The Partonic Nature of Instantons

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Happy Birthday Misha!!
What are Instantons Made of?

- Instantons have a number of collective coordinates:
  
  - SU$(N)$ Yang-Mills
    - 4 translations, 1 scale size, 4N-5 orientation
    - 4N in total

  - CP$^N$ Sigma-Model
    - 2 translations, 1 scale size, 2N-3 orientations
    - 2N in total

- An old idea: Instantons are composed of N partons

Belavin et al ’79
The conjecture of partons usually framed for
- $d=3+1$ dimensions (for Yang-Mills)
- $d=1+1$ dimensions (for sigma-models)

Here we revisit this idea in context of
- $d=4+1$ dimensions (for Yang-Mills)
- $d=2+1$ dimensions (for sigma-models)

Theories are non-renormalizable (effective theories)
- Instantons are particle-like excitations
- Is the single instanton a multi-particle state?
  - ...and how can we tell?
There exists a UV completion of gauge theories in $d=4+1$ dimensions
- (at least with supersymmetry)
- ...but we don’t know much about it

With $N=2$ supersymmetry, the UV completion is the $(2,0)$ theory in $d=5+1$ dimensions

The UV completion has $N^3$ degrees of freedom.
The Instanton

- The instanton is the KK mode from the sixth dimension

\[ M_{\text{inst}} = \frac{8\pi^2}{e^2} = \frac{1}{R} \]

Rozali, ‘97

- Proposal: The instanton is an N-particle state

- The N partons inside the instanton are the remnant of the UV degrees of freedom which comprise the (2,0) theory.
Circumstantial Evidence

- Turn up the heat and look at cross-over in free-energy

\[ F \sim N^2 T^5 \longrightarrow RN^3 T^6 \]

- The transition happens at the temperature

\[ T \sim \frac{1}{NR} \sim \frac{1}{e^2 N} \]

- This had to be the case: this is where the 5d theory is strongly coupled, so this is where we need new UV degrees of freedom.

Itzhaki et al '98
More Circumstantial Evidence

- Anomaly coefficient for $G = ADE$ is conjectured to be $c_2(G') \times |G|$ (Intriligator, ‘00)

- Are the partons in the adjoint of $G$?
  - What is the mechanism that confines them?

- Quantizing the scale size of the instanton gives a continuous spectrum...natural for a multi-particle state.
  - But the moduli space is not that of $N$ free objects...does this contain clues about confining mechanism?
A Toy Model

- These questions are difficult to answer in case of Yang-Mills

- The $\mathbb{CP}^n$ sigma-model provides (as always!) a nice toy-model where we can see how some of these issues are resolved.
A Toy Model

- Our toy will be in $d=2+1$ dimensions
- It is a gauge theory with $N=4$ supersymmetry

- Vector multiplet: $V = (A_\mu, \phi_i, \text{fermions})$  \( i = 1, 2, 3 \)
- Hypermultiplet: $Q = (q, \bar{q}, \text{fermions})$

- $U(1)^N + N$ hypermultiplets

- gauge coupling = $g^2$
- mass of hypers = $m$
The Low-Energy Dynamics

- The hypermultiplets have physical mass

\[ M^a = (\phi^a - \phi^{a+1} + m) \]

- At low energies, we want an effective action for the massless vector multiplets

\[ \phi^a_i \quad \text{and} \quad F_{\mu\nu}^a = g^2 \epsilon_{\mu\nu\rho} \partial_\rho \sigma^a \]
Low-Energy Dynamics

- We integrate out the hypermultiplets
- This induces interactions for vector multiplets
- The low-energy dynamics is a sigma-model with target space

\[ \mathbb{R}^4 \times T^* \mathbb{C}P^{N-1} \]
**An Example: \( \mathbb{CP}^1 \)**

Diagonal U(1) decouples

Axial U(1) has two hypers, with charge +1 and -1

Integrate out hypers:

\[
\mathcal{L}_{\text{eff}} = \frac{1}{g_{\text{eff}}^2} (\partial \phi)^2 + g_{\text{eff}}^2 (\partial \sigma)^2
\]

\[
\frac{1}{g_{\text{eff}}^2} = \frac{1}{g^2} + \frac{1}{m-\phi} + \frac{1}{m+\phi}
\]
The Soliton

- The low-energy theory has a soliton: it is a sigma-model lump
  - 2 translation modes
  - 1 scale size
  - 2N-3 orientation modes

- What is this soliton in the microscopic theory?
  - It is BPS
  - Mass = Nm

- It is an N-particle state.
  - The soliton is made of the objects that we thought we’d integrated out!

\[ \phi = g_{\text{eff}}^2(\phi) \epsilon_{\mu \nu} \nabla^\nu \sigma \]

Aharony, Hanany, Intriligator, Seiberg and Strassler, ‘97
Question: Suppose we just have access to the IR physics on the Coulomb branch. What does the soliton tell us about the UV completion?

Answer: Pretty much everything!
Seeing the Partons

- We get the round metric on $\mathbb{C}P^{N-1}$ only in the limit $g^2 \to \infty$

- If we study solitons on the squashed target space, the solitons dramatically reveal themselves. Here’s a soliton on $\mathbb{C}P^1$

\[
m/g^2 = 0 \quad m/g^2 = 1 \quad m/g^2 = 2
\]
Seeing the Partons

- Here’s a single soliton on a squashed $\mathbb{CP}^2$
Changing the Orientation

- Keep the “scale size” of the lump fixed
- Change the “orientation modes”
- Watch the partons move
Confinement of Partons

- Why does the low-energy theory only include the N-particle state?

\[ Q_1 Q_2 \ldots Q_N \]

- Answer from microscopic theory: logarithmic confinement
  - In d=2+1 dimensions, electric charges have \( E \sim \frac{1}{r} \)
  - This gives log divergent mass
  - Only gauge singlet states have finite mass

- This log divergence re-appears when partons move.
  - Seen in IR theory as a log divergence in moduli space metric. Only modes which don’t change dipole moments have finite norm
Can we reconstruct the quantum numbers of the partons?
- In other words, can we reconstruct the full UV theory?
Dual Bogomolnyi Equations

- The Bogomonyi equations for $\mathbb{CP}^1$ are

$$\partial_\mu \phi = g_{\text{eff}}^2(\phi) \epsilon_{0\mu\nu} \partial^\nu \sigma$$

- They have a moduli space of solutions: dimension $2kN$ for $k$ soliton sector

- Can rewrite this equation in dual variables

$$F_{\mu\nu} = g_{\text{eff}}^2(\phi) \epsilon_{\mu\nu\rho} \partial^\rho \sigma$$

$$F_{0\mu} = \partial_\nu \phi$$
The dual Bogomolnyi equations have no smooth solutions.

But we can reproduce the exact soliton solution if we introduce electric sources:

\[ F_{0\mu} = \partial_\nu \phi \]

\[ \partial_\mu \left( \frac{1}{g^2_{\text{eff}}} F_{0\mu} \right) = \sum_{n=1}^{k} \delta(z - z_n^+) - \delta(z - z_n^-) \]

In soliton description, these are collective coordinates. Here, they are sources.
The dual formulation of the Bogomolnyi equation provides an explicit map between a soliton and fundamental fields.

It also works for $\mathbb{CP}^{N-1}$.

Can reconstruct quantum numbers of partons which determines the UV microscopic theory.
Calorons: A Red Herring?

- There is one way that instantons are known to split into N partons
  - Calorons

- Put $d=4+1$ theory on circle. Add a Wilson line.
  - Instantons $\rightarrow$ N monopole strings

- Doesn’t seem possible that this can happen in non-compact space
- Also happens for lumps in the toy model
  - But the calorons have nothing to do with the true partons
Summary: Questions, not Answers

- Toy model in d=2+1
  - Explicit demonstration that solitons can be thought of as multi-particle states
  - A study of the soliton allows us to reconstruct the UV behaviour

- Real Interest: d=4+1 Yang-Mills
  - Does the instanton solution hold clues about the constituents of the (2,0) theory?
  - What is the confinement mechanism?
    - No hint of log confinement...partons are probably not merons
Happy Birthday Misha!!