Inching Towards Strange Metallic Holography

Based on “Towards Strange Metallic Holography”, 0912.1061 with Sean Hartnoll, Joe Polchinski and Eva Silverstein

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Plan of the Talk

- **Motivation**
  - Remedial Introduction to Conductivity
  - Anomalous Properties of Strange Metals

- **Conductivity from Lifshitz Geometry**
  - Lifshitz Geometry and D-Brane Probes
  - DC and Hall Conductivity
  - Optical Conductivity
Drude Model: DC Conductivity

- Ohm’s Law: \[ \vec{E} = \rho \vec{j} \]

  Resistivity: \( \rho \)

  Conductivity: \( \sigma = \frac{1}{\rho} \)

\[ \rho = \frac{m}{ne^2 \tau} \]

Scattering Time
Scattering Mechanisms

- The temperature dependence of the resistivity sits in the scattering time:

\[
\begin{align*}
\text{Impurities:} & \quad \rho \sim T^0 \\
\text{Phonons:} & \quad \begin{cases} 
\rho \sim T & (T > \Theta_D) \\
\rho \sim T^5 & (T < \Theta_D) 
\end{cases} \\
\text{Electrons:} & \quad \rho \sim T^2
\end{align*}
\]

- Resistivity in metals is typically due to phonons and impurities.
Drude Model: Hall Conductivity

\[ \vec{j} = \sigma \vec{E} \]

\[ \sigma_{xx} = \frac{1}{\rho} \frac{1}{1 + \omega_c^2 \tau^2} \quad \sigma_{xy} = \frac{1}{\rho} \frac{\omega_c \tau}{1 + \omega_c^2 \tau^2} \quad (\omega_c = \frac{eB}{mc}) \]

\[ \frac{\sigma_{xx}}{\sigma_{xy}} = \frac{1}{\omega_c \tau} \]
Drude Model: AC Conductivity

- Fourier Transform:
  \[ \vec{j}(\omega) = \sigma(\omega) \vec{E}(\omega) \]
  \[ \vec{E} = \vec{E}(\omega) e^{-i\omega t} \]
  \[ \vec{j} = \vec{j}(\omega) e^{-i\omega t} \]

\[
\sigma(\omega) = \frac{1}{\rho} \frac{1}{1 - i\omega \tau}
\]

- Note: \( \sigma(\omega) \to i/\omega \) as \( \omega \to \infty \)
Strange Metals

- High Tc Superconductors at optimal doping
  - e.g., cuprates
- Suggestions that behaviour is governed by quantum critical point.
- Heavy Fermion materials have similar properties
Anomalous Properties

- Resistivity:
  \[ \rho \sim T \]

- Hall Conductivity
  \[ \frac{\sigma_{xx}}{\sigma_{xy}} \sim T^2 \]

- AC Conductivity
  \[ \sigma(\omega) \rightarrow \frac{i}{\omega}^{\nu} \]
  \[ \nu \approx 0.65 \]
Challenge

- Reproduce this anomalous behaviour in conductivity from a (presumably strongly coupled) field theory.
Strategy

- Use AdS/CFT
  - Build up ingredients we want by writing an effective theory in the bulk. e.g.

Pros:
- Very simple to compute transport properties in a strongly interacting theory

Cons:
- Can’t build microscopic theories to order
- But even worse…don’t know microscopic degrees of freedom!
The Set-Up

- **Charged particles** moving through a **strongly coupled soup**

  ![Diagram]

  - Probe Brane
  - Gravitational Background

- Our boundary theory will live in $d=2+1$ dimensions
- We want a current with *massive* charge carriers
- We will take the strongly coupled soup to obey *Lifshitz Scaling*
Lifshitz Scaling

- Non-Relativistic Conformal Invariance

\[ \vec{x} \rightarrow \lambda \vec{x} \quad t \rightarrow \lambda^z t \]

\( \vec{x} = (x, y) : d=2 \) spatial dimensions

- A weakly coupled example with \( z=2 \):

\[ S = \int dt d^d x \left[ \dot{\phi}^2 - (\nabla^2 \phi)^2 \right] \]
\[ ds^2 = L^2 \left( -\frac{dt^2}{r^2 z} + \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} \right) \]
Hot Lifshitz Geometry

\[ ds^2 = L^2 \left( -\frac{f(r)}{r^{2z}} \, dt^2 + \frac{dr^2}{f(r) r^2} + \frac{dx^2 + dy^2}{r^2} \right) \]

\[ f(r_+) = 0 \quad \Rightarrow \quad T \sim \frac{f'(r_+)}{r_+^{z-1}} \sim \frac{1}{r_+^z} \]

When needed, we take \( f(r) = 1 - \left( \frac{r}{r_+} \right)^{z+2} \)
Adding Probe Branes

- Brane wraps contractible cycle in internal space
- $\theta(r)$ profile looks like cigar
- When needed, we take this space to be a sphere
Adding Probe Branes

\[ S_{\text{DBI}} = -\tau \int dt d^3\sigma \sqrt{-\det [g_{ab} + \partial_a \theta \partial_b \theta + F_{ab}]} \]

In this talk, we’ll drop powers of \( \tau, L^2, \alpha' \)
Charge Carriers

- Charge carrier = string: \[ m = E_{\text{gap}} \sim 1/r_0^2 \]
- Note: \( m \gg T \quad (E_{\text{gap}} \sim 1\text{eV}, \quad T \sim 10 \rightarrow 10^3\text{K}) \)
Charge Carriers

- Finite charge density: $A_0 \rightarrow \mu - J^t r^{2-z} + \ldots$

- Brane now stretches to horizon. $\theta(r)$ profile looks like tube.

\[r = 0\]

\[r = r_0\]

\[r = r_+\]

c.f. Mateos, Myers et. al. ‘06
DC Conductivity

\[ A_0 \rightarrow \mu - J^t r^{2-z} + \ldots \]
\[ A_x \rightarrow Et + J^x r^z + \ldots \]

- Compute full non-linear conductivity
  \[ J^x = \sigma(E, T) E \]
- Without doing any work....!
**DC Conductivity**

\[
\sigma(E, T) = \sqrt{\text{const.}} + r_\star^4 (J^t)^2
\]

due to pair creation
due to background charge

- where \( f(r_\star) = E^2 r_\star^{2z+2} \)
- when \( E_{\text{gap}}, (J^t)^{z/2} \gg T \)

\[
\rho = \frac{1}{\sigma} \sim \frac{T^{2/z}}{J^t}
\]

**Comments:**
- Finite DC conductivity only because we’re ignoring backreaction
- Linear for \( z=2 \)
Hall Conductivity

- Similar technique: Find temperature dependence

\[
\frac{\sigma_{xx}}{\sigma_{xy}} \sim \frac{T^2}{Z}
\]

- This is like the Drude result: no sign of anomalous behaviour.

  - Caveat: The term from pair creation actually scales as

\[
\frac{\sigma_{xx}}{\sigma_{xy}} \sim \frac{T^4/z}{J^t B}
\]

  - Good for $z=2$, but unclear why this term would dominate
AC Conductivity

\[ A_x(\omega) = \frac{E_x(\omega)}{i\omega} + J^x(\omega) r^z + \ldots \]

- Boundary conditions:
  - \( E_x(\omega) \) at \( r=0 \)
  - Ingoing boundary conditions at black hole horizon

- Compute
  - Current \( J^x(\omega) \)
  - Numerical results for all \( \omega \)
  - Analytic results for \( T \ll \omega \ll E_{\text{gap}} \)
  - \( \omega \sim 0.1 \rightarrow 1 \text{ eV} \)
AC Conductivity

\[ \sigma(\omega) \sim \begin{cases} 
(J^t)^{z/2} \omega^{-1} & z < 2 \\
J^t (\omega \log \omega)^{-1} & z = 2 \\
J^t \omega^{-2/z} & z > 2 
\end{cases} \]

- Note: \( z=3 \) matches data!
- Linearity in \( J^t \) is now dynamical
Comment on the $z=2$ Crossover

- **Dimensional Analysis:**
  \[ [x] = -1 \quad [t] = -z \]

- Operators are relevant if \( [O] \leq d + z \)

\[
\int dt d^d x \quad O
\]

- **Examples:**
  - Charge density: \( [J^t] = d \quad \Longleftrightarrow \quad (J^t)^2 \) relevant for \( z>d \)
  - Probe Inertia: \( \int dt \quad \dot{x}^2 \quad \Longleftrightarrow \quad \) irrelevant for \( z>2 \)

- For \( z>2 \), all inertia due to stuff that particle drags around with it
Summary

- Charge carriers moving in strongly coupled Lifshitz backgrounds yields:
  - DC conductivity: $\rho \sim T^{2/z}$
  - Hall conductivity: $\frac{\sigma_{xx}}{\sigma_{xy}} \sim \rho$
  - AC conductivity $\sigma(\omega) \sim \frac{1}{\omega^{2/z}}$ for $z>2$

- Starting point for model building?