# What is the Gauge Group of the Standard Model?

David Tong

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# The Standard Model

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All standard model fields are invariant under $\mathbf{Z}_6$ generated by

$$\eta = e^{2\pi i Y/6} \otimes \begin{pmatrix} -1 & \\ & -1 \end{pmatrix} \otimes \begin{pmatrix} \omega & & \\ & \omega & \\ & & \omega \end{pmatrix} \qquad \omega^3 = 1$$

i.e.

$$l_L : \quad (\mathbf{2}, \mathbf{1})_{-3}$$

$$q_L : \quad (\mathbf{2}, \mathbf{3})_{+1}$$

$$e_R : \quad (\mathbf{1}, \mathbf{1})_{-6}$$

$$d_R : \quad (\mathbf{1}, \mathbf{3})_{-2}$$

$$u_R : \quad (\mathbf{1}, \mathbf{3})_{+4}$$

$$H : \quad (\mathbf{2}, \mathbf{1})_{3}$$

# The Standard Model

The gauge group of the standard model is

$$G = \frac{U(1) \times SU(2) \times SU(3)}{\Gamma}$$

with $\Gamma = \mathbf{Z}_6,\ \mathbf{Z}_3,\ \mathbf{Z}_2 \text{ or } \mathbf{1}$

Which describes our world? How can we tell?

# A Warm Up: SU(N) vs SU(N)/$\mathbf{Z}_N$

Following Aharony, Seiberg and Tachikawa, 2013

# Line Operators in SU(N) and SU(N)/$\mathbf{Z}_N$

- Wilson line operators: probe electric particles
  - Labeled by transformation under center of SU(N)

$$z^e = 0, 1, \ldots, N-1$$

- ‘t Hooft line operators: probe magnetic particles
  - Also labeled by transformation under center

$$z^m = 0, 1, \ldots, N-1$$

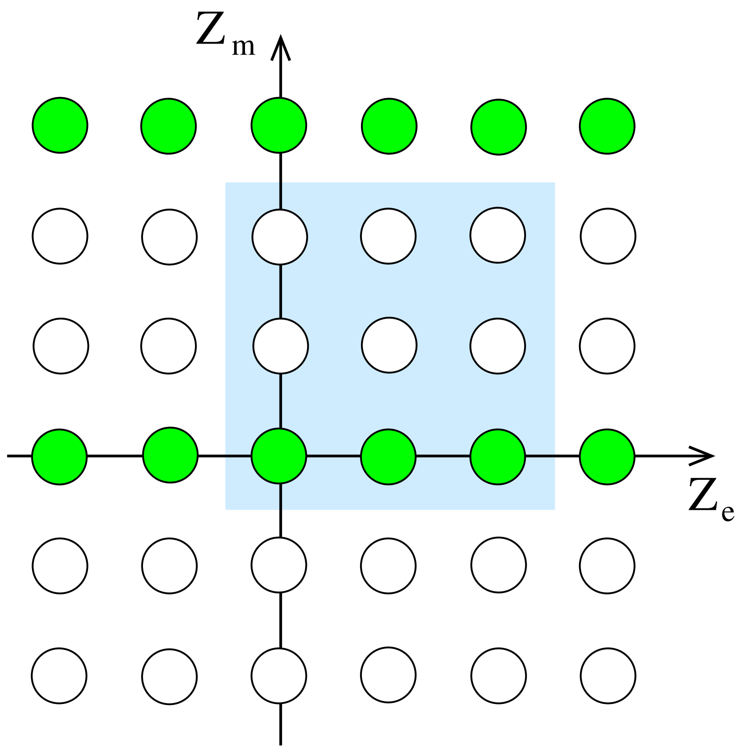
- Dirac quantisation requires

$$z^e z'^m - z^m z'^e = 0 \mod N$$

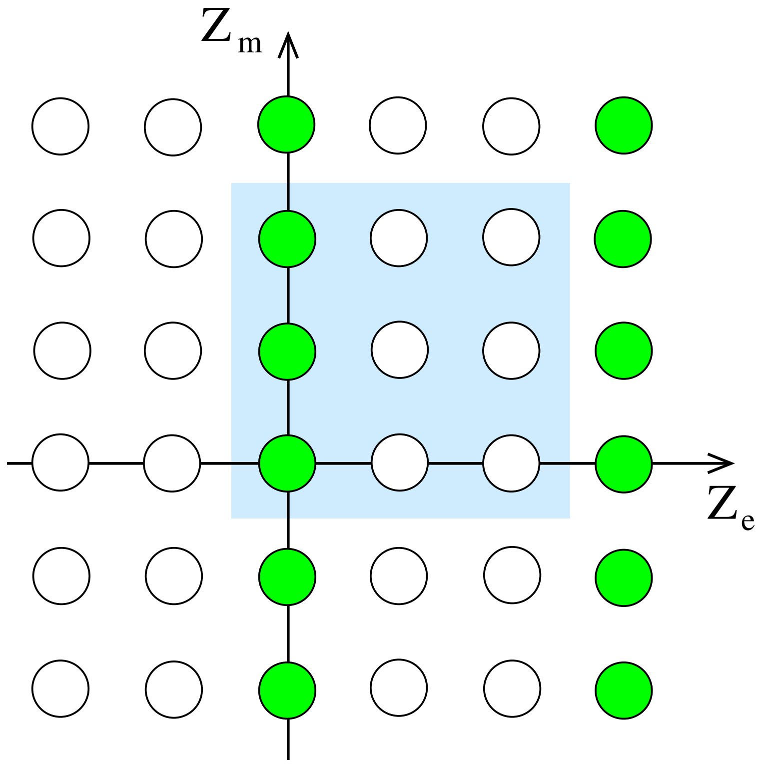
Corrigan and Olive 1976

# Line Operators in SU(N) and SU(N)/$\mathbf{Z}_N$

SU(3)



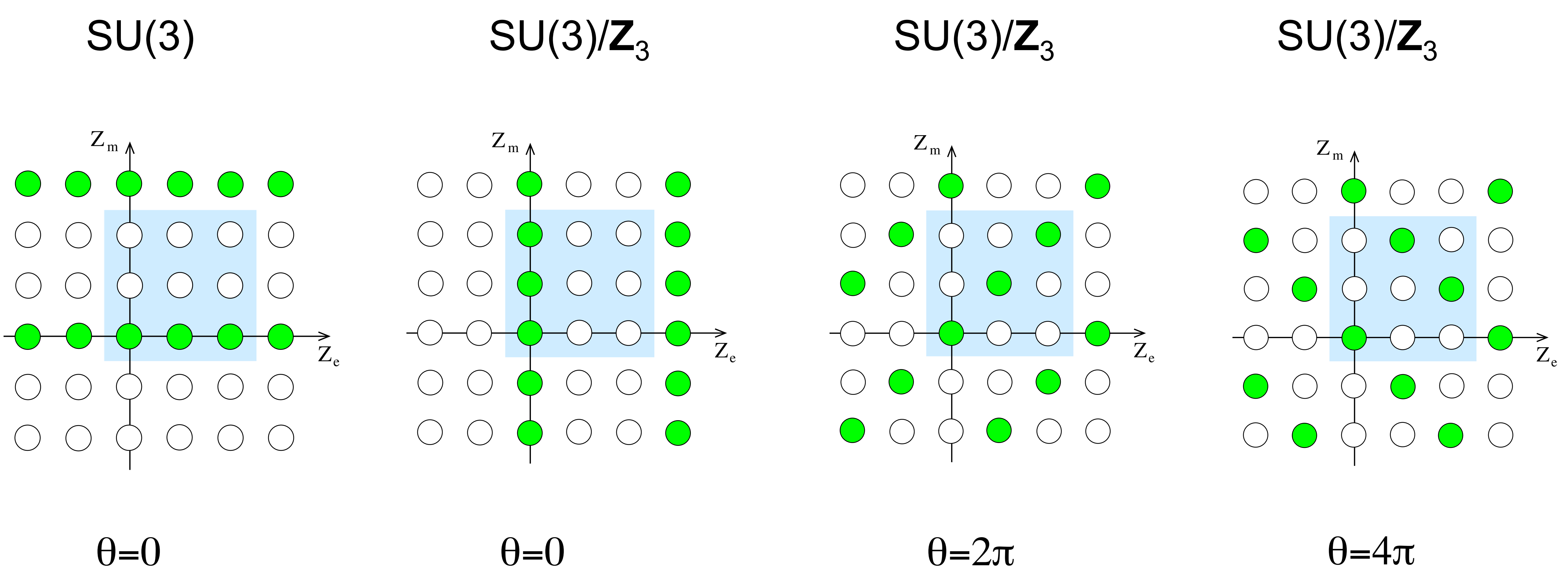

SU(3)/$\mathbf{Z}_3$



# Periodicity of Theta Angle for SU(N) and SU(N)/$\mathbf{Z}_N$

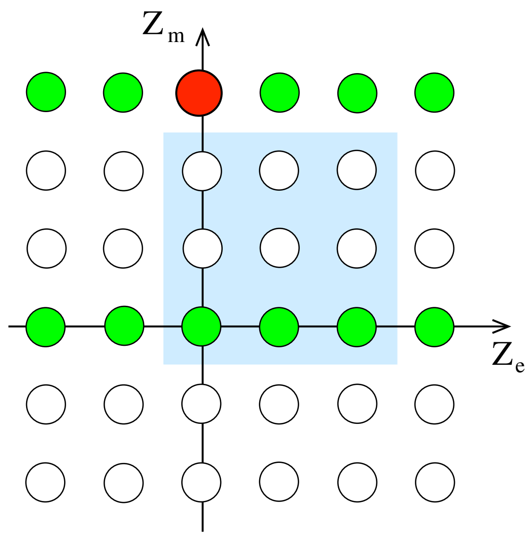
- For SU(N), $\theta$ has periodicity $2\pi$
- For SU(N)/$\mathbf{Z}_N$, $\theta$ has periodicity $2\pi N$

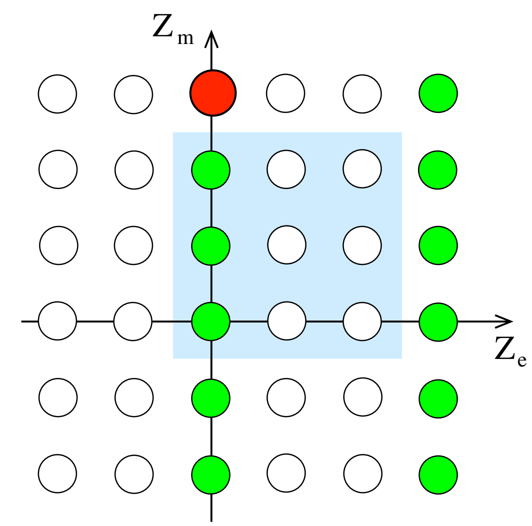
This can be seen by looking at the Witten effect on line operators



# Emergent Magnetic $\mathbf{Z}_N$ Symmetry

Both SU(N) and SU(N)/$\mathbf{Z}_N$ confine through condensation of magnetic monopoles



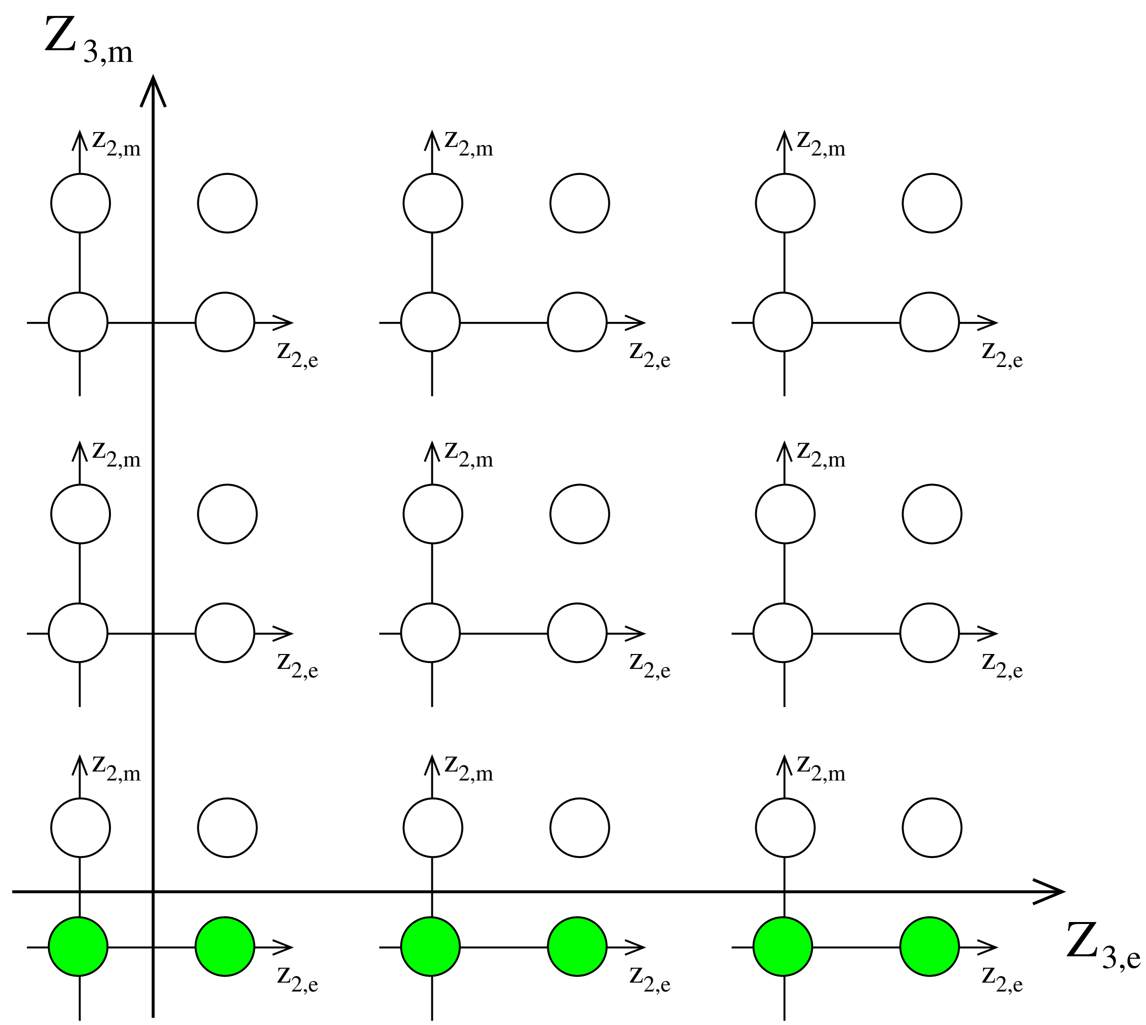

- For SU(N)/$\mathbf{Z}_N$ this is *not* the minimal monopole.
- There is an emergent $\mathbf{Z}_N$ magnetic gauge symmetry.
  - This changes the local dynamics when the theory is compactified on, say, $\mathbf{S}^1$

# Line Operators in the Standard Model

The spectrum of line operators depends on the choice of $\Gamma = \mathbf{Z}_6,\ \mathbf{Z}_3,\ \mathbf{Z}_2 \text{ or } \mathbf{1}$

# Line Operators

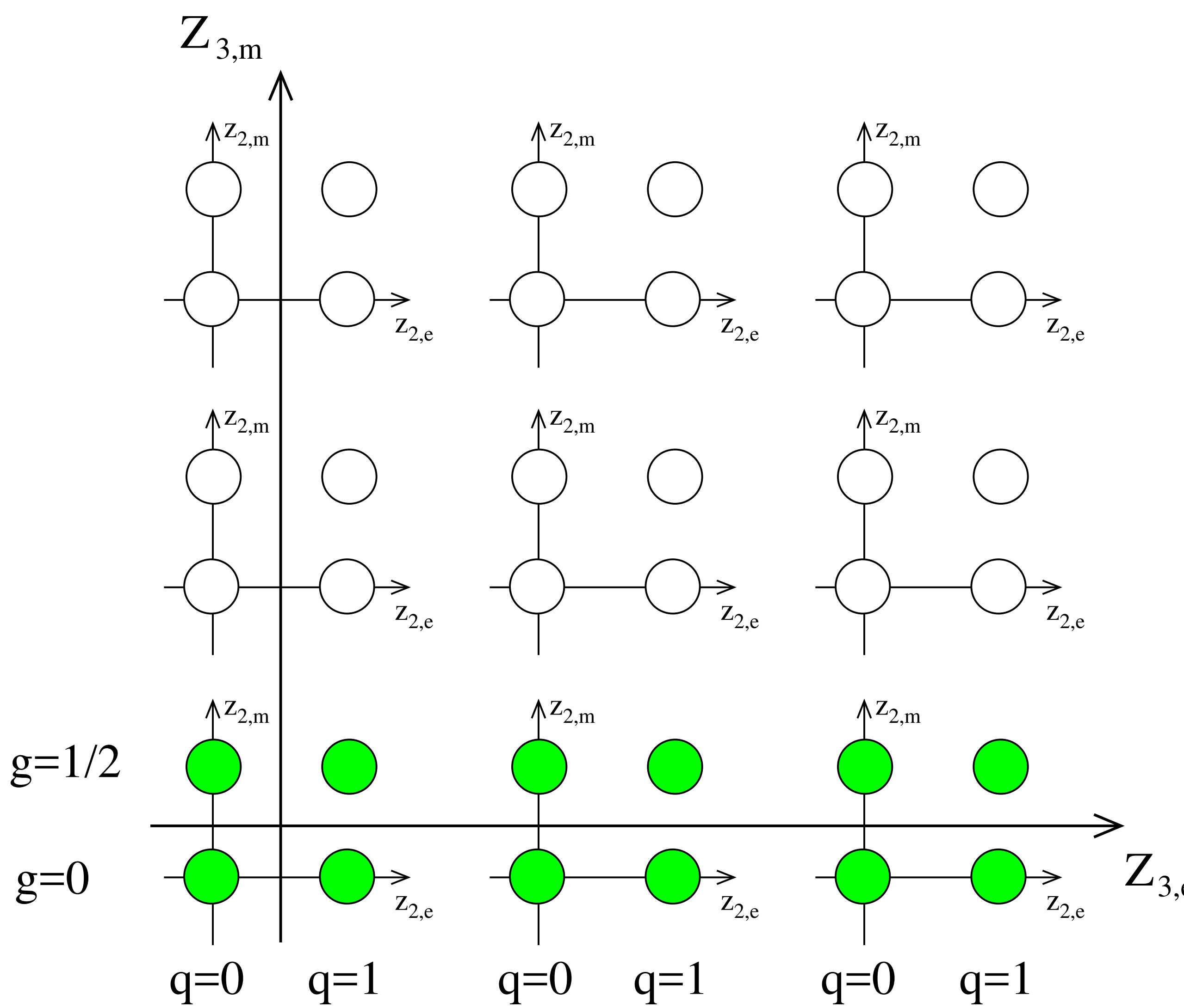
$\underline{\Gamma = \mathbf{1}:}$




Together with U(1) line operators with (q,g) = (1,0) and (0,1)

# Line Operators

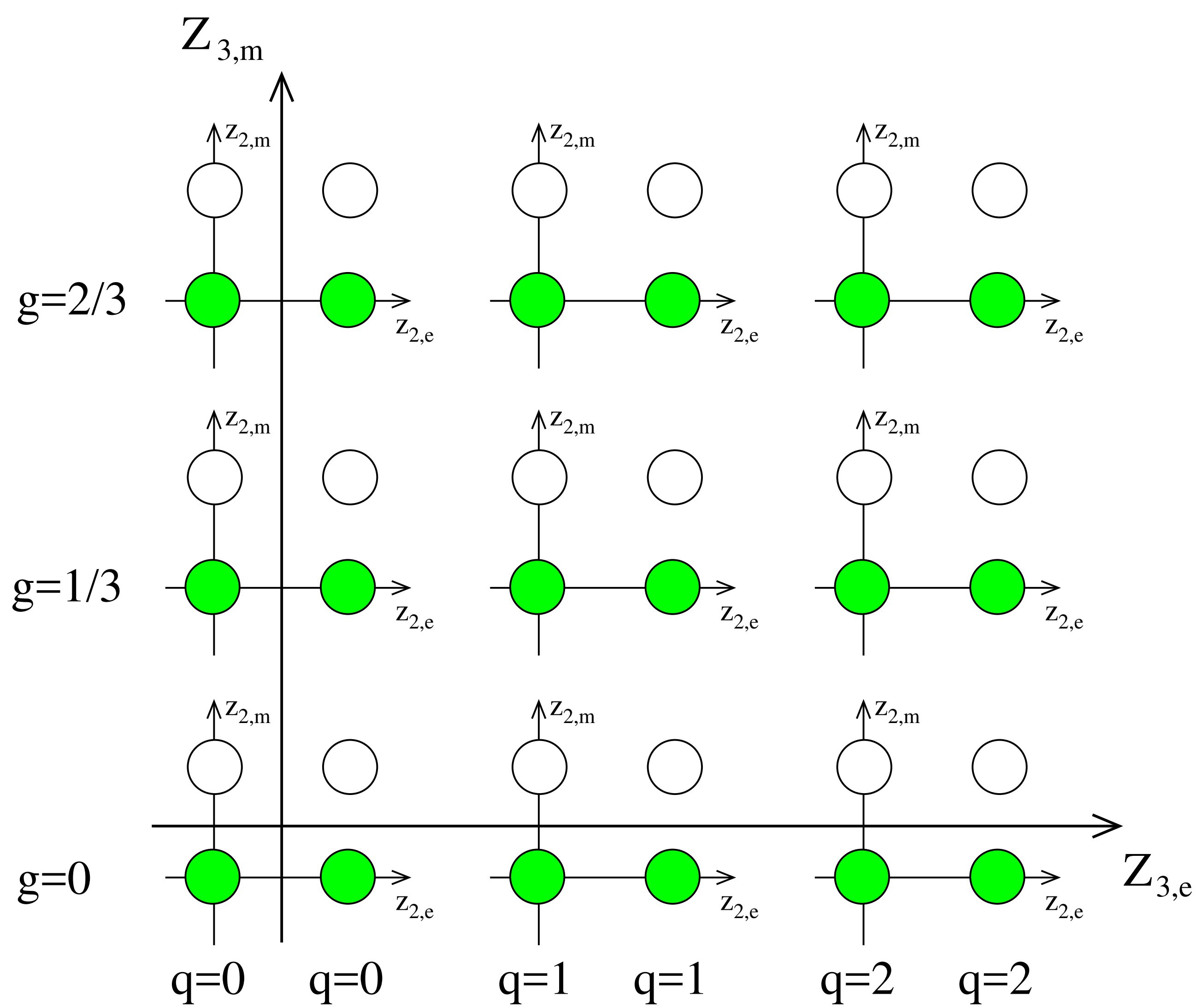
$\underline{\Gamma = \mathbf{Z}_2:}$




Together with U(1) line operators with (q,g) = (2,0) and (0,1)

# Line Operators

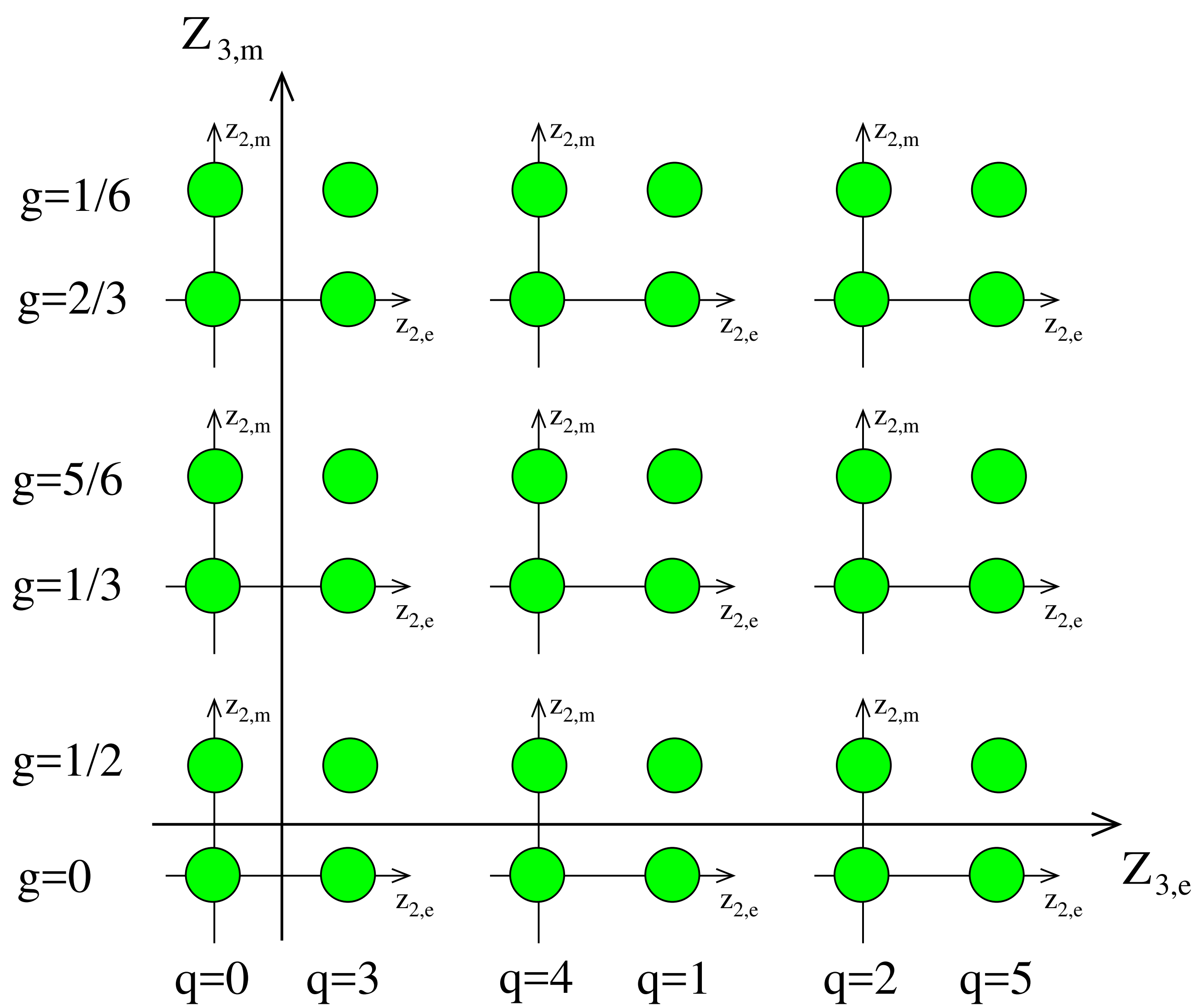
$\underline{\Gamma = \mathbf{Z}_3:}$




Together with U(1) line operators with (q,g) = (3,0) and (0,1)

# Line Operators

$\underline{\Gamma = \mathbf{Z}_6:}$



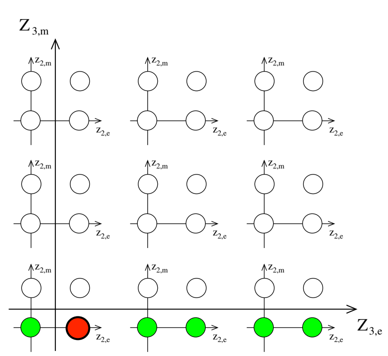

Together with U(1) line operators with (q,g) = (6,0) and (0,1)

# Electroweak Symmetry Breaking

$H : \quad (\mathbf{2}, \mathbf{1})_3$ condenses and $SU(2) \times U(1) \rightarrow U(1)_{em}$

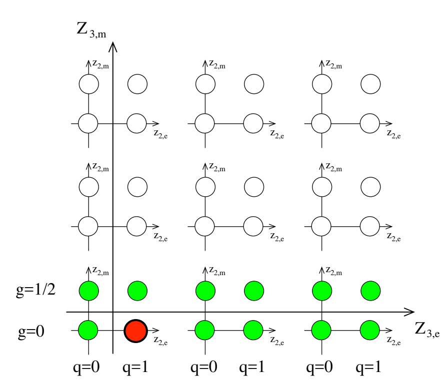
Denote electric and magnetic charges as Q and G (such that electron has Q=-1)

$\underline{\Gamma = \mathbf{1}:}$



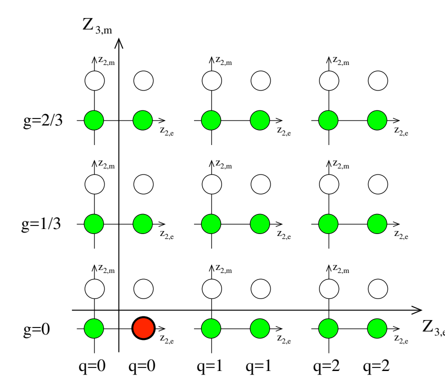

$Q_{min}$=1/6
$G_{min}$=6

$\underline{\Gamma = \mathbf{Z}_2:}$



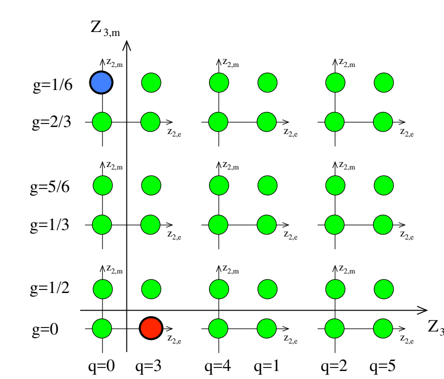

$Q_{min}$=1/3
$G_{min}$=3

$\underline{\Gamma = \mathbf{Z}_3:}$




$Q_{min}$=1/6
$G_{min}$=2

$\underline{\Gamma = \mathbf{Z}_6:}$




$Q_{min}$=1/3
$G_{min}$=1

Comments

- For $\mathbf{Z}_3$ and $\mathbf{Z}_6$, the naïve Dirac quantisation, QG integer, does not hold
  - Magnetic monopole also carries colour magnetic charge
- Black hole physics suggests all possible charges arise as dynamical objects

(Banks and Seiberg, 2010)

# Theta Angles in the Standard Model

The periodicity of theta angles depends on the choice of $\Gamma = \mathbf{Z}_6,\ \mathbf{Z}_3,\ \mathbf{Z}_2 \text{ or } \mathbf{1}$

# Theta Angles

For $\Gamma = \mathbf{Z}_p$: $\theta_2, \theta_3 \in [0, 2\pi)$ and $\theta_Y \in [0, 2\pi p^2)$

Moreover, T-invariant states depend on the choice of $\Gamma$

e.g: $\Gamma = \mathbf{1}$ $\Longrightarrow$ $\theta_2, \theta_3, \theta_Y = 0$ or $\pi$

$\Gamma = \mathbf{Z}_6$ $\Longrightarrow$ $\theta_2 = \theta_3 = 0$ and $\theta_Y = 0$ or $36\pi$

$\theta_2 = 0$ and $\theta_3 = \pi$ and $\theta_Y = 12\pi$ or $48\pi$

$\theta_2 = \pi$ and $\theta_3 = 0$ and $\theta_Y = 18\pi$ or $54\pi$

$\theta_2 = \theta_3 = \pi$ and $\theta_Y = 30\pi$ or $66\pi$

Note: Anomalous *B+L* means that we can rotate away one of the three theta angles.

The surviving angles are $\theta_3$ and $\theta_{em} = \dfrac{\theta_Y + 18\theta_2}{4}$

# The QCD Theta Angle

There is no observed CP violation from the strong force. Experiment bounds

$$\theta_3 \lesssim 10^{-10}$$

How do we know that $\theta_3$ is not close to $\pi$? The chiral Lagrangian tells us

$$\frac{m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2}{m_\pi^2} \approx \frac{m_d \mp m_u}{m_d \pm m_u}$$

(Crewther, Di Vecchia, Veneziano and Witten 1979)

Is there a cleaner argument?

# Yang-Mills at $\theta=\pi$

| SU(N) | | SU(N)/$\mathbf{Z}_N$ |
|---|---|---|
| T-invariant at $\theta=\pi$ | ⟷ Gauge $\mathbf{Z}_N$ 1-form symmetry | T-invariant at $\theta=N\pi$ |

This is a mixed anomaly in time reversal and $\mathbf{Z}_N$ center symmetry at $\theta=\pi$

't Hooft anomaly matching in infra-red ⟹

- Time reversal is spontaneously broken

or
- Theory is a TQFT

or
- Theory is not gapped

(Gaiotto, Kapustin, Komargodski, Seiberg 2017)

# QCD at $\theta=\pi$

U(1) x SU(N)
with quarks

T-invariant at
$\theta_N = 0$ and $\theta_1 = \pi$

T-invariant at
$\theta_N = \pi$ and $\theta_1 = 0$

⟷

Gauge $\mathbf{Z}_N$
1-form symmetry

U(N)
with quarks

T-invariant at
$\theta_N = 0$ and $\theta_1 = N^2\pi$

T-invariant at
$\theta_N = \pi$ and $\theta_1 = N\pi$

- There is again an anomaly when $\theta_N = \pi$
- There is also, now, a massless photon
  - This does not appear to be sufficient to absorb the anomaly

Work in progress with Zohar Komargodski

# Further Questions

What are the physical implications of the different quotients?

How can we tell which gauge group describes our world?