Nobel Prize laureates David Thouless, Duncan Haldane and Michael Kosterlitz. Illustration: Niklas Elmehed. © Nobel Media AB 2016.

The 2016 Nobel Prize in Physics

The 2016 Nobel Prize in Physics was awarded to Duncan Haldane, Michael Kosterlitz and David Thouless, three British theoretical physicists working in the US. Their citation reads 'for theoretical discoveries of topological phase transitions and topological phases of matter'.

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Haldane, Kosterlitz and Thouless are condensed matter physicists. This is the study of 'stuff', of solids and liquids, and how their wonderfully diverse properties can emerge from the simple laws that govern the underlying atoms and electrons.

Topology is the crudest way to characterise shapes and surfaces. It is not easy to explain why topology is a deep concept, nor why it is important in condensed matter physics. In the Nobel announcement, the Swedish academy took to waving around various baked goods – a bun, a bagel and a pretzel – encouraging the assembled journalists to count the number of holes in each and receiving, in return, somewhat bemused stares. Here I will try to do marginally better. But, in case you get lost, here's a picture of a bagel.

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The prize recognises two rather different topics in condensed matter physics, with Thouless involved in both. The first of these is the study of 'phase transitions', the discontinuous manner in which the properties of matter change when heated: how solid ice becomes liquid water which, in turn, becomes gaseous steam. The basic theory of phase transitions was laid down in the 1940s by the great Russian physicist Landau. In 1973, Kosterlitz and Thouless discovered a novel form of phase transitions that does not fall into Landau's classification. This involves the creation and liberation of vortices, swirling patterns of the underlying material which, at sufficiently high temperatures, can roam unimpeded through the sample, changing its properties in a surprising and subtle way. The Kosterlitz-Thouless transition underlies our understanding of a number of materials, including thin films of superfluids and two-dimensional magnets.

The second topic is where the deeper and more profound connections to the mathematics of topology arise. The question is: how do electrons move in solids? This is one of the most basic and important questions in condensed matter physics because its answer tells us how materials conduct electricity. A detailed understanding was developed in the early part of the twentieth century but, once again, something subtle was missed.

In quantum mechanics, the state of electrons is described by a complex function called the *wavefunction*. Thouless and collaborators realised that the phase of this function can twist and wind as the electron moves in different directions. This twisting and winding is where the notion of topology comes in. Importantly, the twisting can only occur an integer number of times. This means that the motion of electrons in certain materials is characterised by an integer C.

(A technical aside: the 'phase of the wavefunction' is really the Berry phase, named after the mathematical physicist Sir Michael Berry at the University of Bristol. The 'twisting and winding' occurs over the Brillouin zone, a torus which parameterises the momentum of an electron moving on a lattice. Finally, the integer C is the first Chern number, defined as the Berry curvature integrated over the Brillouin zone.)

In 1982, Thouless and collaborators applied these ideas to the motion of electrons in a two-dimensional material in the presence of a magnetic field. The 'Hall conductivity', σ_H, describes the propensity for the electrons to bend in the magnetic field. Their Nobel-prize winning result is a simple expression known as the TKNN formula

$$\sigma_H = \frac{e^2}{h} C$$

where e is the electric charge of the electron and h is Planck's

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constant which appears whenever quantum mechanics plays an important role. This is an astonishing equation. The topological integer C is something pure and simple. But the conductivity σ_H should be messy and complicated; it is a property of billions upon billions of electrons swarming through the material, bouncing off themselves, the underlying lattice, and any dirt that happens to have fallen into the sample. You would naively expect the conductivity to depend on everything, from the temperature of the room to the amount of rust on your crocodile clips. The TKNN formula says that this isn't the case. None of these details matter. The power of topology means that the marauding army of electrons will always conspire to give a conductivity which is exactly an integer. This Hall conductivity has now been measured to be an integer to nine decimal places! It is one of the most precise measurements in all of science.

The work of Haldane also falls into the second category of understanding the properties of electrons in material. He is recognised as one of the visionaries of the field and his contributions are too numerous to describe in detail here. Instead I will mention only a small, but ultimately influential, insight from 1988 which extends the TKNN formula to systems which do not have magnetic fields. In subsequent years his paper picked up only a trickle of citations. This changed around a decade ago when the field suddenly exploded. At that time, it became clear that the topology underlying the TKNN formula applies to a much greater class of materials than previously imagined, and both theoretical and experimental developments came thick and fast. The resulting materials are known as 'topological insulators'. They are spectacularly superficial, dull on the inside but alive on the surface where they play host to some of the most beautiful ideas in theoretical physics, from relativity to quantum field theory. It is expected that these materials will have important technological applications, including in the development of quantum computers.

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Not all Nobel prizes are equal. Some are awarded for an acute insight, for a brilliantly executed experiment, or simply for a surprising discovery. But some are awarded for a paradigm shift in the way we think about Nature. This year's Nobel prize falls into the latter camp. It celebrates the interplay of ideas between theoretical physics and modern mathematics, ideas which have revolutionised our understanding of condensed matter and are playing an increasingly important role in other areas, from high-energy physics to quantum information.

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