# The real number line is a good model for the physical continuum

David Tong
University of Cambridge

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# The Punchline

No one knows how to write down a discrete version of the laws of physics

# Quantum Field Theory

$$\phi(x) \qquad \psi_\alpha(x) \qquad A_\mu(x)$$

Continuum spacetime $\Longrightarrow$ Interesting physics

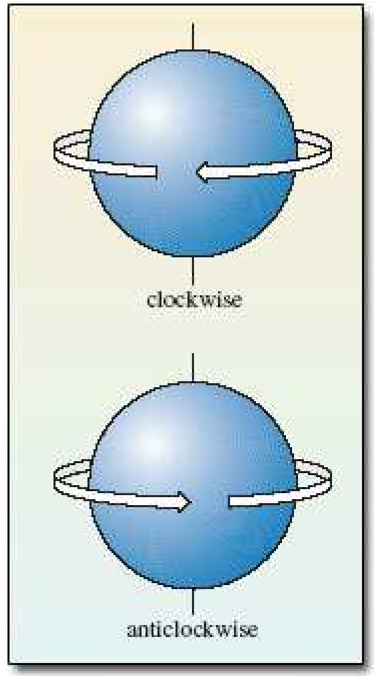
# The Trouble: Chiral Fermions

$$\{\gamma^5, \not{D}\} = 0$$

$$P_\pm = \frac{1}{2}(1 \pm \gamma^5)$$

$$\psi_L = P_+ \psi$$

$$\psi_R = P_- \psi$$



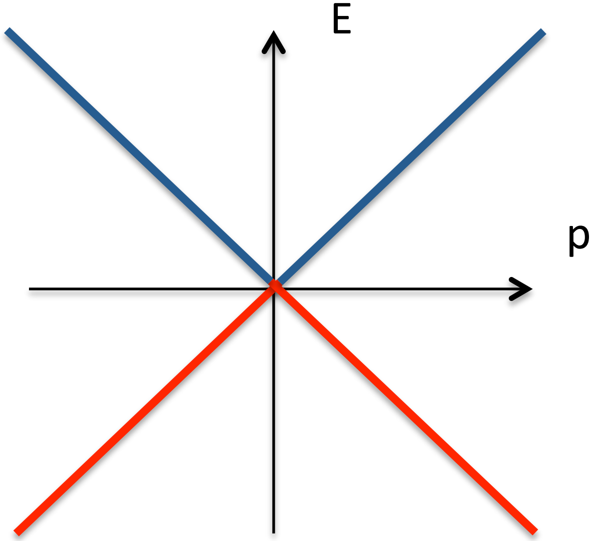

- A massive fermion needs both chiralities
- A massless fermion needs only one

# Anomalies

Symmetries that act differently on fermions of opposite chirality often don’t exist in the quantum theory.

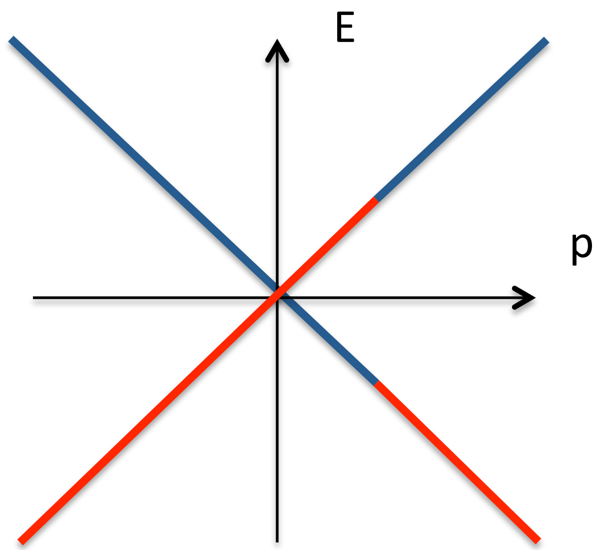
# An Example in d=1+1

d=1+1 dimensional Maxwell coupled to a massless Dirac fermion



# An Example in d=1+1

Turn on an electric field *E(t)* for some time ⇨ momenta shift




Extra right-moving particles

Extra left-moving anti-particles

⇨ Axial charge is violated

# Anomalies

Where did the extra charge come from?

From infinity!

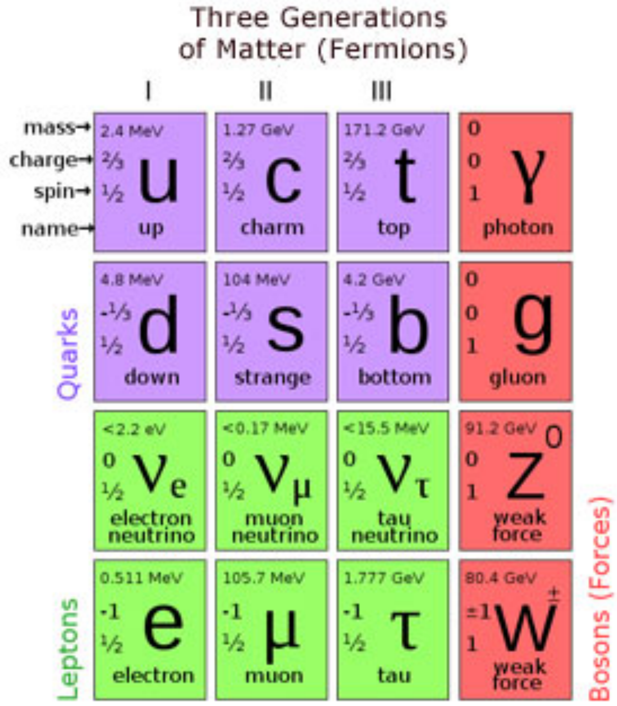
The anomaly is an effect arising from the continuum

# Gauge Anomalies

Anomalies in global symmetries are merely interesting.

Anomalies in gauge symmetries are fatal.

# Chiral Gauge Theories



# A Discrete World

- Replace the continuum with a discretized lattice
- Now quantum field theory = quantum mechanics
- No infinities. No anomalies.

# Anomalies on the Lattice

Anomalous global symmetries are explicitly broken by the discretization procedure.

What about gauge symmetries?

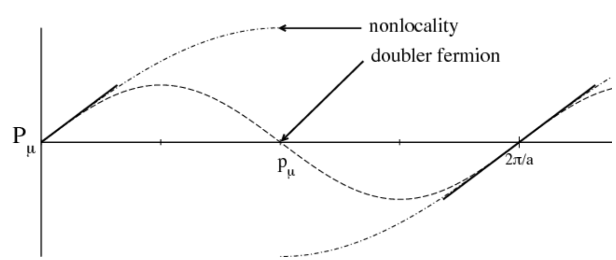
# Fermions on the Lattice

$$S = \int d^4x\, \bar{\psi}(i\partial_\mu \gamma^\mu - m)\psi$$

$$= \int_{-\pi/a}^{\pi/a} d^4p \left[\frac{i}{a}\sin(p_\mu)\gamma^\mu - m\right] \bar{\psi}_{-p}\psi_p$$

BZ

lattice spacing




⇒ $2^4$ fermions!

- This is fermion doubling.
- Try to naively discretize a chiral theory and you get a non-chiral theory

# Nielsen-Ninomiya Theorem

You can't do it*.

* up to certain assumptions

# Overlap Fermions

Kaplan, Neuberger, Narayan, Luscher, late 1990s

Project onto left/right fermions with

$$\hat{\gamma}_5 = \gamma_5 \left(1 - a \not{D}\right)$$

depends on momentum *and* gauge field.

with $\{\gamma_5, \not{D}\} = a \not{D} \gamma^5 \not{D}$

# Just One Last Thing…

Is this definition of chirality gauge invariant?

(i.e. does the theory exist?)

# Abelian Theories

Yes, if and only if the continuum theory is non-anomalous.

# Non-Abelian Theories

No one knows.

# The Punchline

The Standard Model is a non-Abelian chiral gauge theory.

No one knows how to write down a discrete version of the Standard Model.