On Particle-Vortex Duality

David Tong

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Based on work with Andreas Karch

Also related work by Seiberg, Senthil, Witten and Wang

Particle-Vortex Duality for Bosons

Theory A:

$$S = \int d^3x \; |(\partial_\mu - iA_\mu)\phi|^2 - V(\phi)$$

Theory B:

$$S = \int d^3x \; |(\partial_\mu - ia_\mu)\Phi|^2 - \tilde{V}(\Phi) + \frac{1}{2\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu a_\rho$$

dynamical gauge field (a_μ) — background gauge field (A_μ)

Boson $\Longleftrightarrow$ Monopole operator $\int f = 2\pi$

Peskin '78, Dasgupta and Halperin '81

Particle-Vortex Duality for Fermions

Theory A:

$$S = \int d^3x \; i\bar{\psi}\gamma^\mu(\partial_\mu - iA_\mu)\psi$$

Theory B:

$$S = \int d^3x \; i\bar{\Psi}\gamma^\mu(\partial_\mu - ia_\mu)\Psi + \frac{1}{4\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu a_\rho$$

Fermion $\Longleftrightarrow$ Monopole operator $\int f = 4\pi$

Parity anomaly means that theory really makes sense only on boundary of topological insulator

Son; Wang and Senthil; Metlitski and Vishwanath '15

Some notation

$$S_{\text{scalar}}[\phi; A] = \int d^3x \; |(\partial_\mu - iA_\mu)\phi|^2 + \ldots$$

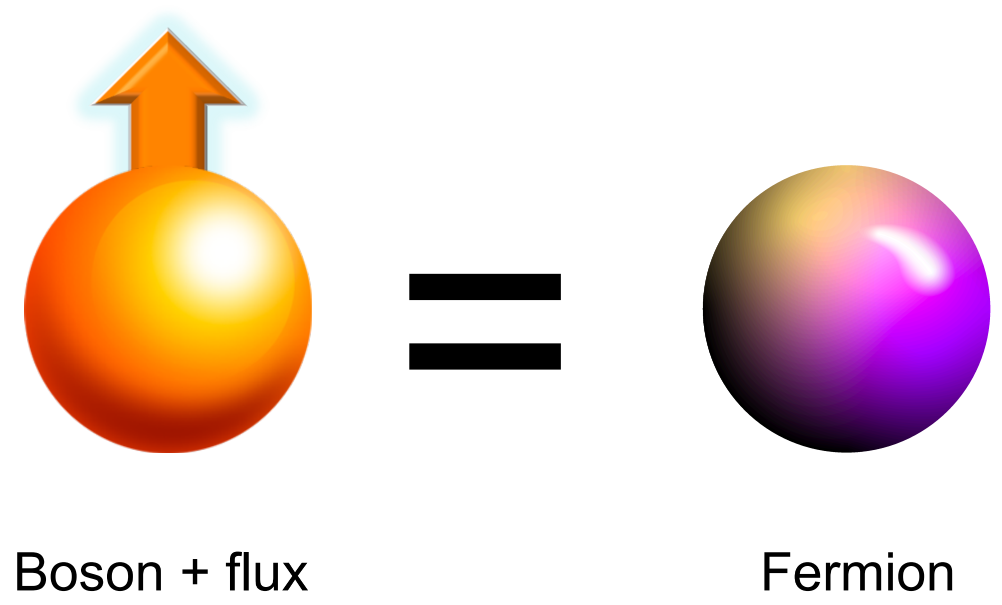
and $\quad Z_{\text{scalar}}[A] = \int \mathcal{D}\phi \;\; \exp\Big(iS_{\text{scalar}}[A]\Big)$

$$S_{\text{fermion}}[\psi; A] = \int d^3x \; i\bar{\psi}\gamma^\mu(\partial_\mu - iA_\mu)\psi + \ldots$$

and $\quad Z_{\text{fermion}}[A] = \int \mathcal{D}\psi \;\; \exp\Big(iS_{\text{fermion}}[A]\Big)$

$$S_{CS}[a] = \frac{1}{4\pi}\int d^3x \; \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \qquad\qquad S_{BF}[a; A] = \frac{1}{2\pi}\int d^3x \; \epsilon^{\mu\nu\rho} a_\mu \partial_\nu A_\rho$$

Something Simple…Flux Attachment



Wilczek, Jain and many others

Relativistic Flux Attachment

$$S_{\rm scalar+flux} = \int d^3x \; |(\partial_\mu - ia_\mu)\phi|^2 + \frac{1}{4\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho + \frac{1}{2\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu a_\rho$$

When we turn off the background field *A*, the equation of motion for *a* attaches one unit of flux

$$\rho_{\rm scalar} + \frac{f}{2\pi} = 0$$

This is a fermion

Polyakov '88 and many others

A Seed Duality

Theory A:

$$S_{\text{scalar+flux}} = S_{\text{scalar}}[\phi; a] + S_{CS}[a] + S_{BF}[a; A]$$

Theory B:

$$S = S_{\text{fermion}}[A] - \frac{1}{2} S_{CS}[A]$$

ensures Hall conductivities agree

Polyakov ’88 and many others

(More recently, many works on non-Abelian versions of this duality, notably by Minwalla et. al. and Aharony et.al.)

A Seed Duality

Theory A: $$S_{\text{scalar+flux}} = S_{\text{scalar}}[\phi; a] + S_{CS}[a] + S_{BF}[a; A]$$

Theory B: $$S = S_{\text{fermion}}[A] - \frac{1}{2} S_{CS}[A]$$

$$Z_{\text{scalar+flux}}[A] = \int \mathcal{D}a \; Z_{\text{scalar}}[a] \exp\Big(iS_{CS}[a] + iS_{BF}[a; A]\Big) = Z_{\text{fermion}}[A] \, e^{-\frac{i}{2} S_{CS}[A]}$$

Basic Idea

$$Z_{\text{scalar+flux}}[A] = \int \mathcal{D}a \; Z_{\text{scalar}}[a] \exp\Big(iS_{CS}[a] + iS_{BF}[a;A]\Big) = Z_{\text{fermion}}[A]\, e^{-\frac{i}{2}S_{CS}[A]}$$

Manipulate path integral to derive other dualities

- c.f. Kapustin and Strassler ‘99 for supersymmetric theories

Example 1: Another Flux Attachement

$$Z_{\text{scalar+flux}}[A] = \int \mathcal{D}a \; Z_{\text{scalar}}[a] \exp\Big(iS_{CS}[a] + iS_{BF}[a;A]\Big) = Z_{\text{fermion}}[A]\, e^{-\frac{i}{2}S_{CS}[A]}$$

Promote *A* to dynamical gauge field. Right-hand side becomes

$$Z_{\text{fermion+flux}}[C] = \int \mathcal{D}A \; Z_{\text{fermion}}[A] \exp\Big(-\frac{i}{2}S_{CS}[A] - iS_{BF}[A;C]\Big)$$

new background field

Do the same on left-hand side.

$$\int \mathcal{D}A \; Z_{\text{scalar+flux}}[A]\, e^{-iS_{BF}[A;C]} = Z_{\text{scalar}}[C]\, e^{iS_{CS}[C]}$$

integrate out *A;* equation of motion also eliminates *a* through $da = dC$

$$\Longrightarrow \quad \boxed{Z_{\text{fermion+flux}}[C] = Z_{\text{scalar}}[C]\, e^{iS_{CS}[C]}}$$

Chen, Fisher, Wu ‘93

Example 2: Bosonic Particle-Vortex Duality

$$Z_{\rm fermion+flux}[C] = Z_{\rm scalar}[C]\, e^{iS_{CS}[C]} \quad \Longrightarrow \quad Z_{\rm fermion+flux}[C]\, e^{-iS_{CS}[C]} = Z_{\rm scalar}[C]$$

Promote *C* to a dynamical gauge field. Right-hand side becomes

$$Z_{\rm scalar-QED}[A] = \int \mathcal{D}C\ Z_{\rm scalar}[C]\, e^{iS_{BF}[C;A]}$$

On the left-hand-side something nice happens. After integrating out *C*, we find

$$\int \mathcal{D}a\ Z_{\rm fermion}[a]\ \exp\left(\frac{i}{2}S_{CS}[a] - iS_{BF}[a;A] + iS_{CS}[A]\right)$$

But this is just our original "fermion+flux" partition function (or, more precisely, a time-reversed version)

$$\Longrightarrow \quad \boxed{Z_{\rm scalar-QED}[A] = Z_{\rm scalar}[A]}$$

Example 3: Fermionic Particle-Vortex Duality

$$Z_{\text{scalar+flux}}[C] = Z_{\text{fermion}}[C]\, e^{-\frac{i}{2}S_{CS}[C]} \quad \Longrightarrow \quad Z_{\text{scalar+flux}}[C]\, e^{+\frac{i}{2}S_{CS}[C]} = Z_{\text{fermion}}[C]$$

Again, promote *C* to a dynamical gauge field, but now restricted to have even flux sector. Left-hand side is

$$Z_{\text{QED}}[A] = \int \mathcal{D}A\ Z_{\text{fermion}}[A] e^{\frac{i}{2}S_{BF}[a;A]}$$

On the left-hand-side something nice happens. After integrating out *C*, we find

$$\int \mathcal{D}a\ Z_{\text{scalar}}[a]\ \exp\Big(- iS_{CS}[a] - iS_{BF}[a;A] - \frac{i}{2}S_{CS}[A]\Big)$$

This is again the time-reversed version of our original scalar+flux. We can then use duality to derive

$$\Longrightarrow \quad \boxed{Z_{\text{QED}}[A] = Z_{\text{fermion}}[A]}$$

Example 4: A Self-Dual Theory

- Take two copies of the scalar+flux = fermion duality.
- Gauge an overall symmetry to get N_f=2 QED

$$Z_{\mathrm{QED}[N_f=2]}[A;C] = \int \mathcal{D}a\ Z_{\mathrm{fermion}}[a+C] Z_{\mathrm{fermion}}[a-C]\, e^{+iS_{BF}[a;A]}$$

Two background gauge fields: *A* for the topological current, and *C* for the Cartan of the *SU(2)* flavour

The dual theory of 2 scalars takes the schematic form

$$\int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \mathcal{D}c_+\ \exp\left(iS[\phi_1,\phi_2,c_+;A+C] - \frac{i}{2} S_{BF}[c_+;A-C]\right)$$

But this is invariant under swapping *A* and *C*, together with time-reversal

$\Longrightarrow$

$$Z_{\mathrm{QED}[N_f=2]}[A;C] = \bar{Z}_{\mathrm{QED}[N_f=2]}[C;A]$$

Time reversed theory

Xu and You ‘15

Summary

Many dualities follow from relativistic version of flux attachement