Numerical simulations of astrophysical BHBs

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CALTECH

Overview

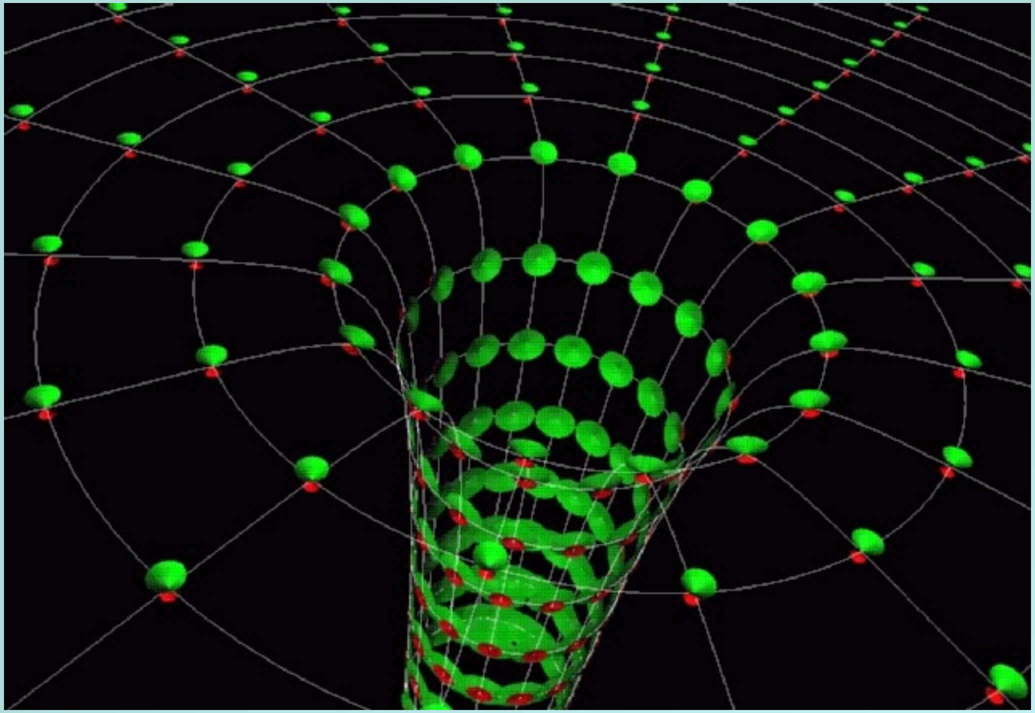
- Motivation
- Introduction
- Ingredients of numerical relativity
- Results
  - A brief history of BH simulations
  - Results following the recent breakthrough
- Summary

1. Black holes in physics

Black Holes predicted by GR

- Black holes predicted by Einstein's theory of relativity
- Term "Black hole" by John A. Wheeler 1960s
- Vacuum solutions with a singularity
- For a long time: mathematical curiosity
  - valuable insight into theory
  - but real objects in the universe?
- That picture has changed dramatically!

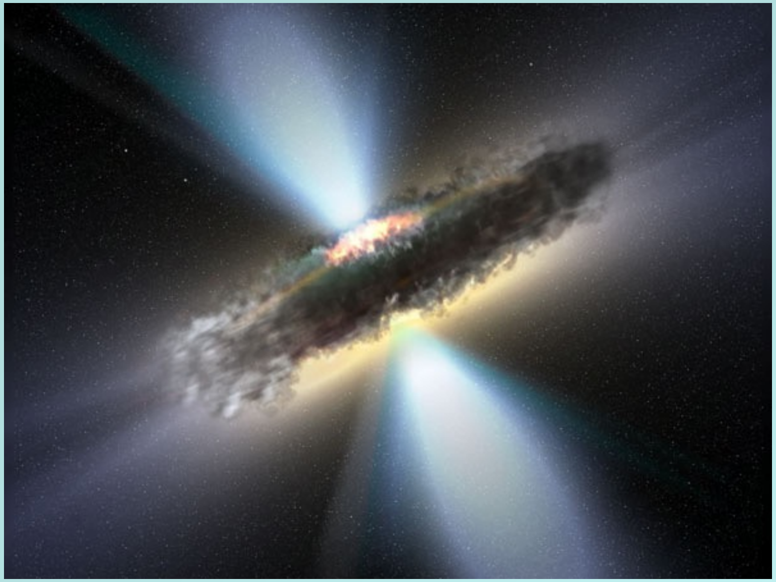
How to characterize a black hole?

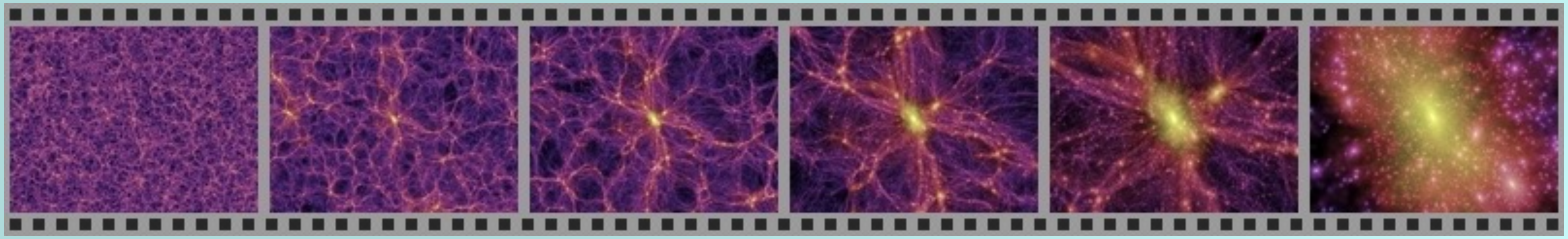


- Consider light cones
- Outgoing, ingoing light
- Calculate surface area of outgoing light
- Expansion:=Rate of change of that area
- Apparent horizon:= Outermost surface with zero expansion
- “Light cones tip over” due to curvature

Black Holes in astrophysics

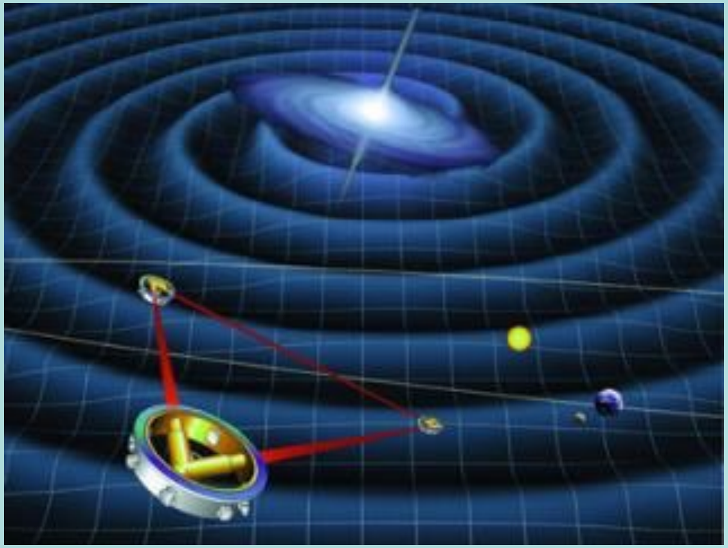
- Black holes are important in astrophysics
  - Black holes found at centers of galaxies
  - Structure of galaxies
  - Important sources of electromagnetic radiation
  - Structure formation in the early universe

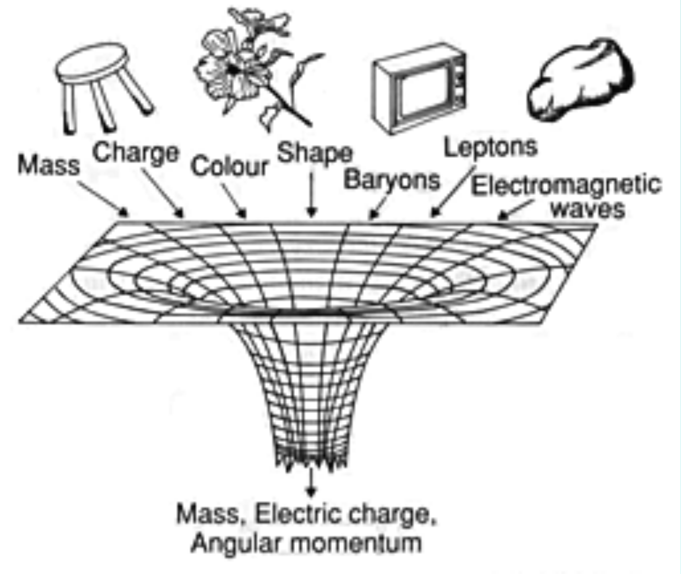




Fundamental physics of black holes

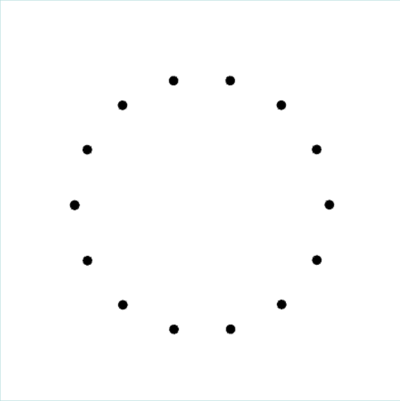
- Allow for unprecedented tests of fundamental physics
  - Strongest sources of Gravitational Waves (GWs)
  - Test alternative theories of gravity
  - No-hair theorem of GR
  - Production in Accelerators



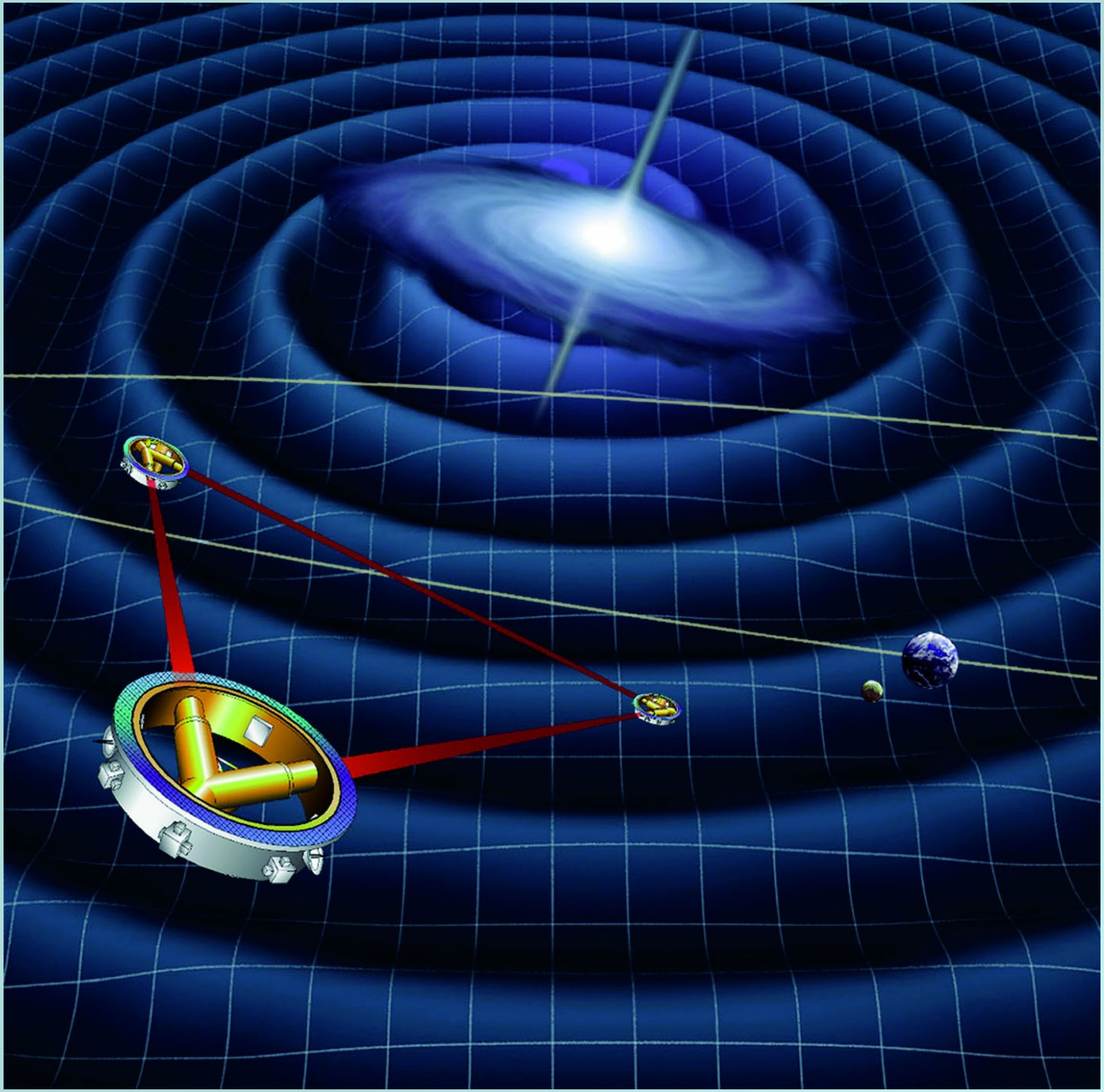


Gravitational wave physics

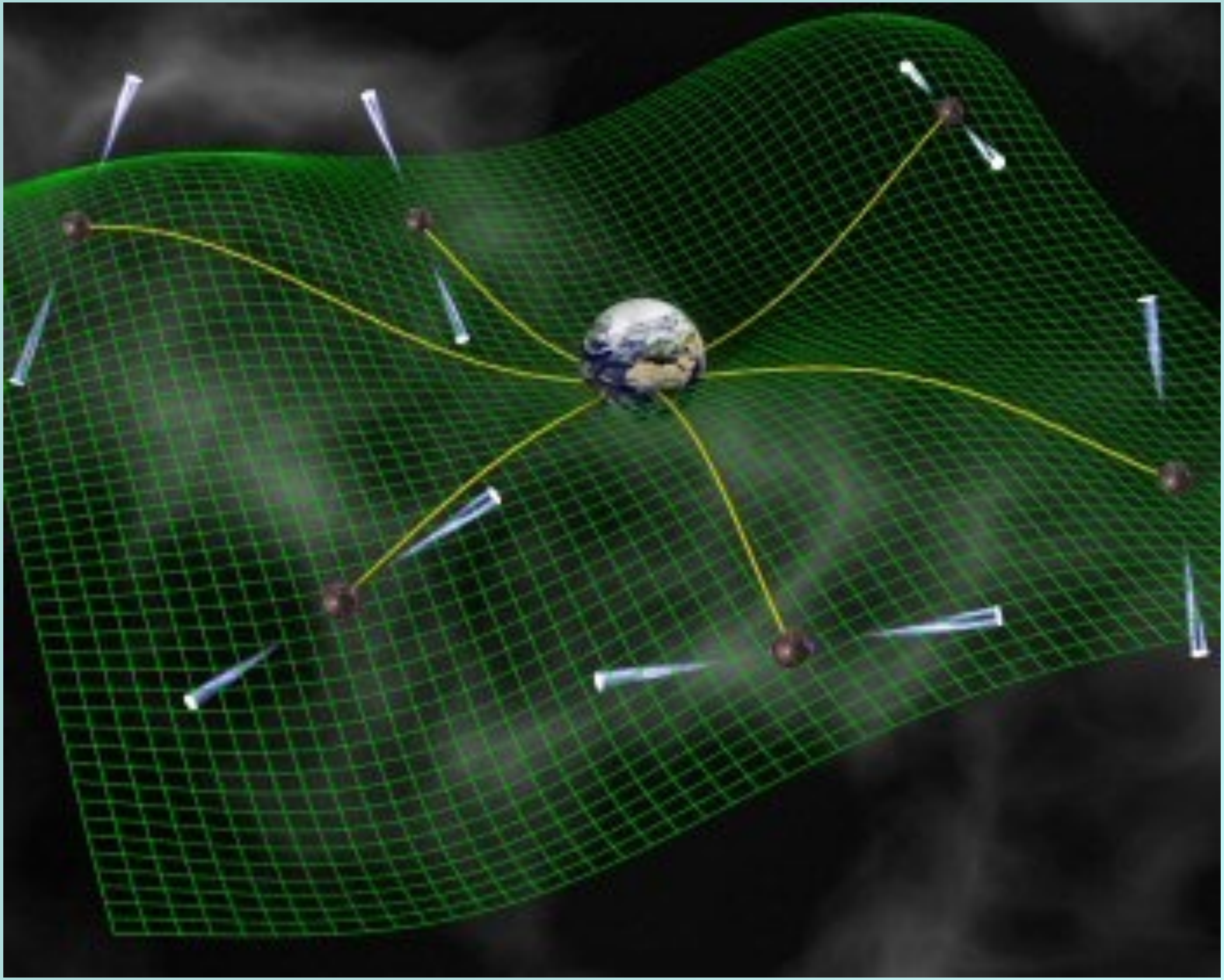
- Accelerating bodies produce GWs
- Weber 1960s
  - Bar detector
  - Claimed detection probably not real
- GWs displace particles
- GW observatories: GEO600, LIGO, TAMA, VIRGO
  Bar detectors



Space interferometer LISA

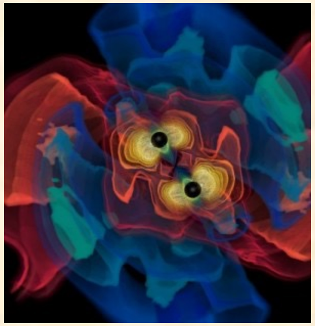


Pulsar timing arrays



The big picture

Physical system



observe

Detectors

describes

test

Help detection

Provide info

Model

- GR (NR)
- PN
- Perturbation theory
- Alternative Theories?

External Physics

- Astrophysics
- Fundamental Physics
- Cosmology

2. General relativity

The framework: General Relativity

- Curvature generates acceleration

  "geodesic deviation"

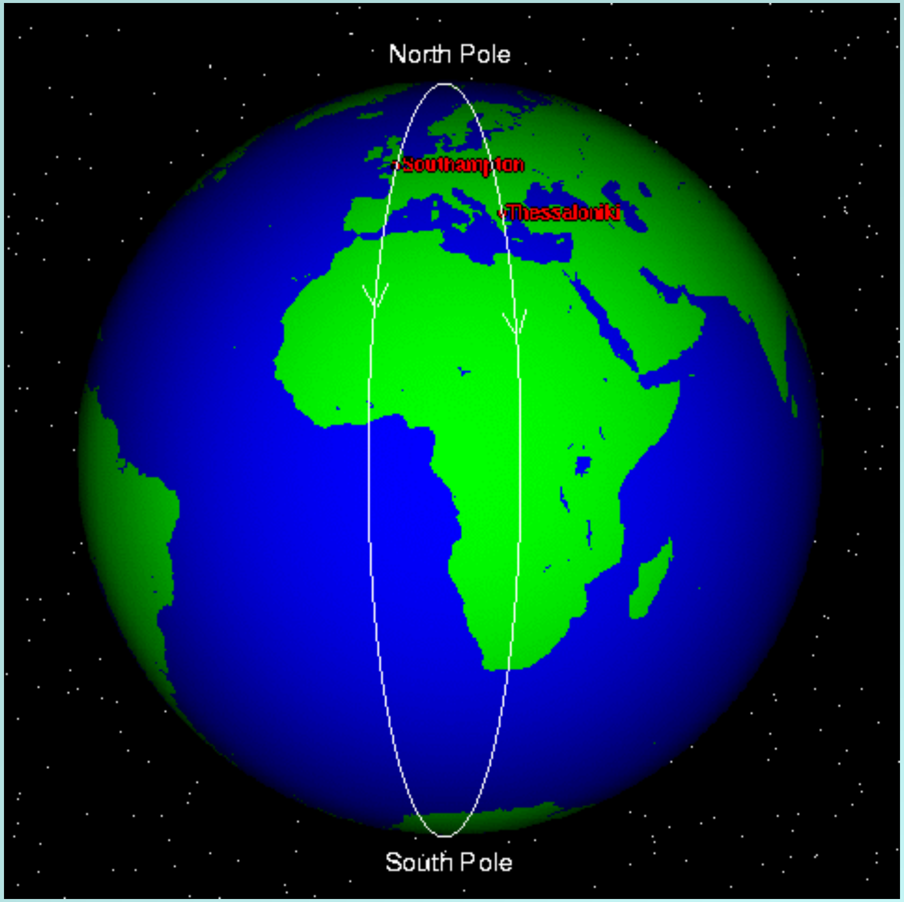
  No "force" !!

- Description of geometry

  Metric $g_{\alpha\beta}$

  Connection $\Gamma^{\alpha}_{\beta\gamma}$

  Riemann Tensor $R^{\alpha}{}_{\beta\gamma\delta}$



The metric defines everything

- Christoffel connection

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left(\partial_{\beta} g_{\gamma\mu} + \partial_{\gamma} g_{\mu\beta} - \partial_{\mu} g_{\beta\gamma} \right)$$

- Covariant derivative

$$\nabla_{\alpha} T^{\beta}{}_{\gamma} = \partial_{\alpha} T^{\beta}{}_{\gamma} + \Gamma^{\beta}_{\mu\alpha} T^{\mu}{}_{\gamma} - \Gamma^{\mu}_{\gamma\alpha} T^{\beta}{}_{\mu}$$

- Riemann Tensor

$$R^{\alpha}{}_{\beta\gamma\delta} = \partial_{\gamma} \Gamma^{\alpha}_{\beta\delta} - \partial_{\delta} \Gamma^{\alpha}_{\beta\gamma} + \Gamma^{\alpha}_{\mu\gamma} \Gamma^{\mu}_{\beta\delta} - \Gamma^{\alpha}_{\mu\delta} \Gamma^{\mu}_{\beta\gamma}$$

  - Geodesic deviation
  - Parallel transport
  - …

How to get the metric?

- The metric must obey the Einstein Equations
- Ricci-Tensor, Einstein-Tensor, Matter tensor

$$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\mu}{}_{\mu}$$ "Trace-reversed" Ricci

$T_{\alpha\beta}$ "Matter"

- Einstein Equations $$\boxed{G_{\alpha\beta} = 8\pi\, T_{\alpha\beta}}$$

- Solutions: Easy!
  - Take metric
  - Calculate $G_{\alpha\beta}$
  - Use that as matter tensor

- Physically meaningful solutions: Difficult!

The Einstein equations in vacuum

- "Spacetime tells matter how to move, matter tells spacetime how to curve"
- Field equations: $R_{\alpha\beta} = 0$

  Second order PDEs for the metric components

  Invariant under coordinate (gauge) transformations

- System of equations extremely complex: Pile of paper!

  Analytic solutions: Minkowski, Schwarzschild, Kerr,
  Robertson-Walker,…

Numerical methods necessary for general scenarios!

3. The basics of numerical relativity

A list of tasks

- Target: Predict time evolution of BBH in GR
- Einstein equations:
  - Cast as evolution system
  - Choose specific formulation
  - Discretize for Computer
- Choose coordinate conditions: Gauge
- Fix technical aspects:
  - Mesh-refinement / spectral domains
  - Excision
  - Parallelization
  - Find large computer
- Construct realistic initial data
- Start evolution and wait…
- Extract physics from the data

Gourgoulhon gr-qc/0703035

3.1. The Einstein equations

Split spacetime

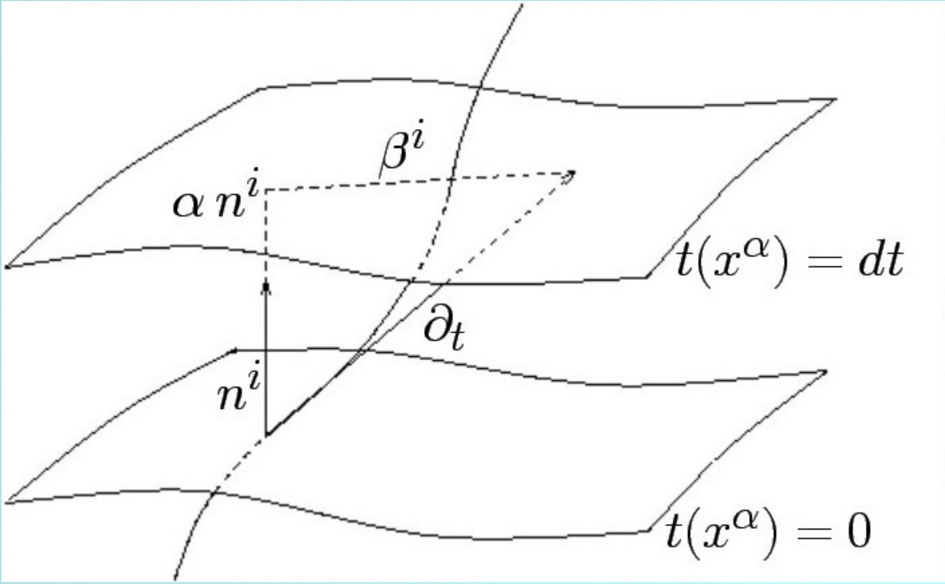
- GR: "Space and time exist as Spacetime"
- NR: Split spacetime
  - Characteristic / null split using Lightrays (not this lecture)
  - "3+1" split: most common approach

Arnowitt, Deser, Misner '62, York '79

- Foliation

Let (M, g) be a spacetime with coordinates x^α

Introduce scalar field t on M with gradient $\mathrm{d}t$ that satisfies $\langle \mathrm{d}t, \mathrm{d}t \rangle < 0$



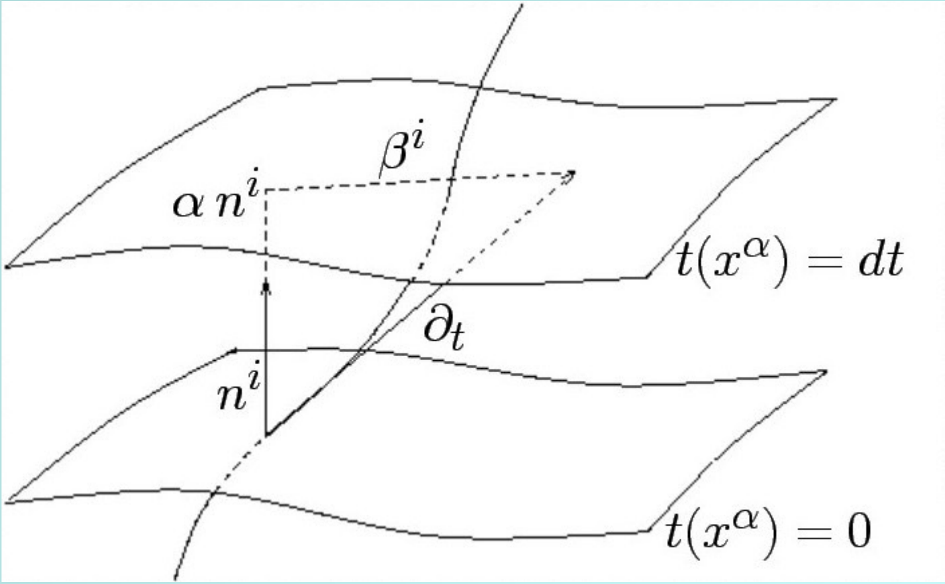
"The hypersurfaces $t = \mathrm{const}$ are spacelike"

Unit normal field

- Unit normal field

For any given hypersurface Σ_t the gradient $\mathrm{d}t$ has vanishing inner product with vectors tangential to Σ_t.

$$\Rightarrow \quad n = \frac{\mathrm{d}t}{\sqrt{-\langle \mathrm{d}t, \mathrm{d}t \rangle}}$$ is the unit normal field



- Adapted coordinates

(t, x^i)

- Tangential vector

∂_t is the vector along
The curves $x^i = \mathrm{const}$

In general ∂_t is NOT normal to Σ_t !!

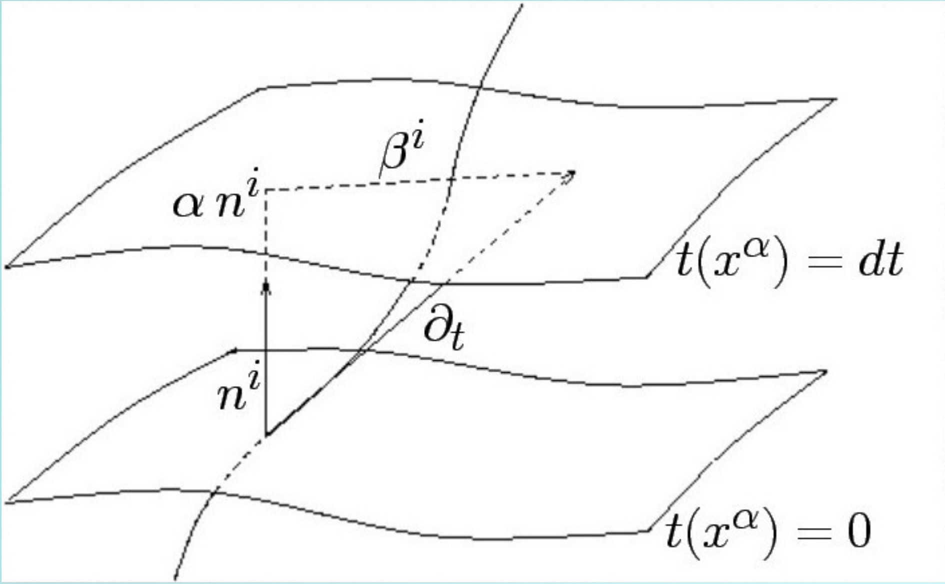
Unit normal field

- Lapse function

The norm of $\mathrm{d}t$ is important and has its own name: lapse

$$\alpha = \sqrt{-\langle \mathrm{d}t, \mathrm{d}t \rangle}$$

The lapse measures the advance of proper timealong n



- Shift vector

The vector

$$\beta := \partial_t - \alpha\, n$$

Is tangent to Σ_t

The shift vector measures
How points with constant x^i on different slices are related

- Lapse and shift represent coordinate choices

Projections

- Spatial projection operator $\perp^{\mu}{}_{\alpha} = \delta^{\mu}{}_{\alpha} + n^{\mu} n_{\alpha}$

  For any given tensor $T^{\lambda}{}_{\mu\nu}$ we obtain the spatial projection

  $$(\perp T)^{\alpha}{}_{\beta\gamma} = \perp^{\alpha}{}_{\lambda} \perp^{\mu}{}_{\beta} \perp^{\nu}{}_{\gamma} T^{\lambda}{}_{\mu\nu}$$

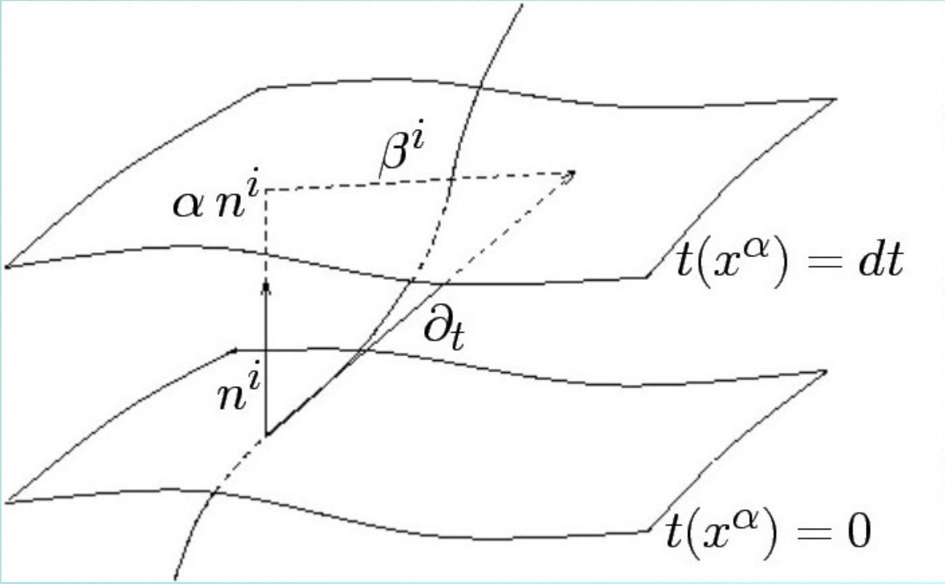
- Time projection

  $$T^{\lambda}{}_{\mu\nu} n_{\lambda} n^{\mu} n^{\nu}$$

- Mixed projection

  For example:

  $$\perp^{\mu}{}_{\beta} T^{\lambda}{}_{\mu\nu} n_{\lambda} n^{\nu}$$



First fundamental form: 3-metric

- View the hypersurface Σ_t as a manifold in its own right
- It has its own "3-metric" $\gamma_{\alpha\beta}$
- The components are $\boxed{\gamma_{\alpha\beta} = \perp_{\alpha\beta}}$

Raising and lowering of indices with $\gamma_{\alpha\beta}$

- The complete machinery of
  - Connection
  - Riemann tensor
  - Ricci tensor

Works in 3 dimensions with $\gamma_{\alpha\beta}$ as in 4 dimensions with $g_{\alpha\beta}$

- For each of these we have a 3-dim. and a 4-dim. version

Second fundamental form: Extrinsic curvature

- An embedded hypersurface Σ_t has two types of curvatures

  1) Intrinsic curvature: Riemann tensor of $\gamma_{\alpha\beta}$

  2) Extrinsic curvature $K_{\alpha\beta}$

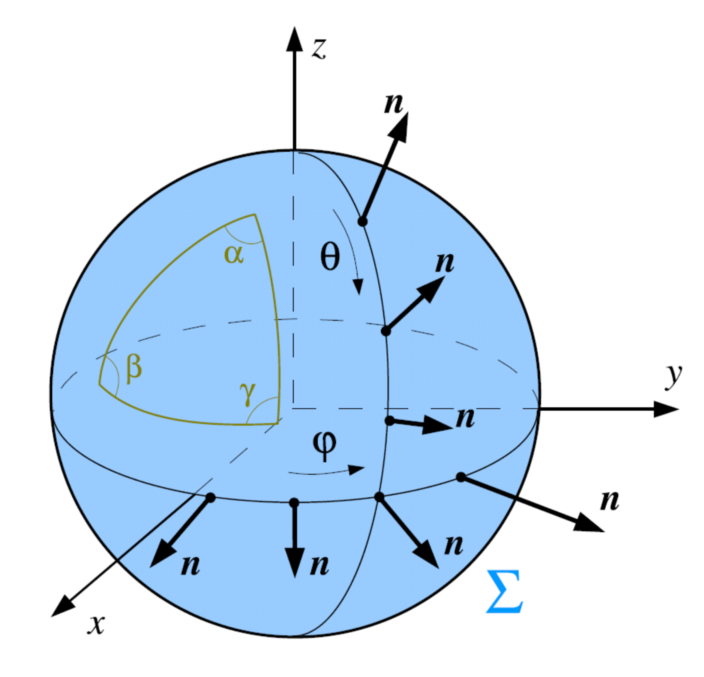
  The embedding of Σ_t in the 4-dim spacetime (M, g)

- Interpretations of extr. curvature:
  - Change of n_α :

$$K_{\alpha\beta} = -\nabla_\beta n_\alpha - n_\beta n^\mu \nabla_\mu n_\alpha$$

  - Evolution of 3-metric:

$$\mathbf{K} = -\frac{1}{2} L_{\mathbf{n}} \boldsymbol{\gamma}$$



Projections of Riemann

- How is the 4-dim. Curvature related to the 3-dim. intrinsic and extrinsic curvature?

- Answer: Project Riemann tensor

- Gauss Equation

$$\perp^{\mu}{}_{\alpha} \perp^{\nu}{}_{\beta} \perp^{\gamma}{}_{\rho} \perp^{\sigma}{}_{\delta} \; {}^{4}R^{\rho}{}_{\sigma\mu\nu} = R^{\gamma}{}_{\delta\alpha\beta} + K^{\gamma}{}_{\alpha} K_{\delta\beta} - K^{\gamma}{}_{\beta} K_{\alpha\delta}$$

Gauss-Codacci Equation

$$\perp^{\mu}{}_{\alpha} \perp^{\nu}{}_{\beta} \perp^{\gamma}{}_{\rho} \; n^{\sigma} \; {}^{4}R^{\rho}{}_{\sigma\mu\nu} = D_{\beta} K^{\gamma}{}_{\alpha} - D_{\alpha} K^{\gamma}{}_{\beta}$$

Fully mixed projection

$$\perp_{\rho\alpha} \perp^{\mu}{}_{\beta} \; n^{\sigma} n^{\nu} \; {}^{4}R^{\rho}{}_{\sigma\mu\nu} = L_{\mathbf{n}} K_{\alpha\beta} + \frac{1}{\alpha} D_{\alpha} D_{\beta} \alpha + K_{\alpha\mu} K^{\mu}{}_{\beta}$$

Projections of the energy momentum tensor

- Energy momentum tensor $T_{\alpha\beta}$ defined such that

$E := T_{\mu\nu} n^{\mu} n^{\nu}$ Energy density for observer with $u^{\alpha} = n^{\alpha}$

$p_{\alpha} := -T_{\mu\nu} n^{\mu} \perp^{\nu}{}_{\alpha}$ Momentum density

$S_{\alpha\beta} := T_{\mu\nu} \perp^{\mu}{}_{\alpha} \perp^{\nu}{}_{\beta}$ Matter stress tensor

$S(e, \hat{e}) :=$ Force in direction of e acting on surface normal to $\hat{e}$

- With that: $T_{\alpha\beta} = S_{\alpha\beta} + n_{\alpha} p_{\beta} + p_{\alpha} n_{\beta} + E n_{\alpha} n_{\beta}$

$$T := T^{\mu}{}_{\mu} = g^{\mu\nu} S_{\mu\nu} + g^{\mu\nu} \left(n_{\alpha} p_{\beta} + p_{\alpha} n_{\beta} \right) + E n^{\mu} n_{\mu} = S - E$$

Projections of the Einstein equations

- Einstein equations: $$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} = 8\pi T_{\alpha\beta}$$
- Projections follow from Gauss-Codacci and Mainardi
- Notes:
  - In 3-dim. objects we can ignore time components

    $\Rightarrow$ Spatial indices $i = 1,2,3$
  - 3-dim. Covariant derivative: D_i
  - With matter there would be additional terms!

The ADM equations

- Time projection $R_{\alpha\beta} n^{\alpha} n^{\beta} = 0$

$$\Rightarrow \boxed{R + K^2 - K_{ij} K^{ij} = 16E}$$

Hamiltonian constraint

- Mixed projection $\perp^{\mu}{}_{\alpha} \; R_{\mu\beta} n^{\beta} = 0$

$$\Rightarrow \boxed{-D^i K + D_j K^{ij} = 8\pi p^i}$$

Momentum constraints

- Spatial projection $\perp^{\mu}{}_{\alpha} \perp^{\nu}{}_{\beta} \; R_{\mu\nu} = 0$

$$\Rightarrow \boxed{\begin{aligned}(\partial_t - L_\beta) K_{ij} = -D_i D_j \alpha + \alpha [R_{ij} - 2K_{im} K^m{}_j + K_{ij} K] \\ + 4\pi [(S - E)\gamma_{ij}] - 2S_{ij}\end{aligned}}$$

Evolution equations

- Matter evolution: $\boxed{\nabla_\mu T^{\mu\alpha} = 0}$

The structure of the ADM equations

- Constraints:
  - They do not contain time derivatives
  - They must be satisfied on each slice Σ_t
  - The Bianchi identities propagate the constraints:

  If they are satisfied initially, they are always satisfied

- Evolution equations:
  - Commonly written as first order system

$$(\partial_t - L_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

$$(\partial_t - L_\beta)K_{ij} = -D_i D_j \alpha + \alpha[R_{ij} - 2K_{im}K^m{}_j + K_{ij}K] + 4\pi[(S-E)\gamma_{ij}] - 2S_{ij}$$

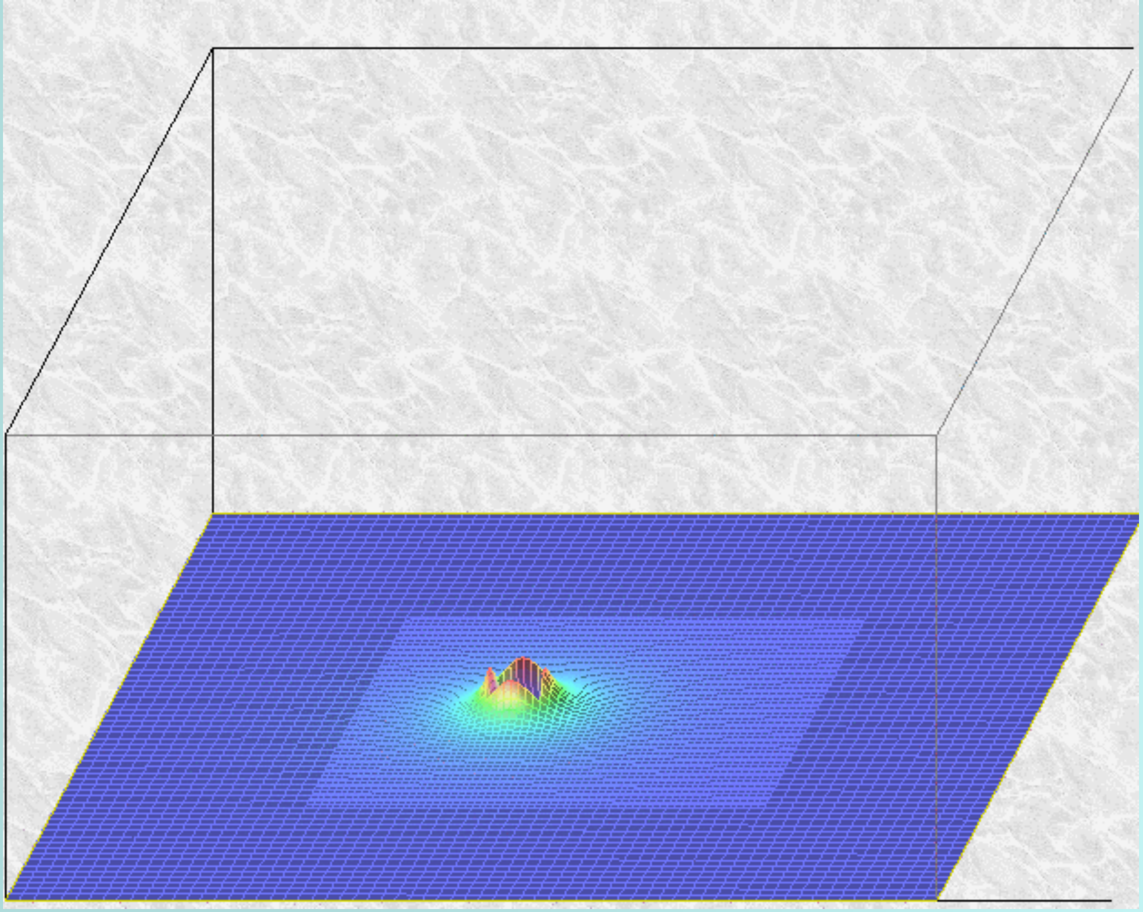
- Gauge:
  - Equations say nothing about lapse α and shift β !

The ADM equations as an initial value proble

- Entwicklungsgleichungen (from now on vacuum)

$$(\partial_t - L_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

$$(\partial_t - L_\beta)K_{ij} = -D_i D_j \alpha + \alpha[R_{ij} - 2K_{im}K^m{}_j + K_{ij}K]$$



Alternatives to the ADM equations

- Unfortunately the ADM eqs. do not seem to work in NR !!

  Weak hyperbolicity: Nearby initial data can diverge

  From each other super-exponentially

- Many alternative formulations have been suggested

- Two successful families so far
  - ADM based formulations: BSSN

    Shibata & Nakamura ’95, Baumgarte & Shapiro ‘99
  - Generalized harmonic formulations

    Choque-Bruhat ’62, Garfinkle ’04, Pretorius ‘05

BSSN

- One can easily change variables. E.g. wave equation:

$$\boxed{\partial_{tt} u - c^2 \partial_{xx} u = 0} \quad \Leftrightarrow \quad \boxed{\begin{array}{rl} & \partial_t F - c^2 \partial_x G = 0 \\ \wedge & \partial_x F - \partial_t G = 0 \end{array}}$$

- BSSN: rearrange degrees of freedom

$$\phi = \frac{1}{12} \ln\left(\det \gamma\right) \qquad \tilde{\gamma}_{ij} = e^{-4\phi} \gamma_{ij}$$

$$K = \gamma_{ij} K^{ij} \qquad \tilde{A}_{ij} = e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij} K\right)$$

$$\tilde{\Gamma}^i = \tilde{\gamma}^{mn} \tilde{\Gamma}^i_{mn} = -\partial_m \tilde{\gamma}^{im}$$

Shibata, Nakamura ’95, Baumgarte, Shapiro ‘99

The BSSN equations

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\boxed{\begin{array}{ll} \phi = \frac{1}{12}\ln\gamma & \hat{\gamma}_{ij} = e^{-4\phi}\gamma_{ij} \\ K = \gamma_{ij}K^{ij} & \hat{A}_{ij} = e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij}K\right) \\ \hat{\Gamma}^i = \gamma^{ij}\hat{\Gamma}^i_{jk} = -\partial_j\hat{\gamma}^{ij} & \end{array}}$$

$$(\partial_t - \mathcal{L}_\beta)\hat{\gamma}_{ij} = -2\alpha\hat{A}_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\frac{1}{6}\alpha K$$

$$(\partial_t - \mathcal{L}_\beta)\hat{A}_{ij} = e^{-4\phi}\left(-D_iD_j\alpha + \alpha R_{ij}\right)^{\mathrm{TF}} + \alpha(K\hat{A}_{ij} - 2\hat{A}_{ik}\hat{A}^k{}_j)$$

$$(\partial_t - \mathcal{L}_\beta)K = -D^iD_i\alpha + \alpha\left(\hat{A}_{ij}\hat{A}^{ij} + \frac{1}{3}K^2\right)$$

$$\partial_t\hat{\Gamma}^i = 2\alpha\left(\hat{\Gamma}^i_{jk}\hat{A}^{jk} + 6\hat{A}^{ij}\partial_j\phi - \frac{2}{3}\hat{\gamma}^{ij}\partial_jK\right) - 2\hat{A}^{ij}\partial_j\alpha + \hat{\gamma}^{jk}\partial_j\partial_k\beta^i$$

$$+\frac{1}{3}\hat{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\hat{\Gamma}^i + \frac{2}{3}\hat{\Gamma}^i\partial_j\beta^j \underbrace{- \left(\chi + \frac{2}{3}\right)\left(\hat{\Gamma}^i - \hat{\gamma}^{jk}\hat{\Gamma}^i_{jk}\right)\partial_l\beta^l}_{\text{Yo et al. (2002)}}$$

Generalized harmonic (GHG)

- Harmonic gauge: choose coordinates so that

$$\nabla_\mu \nabla^\mu x^\alpha = 0$$

- 4-dim. Version of Einstein equations

$$R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \dots$$ (no second derivatives!!)

Principal part of wave equation

- Generalized harmonic gauge: $H_\alpha := g_{\alpha\nu} \nabla_\mu \nabla^\mu x^\nu$

$$\Rightarrow R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \dots - \frac{1}{2} \left(\partial_\alpha H_\beta + \partial_\beta H_\alpha \right)$$

Still principal part of wave equation!!!

The gauge in GHG

- Relation between H_α and lapse α and shift β^i :

$$H_\mu n^\mu = -K - \frac{1}{\alpha^2}\left(\partial_0\alpha - \beta^i\partial_i\alpha\right)$$

$$\perp^i{}_\mu H^\mu = \frac{1}{\alpha}\gamma^{ik}\partial_k\alpha + \frac{1}{\alpha^2}\left(\partial_0\beta^i - \beta^k\partial_k\beta^i\right) - \gamma^{mn}\Gamma^i_{mn}$$

- Auxiliary constraint

$$C_\gamma := H_\gamma - \Gamma^\mu_{\mu\gamma} + g^{\mu\nu}\partial_\mu g_{\nu\gamma}$$

Requires constraint damping

Gundlach et al. 2005

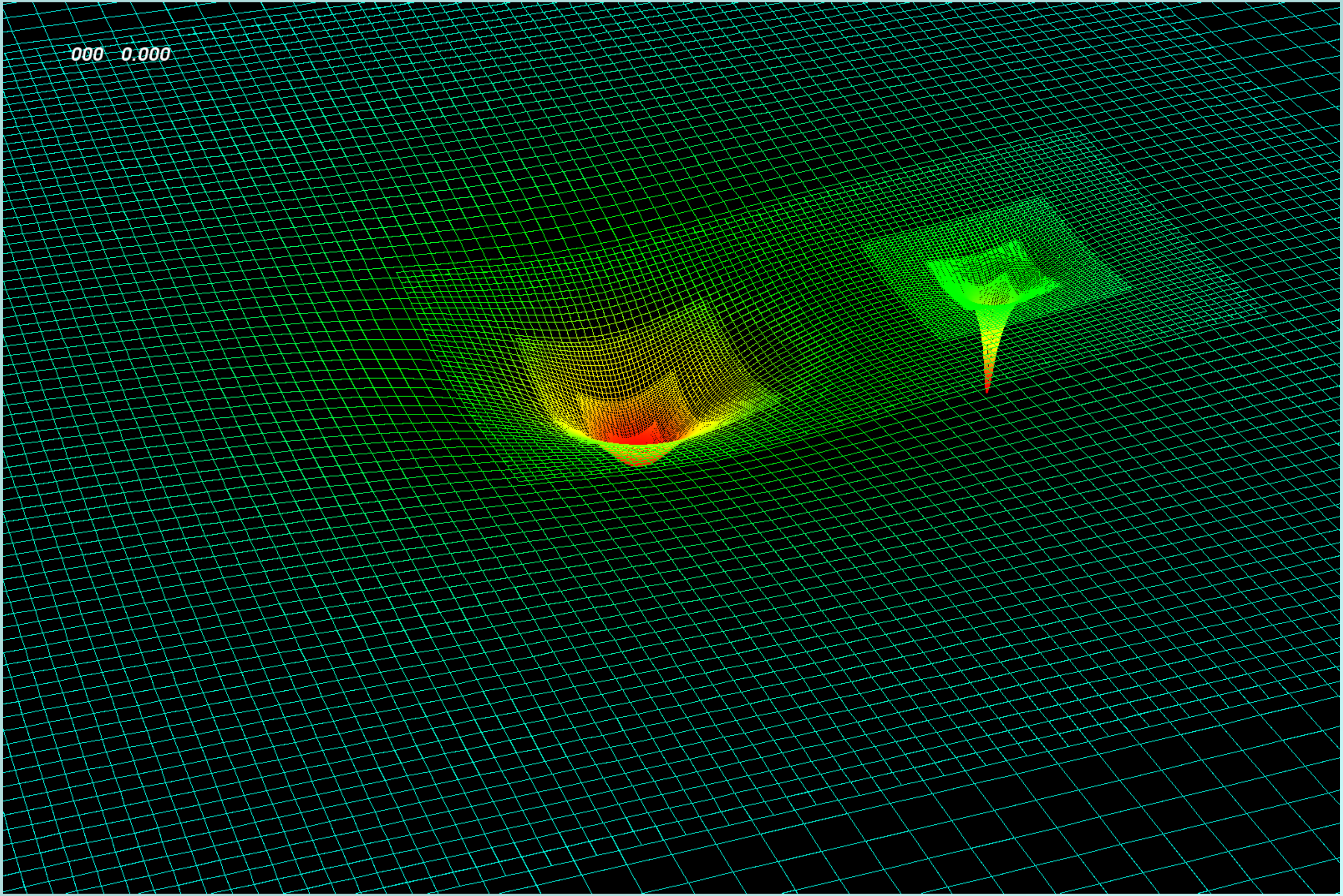
3.2. Gauge choices

The gauge freedom

- Remember: Einstein equations say nothing about $\alpha, \ \beta^i$
- Any choice of lapse and shift gives a solution
- This represents the coordinate freedom of GR
- Physics do not depend on $\alpha, \ \beta^i$

  So why bother?
- Avoid coordinate singularities!
- Stop the code from running into the physical singularity
- No full-proof recipe, but
  - Singularity avoiding slicing
  - Use shift to avoid coordinate stretching

What goes wrong with bad gauge?



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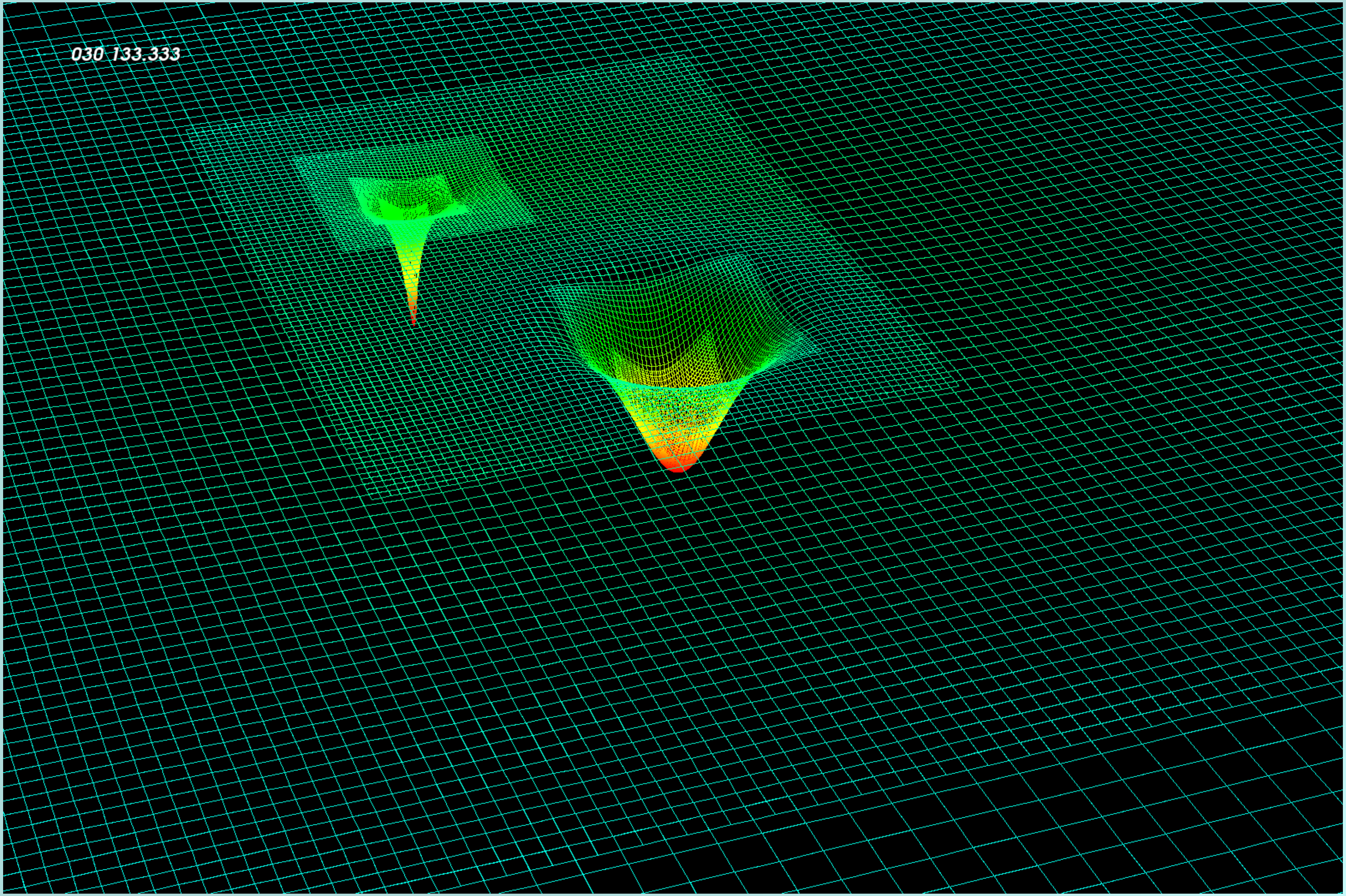

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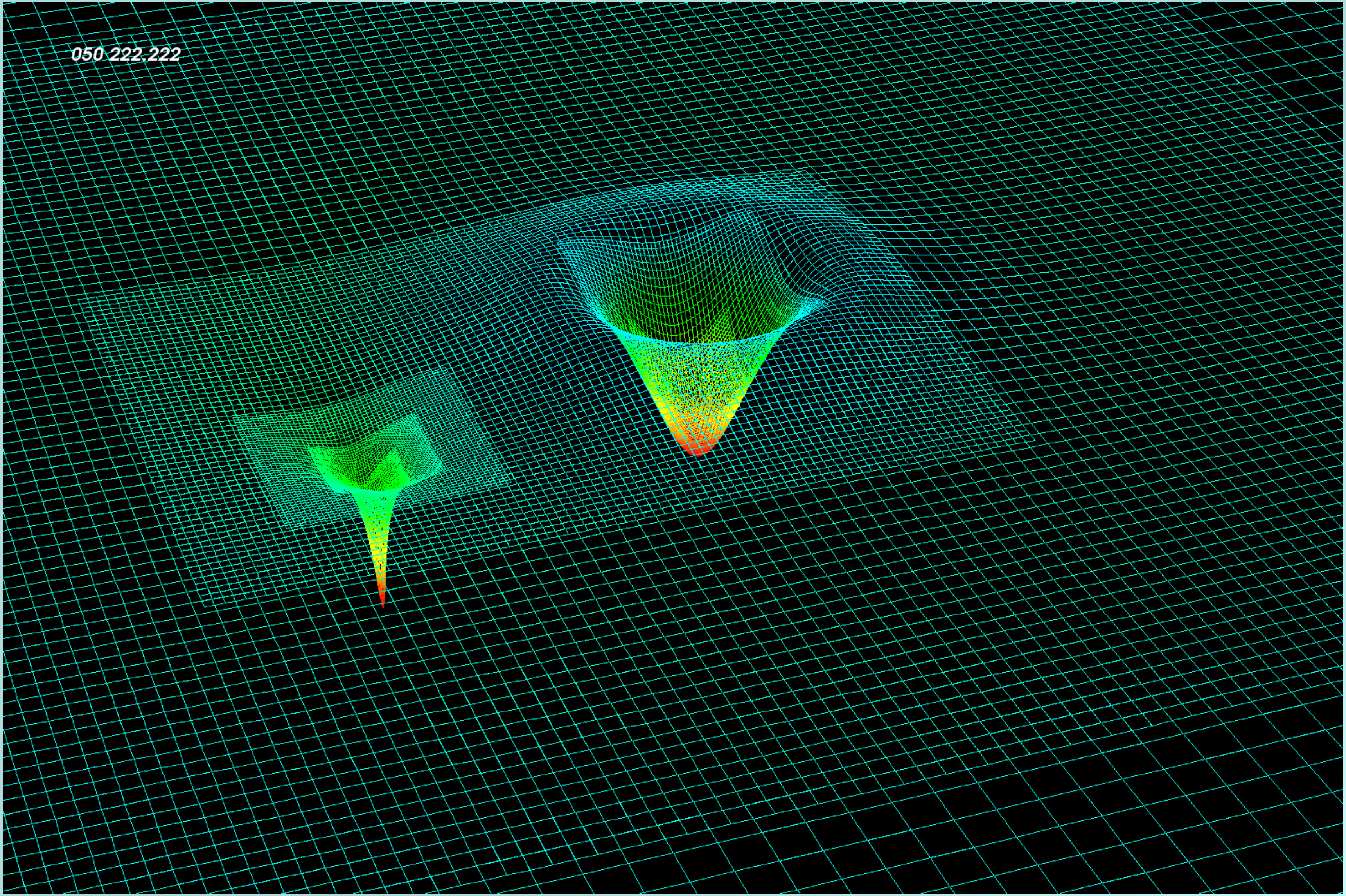

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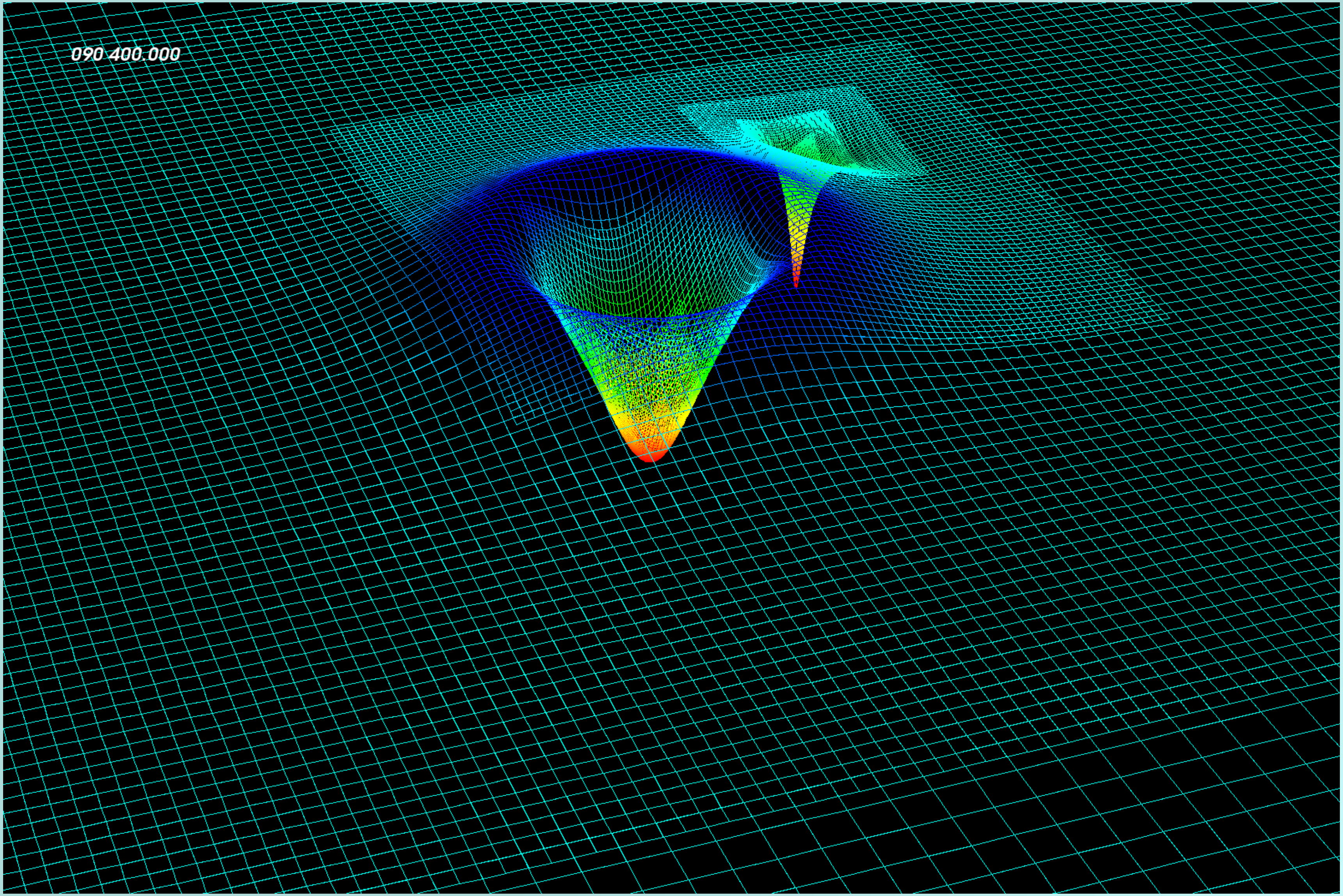

How do we get good gauge?

- Target: Avoid singularities and instabilities
- Methods: Geometric ideas, mathematical structure of equations

Maximal slicing,
min.distortion shift
Smarr, York '78

Harmonic coords.
Choquet-Bruhat'62
▶ Analytic studies

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Driver conditions
Balakrishna et.al.'96

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↓

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↓

1+log, $\tilde{\Gamma}$-driver
AEI
▶ Aim: Stationarity

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1+log, $\tilde{\Gamma}$-driver
AEI
- Aim: Stationarity

Bona-Massó family
Bona, Massó '95

Special case

Harmonic coords.
Choquet-Bruhat'62
- Analytic studies

Special case

Avoid singularities
Alcubierre '03

How do we get good gauge?

- Target: Avoid singularities and instabilities
- Methods: Geometric ideas, mathematical structure of equations

Maximal slicing, min.distortion shift
`Smarr, York '78`

Driver conditions
`Balakrishna et.al.'96`

1+log, $\tilde{\Gamma}$-driver
`AEI`
- Aim: Stationarity

Moving punctures
`UTB, Goddard '06`

Special case

Bona-Massó family
`Bona, Massó '95`

Special case

Avoid singularities
`Alcubierre '03`

Harmonic coords.
`Choquet-Bruhat'62`
- Analytic studies

Generalized harmonic
`Garfinkle '04`
`Pretorius '05`
- gauge sources
- Relation to α, β^i

3.3. Initial data

Initial data problem

Two problems: Constraints, realistic data

- York-Lichnerowicz split $\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$

$$K_{ij} = A_{ij} + \frac{1}{3}\gamma_{ij} K$$

Rearrange degrees of freedom

- Conformal transverse traceless — York, Lichnerowicz
- Physical transverse traceless — O'Murchadha, York
- Thin sandwich — Wilson, Mathews; York

- Conformal flatness: Kerr is NOT conformally flat!

Non-physical GWs: problematic for high energy collisions!

2 families of initial data

- Generalized analytic solutions:

  Isotropic Schwarzschild: $ds^2 = -dt^2 + \left(1 + \frac{M}{2r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$

  $\Rightarrow$ Time-symmetric, N-holes: Brill-Lindquist, Misner (1960s)

  $\Rightarrow$ Spin, Momenta: Bowen, York (1980)

  $\Rightarrow$ Punctures Brandt, Brügmann (1997)

- Excision Data: IH boundary conditions on excision surface

  Meudon group; Cook, Pfeiffer; Ansorg

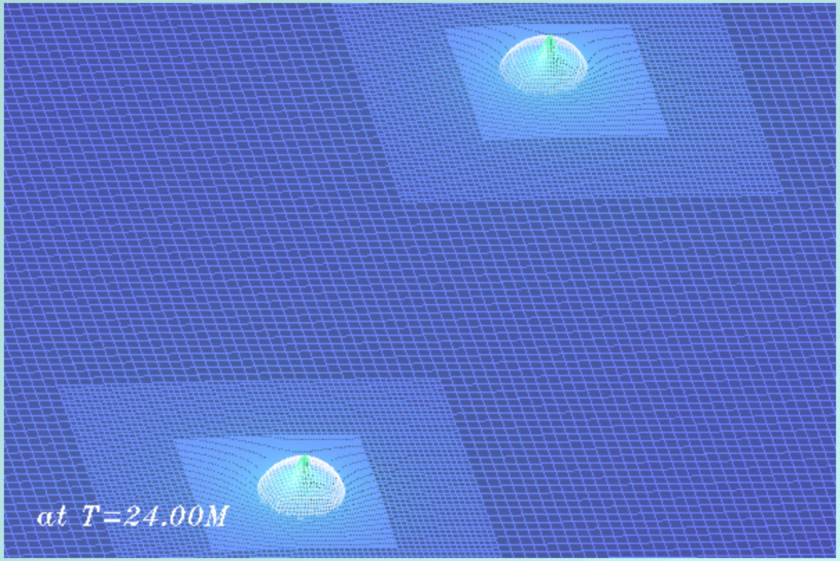
- Quasi-circular:
  - Effective potential
  - PN parameters
  - helical Killing Vektor

3.4. Mesh refinement

Mesh-refinement

- 3 Length scales: BH $\approx 1M$
  Wavelength $\approx 10M$
  Wave zone $\approx 100M$
- Mesh Refinement!
- Choptuik '93
  AMR, Critical phenomena
- Stretch coords.: Fish-eye
  Lazarus, AEI, UTB
- FMR, Moving boxes:
  Berger-Oliger

  BAM Brügmann'96
  Carpet Schnetter et.al.'03



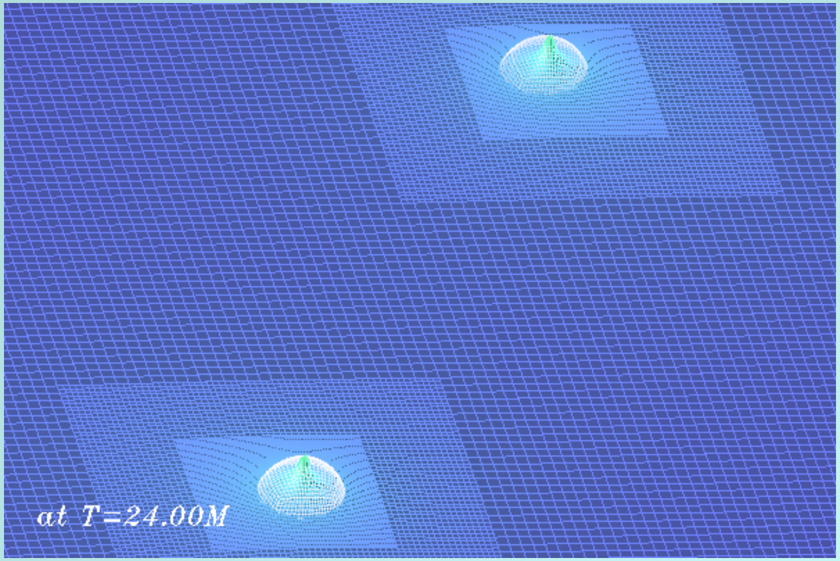
Mesh-refinement

- AMR: Control resolution via curvature

Paramesh:
MacNeice et.al.'00, Goddard

Modified Berger-Oliger:
Pretorius, Choptuik '05

SAMRAI:
OpenGR
UT Austin

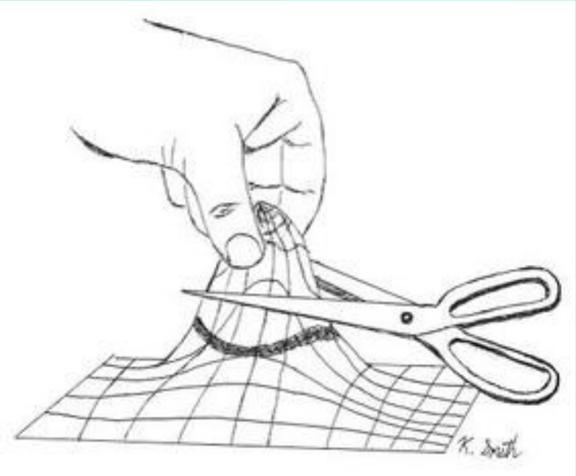


- Refinement boundaries:
reflections, stability
Tapered boundaries
Lehner, Liebling, Reula '05

3.5. Singularity treatment

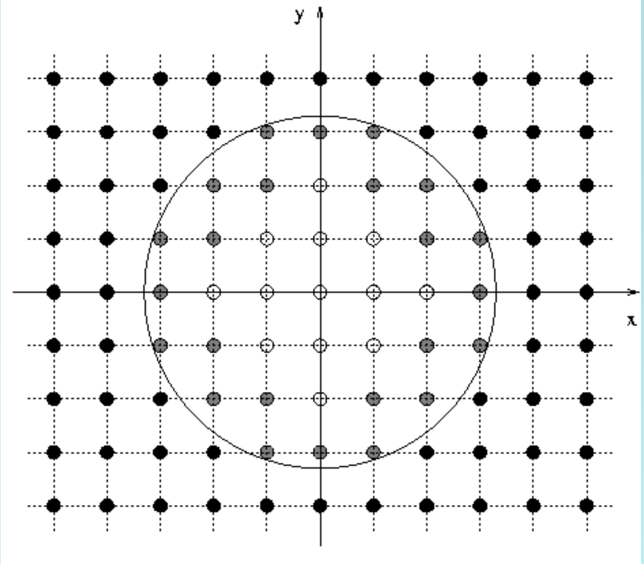
Singularities: Excision

- Cosmic censorship: horizon is causal boundary
- Unruh '84 cited in Thornburg '87
- Grand Challenge: Causale differencing
- "Simple Excision"
  Alcubierre, Brügmann '01
- Dynamic "moving" excision
  - Pitt-PSU-Texas
  - PSU-Maya
  - Pretorius
  - LEAN (U.S.'06)
- Combined with "Dual coordinate frame" Caltech-Cornell
- Mathematic properties: Wealth of literature



Singularities: Excision

- Cosmic censorship: horizon is causal boundary
- Unruh '84 cited in Thornburg '87



- Grand Challenge: Causale differencing
- "Simple Excision"
  Alcubierre, Brügmann '01
- Dynamic "moving" excision
  - Pitt-PSU-Texas
  - PSU-Maya
  - Pretorius
  - LEAN (U.S.'06)
- Combined with "Dual coordinate frame" Caltech-Cornell
- Mathematic properties: Wealth of literature

4. Extracting physics

Basic assumptions

- Extracting physics in NR is non-trivial !!
- Newtonian quantities are not always well-defined !!
- We assume that the ADM variables

Lapse	α
Shift	β^i
3-metric	γ_{ij}
Extrinsic curvature	K_{ij}

are given on each hypersurface Σ_t

- Even when using other formulations, the ADM variables are straightforward to calculate

Global quantities

- ADM mass: Global energy of the spacetime

$$M_{\mathrm{ADM}} = \frac{1}{16\pi} \lim_{r\to\infty} \int_{S_r} \sqrt{\gamma}\, \gamma^{ij} \gamma^{kl} \left(\gamma_{ik,j} - \gamma_{ij,k} \right) dS_l$$

- Total angular momentum of the spacetime

$$P_i = \frac{1}{8\pi} \lim_{r\to\infty} \int_{S_r} \sqrt{\gamma} \left(K^m{}_i - \delta^m{}_i K \right) dS_m$$

$$J_i = \frac{1}{8\pi} \varepsilon_{il}{}^{m} \lim_{r\to\infty} \int_{S_r} \sqrt{\gamma}\, x^{l} \left(K^n{}_m - \delta^n{}_m K \right) dS_n$$

By construction all of these are time-independent !!

Local quantities

- Often impossible to define !!
- Isolated horizon framework Ashtekar and coworkers
  - Calculate apparent horizon
  - irreducible mass, momenta associated with horizon

$$M_{\rm irr} = \sqrt{\frac{A_{\rm AH}}{16\pi}}$$

- Total BH mass Christodoulou

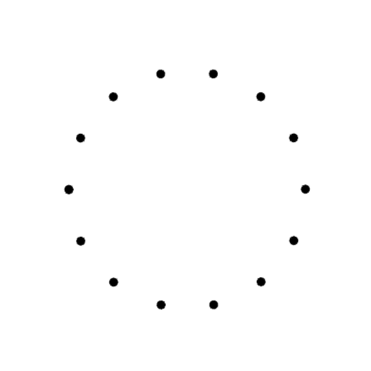
$$M^2 = M_{\rm irr}^2 + \frac{S^2}{4M_{\rm irr}^2} + P^2$$

- Binding energy of a binary: $E_{\rm b} = M_{\rm ADM} - M_1 - M_2$

Gravitational waves

- Most important result: Emitted gravitational waves (GWs)

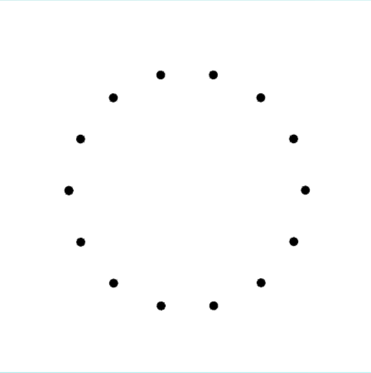
- Newman-Penrose scalar



$$\Psi_4 = C_{\alpha\beta\gamma\delta}\, n^{\alpha}\, \overline{m}^{\beta}\, n^{\gamma}\, \overline{m}^{\delta}$$

Complex $\Rightarrow$ 2 free functions

- GWs allow us to measure
  - Radiated energy E_{rad}
  - Radiated momenta $P_{\text{rad}}, J_{\text{rad}}$
  - Angular dependence of radiation
  - Predicted strain $h_{+}, h_{\times}$



Angular dependence

- Waves are normally extracted at fixed radius r_{ex}

$$\Rightarrow \Psi_4 = \Psi_4(t, \theta, \phi)$$

θ, ϕ are viewed from the source frame !!

- Decompose angular dependence

$$\Psi_4 = \sum_{\ell m} \psi_{\ell m}(t) Y_{\ell m}^{-2}(\theta, \phi)$$

  - Modes $\Psi_4(t) = A_{\ell m}(t)\, e^{i\phi(t)}$
  - Spin-weighted spherical harmonics

5. A brief history

A brief history of BH simulations

- Pioneers: Hahn, Lindquist '60s, Eppley, Smarr et.al. '70s
- Grand Challenge: First 3D Code Anninos et.al. '90s
- Further attempts: Bona & Massó, Pitt-PSU-Texas, …

  AEI-Potsdam Alcubierre et al.

  PSU: first orbit Brügmann et al. '04

  Codes unstable

- Breakthrough: Pretorius '05 "GHG"

  UTB, Goddard '05 "Moving Punctures"
- Currently: ≈ 10 codes, a.o. BAM Brügmann

  LEAN Sperhake '07

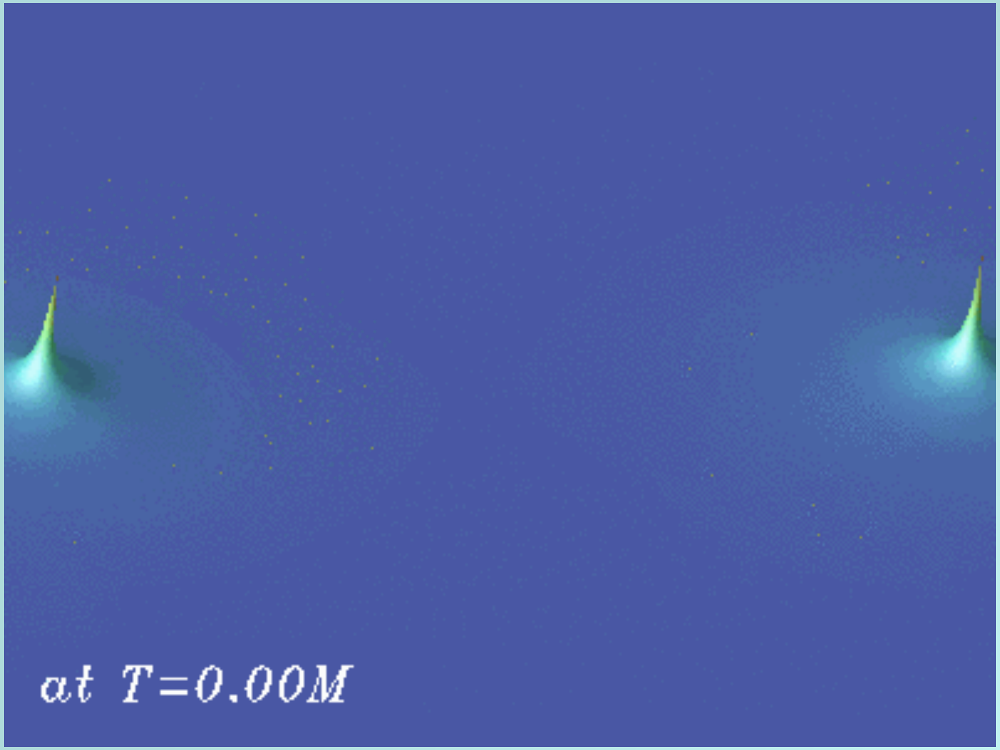
6. Animations

Animations

- Lean Code representative for other codes
- Extrinsic curvature

  $\mathrm{tr}K$

- Apparent horizon



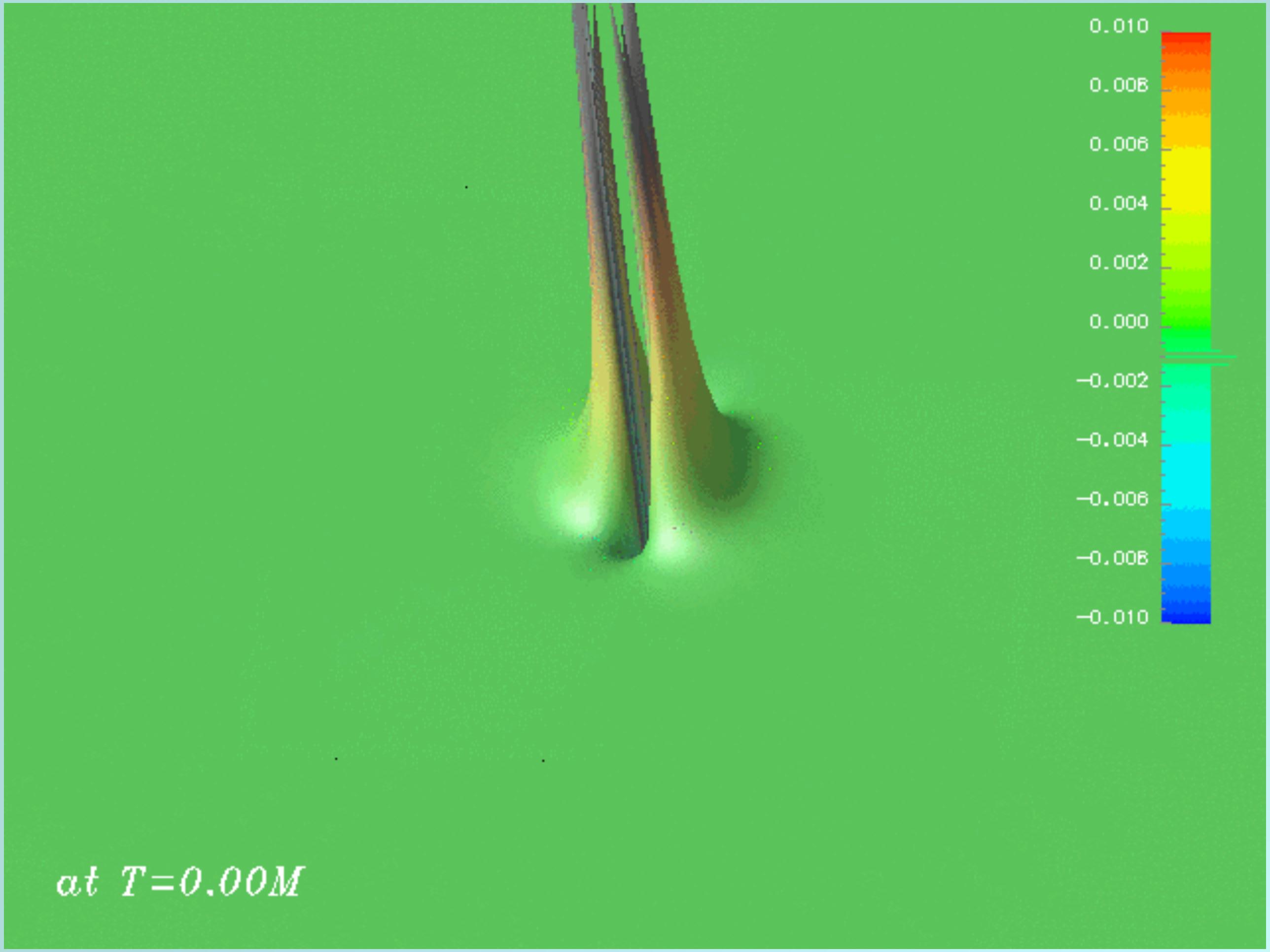
Animations

- $\mathrm{Re}[\Psi_4]$
- Angular dependence

$\Rightarrow$ Spherical harmonics

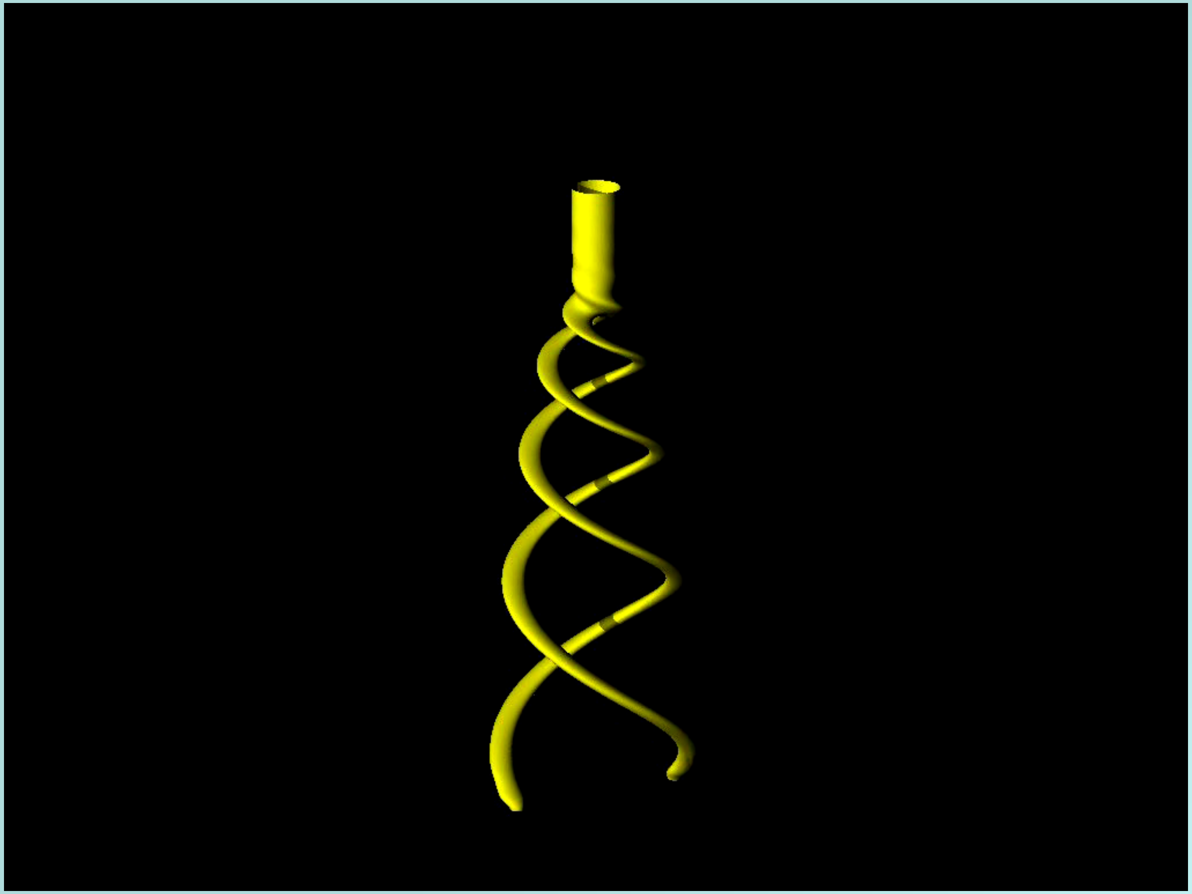
$\ell = 2, \; m = 2$

dominant



Animations

- Event horizon of binary inspiral and merger BAM



Thanks to Marcus Thierfelder

7. Results on black-hole binaries

Free parameters of BH binaries

- Total mass M_{ADM}
  - ➢ Relevant for detection: Frequencies depend on M_{ADM}
  - ➢ Not relevant for source modeling: trivial rescaling
- Mass ratio $q = \dfrac{M_1}{M_2}, \qquad \eta = \dfrac{M_1 M_2}{(M_1 + M_2)^2}$
- Spin $\vec{S}_1, \ \vec{S}_2$
- Initial parameters
  - Binding energy E_{b} Separation
  - Orbital angular momentum L Eccentricity
  - Alternatively: frequency, eccentricity

7.1. Non-spinning equal-mass holes

The BBH breakthrough

- Simplest configuration
- GWs circularize orbit
  $\Longrightarrow$ quasi-circular initial data

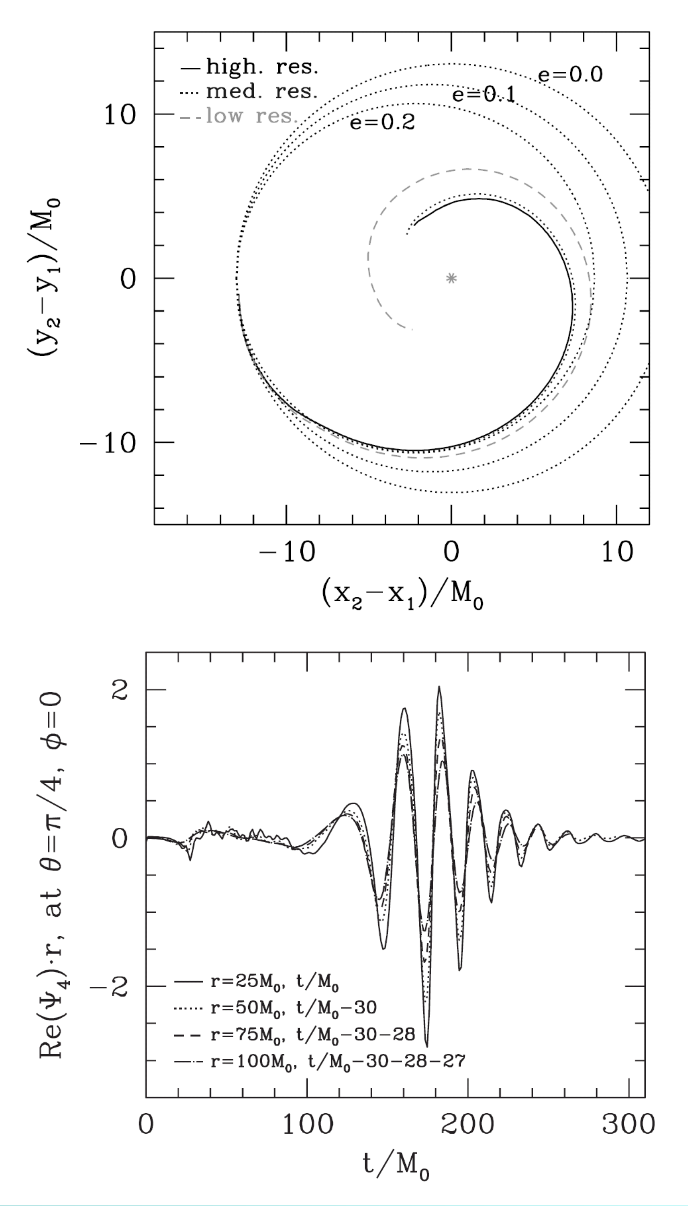
Pretorius PRL '05

- BBH breakthrough
- Initial data: scalar field
- Radiated energy

$R_{ex}\,[M] =$	25	50	75	100
$E\,[\%M] =$	4.7	3.2	2.7	2.3

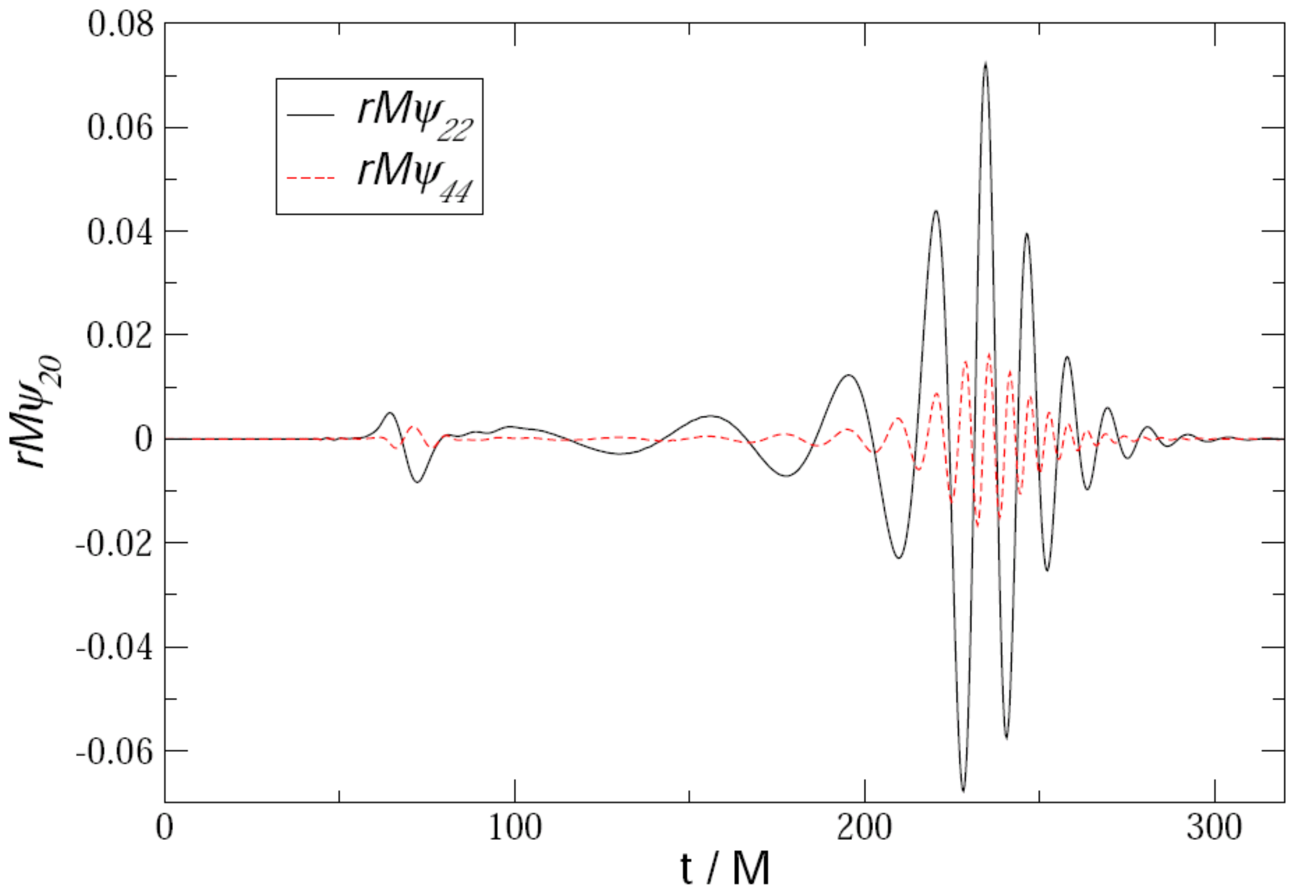
- Eccentricity

  $e = 0 \ldots 0.2$



Non-spinning equal-mass binaries

- Total radiated energy: $3.6\%\, M_{\mathrm{ADM}}$
- $\ell = 2,\ m = 2$ mode dominant: > 98%



The merger part of the inspiral

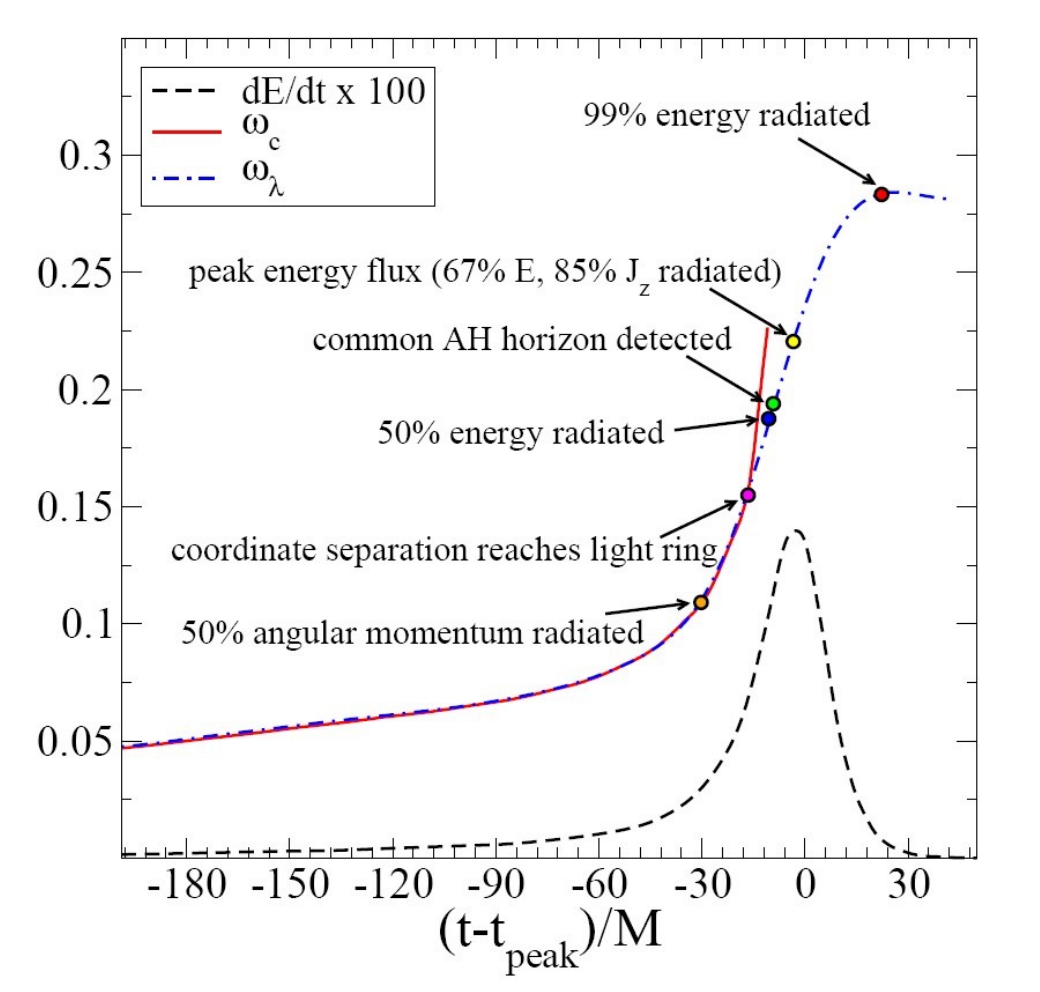
Buonanno, Cook, Pretorius ’06 (BCP)

- merger lasts short: 0.5 – 0.75 cycles
- Eccentricity small

  ≈ 0.01

  non-vanishing

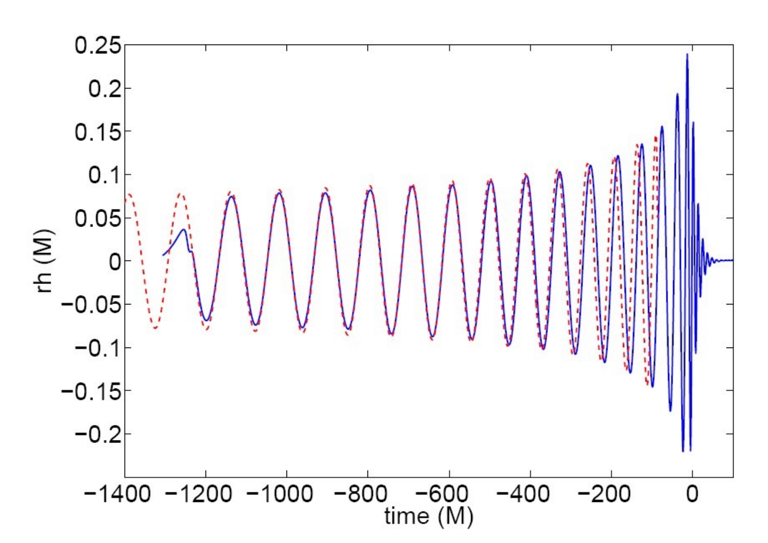
  Initial radial velocity



Comparison with Post-Newtonian

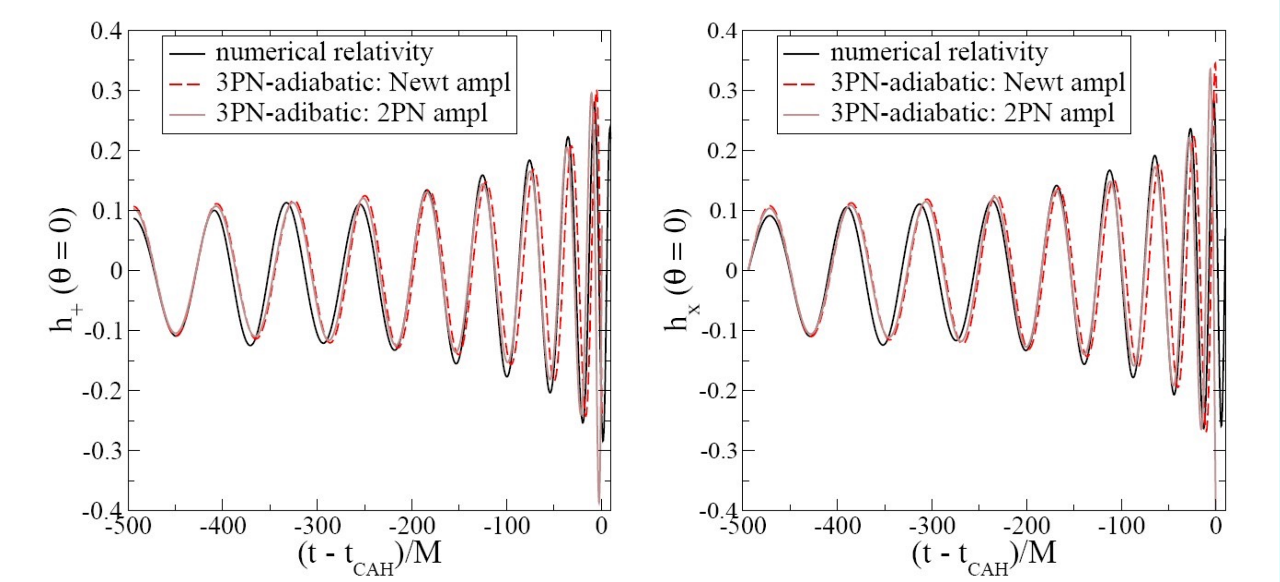
Goddard '07

- 14 cycles, 3.5 PN phasing
- Match waveforms: $\phi, \ \phi^{\mathrm{PN}}$
- Accumulated phase error $1\,\mathrm{rad}$

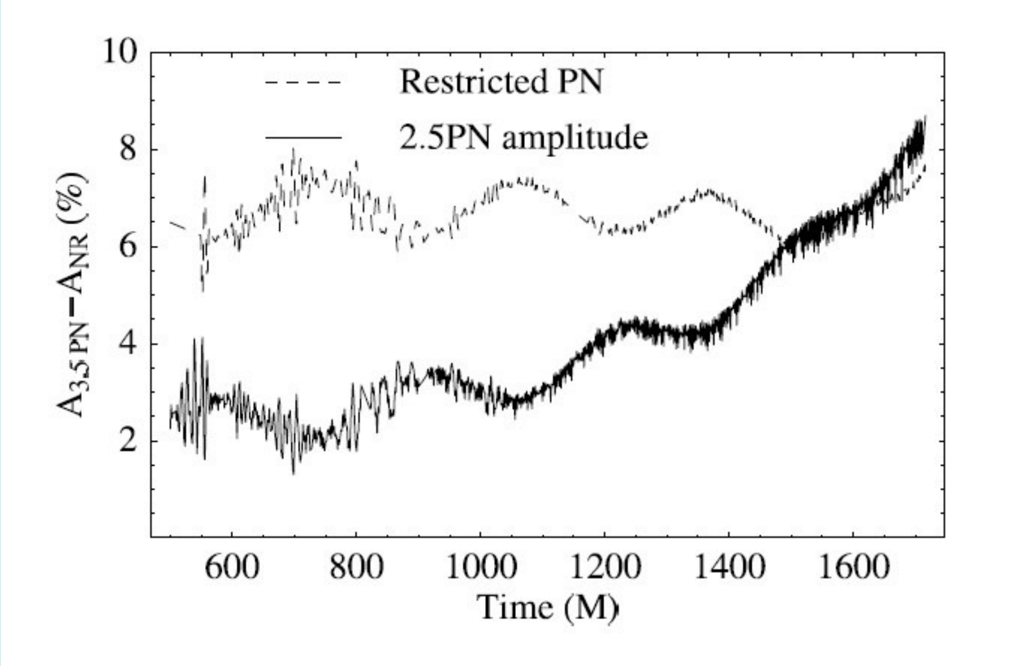


Buonanno, Cook, Pretorius '06 (BCP)

- 3.5 PN phasing
  2 PN amplitude



Comparison with Post-Newtonian



Jena '07

- 18 cycles
- phase error $< 1\,\mathrm{rad}$

  6th order differencing !!
- Amplitude: % range

Cornell/Caltech & Buonanno

- 30 cycles
- phase error $\approx 0.02\,\mathrm{rad}$
- Effective one body (EOB)

RIT

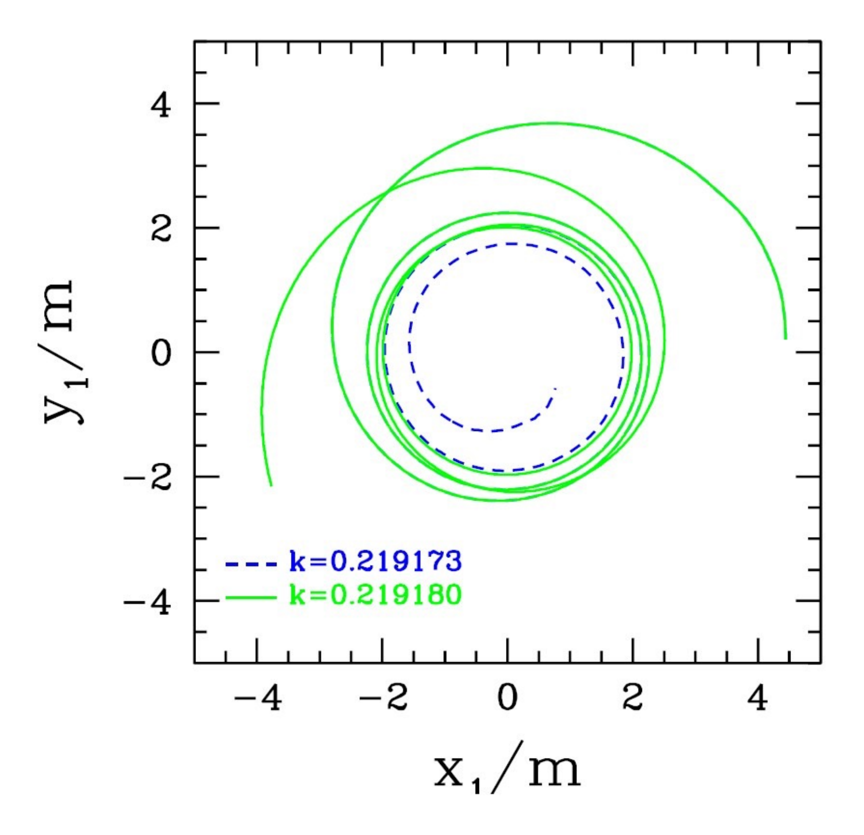
- First comparison with spin; not conclusive yet

Zoom whirl orbits

Pretorius & Khurana '07

- 1-parameter family of initial data: linear momentum
- Fine-tune parameter

  $\Rightarrow$ "Threshold of immediate merger"



- Analogue in gedodesics !
- Reminiscent of

  "Critical phenomena"

- Similar observations by

  PSU

Max. spin $j_{\text{fin}} = 0.78$ for $L \approx M^2$

7.2. Unequal masses

Unequal masses

- Still zero spins
- Astrophysically much more likely !!
- Symmetry breaking
  - Anisotropic emission of GWs
  - Certain modes are no longer suppressed
- Mass ratios
  - Stellar sized BH into supermassive BH $\approx 10^6$
  - Intermediate mass BHs $\approx 10^3$
  - Galaxy mergers $\approx 1 \ldots 10^3$
  - Currently possible numerically: $\approx 1 \ldots 10$

Gravitational recoil

- Anisotropic emission of GWs radiates momentum
  $\Rightarrow$ recoil of remaining system

- Leading order: Overlap of Mass-quadrupole
  with octopole/flux-quadrupole

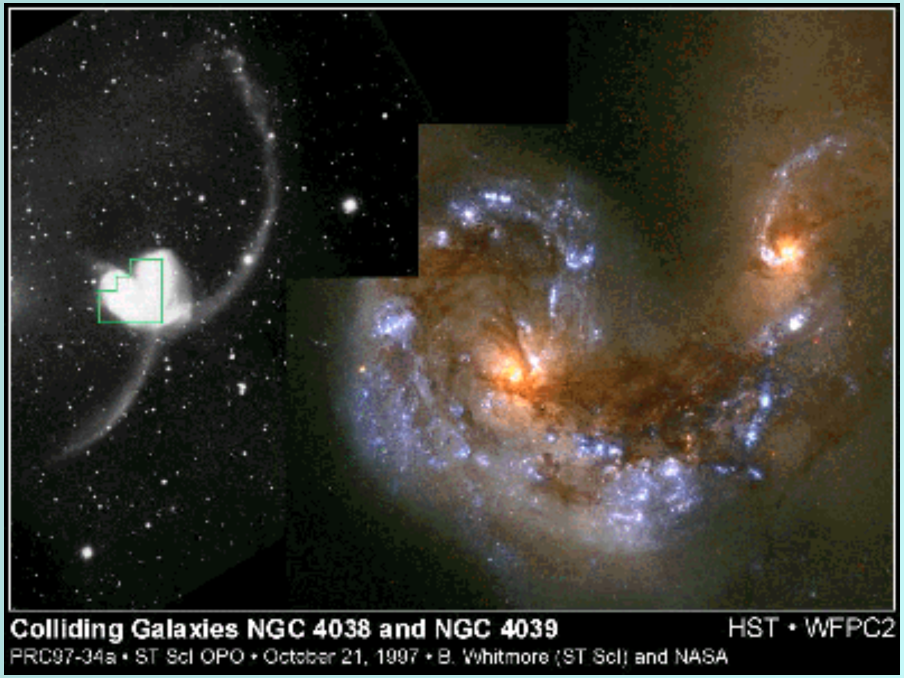
Bonnor & Rotenburg '61, Peres '62, Bekenstein '73

- Merger of galaxies

$\Rightarrow$ Merger of BHs

$\Rightarrow$ Recoil

$\Rightarrow$ BH kicked out?



Gravitational recoil

- Escape velocities

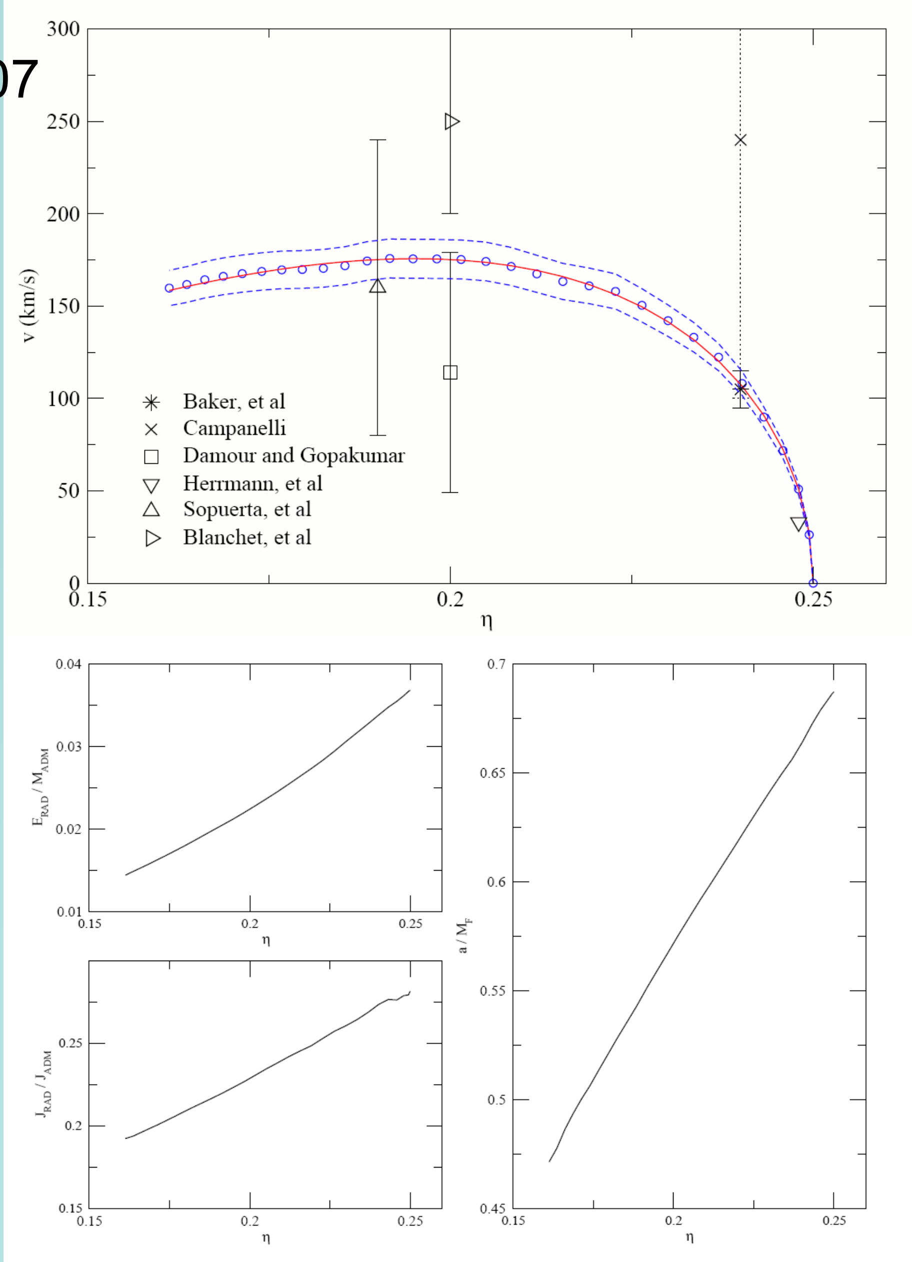
Globular clusters	$30\,\mathrm{km/s}$
dSph	$20-100\,\mathrm{km/s}$
dE	$100-300\,\mathrm{km/s}$
Giant galaxies	$\approx 1000\,\mathrm{km/s}$

Merrit et al '04

- Ejection or displacement of BHs has repercussions on:
  - Structure formation in the universe
  - BH populations
    IMBHs via ejection?
  - Growth history of Massive Black Holes
  - Structure of galaxies

Kicks of non-spinning black holes

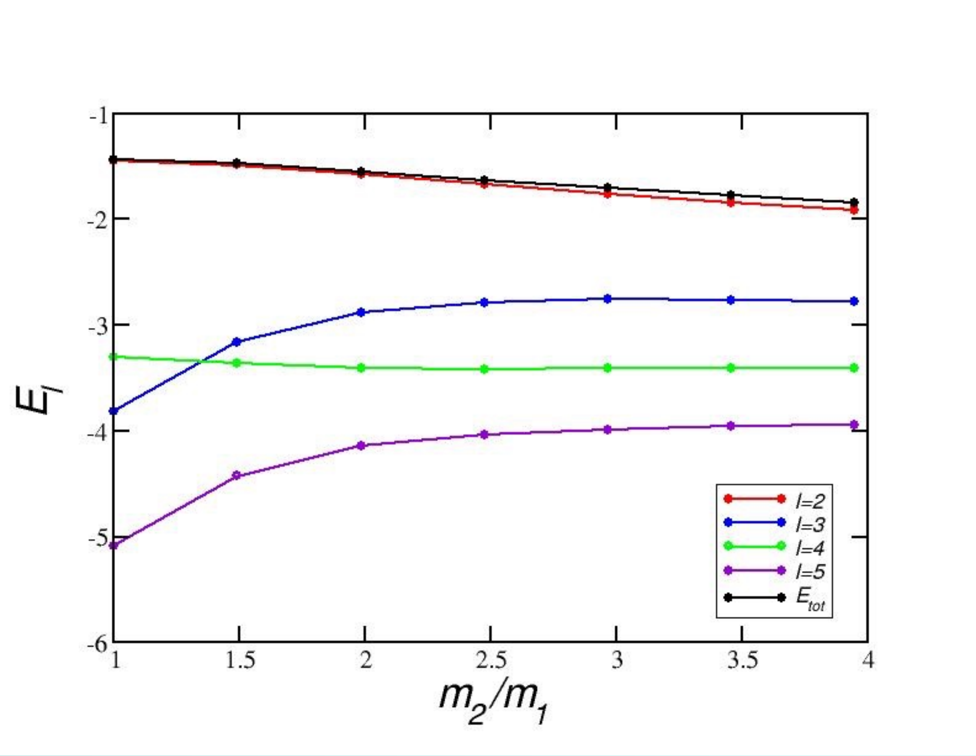
- Simulations PSU '07, Goddard '07
- Parameter study Jena '07
- Target: Maximal Kick
- Mass ratio: $M_1 / M_2 = 1 \ldots 4$
- 150,000 CPU hours
- Maximal kick 178 km/s for $M_1 / M_2 \approx 3$
- Convergence 2nd order
- $E_{\mathrm{rad}} \approx 3\%,\ J_{\mathrm{rad}} \approx 25\%$
- Spin $0.45 \ldots 0.7$



Features of unequal-mass mergers

Berti et al '07

- Distribution of radiated energy
  - More energy in higher modes
  - Odd ℓ modes suppressed for equal masses
- Important for GW-DA

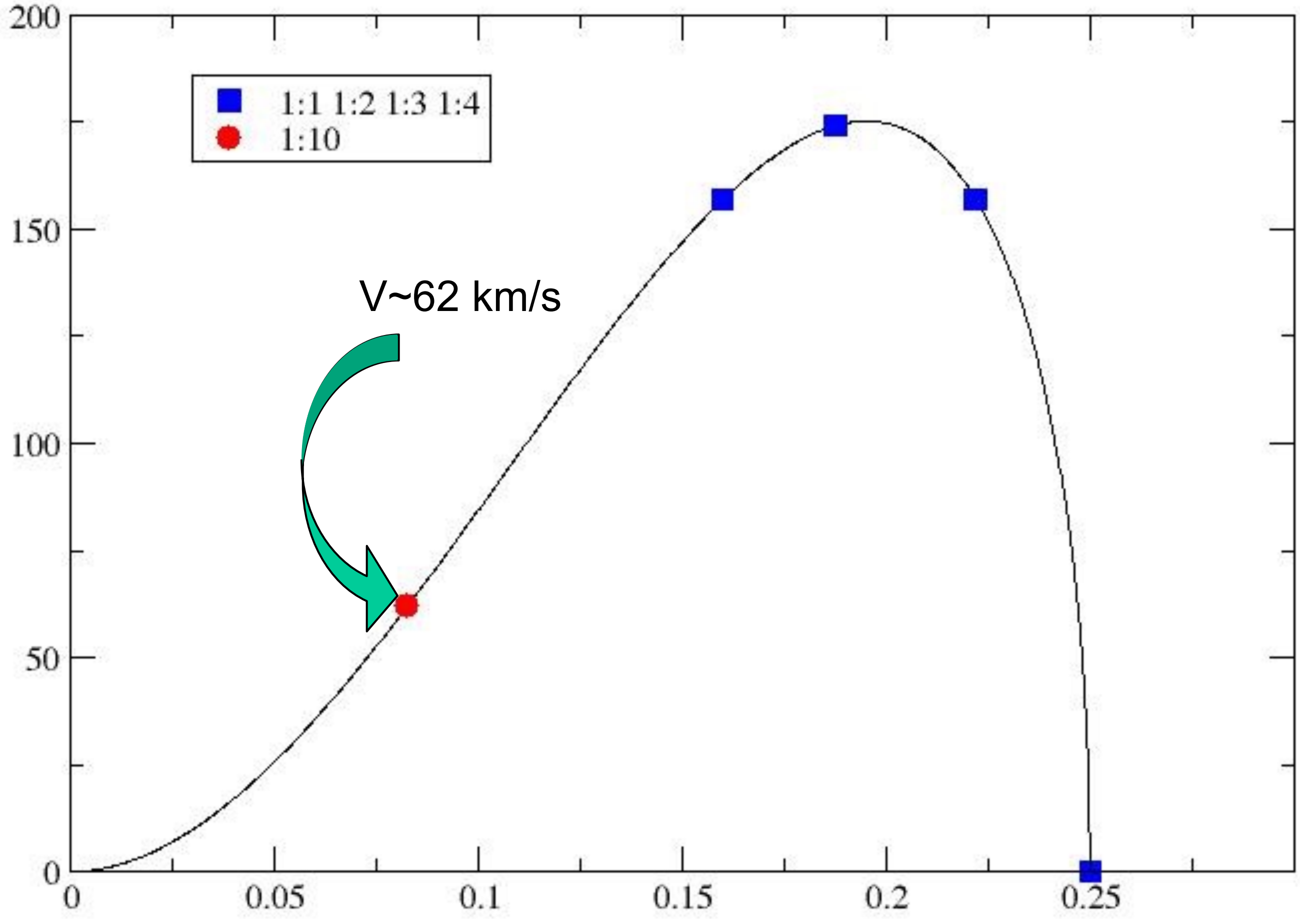


Mass ratio 10:1

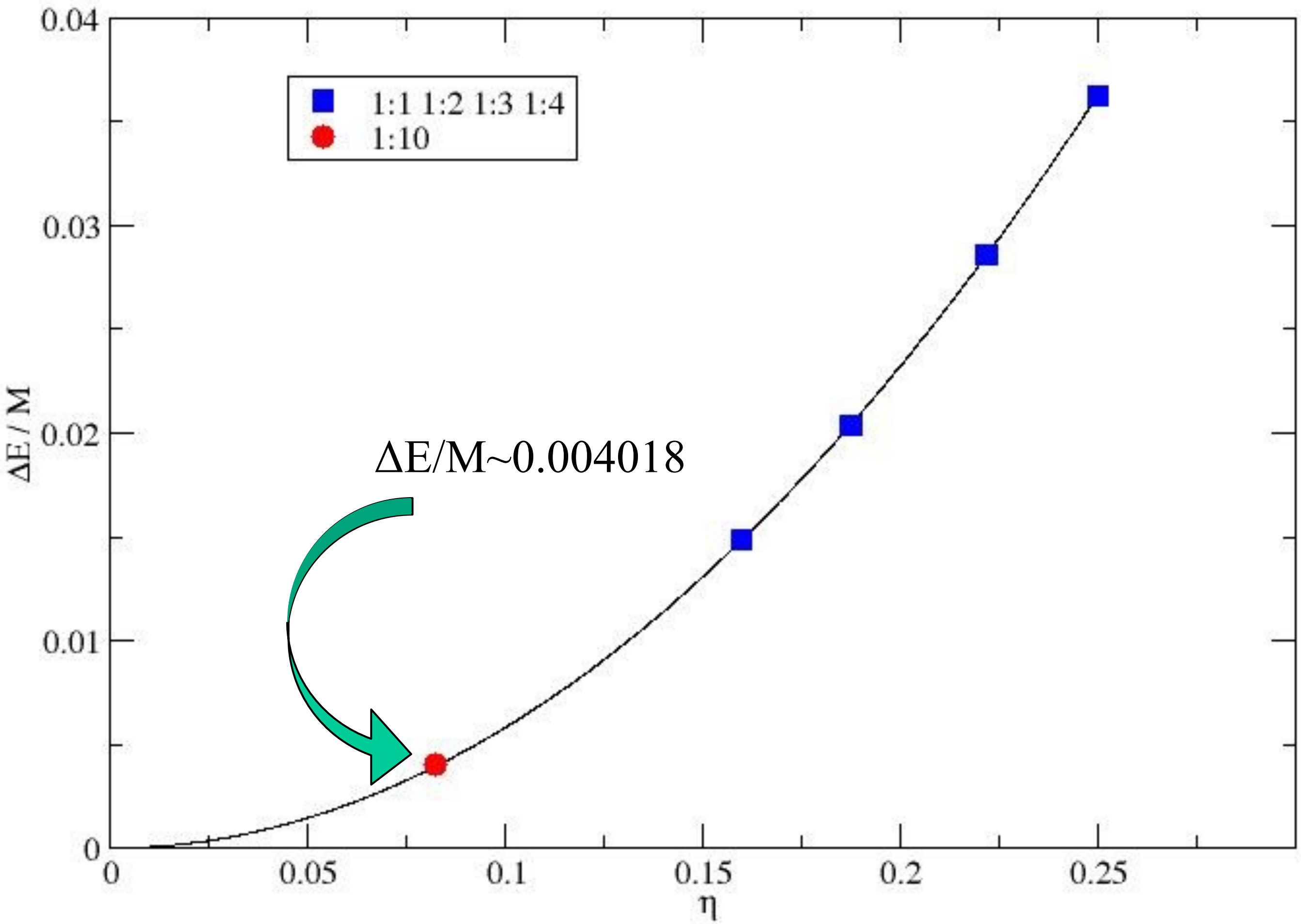
- In preparation: González, U.S., Brügmann
- Mass ratio $q = 10$; Bam
- 4^{th} order convergence

- Astrophysically likely configuration: Sesana et al. '07
- Test fitting formulas for spin and kick!

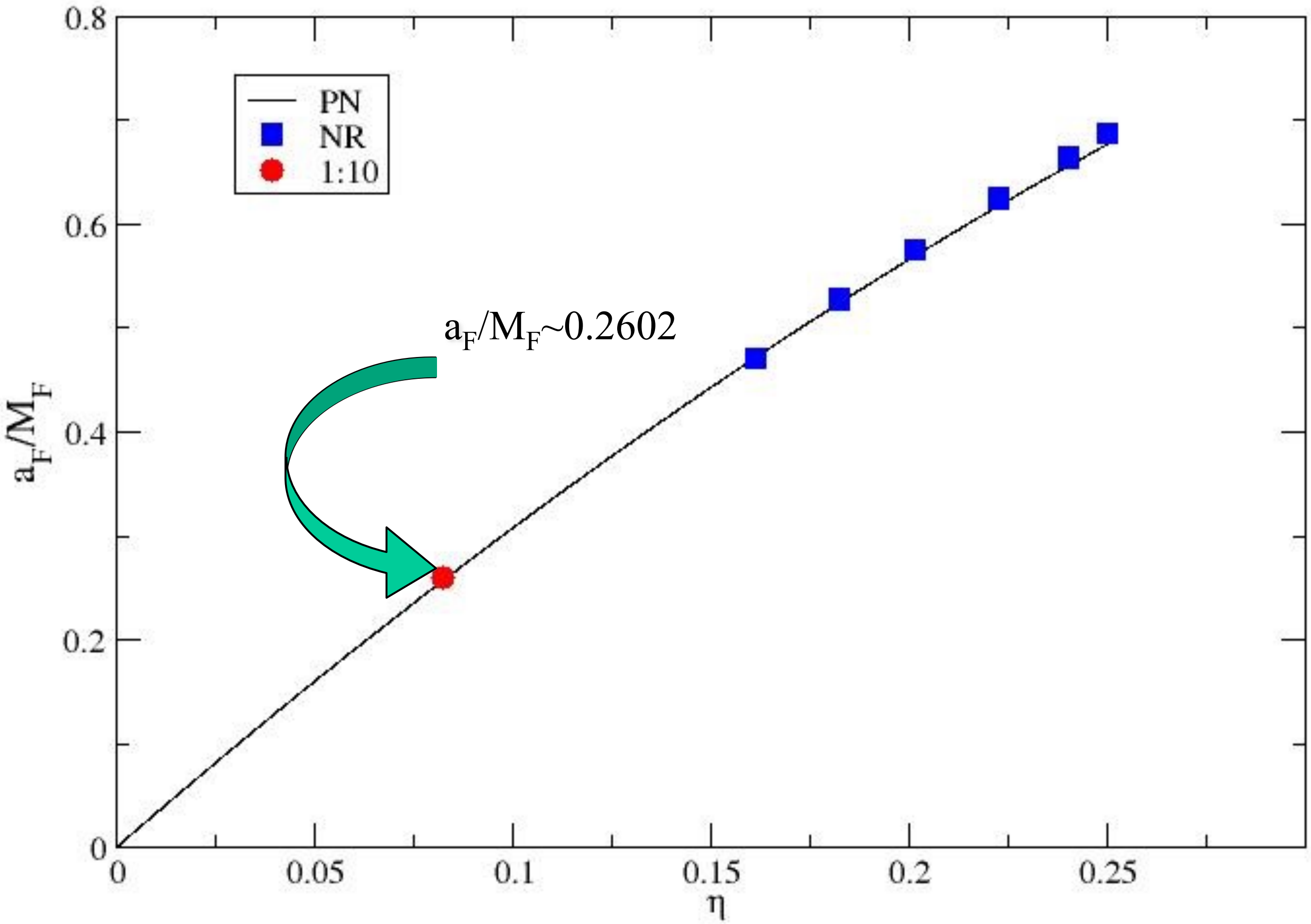
Kick: $v = 1.2 \times 10^4 \eta^2 \sqrt{1-4\eta}\,(1-0.93\eta)$ (Fitchett '83
Gonzalez et al. '07)



Radiated energy: $\frac{\Delta E}{M} = 0.5802\eta^2$ (Berti et al. '07)



Final spin: $a_F/M_F = 3.32\eta - 2.45\eta^2$ (Damour and Nagar 2007)

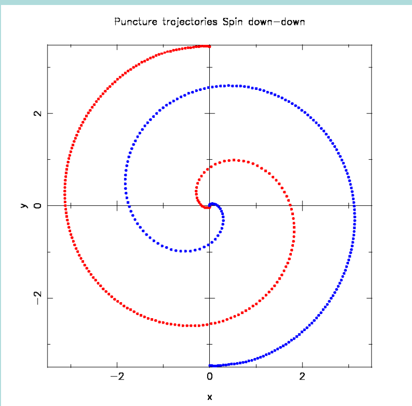


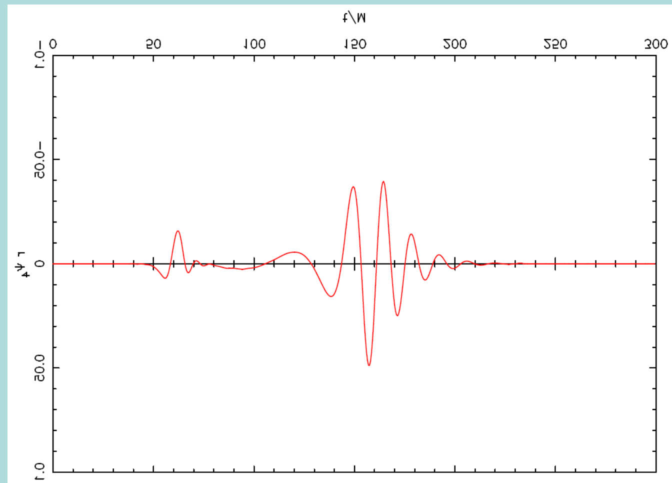
7.3. Spinning black holes

Spinning holes: The orbital hang-up

- ↑↑↑ Spins parallel to $\vec{L} \Rightarrow$ more orbits, E_{rad}, J_{rad} larger

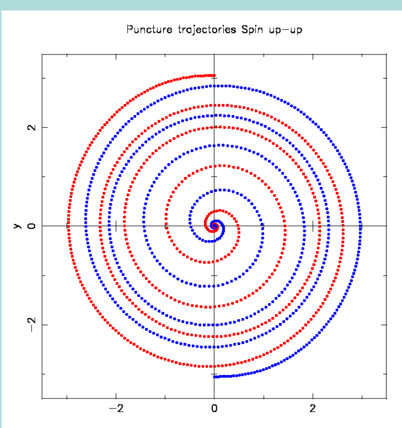
  ↓↑↓ Spins anti-par. to $\vec{L} \Rightarrow$ fewer orbits E_{rad}, J_{rad} smaller

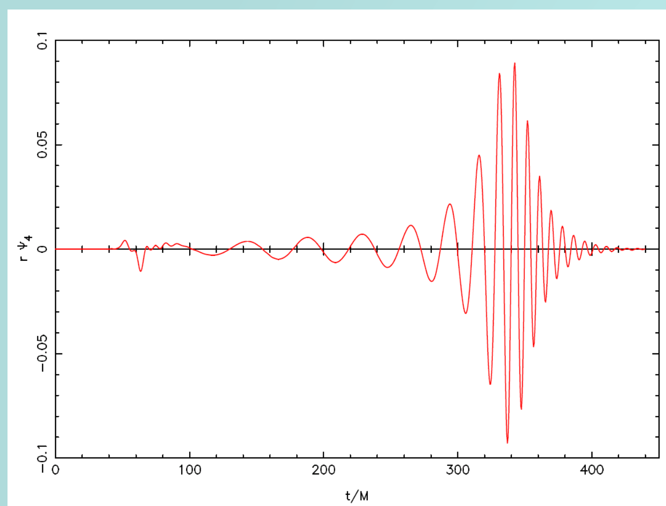




UTB/RIT '07

- no extremal Kerr BHs





Spin precession and flip

- X-shaped radio sources

  Merritt & Ekers '07

- Jet along spin axis

- Spin re-alignment

  $\Rightarrow$ new + old jet

- Spin precession $98°$

  Spin flip $71°$

  UTB, Rochester '06

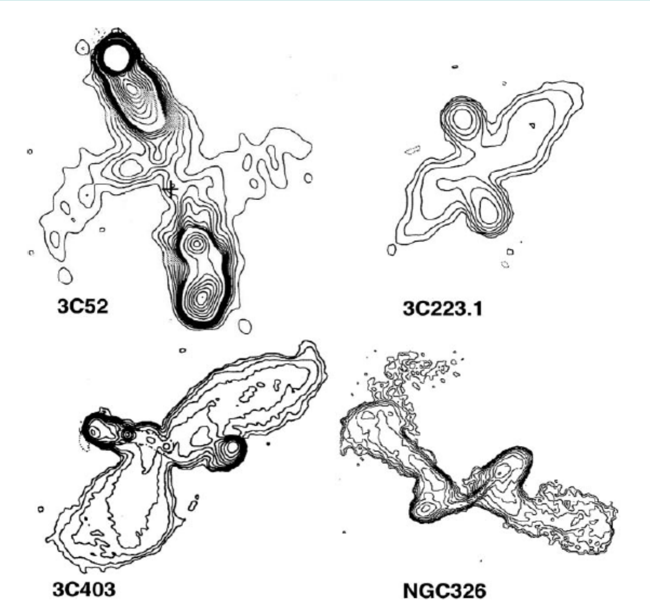
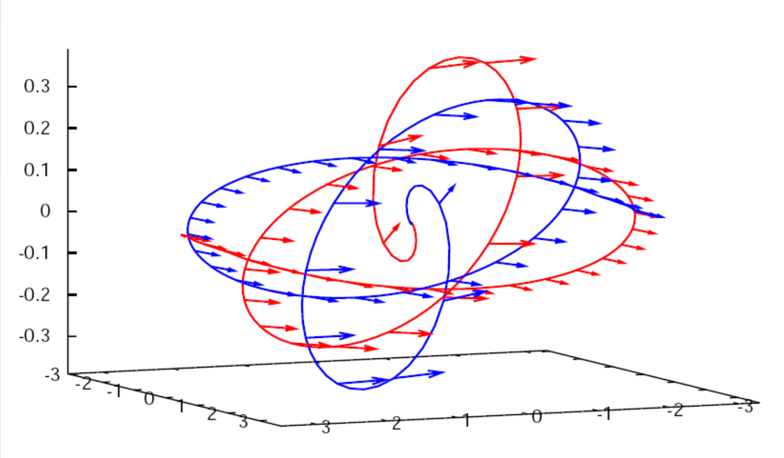


Fig. 2. A composite of four clear examples of the X-shape morphology. VLA radio observations of 3C52 (28), 3C223.1 (29), 3C403 (29), and NGC 326 (12).

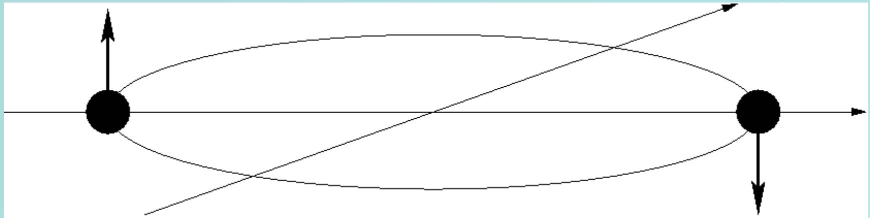


Recoil of spinning holes

- Kidder '95: PN study with Spins

$$\frac{d\mathbf{P}}{dt} = \dot{\mathbf{P}}_{\mathrm{N}} + \dot{\mathbf{P}}_{\mathrm{SO}},$$ = "unequal mass" + "spin(-orbit)"

- Penn State '07: SO-term larger

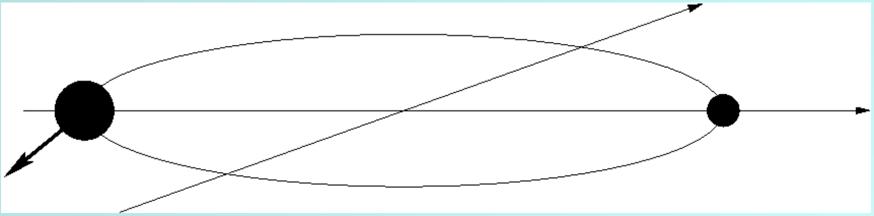


$$\frac{a}{m} = 0.2,\ldots,0.8$$

extrapolated: $v = 475\ \mathrm{km/s}$

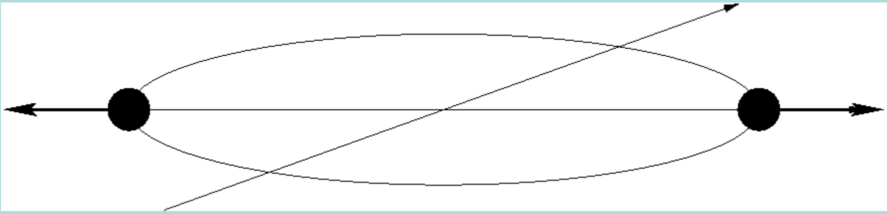
- AEI '07 : One spinning hole, extrapolated: $v = 440\ \mathrm{km/s}$

- UTB-Rochester: $v = 454\ \mathrm{km/s}$



Super Kicks

- Side result RIT '07, Kidder '95: maximal kick predicted for



$$v \approx 1300\,\mathrm{km/s}$$

- Test hypothesis

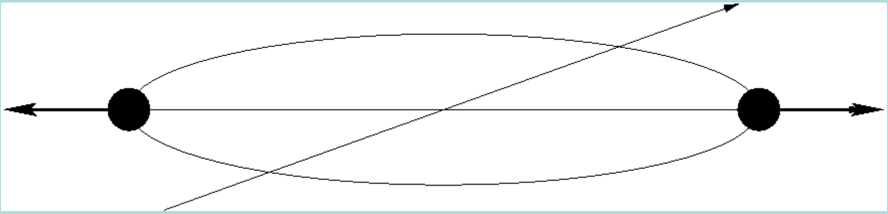
  González, Hannam, US, Brügmann & Husa '07

  Use two codes: Lean, BAM

- Generates kick $v = 2500\,\mathrm{km/s}$ for spin $a \approx 0.75$

Super Kicks

- Side result RIT '07, Kidder '95: maximal kick predicted for



$v \approx 1300\,\mathrm{km/s}$

- Test hypothesis

  González, Hannam, US, Brügmann & Husa '07

  Use two codes: Lean, BAM

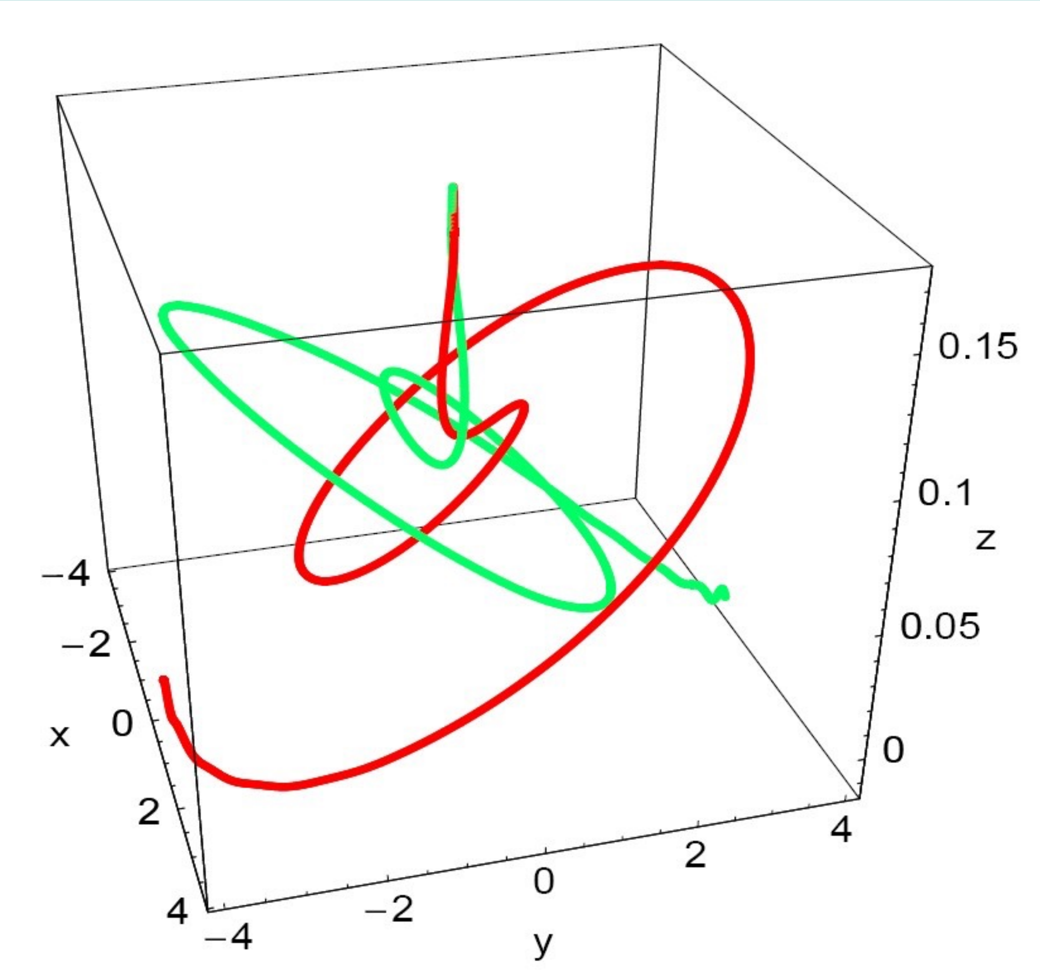
- Generates kick $v = 2500\,\mathrm{km/s}$ for spin $a \approx 0.75$

- Extrapolated to maximal spin $v = 4000\,\mathrm{km/s}$
  RIT '07

- Highly eccentric orbits $v = 10000\,\mathrm{km/s}$
  PSU '08

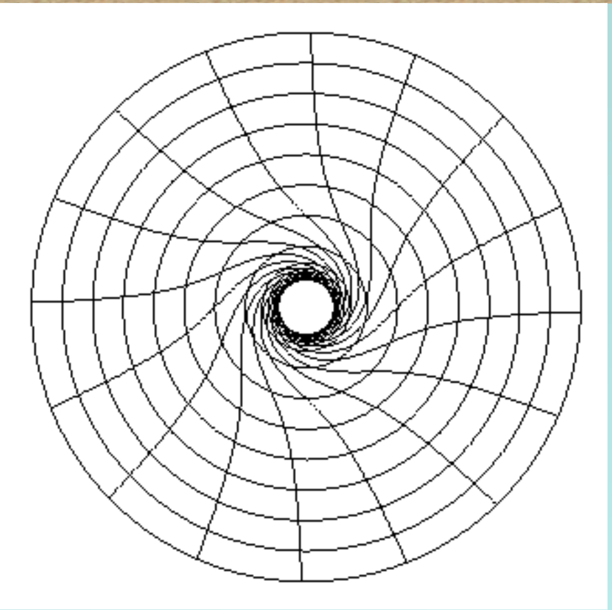
What's happening physically?

- Black holes "move up and down"



A closer look at super kicks

- Physical explanation:
  "Frame dragging"
- Recall: rotating BH drags objects along with its rotation



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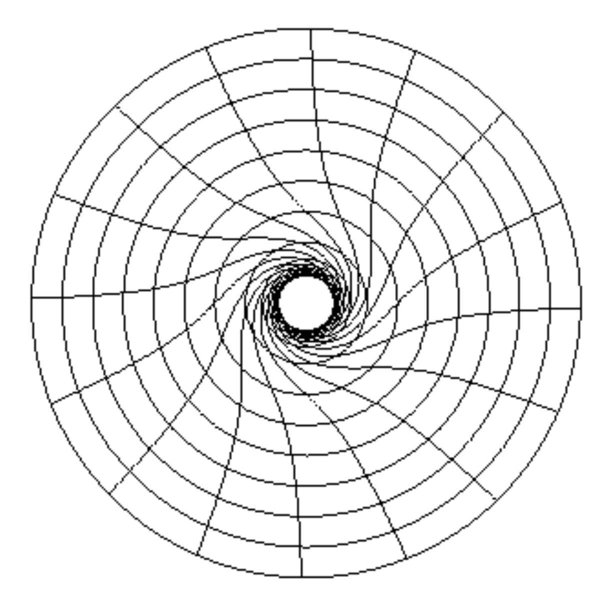
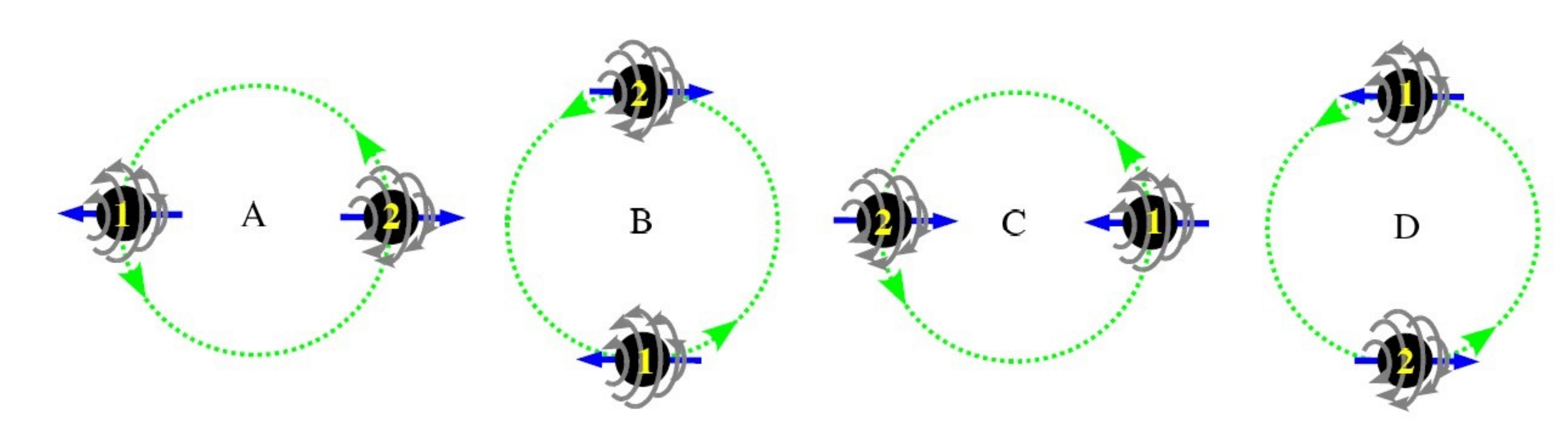




Thanks to F. Pretorius

How realistic are superkicks?

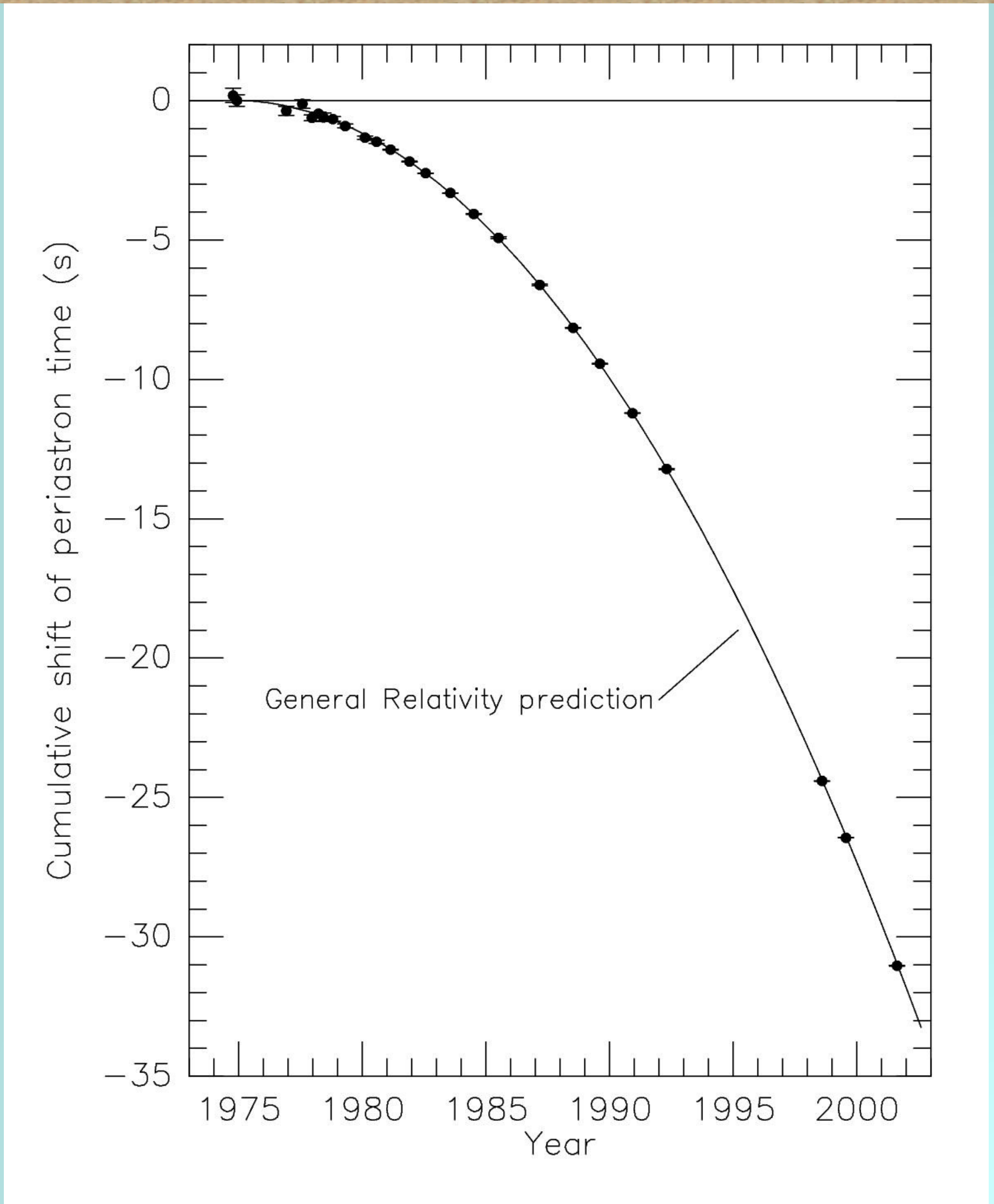
- Observations $\Rightarrow$ BHs are not generically ejected!
- Are superkicks real?
- Gas accretion may align spins with orbit Bogdanovic et al.
- Kick distribution function: $v_{\mathrm{kick}} = v_{\mathrm{kick}}(\vec{S}_1, \vec{S}_2, M_1/M_2)$
- Analytic models and fits: Boyle, Kesden & Nissanke, AEI, RIT, Tichy & Marronetti,…
- Use numerical results to determine free parameters
- 7-dim. Parameter space: Messy! Not yet conclusive…
- EOB study $\Rightarrow$ only 12% of all mergers have $v > 500\,\mathrm{km/s}$ Schnittman & Buonanno '08

7.4. Numerical relativity and data analysis

The Hulse-Taylor pulsar

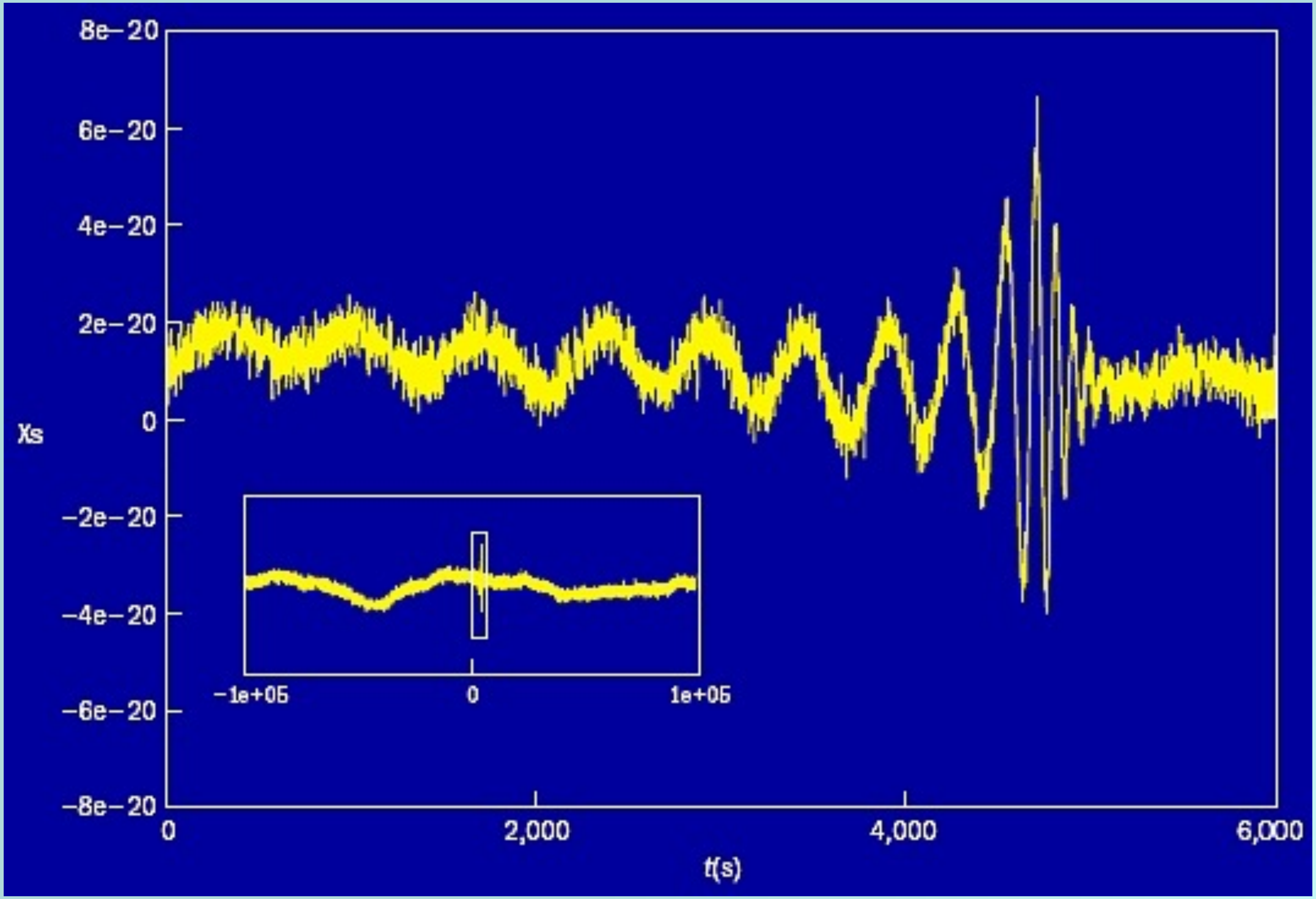
Hulse, Taylor '93

- Binary pulsar 1913+16
- GW emission
- Inspiral
- Change in period
- Excellent agreement with relativistic prediction



The data stream: Strong LISA source

- SMBH binary



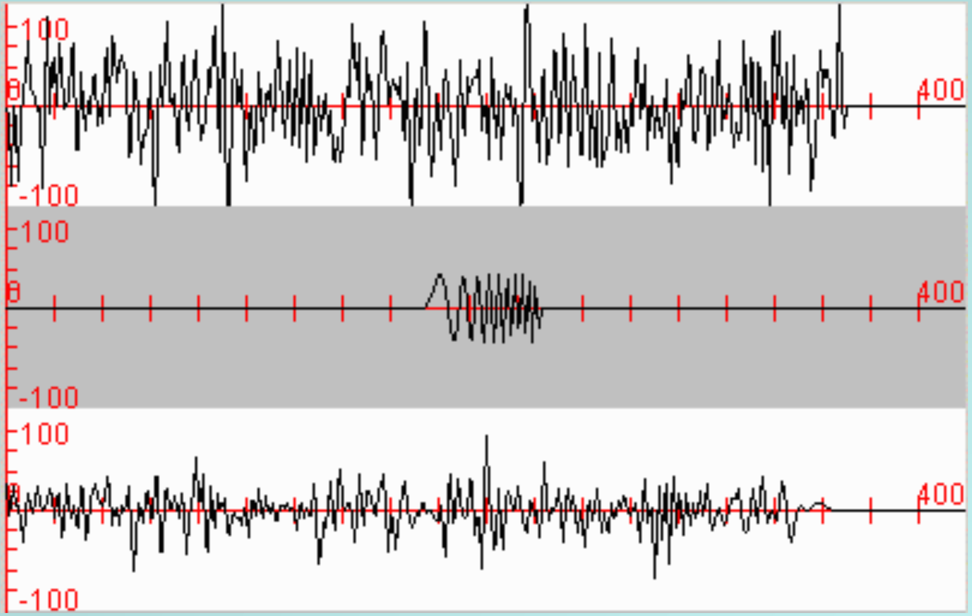
The data stream: Matched filtering

- Matched filtering (not real data)

Noise + Signal

Theoretically
Predicted signal

Overlap

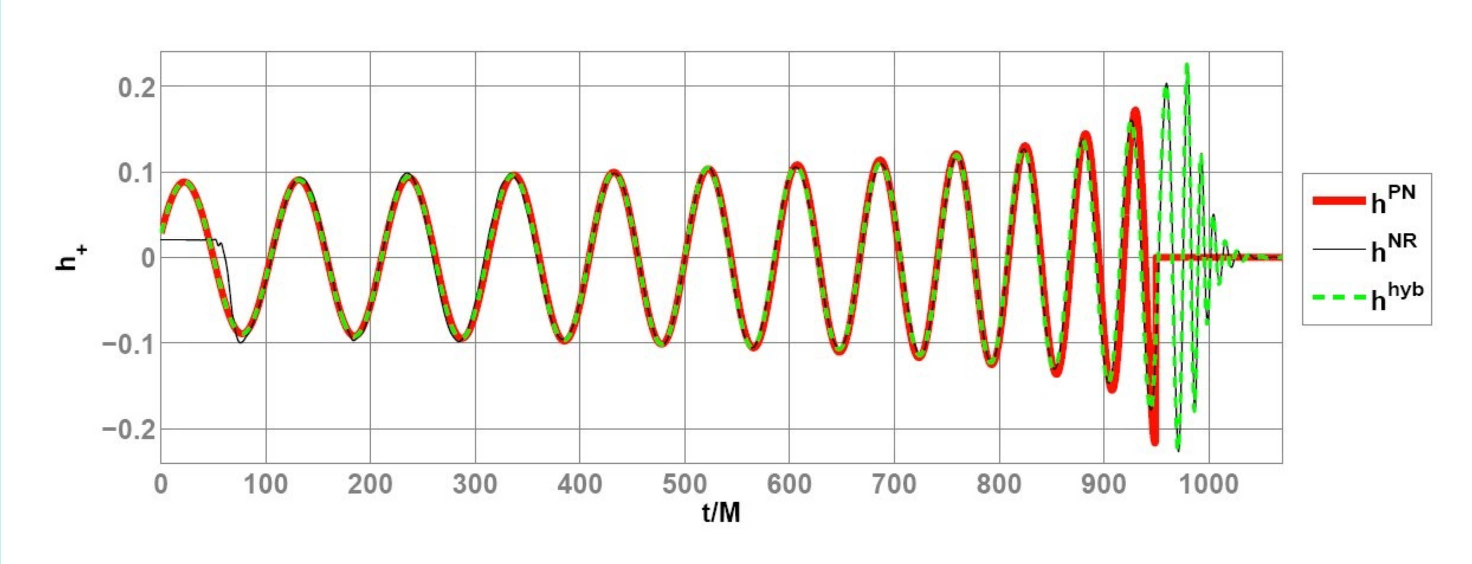


- Filter with one waveform per parameter combination
- Problem: 7-dim parameter space
- We need template banks!

Numerical relativity meets data analysis

Ajith et al. '07

- PN, NR $\Rightarrow$ hybrid waveforms



- Approximate hybrid WFs with phenomenological WFs
- Fitting factors: 0.99
- Create look-up tables to map between phenomenological and physical parameters

Numerical relativity meets data analysis

PSU ‘07

- Investigate waveforms from spinning binaries
- Detection of spinning holes likely to require inclusion of higher order multipoles

Cardiff ‘07

- Higher order multipoles important for parameter estimates

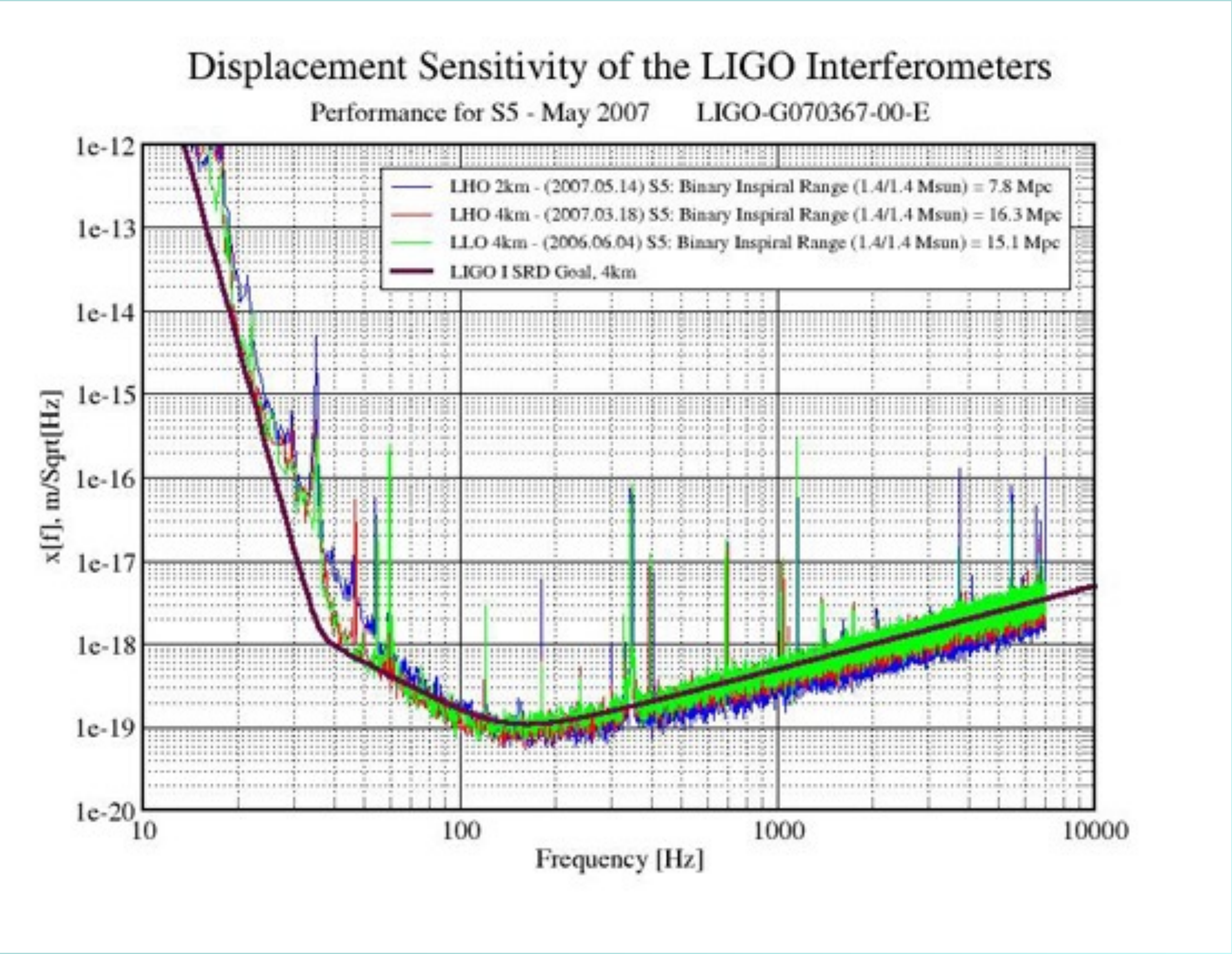
Pan et al. ‘07

- Equal-mass, non-spinning binaries
- Plot combined waveforms for different masses

Ninja

- Large scale effort to use NR in DA

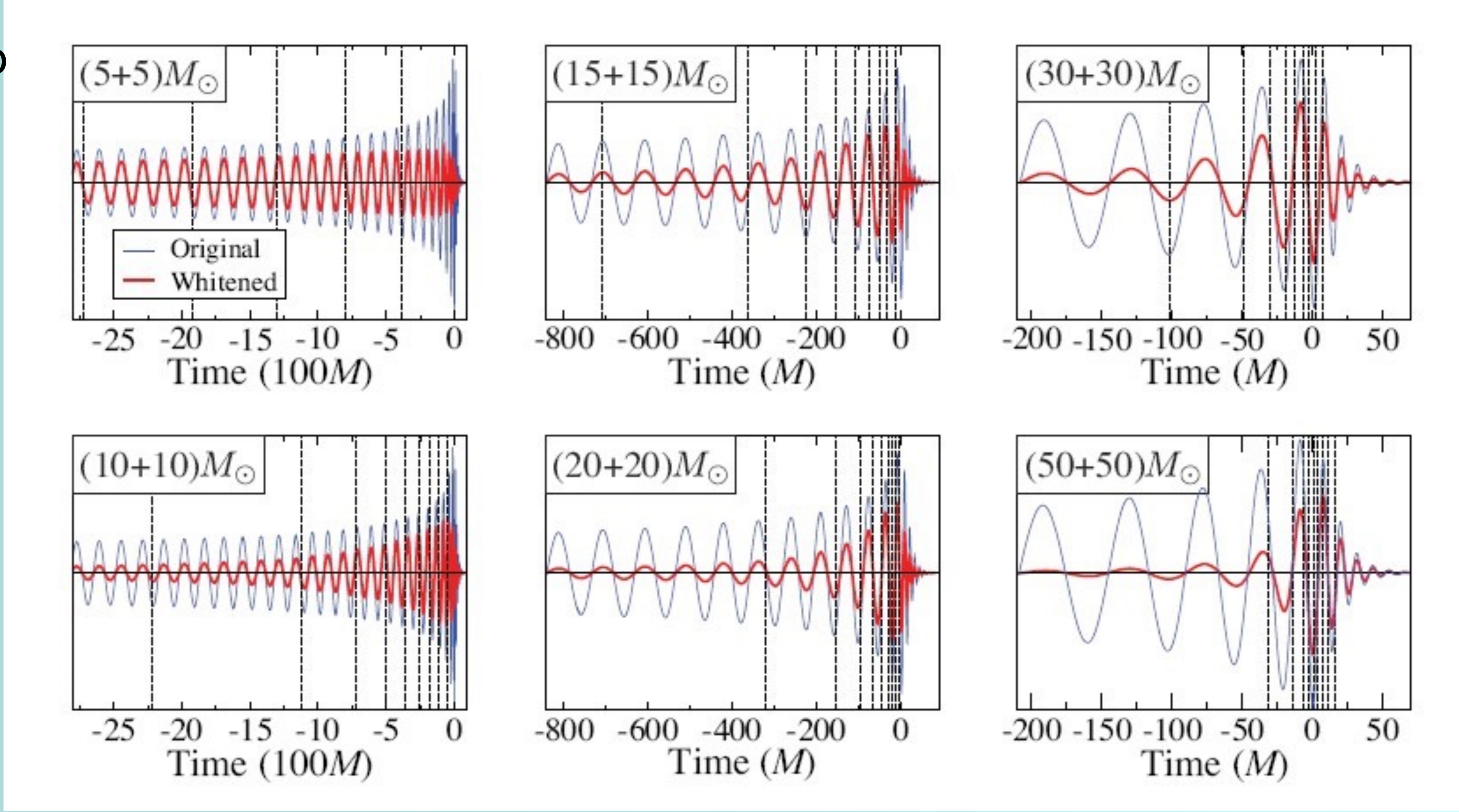
Noise curves



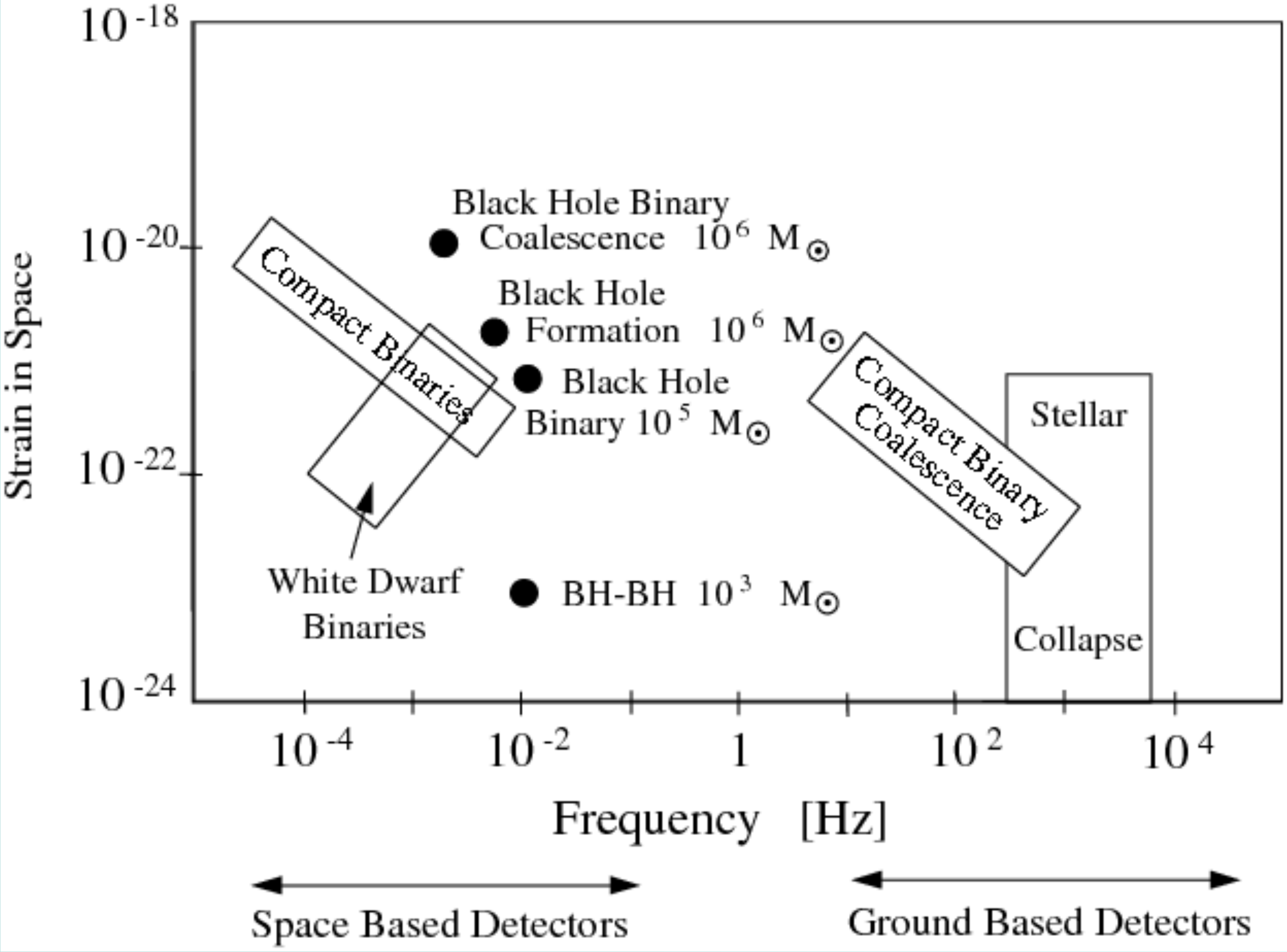
Size doesn't matter… or does it?

- $10\,M_{\rm sol}$ Only $50\,\%$ in last 25 cycles plus Merger and RD
- $20\,M_{\rm sol}$ $>90\,\%$ in last 23 cycles + MRD
- $30\,M_{\rm sol}$ $>90\,\%$ in last 11 cycles + MRD NR can do that!
- $100\,M_{\rm sol}$ $>90\,\%$ in last cycle + MRD Burst!

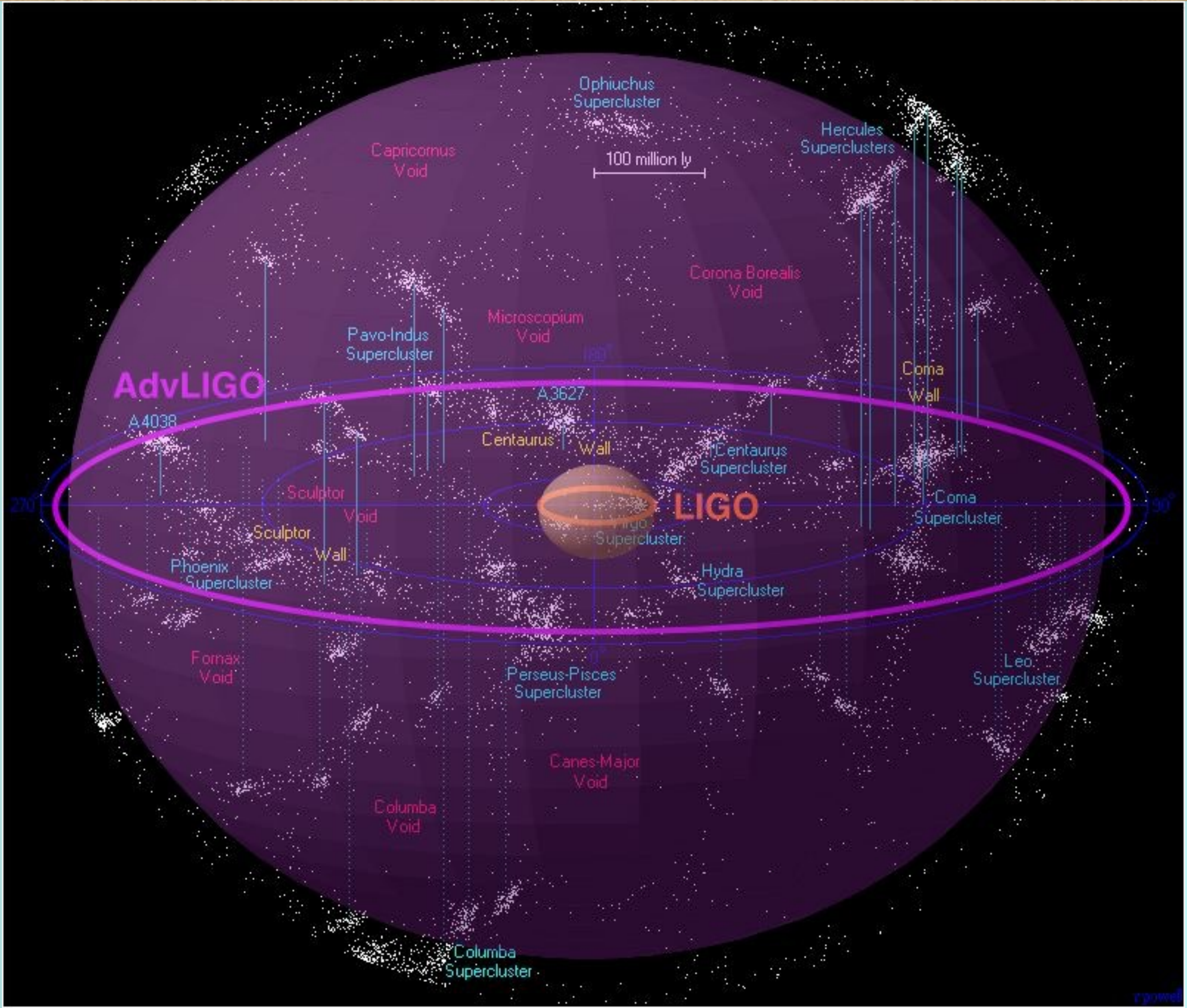
Buonanno et al.'07



Expected GW sources



How far can we observe?



7.4. High energy collisions

Motivation

US, Cardoso, Pretorius, Berti & González '08

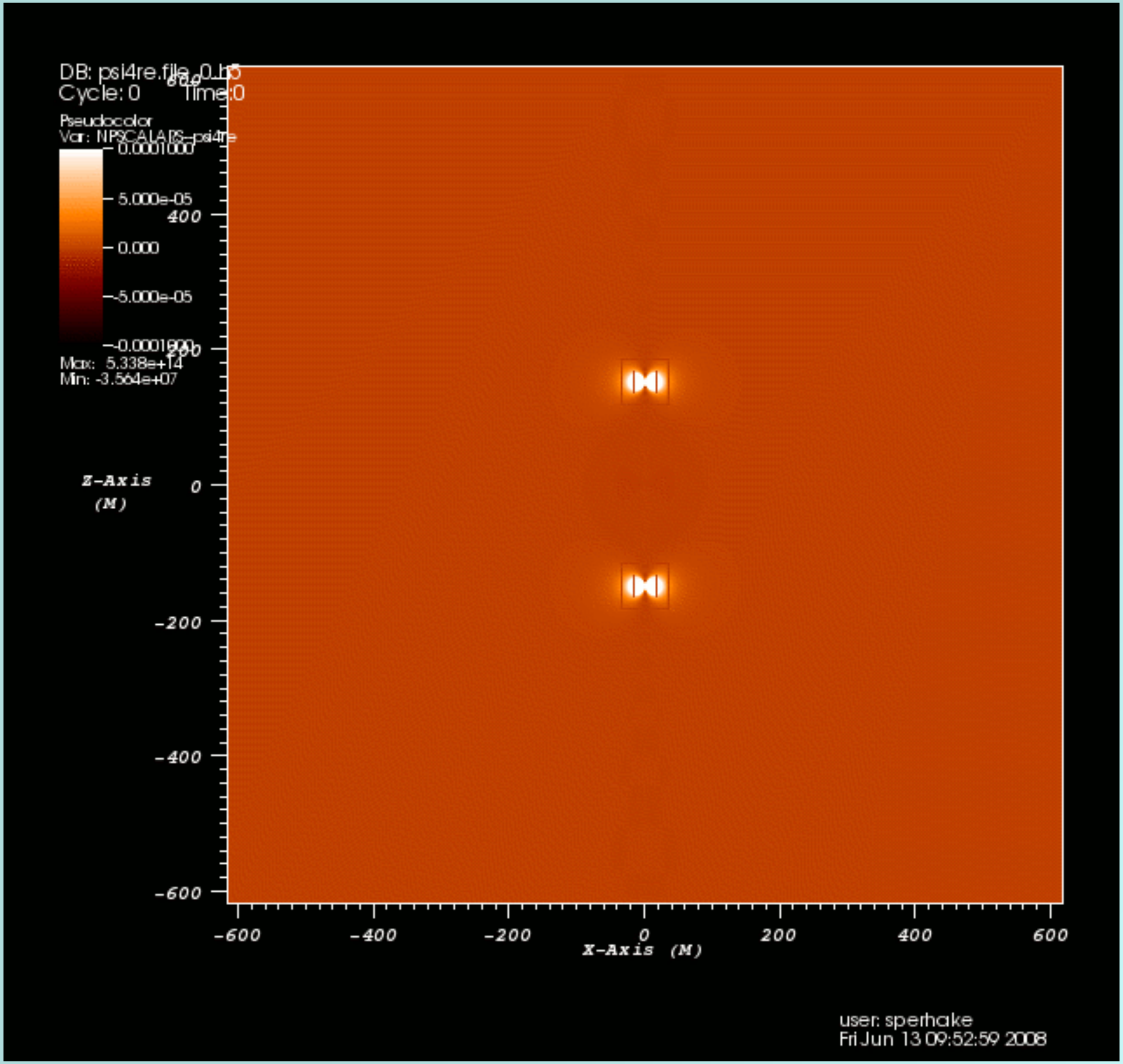
- Head-on collision of BHs near the speed of light
- Test cosmic censorship
- Maximal radiated energy
- First step to estimate GW leakage in LHC collisions
- Model GR in most violent regime
- Numerically challenging

  Resolution, Junk radiation

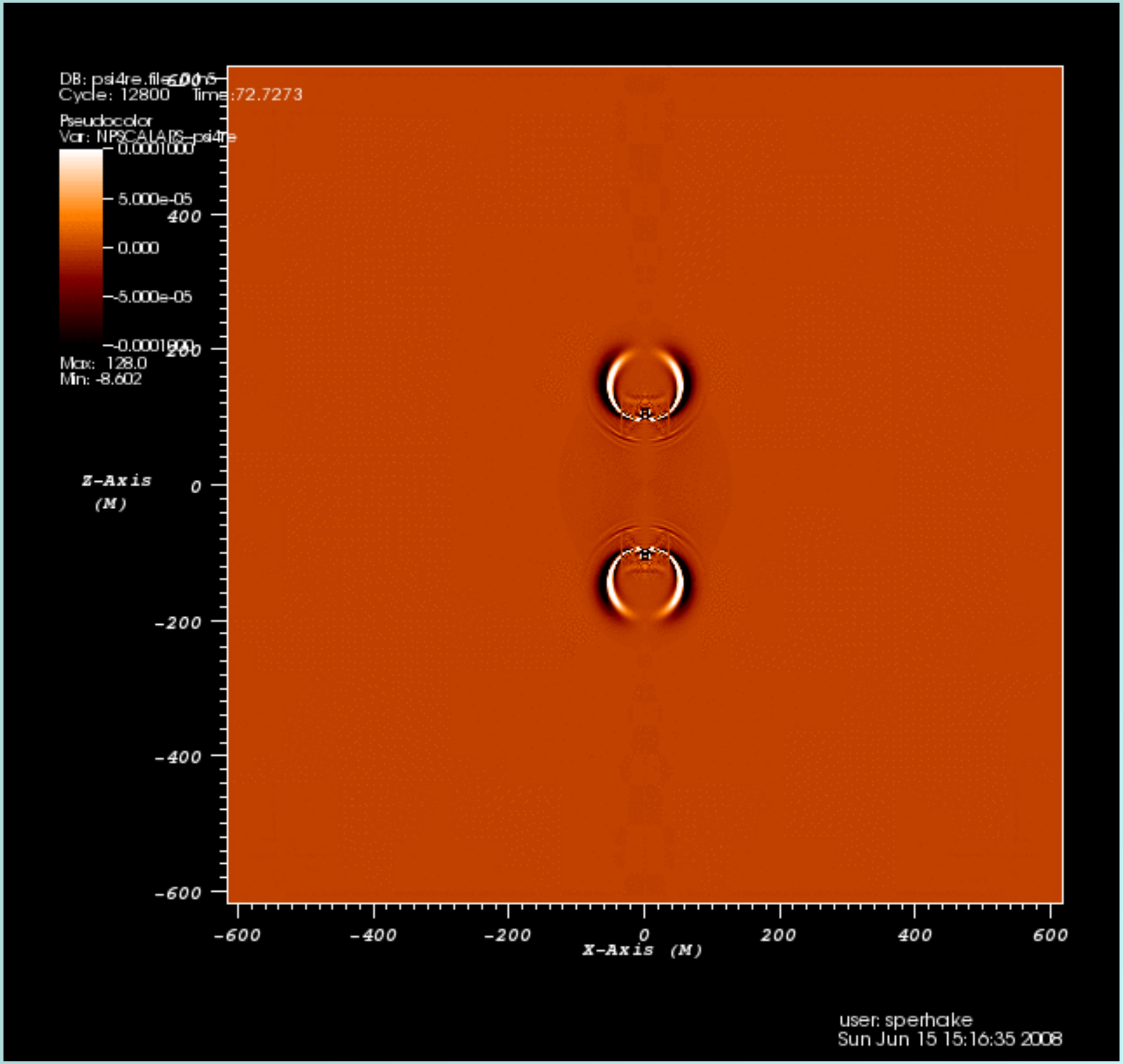
Shibata et al. '08

- Grazing collisions, cross sections
- Radiated energy even larger

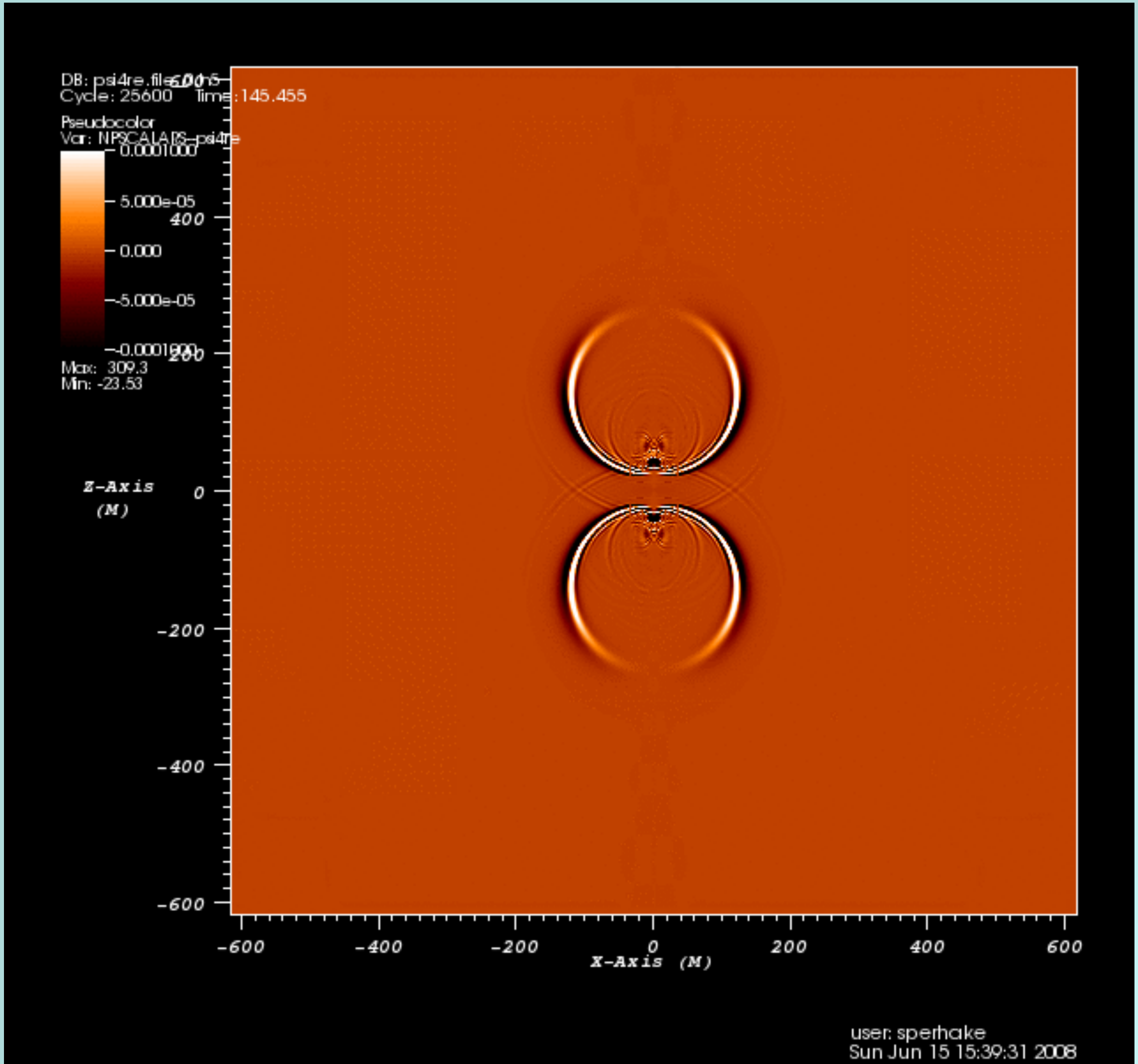
Example: Head-on with $\gamma = 2.75$



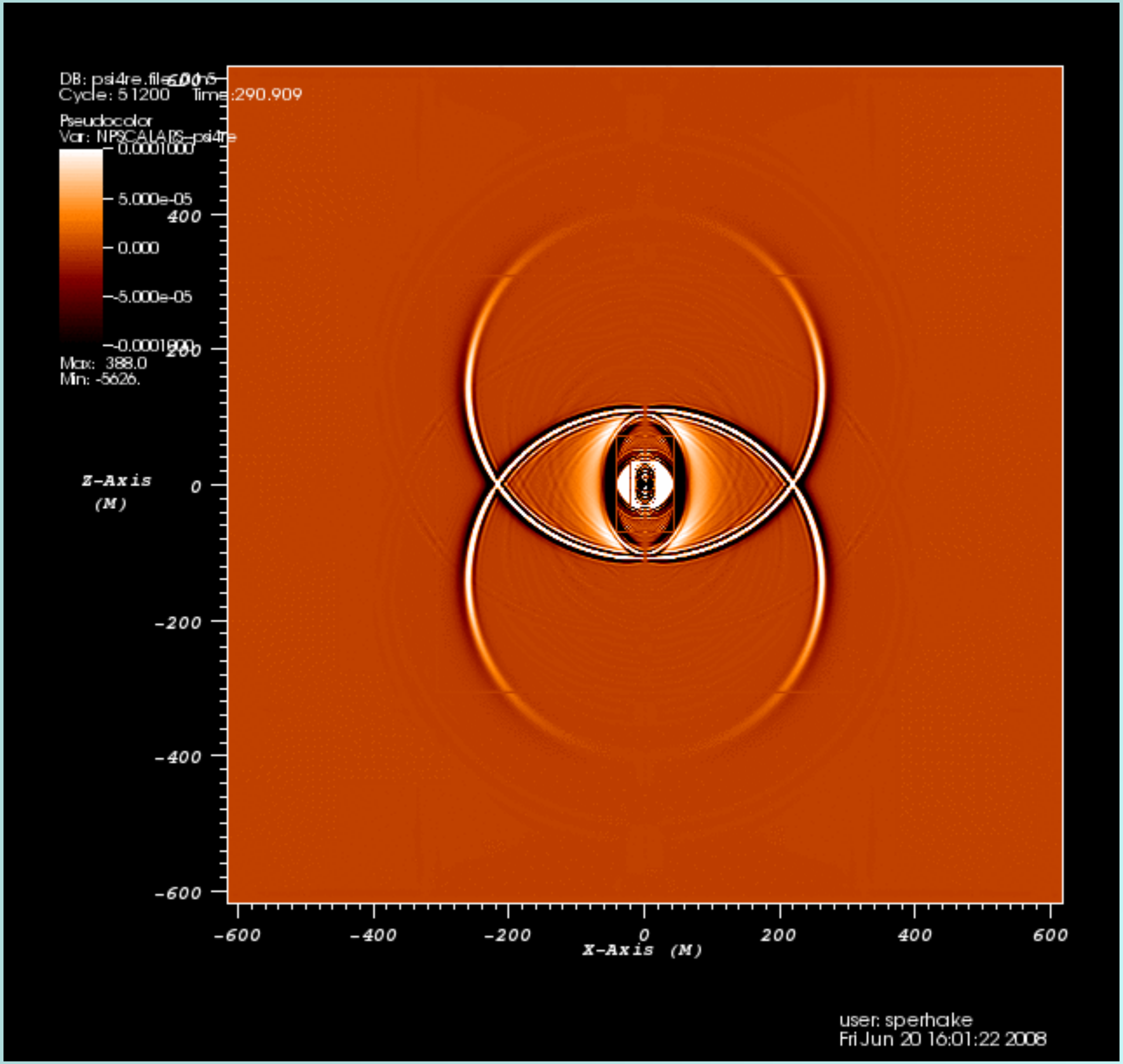
Example: Head-on with $\gamma = 2.75$



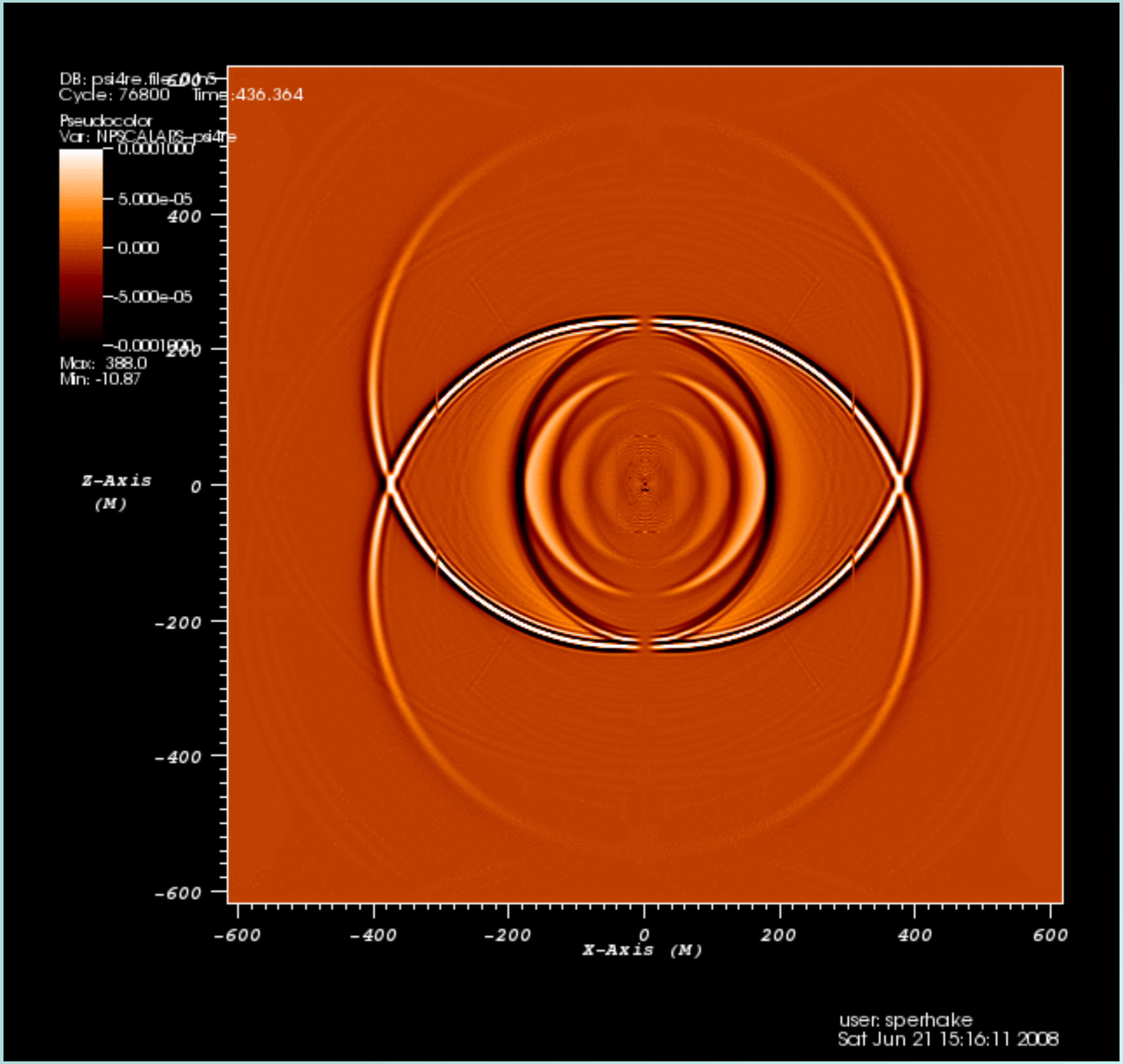
Example: Head-on with $\gamma = 2.75$



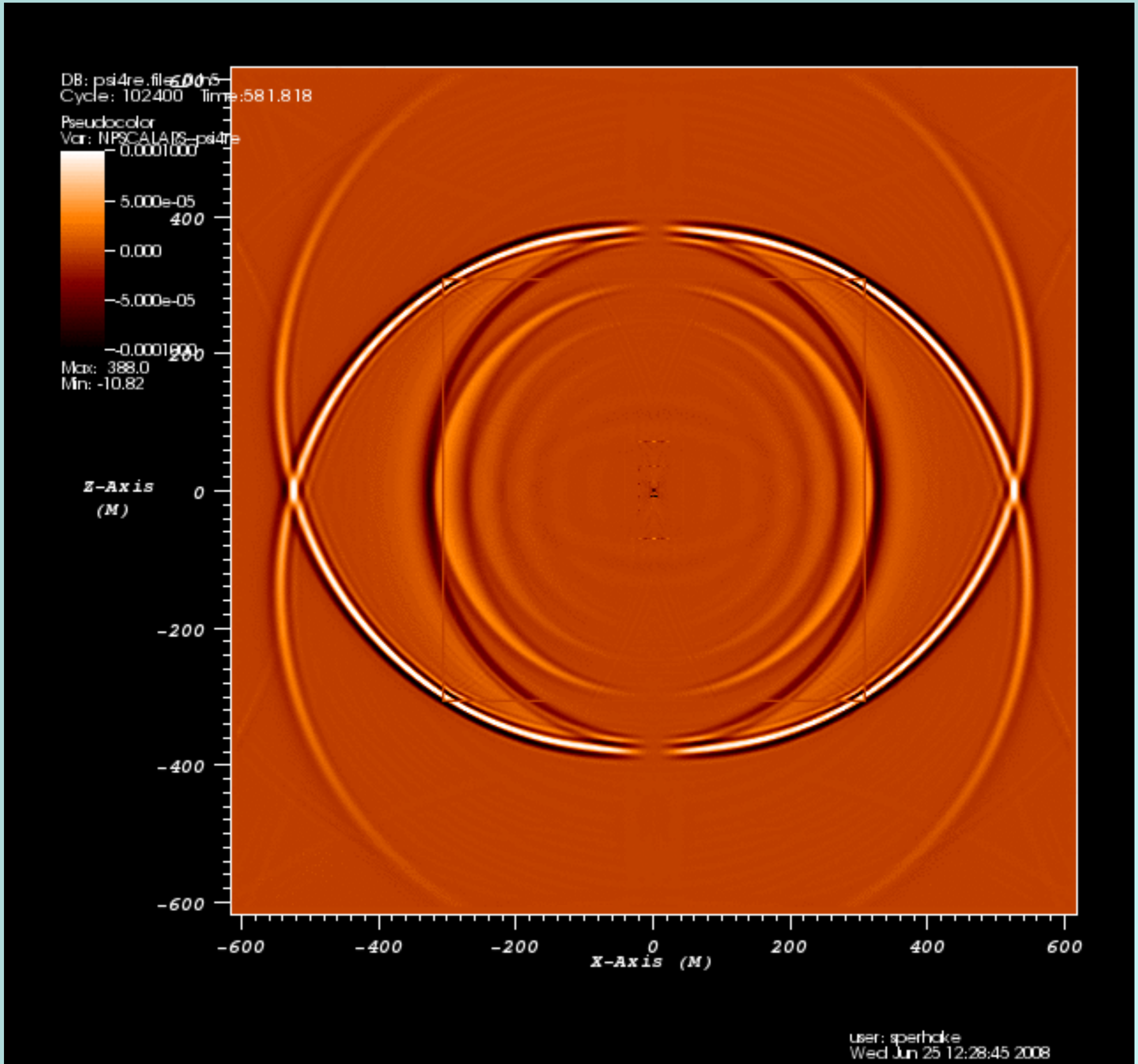
Example: Head-on with $\gamma = 2.75$



Example: Head-on with $\gamma = 2.75$

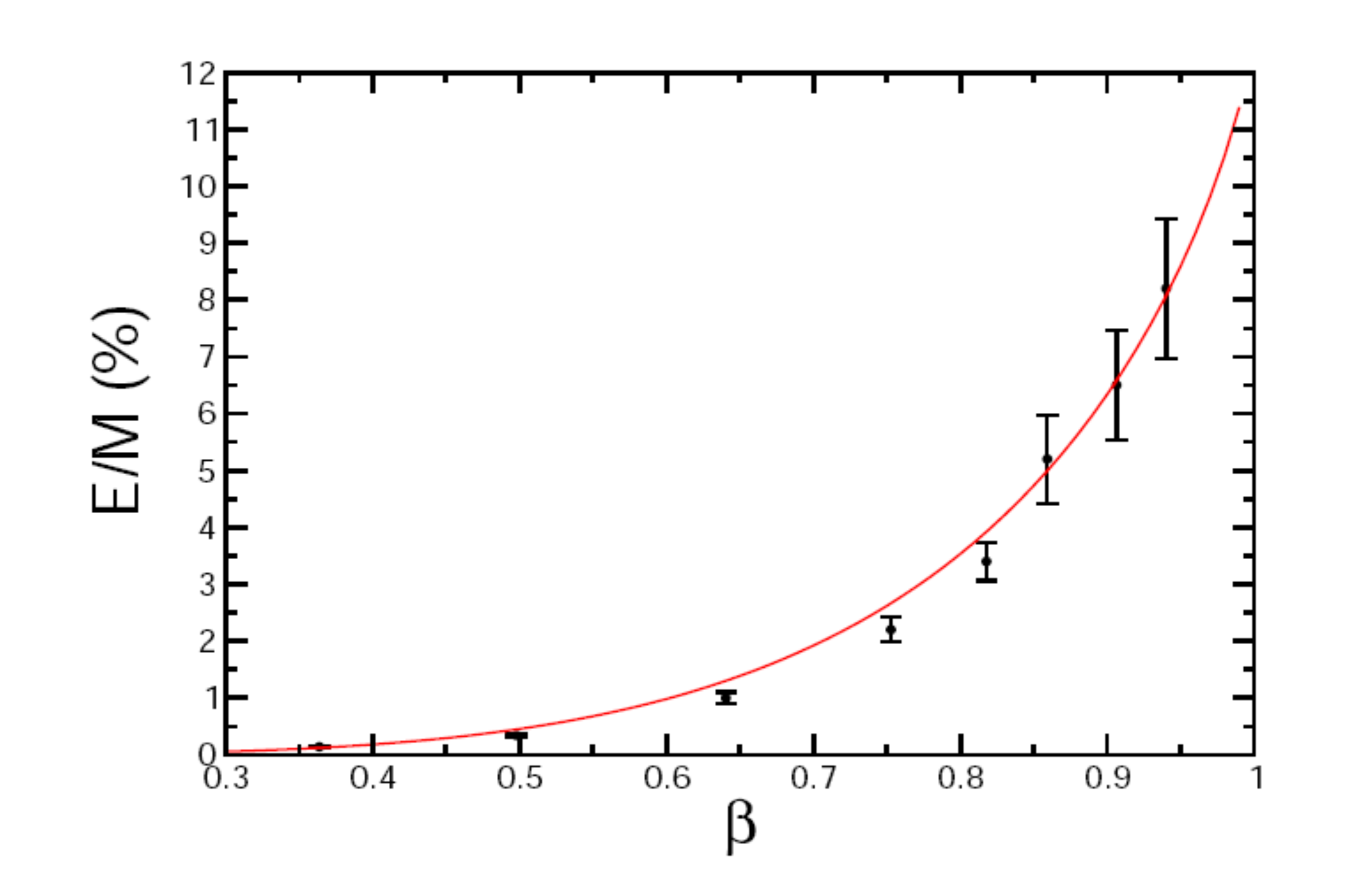


Example: Head-on with $\gamma = 2.75$



Total radiated energy

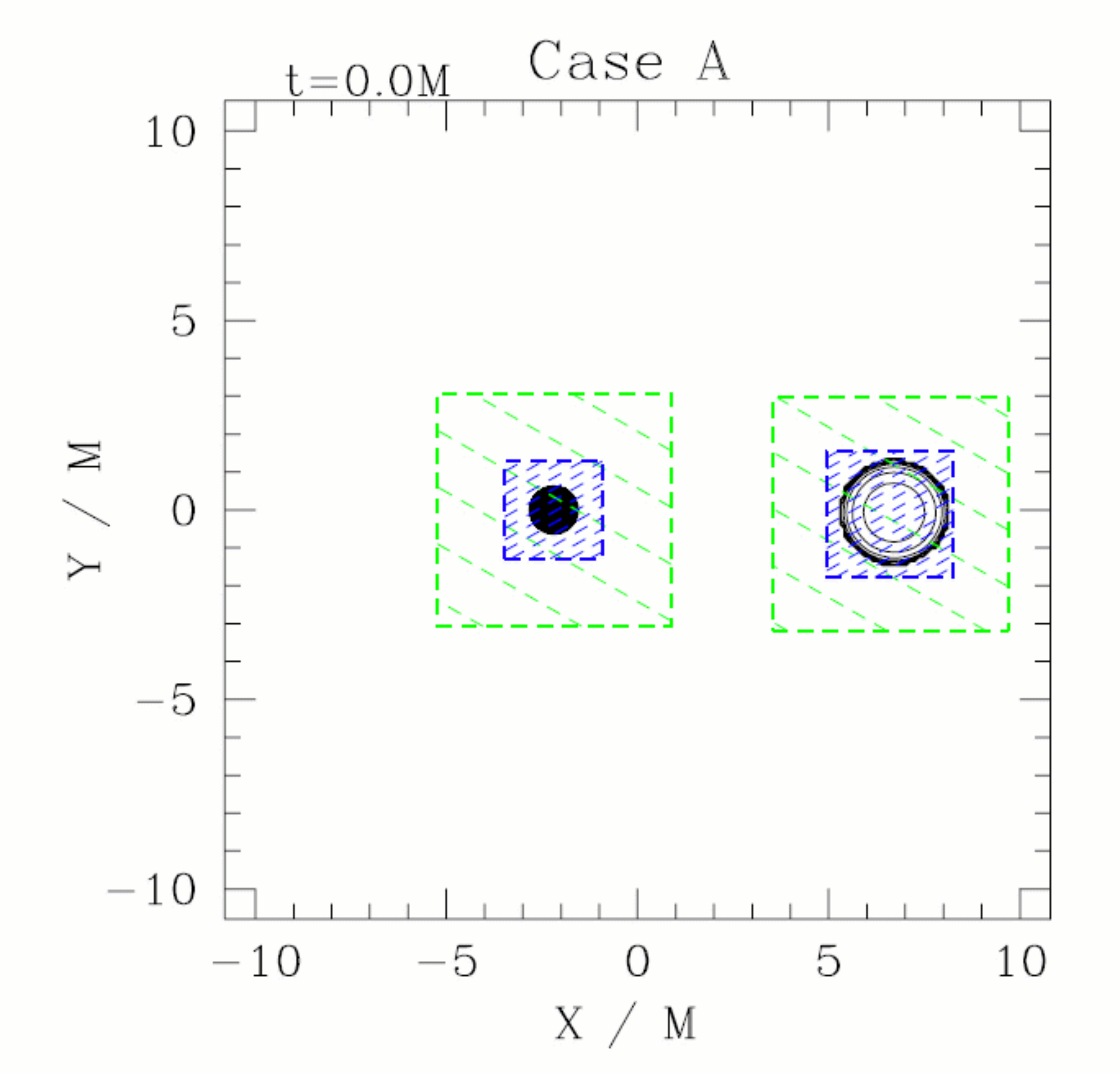
- Total radiated energy: $14 \pm 3\,\%$ about half of Penrose's limit



7.5. Neutron star – BH binaries

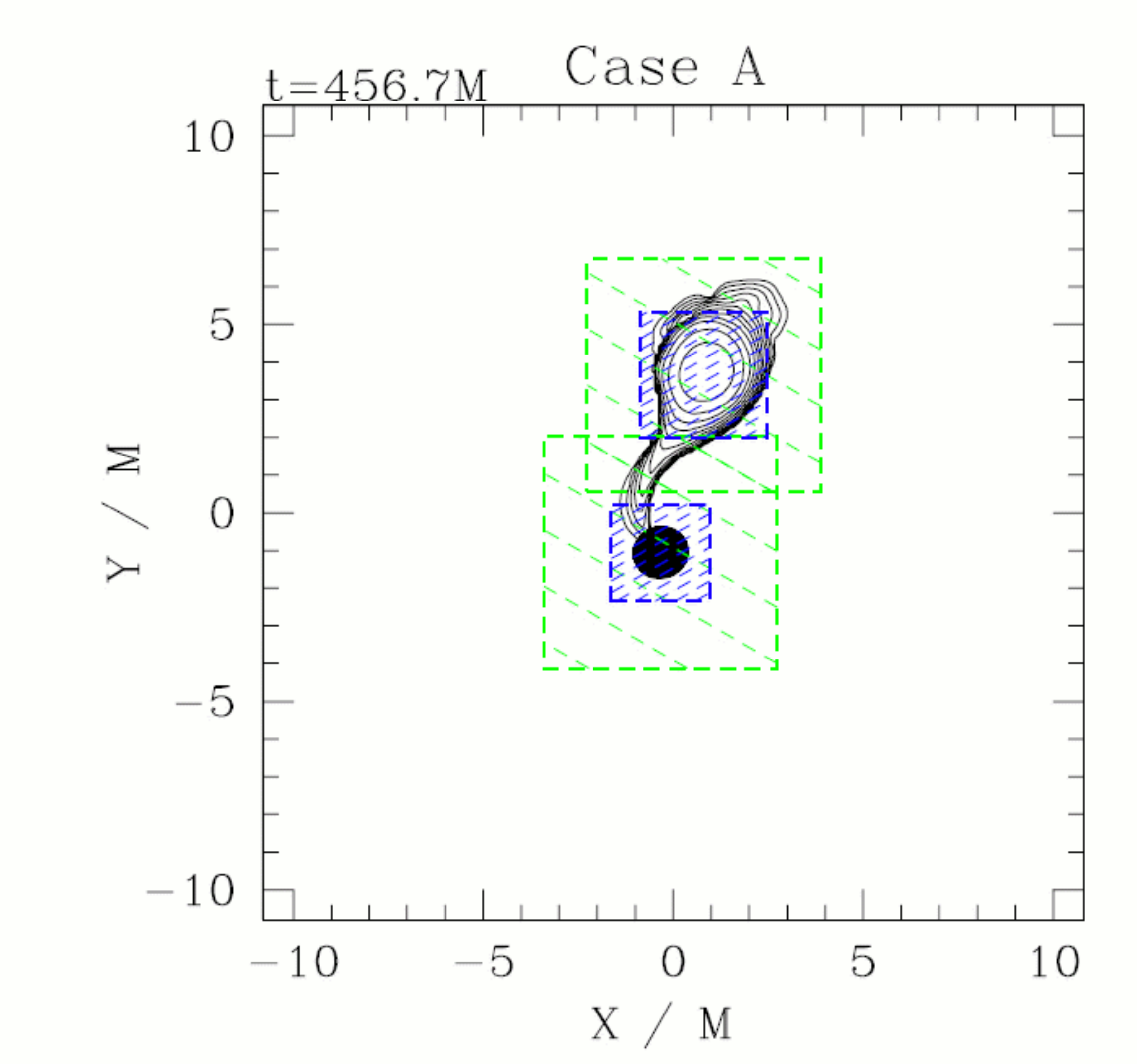
Neutron star is disrupted

Etienne et al. '08



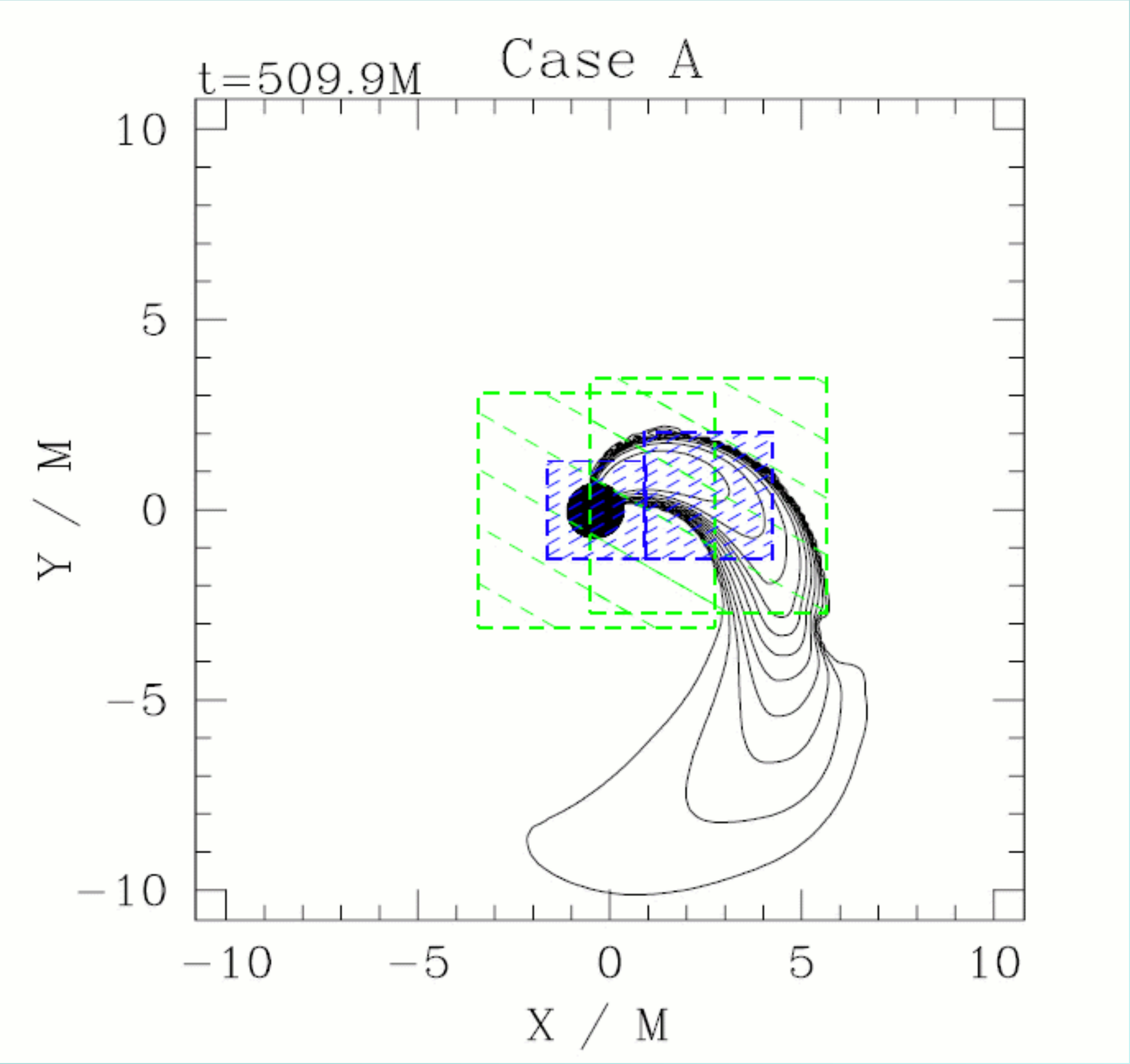
Neutron star is disrupted

Etienne et al. '08



Neutron star is disrupted

Etienne et al. ‘08

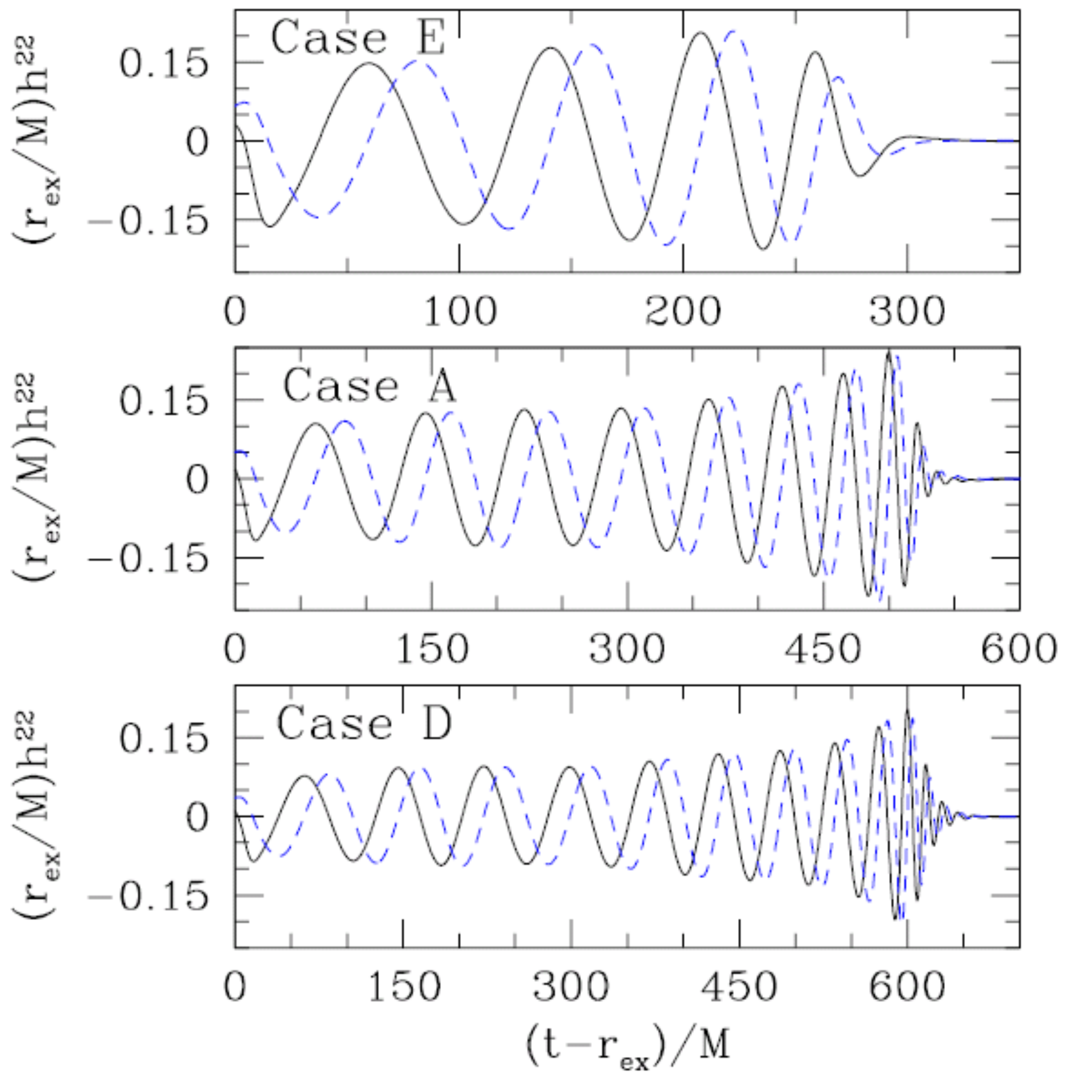


Waveforms

Etienne et al. '08

- Ringdown depends on mass ratio
  $q = 1, 3, 5$

- Active research area:
  UIUC, AEI, Caltech/Cornell



Future research

Main future research directions

- Gravitational wave detection
  - PN comparisons with spin
  - Generate template banks
  - Understand how to best generate/use hybrid wave forms
  - Simulate extreme mass ratios
- Astrophysics
  - Distribution functions for Kick, BH-spin, BH-mass
  - Improve understanding of Accretion, GW bursts,…
- Fundamental physics
  - High energy collisions: radiated energy, cross sections
  - Higher dimensional BH simulations