Holographic Dual of the Lowest Landau Level

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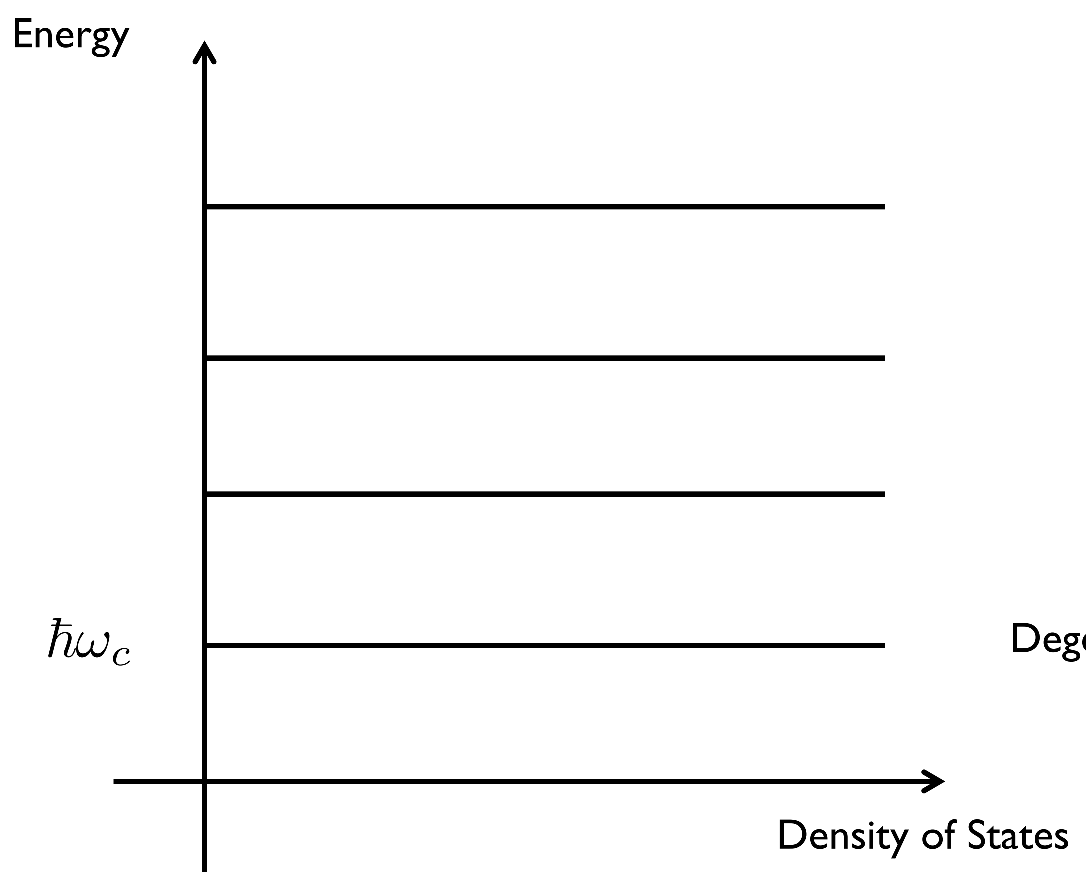
Motivation

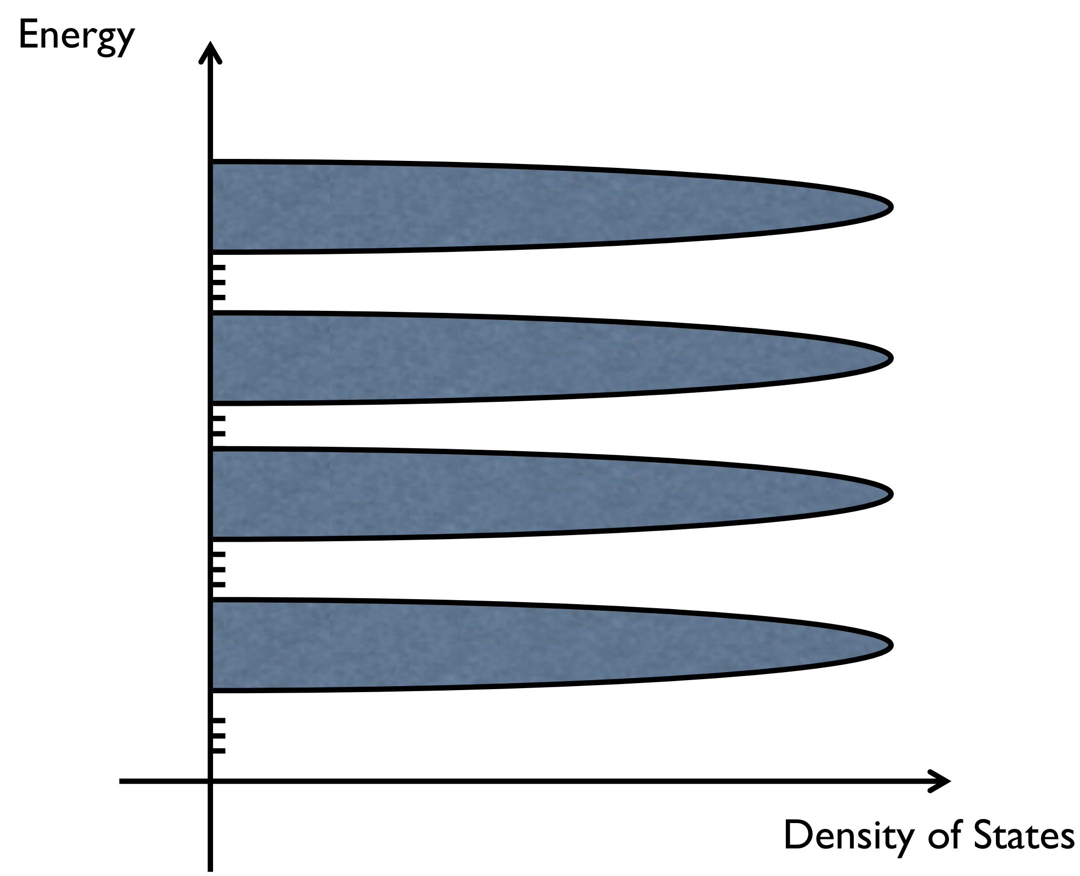
Take free fermions in d=2+1. Placed in a magnetic field, they sit in Landau levels



Degeneracy $\sim \dfrac{BA}{2\pi}$

Motivation

Turning on interactions and/or disorder leads to widely degenerate perturbation theory…

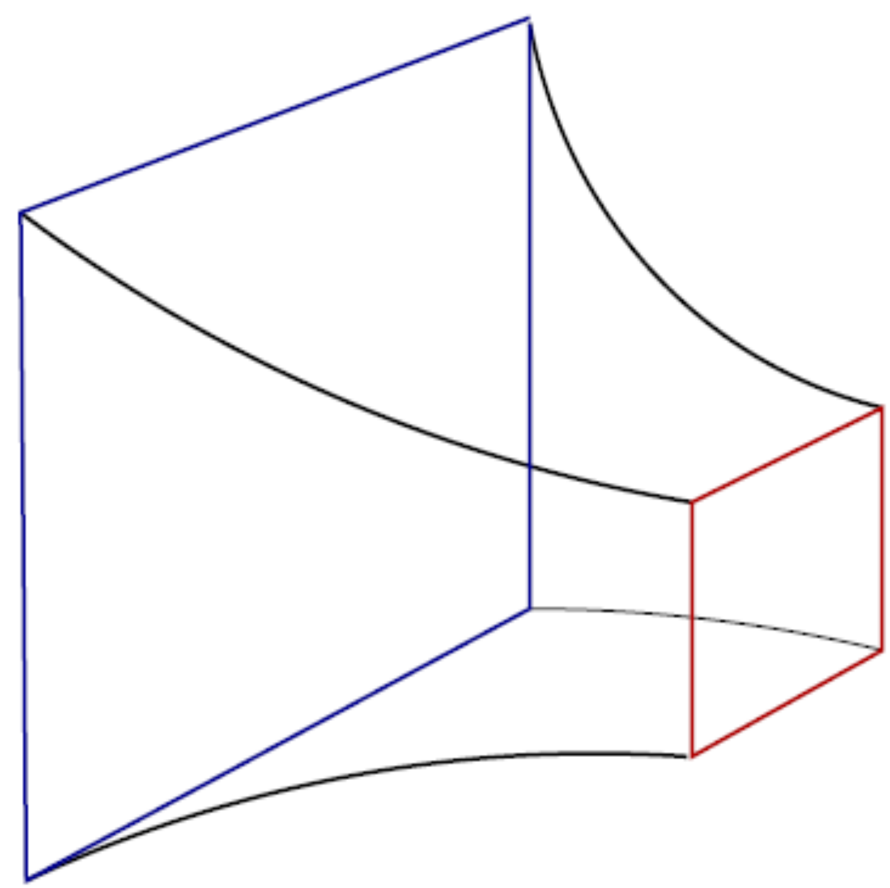


…resulting in the rich story of quantum Hall physics

Motivation

What happens if electrons are strongly coupled to begin with?

Holographic Setting: AdS

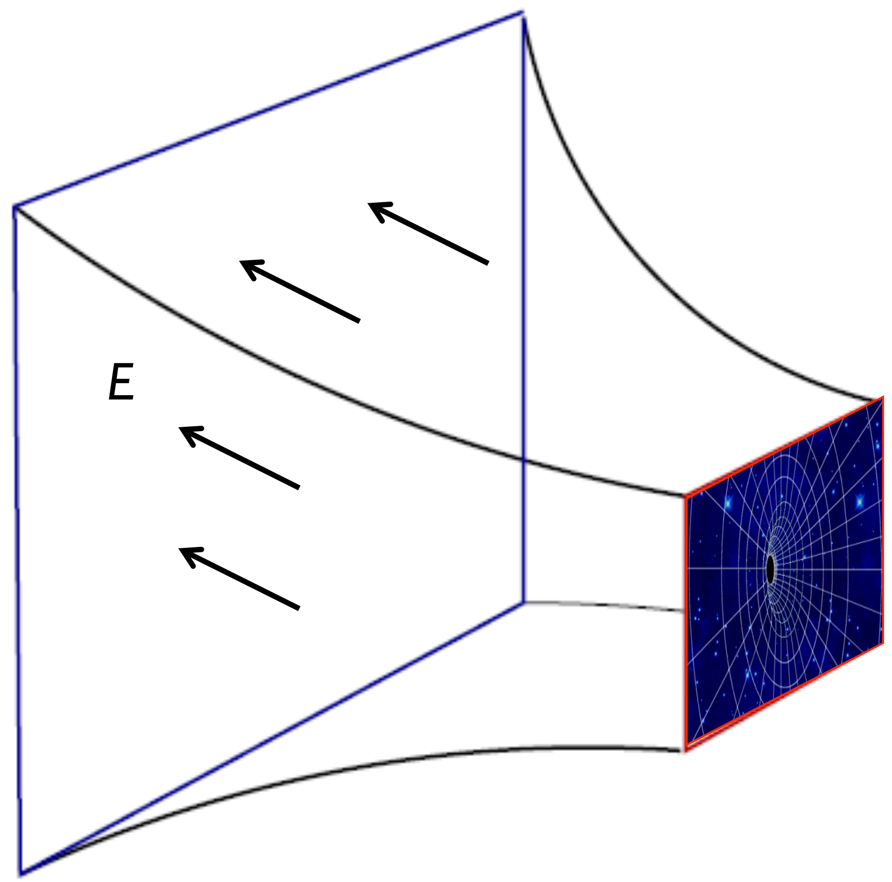


$$S = \int d^4x\sqrt{g}\ \left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - i\bar{\psi}\left(e^\mu_a\Gamma^a\mathcal{D}_\mu - m\right)\psi\right]$$

$$ds^2 = \frac{L^2}{r^2}\left(-dt^2 + dx^2 + dy^2 + dr^2\right)$$

Finite Density

Electric field



Extremal electric Reissner-Nordstrom black hole

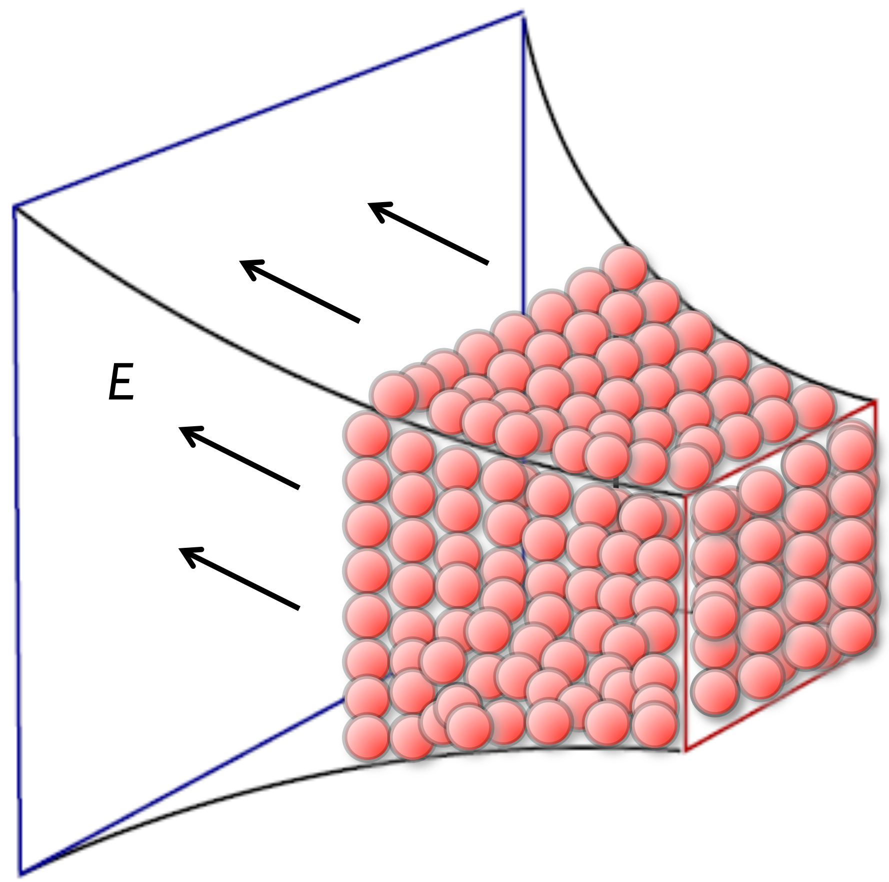
Probe fermions ⟹ Fermi surfaces

Sung-Sik Lee, MIT group, Leiden group

Fermion backreaction in near horizon AdS_2 region ⟹ Electron Star

Polchinski et al.

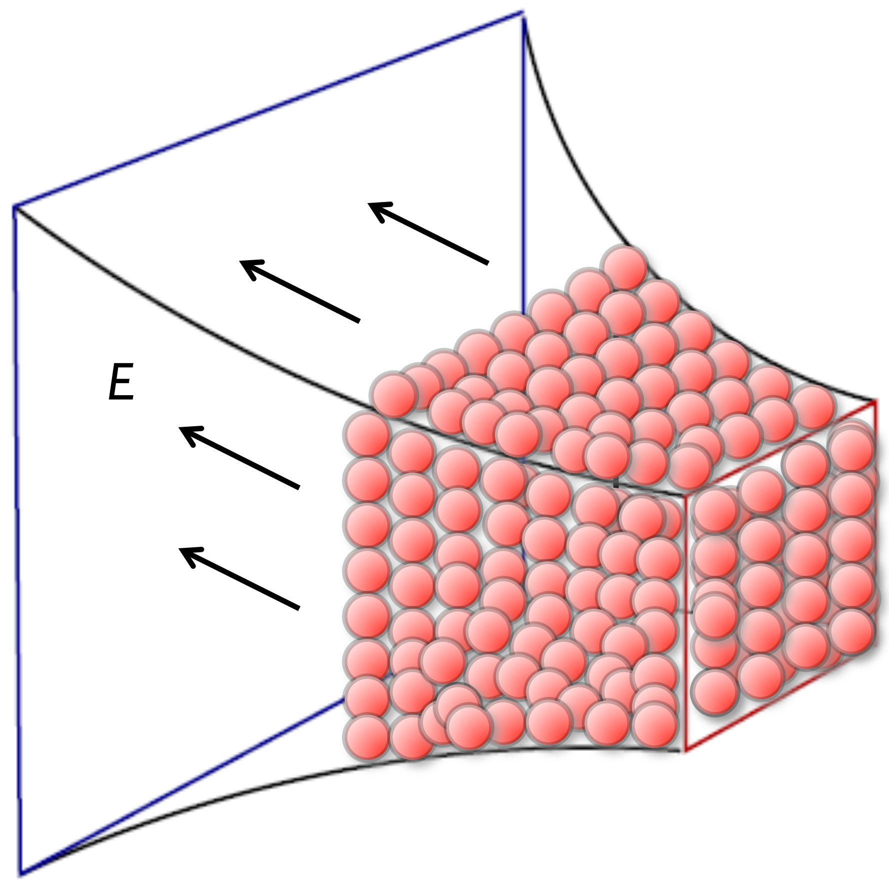
Fermi Surface = Electron Star



- The star is supported by fermionic degeneracy pressure.
  - e.g. white dwarfs or neutron stars
- It differs from stars in the night sky because
  - It lives in AdS
  - Constituents are charged
  - It is a planar (infinite) star

Hartnoll et al.

Building a Quantum Star

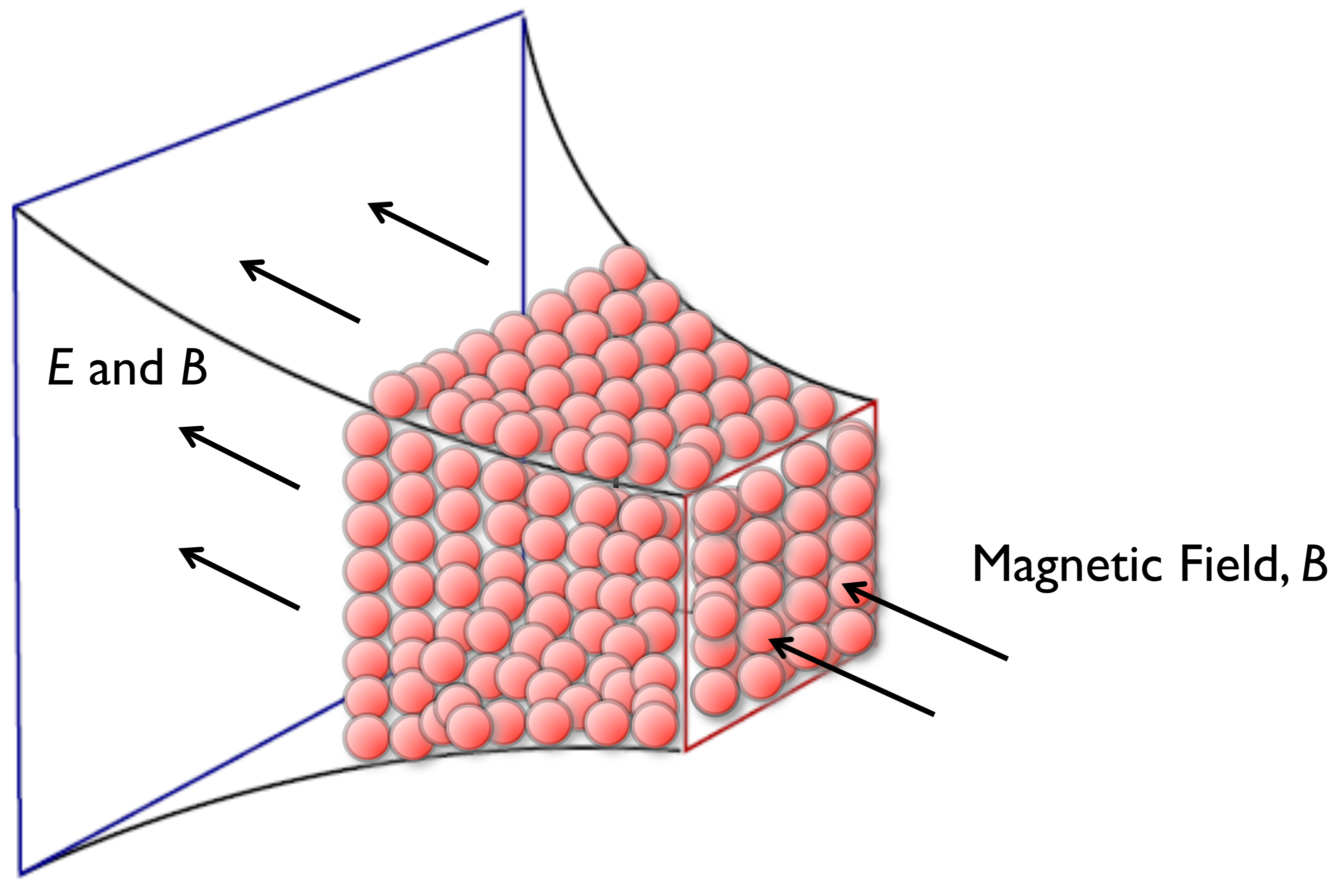


- Standard astrophysical methods only useful in the regime $m \gg \frac{1}{L}$
  - Large number of densely packed Fermi surfaces
- A *quantum electron star* has $m \sim \frac{1}{L}$
- Pauli exclusion means that building a star from fermions is much harder than bosons

Hartnoll et al.
Iqbal et al.

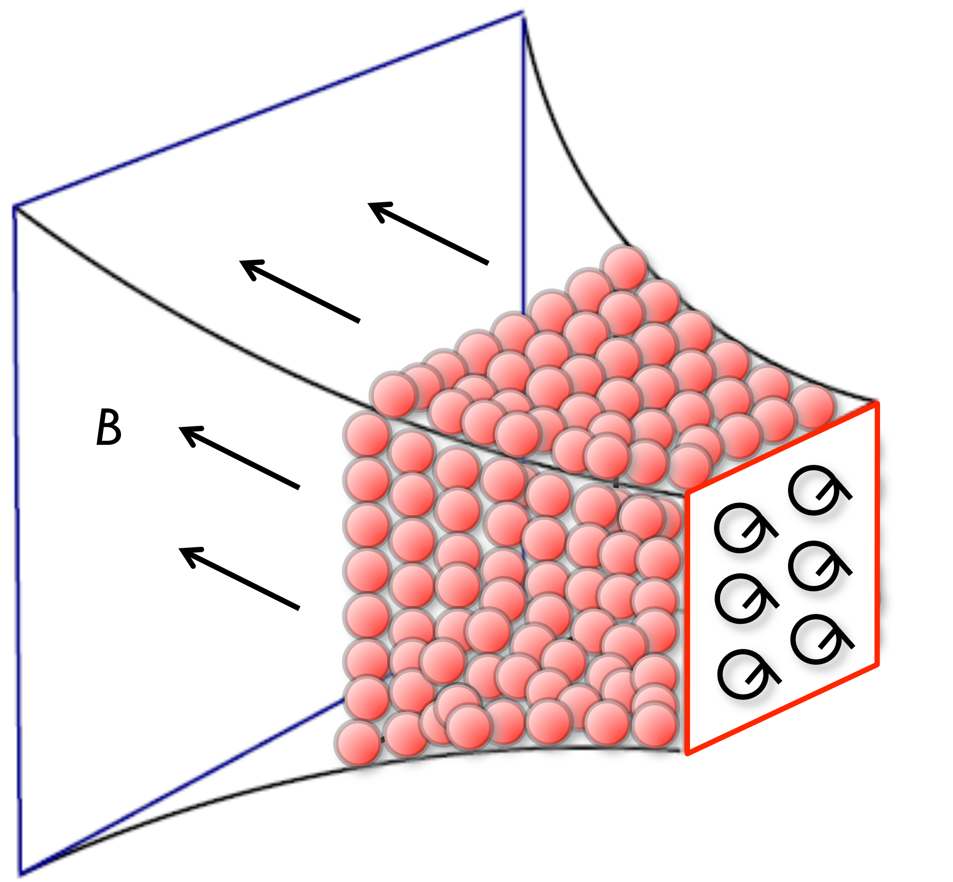
Sachdev
Allais et al.

Now Add a Magnetic Field



What is the true ground state now?

Key Idea



- In a strong magnetic field, the electrons are confined to their lowest Landau level in the *x*-*y* plane
- They move only in the radial, *r*, direction
- But fermions that move in $d=1+1$ dimensions are equivalent to bosons!

Bosonizing the Lowest Landau Level

- The states in the lowest Landau level are described by *d=1+1* dimensional spinors, $\xi_k(r,t)$
- Each of these can be bosonized to a *d=1+1* dimensional boson*

$k = 1, \ldots, BA/2\pi$

$$i\bar{\xi}\gamma^\mu\partial_\mu\xi = \frac{1}{8\pi}\partial_\mu\phi\partial^\mu\phi$$

$$\bar{\xi}\gamma^\mu\xi = \frac{1}{2\pi}\epsilon^{\mu\nu}\partial_\nu\phi$$

$$im\bar{\xi}\xi = \frac{m^2}{\pi}\cos\phi$$

* All the subtleties sit in that m^2 term

Bosonizing the Lowest Landau Level

- Importantly, we can do this bosonization in an arbitrary spacetime background.

$$ds^2 = \Omega^2(\tilde{r})(-dt^2 + d\tilde{r}^2) + \Sigma^2(\tilde{r})(dx^2 + dy^2)$$

- The dynamics of the lowest Landau level fields for *translationally invariant states* is described by

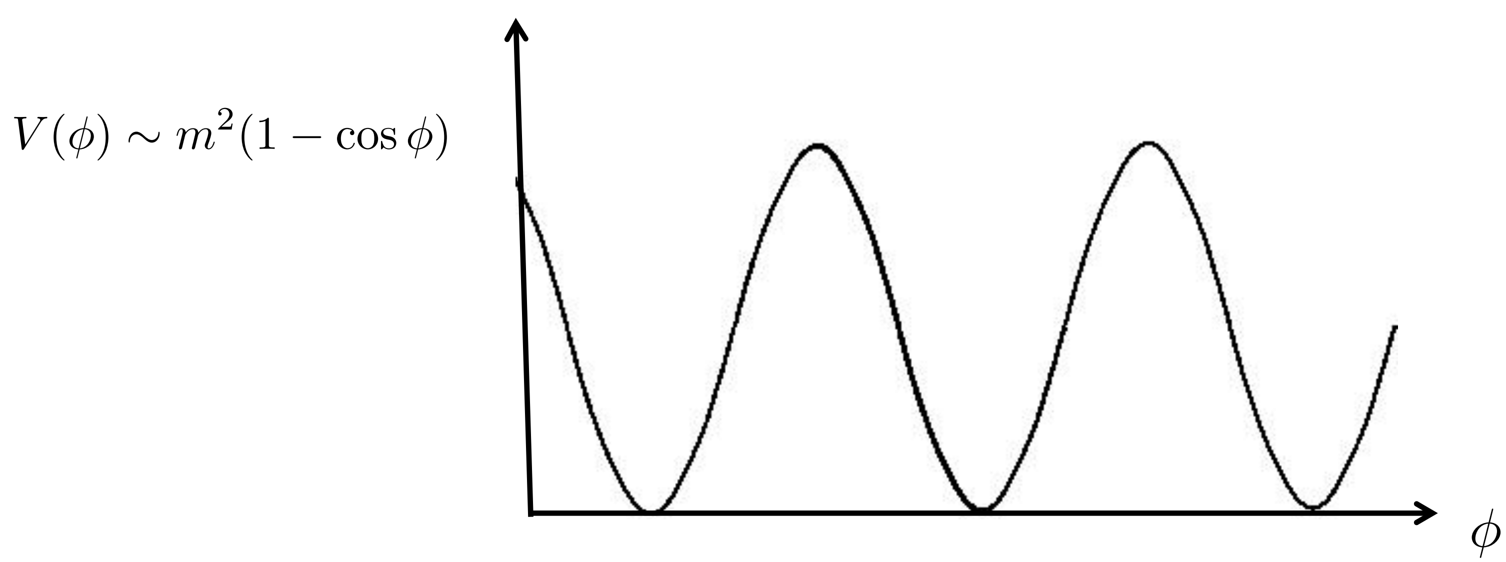
$$S_{LLL} = -\frac{BA}{2\pi}\int d^2x\ \left(\frac{1}{8\pi}\partial^\mu\phi\partial_\mu\phi + \frac{m^2\Omega^2}{4\pi}(1-\cos\phi) + \frac{1}{4\pi}\epsilon^{\mu\nu}\phi F_{\mu\nu}\right)$$

Degeneracy of lowest Landau level

- We then construct the four-dimensional stress tensor from the solutions and use this to solve the Einstein equations to determine the background self-consistently.
  - Requiring a good stress tensor is the best argument for fixing the m^2 term in bosonization

Electrons and Kinks

Ignore the gauge field and background metric for now



$$\bar{\xi}\gamma^\mu\xi = \frac{1}{2\pi}\epsilon^{\mu\nu}\partial_\nu\phi \quad \Longrightarrow \quad \text{Bosonic kinks are original fermions}$$

Bosonization and the Anomaly

- The 2d axial current becomes: $\bar{\xi}\gamma^3\gamma^\mu\xi = \frac{1}{2\pi}\partial^\mu\phi$

- When coupled to a background gauge field, the classical equation of motion is

$$\partial_\mu\partial^\mu\phi = m^2\sin\phi + \epsilon^{\mu\nu}F_{\mu\nu}$$

This captures 2d anomaly

- But, for us, this 2d anomaly is really the 4d anomaly

$$\Delta Q_A^{4d} \sim \frac{BA}{2\pi}\,\Delta Q_A^{2d} \sim \frac{\vec{E}\cdot\vec{B}}{2\pi^2}$$

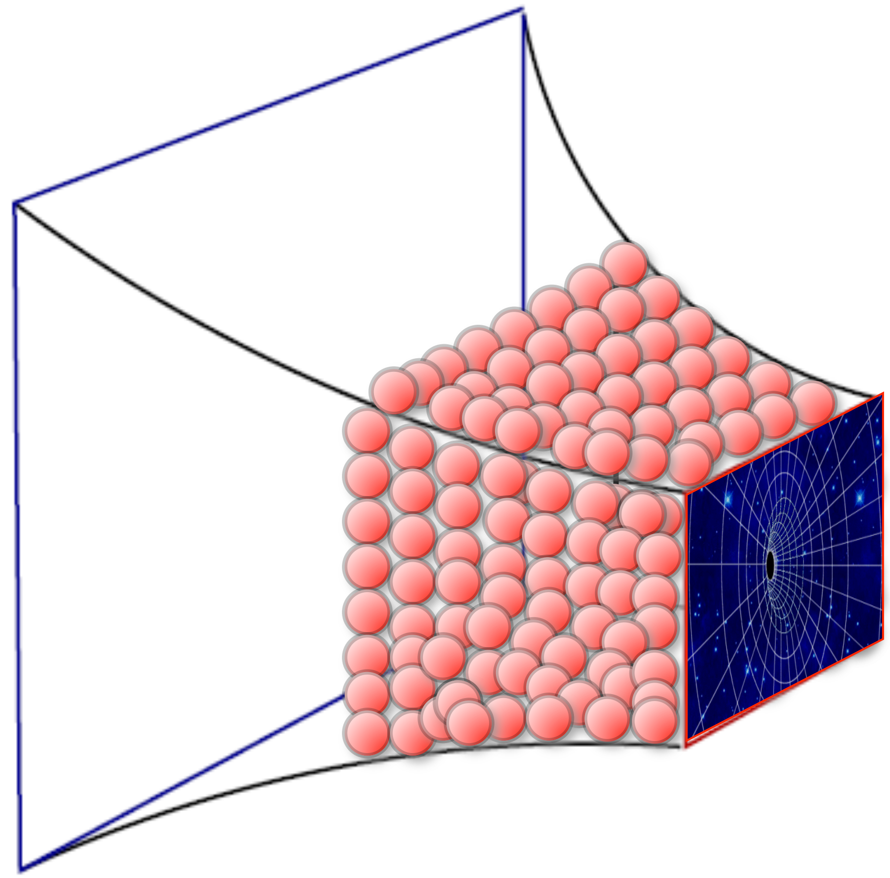
Degeneracy of Landau level

Our New Goal

Solve the Sine-Gordon model

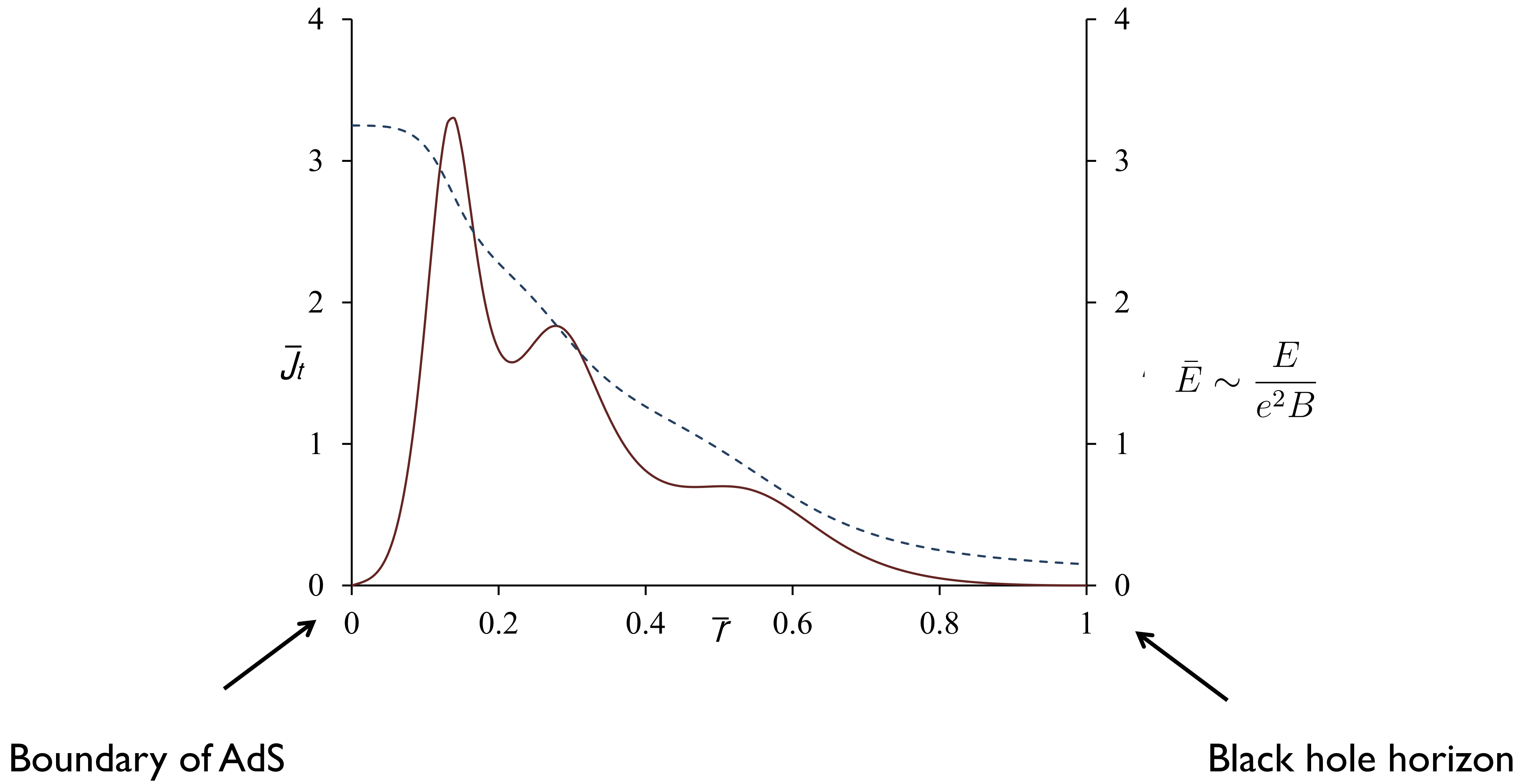
(coupled to a gauge field and gravity with a negative cosmological constant)

The Solution: Background Geometry



- Because we're already in a region with $B > E$, the geometry is dominated by the magnetic field
  - It is simply the magnetic Reissner-Nordstrom black hole + small corrections

The Solution: Background Fields



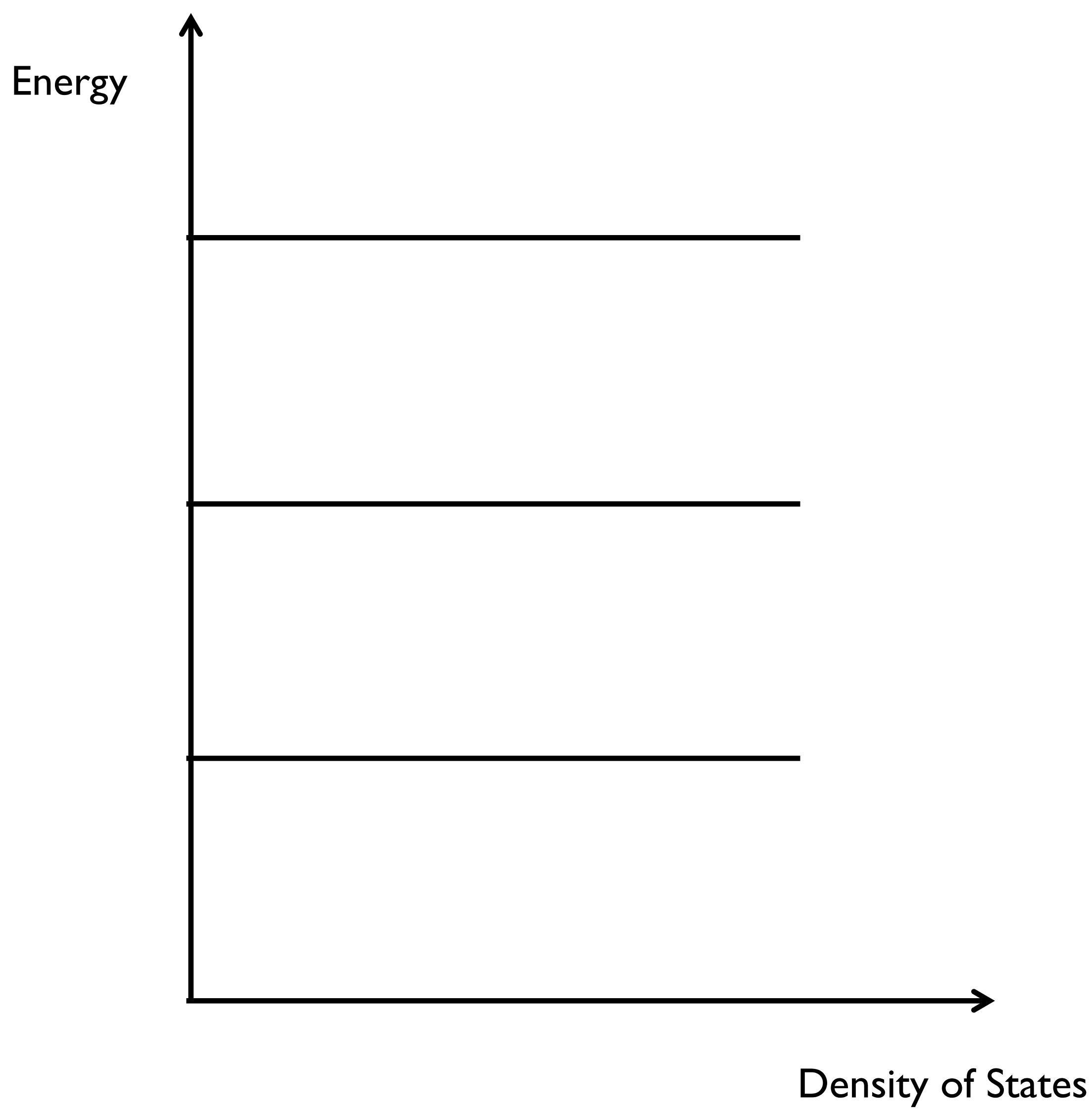
Charge density = “fermion wavefunction” = profile of star

Electric field

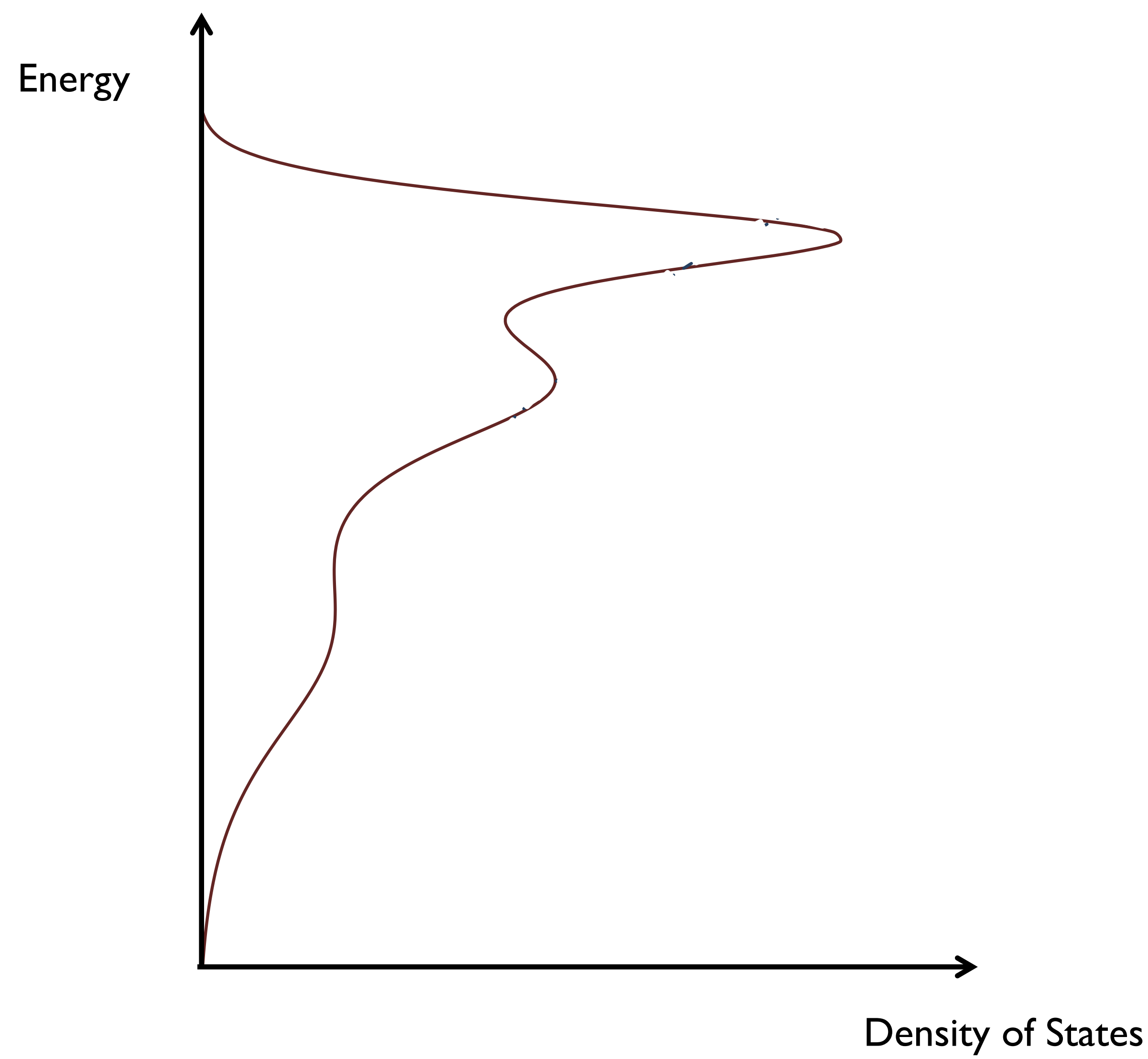
What does this mean for the boundary theory?

“Density of States” Revisited (Roughly!)

Free Theory

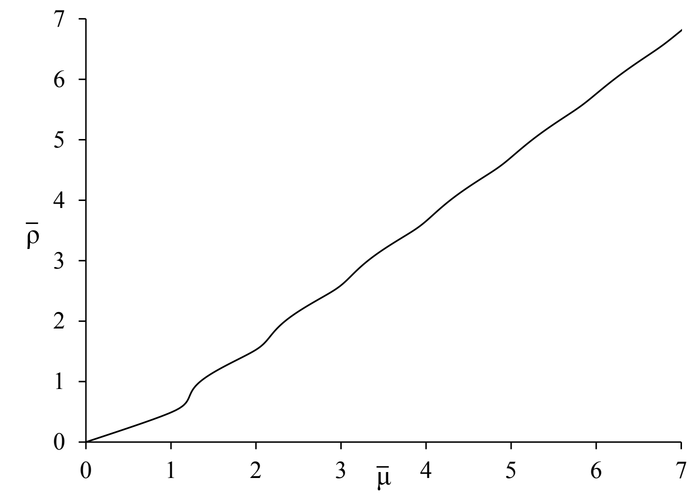


Holographic Model

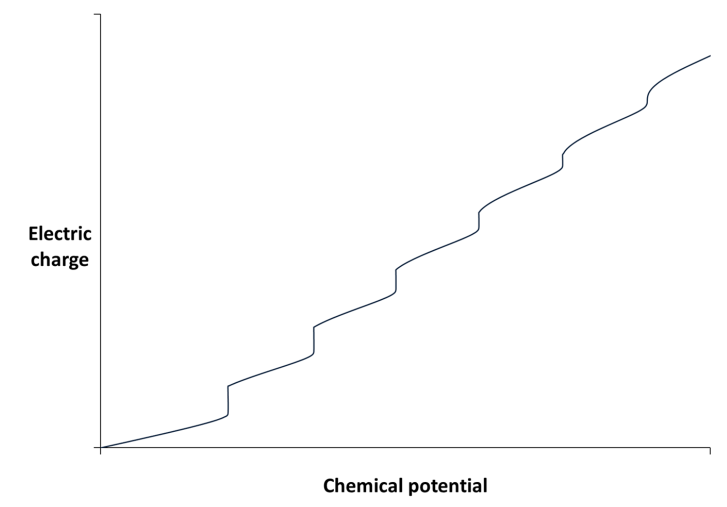


Charge Density vs Chemical Potential

- For small masses, the total charge density is smooth



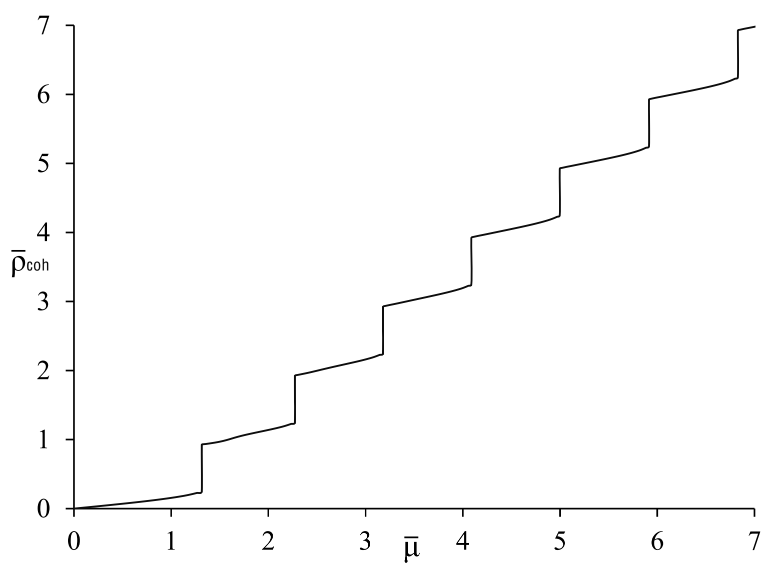
- As the mass is increased, the total charge density shows discrete jumps at low values. It is smooth for higher values.



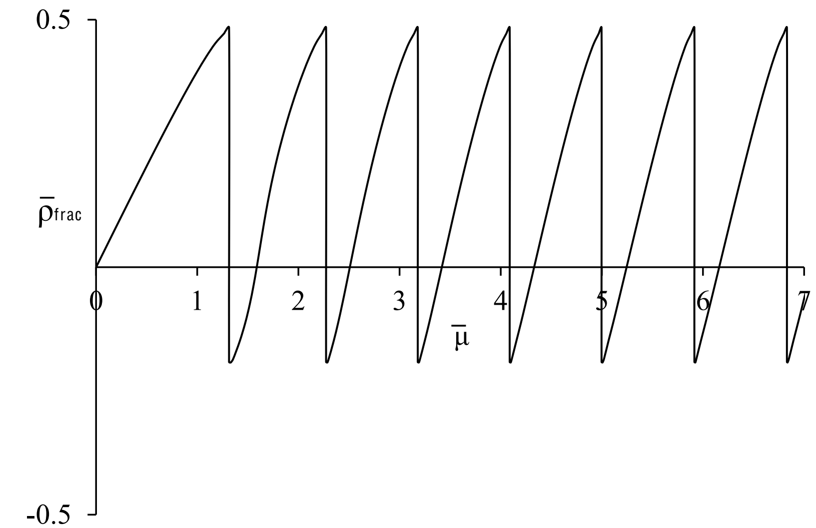
- But even when the total charge density is smooth, a closer look reveals otherwise…

Charge Density vs Chemical Potential

Cohesive charge in the star



Fractionalized charge behind the horizon

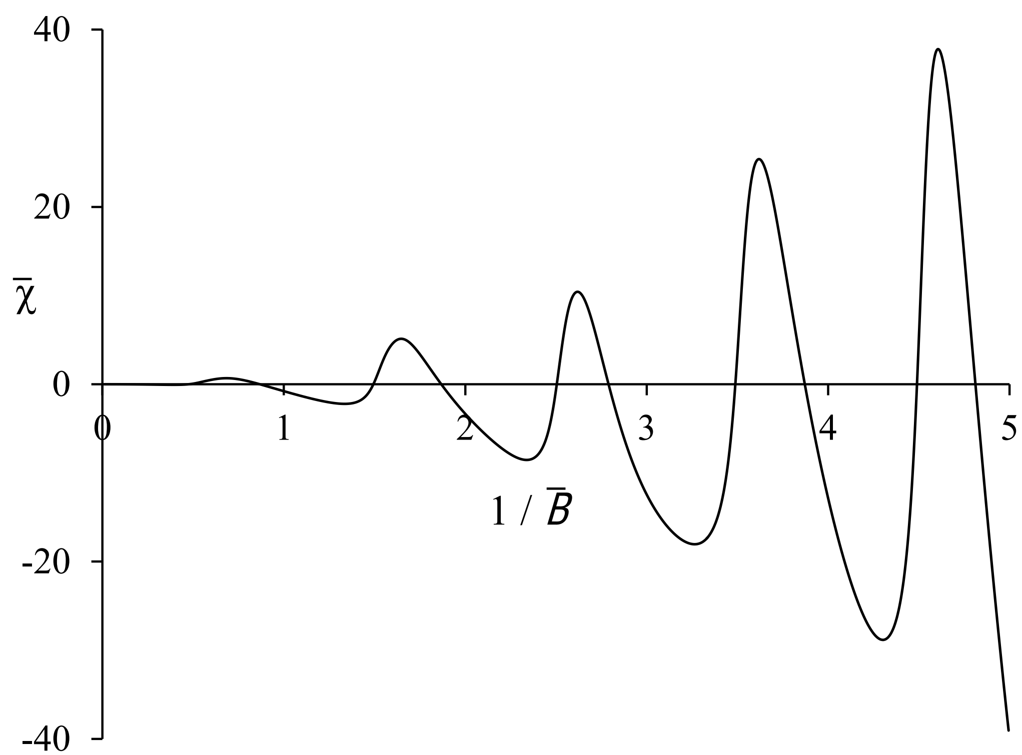


$$m^2L^2 \gtrsim \frac{e^3 L}{\kappa}$$

- Note: The systems sits in its lowest Landau level
  - These jumps are due to different “carrier bands”
- The plateaux are not horizontal; the Landau levels are not flat bands

Magnetic Suscepibility

$$\chi = \frac{\partial^2 F}{\partial B^2}$$



- Analog of de Haas van Alphen oscillations, but now at strong coupling
- Periodicity is again $\Delta\left(\frac{1}{B}\right) = \frac{1}{2\pi\rho}$
- Amplitude appears to scale numerically as Kosovich-Lifshitz formula $\chi \sim 1/B^2$

What Next?

- We have constructed configurations in AdS that are dual to a strongly coupled field theory with its lowest Landau levels populated
- We can compute equilibrium (i.e. thermodynamic) properties such as susceptibilities.
- We really want to compute transport properties
  - This is where quantum Hall physics is hiding (if it is there)
  - This is hard using bosonization.....probably needs a clever idea