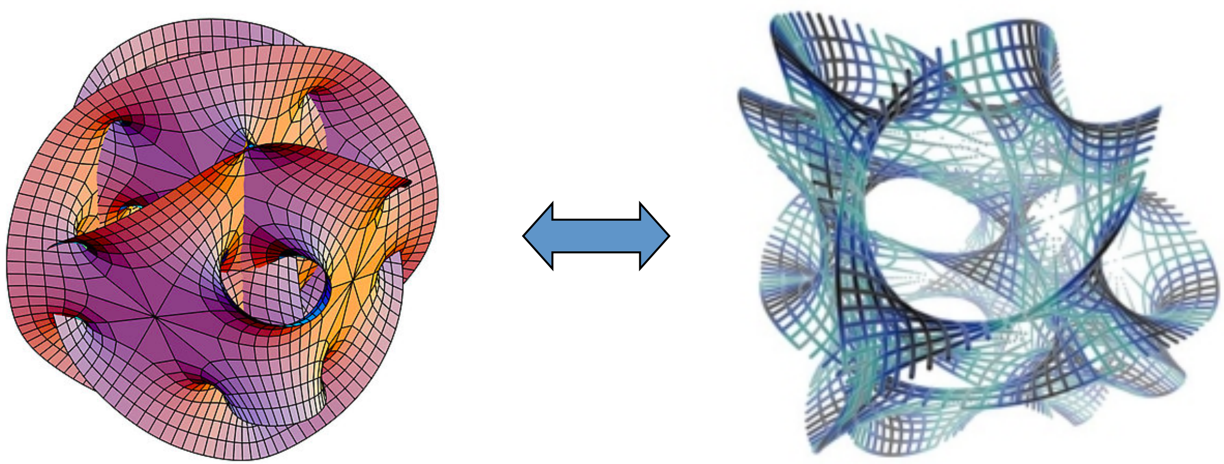
This remains the same if we swap

$$R \to \frac{1}{R}$$

Strings cannot tell the difference between very big circles and very small circles!!

Mirror Symmetry

Strings get confused between other manifolds too! In fact, manifolds come in pairs*.

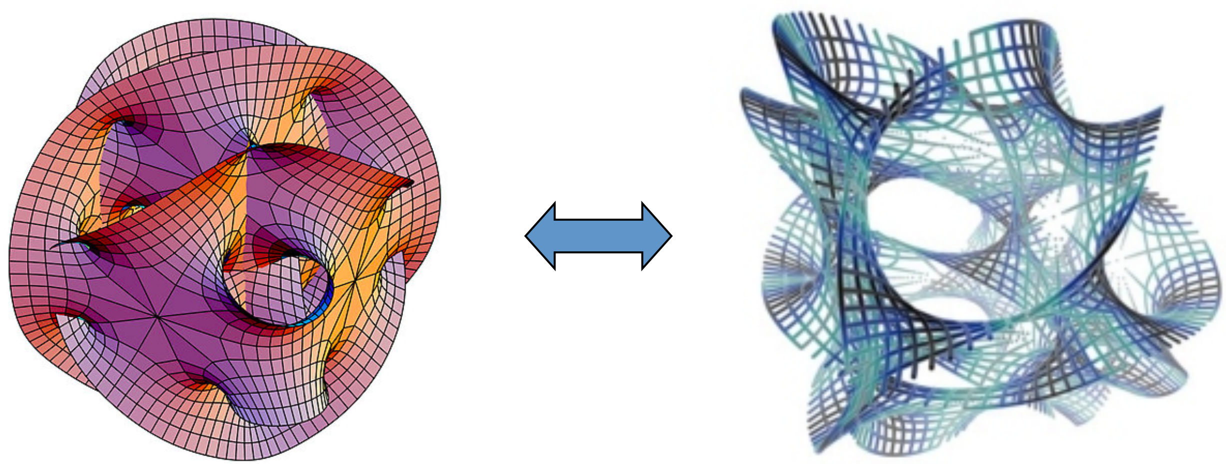


Mathematicians can tell them apart. String theorists can't!

*Calabi-Yau manifolds in particular.

Why Mirror Symmetry is Interesting

The ignorance of string theorists is their strength! In many cases, computations are easy for one manifold, but hard for the other.



String theory says that the answers have to be the same

Summary