Accuracy in numerical simulations of unequal-mass black-hole binaries

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Overview

- Introduction
- The mass ratio 1:10
  - Numerical framework
  - Overall properties
  - Accuracy of GW signal
- Long simulations with
  - Setup
  - Dependence of GW signal on extraction radius
- Discussion…

Introduction

- Equal-mass, non-spinning BHB "under control"

  Scheel et al. '08

- Unequal-mass binaries are harder!
  - Lower degree of symmetry
  - More demanding computationally
  - 1-parameter family; which q necessary?

- Astrophysically relevant!
  - SMBH formation may favor mergers around $q \approx 10$

    Sesana et al.'07
  - Mass distribution of SMBHs predicts most mergers in the range $3 \leq q \leq 10$ Gergely & Biermann '07

- Comparison with: PN, EOB, Perturbation theory

Numerical framework: q=10

- BAM code Brügmann et al. '08
  - Puncture initial data
  - BSSN evolution
  - Moving puncture gauge
  - FD 6th order in space, 4th order in time

- Parameters

$$M := M_1 + M_2 \qquad q := \frac{M_1}{M_2} = 10$$

$$D = 7\,M \qquad \eta := \frac{M_1 M_2}{(M_1 + M_2)^2} = 0.0826$$

$$\boldsymbol{S}_1 = \boldsymbol{S}_2 = 0$$

- Quasi circular Kidder '95

Numerical difficulties: Gauge

- Shift condition

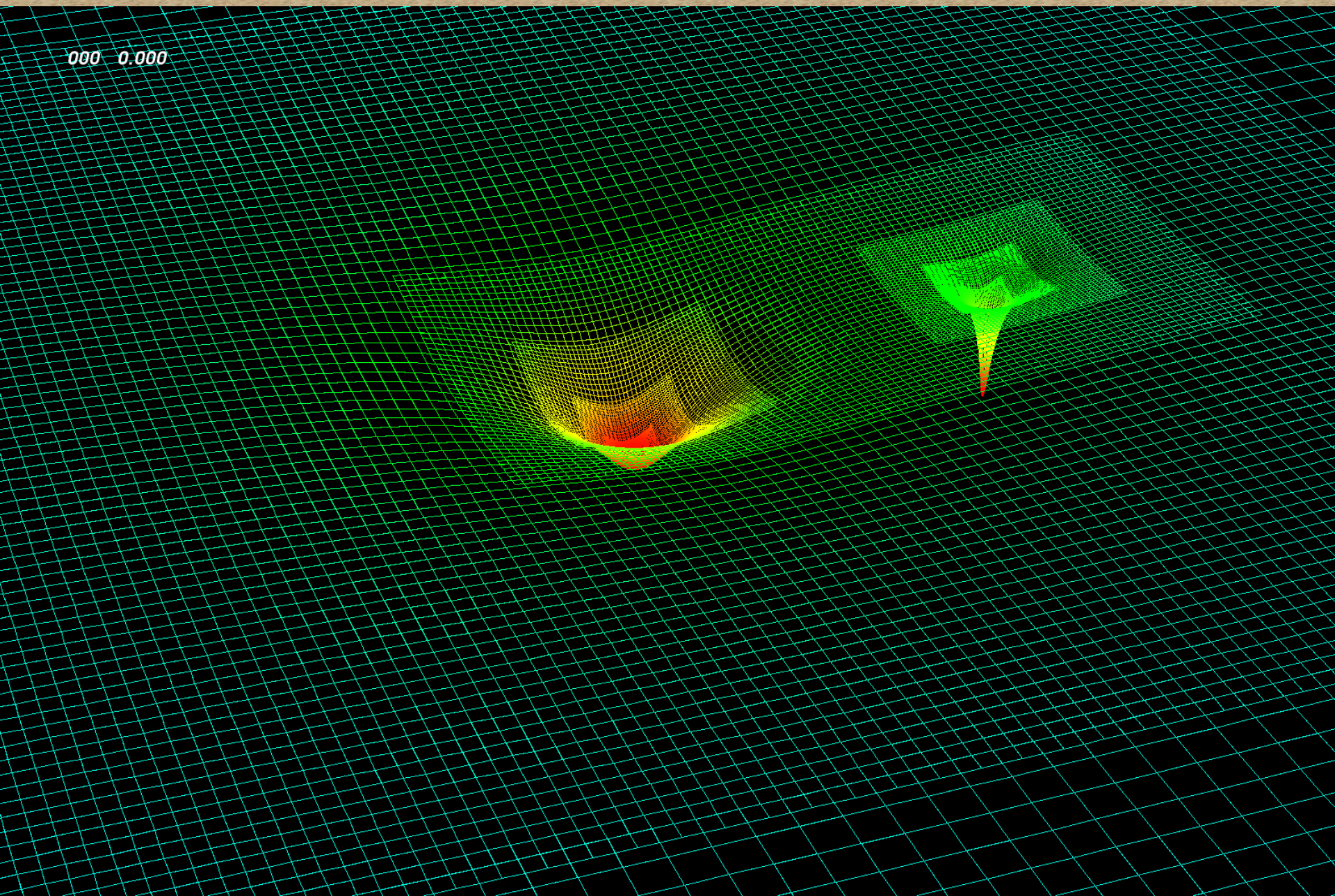
$$\eta_b = \frac{1.375}{M}$$

$$\partial_t \beta^i = \frac{3}{4} B^i, \quad \partial_t B^i = \partial_t \tilde{\Gamma}^i - \eta_b B^i.$$

- Poor choices of η_b
  - Loss of convergence
  - Instabilities
- $q = 10$ more difficult than earlier study $q = 1...4$

González et al. ‘07

What went wrong?



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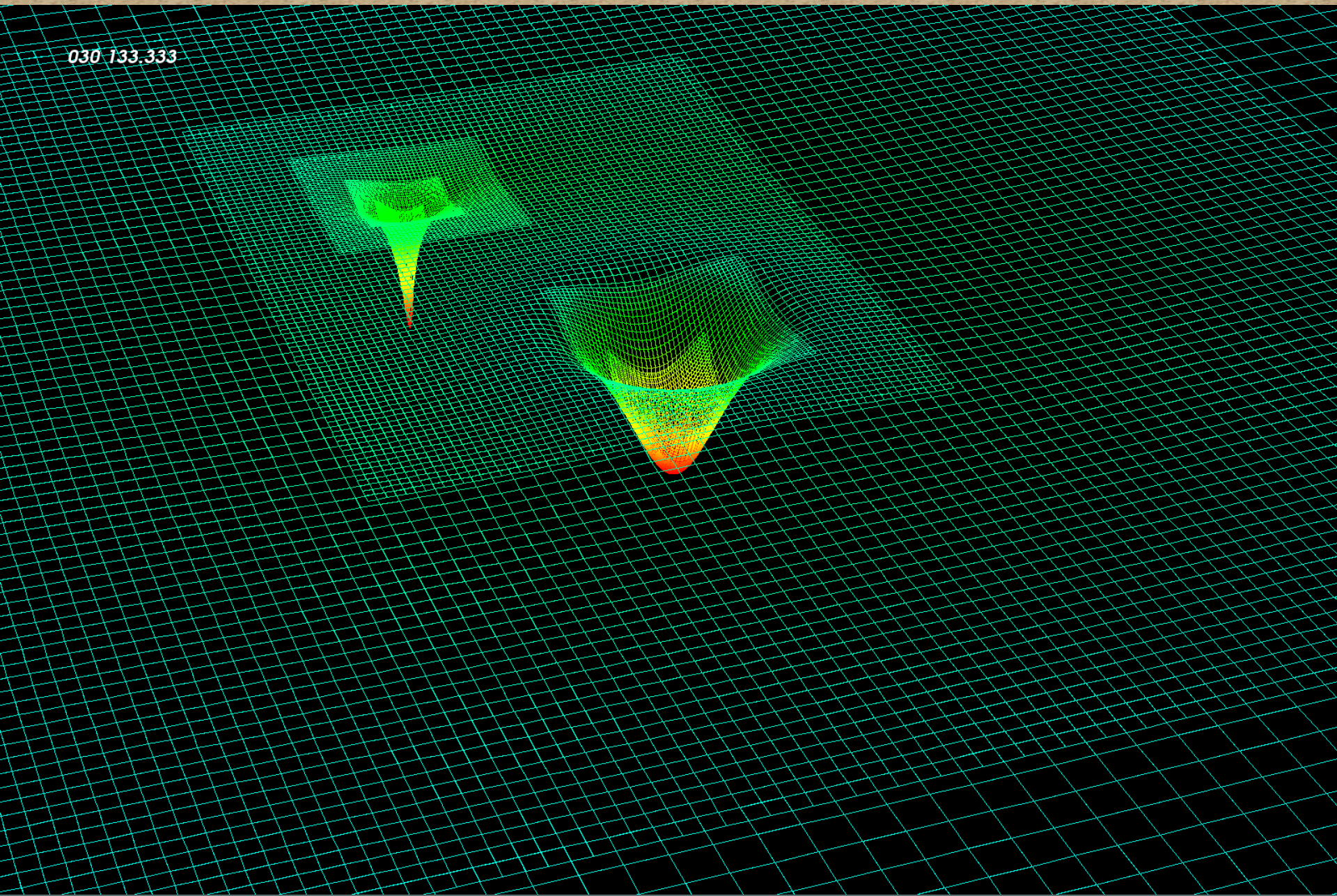

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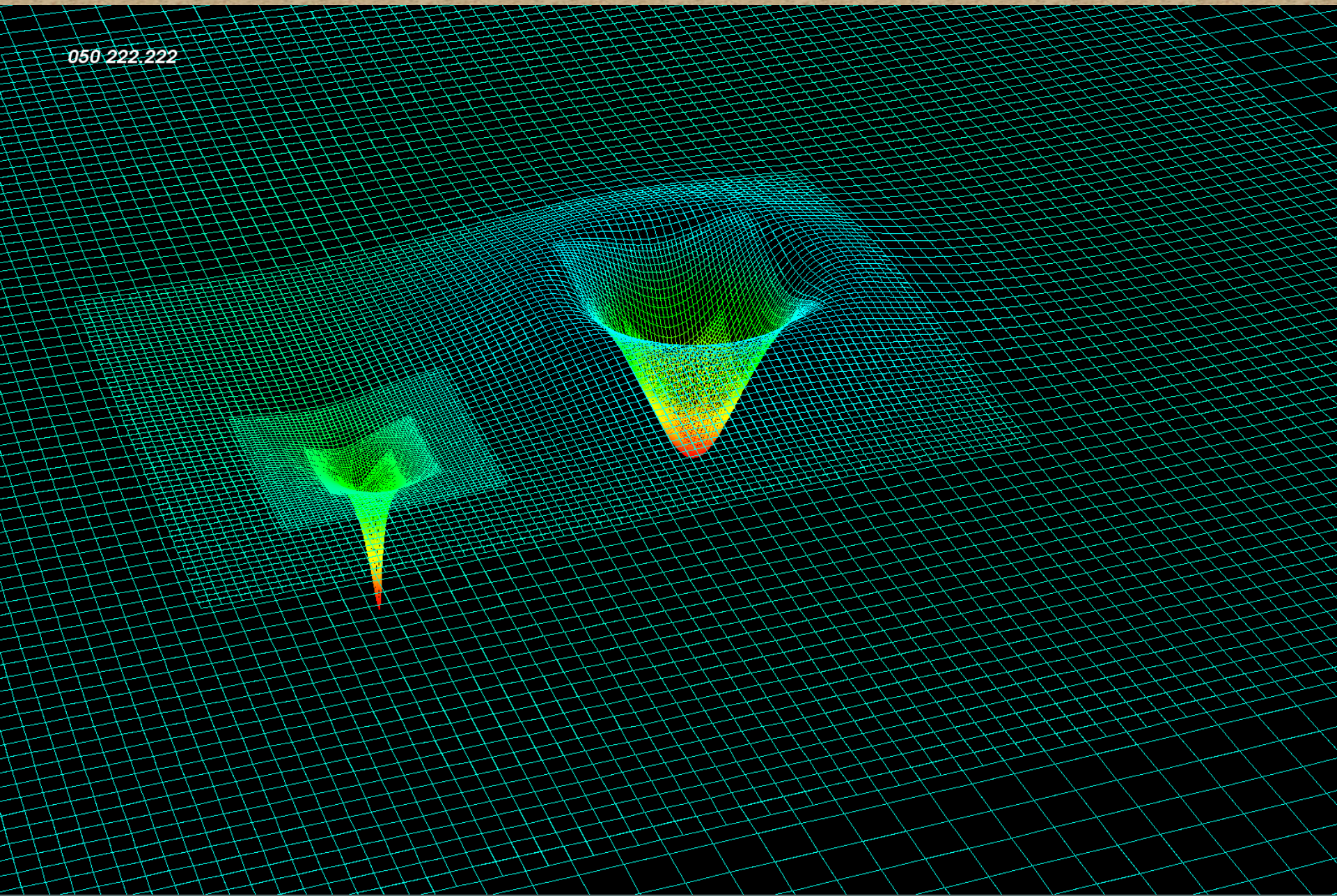

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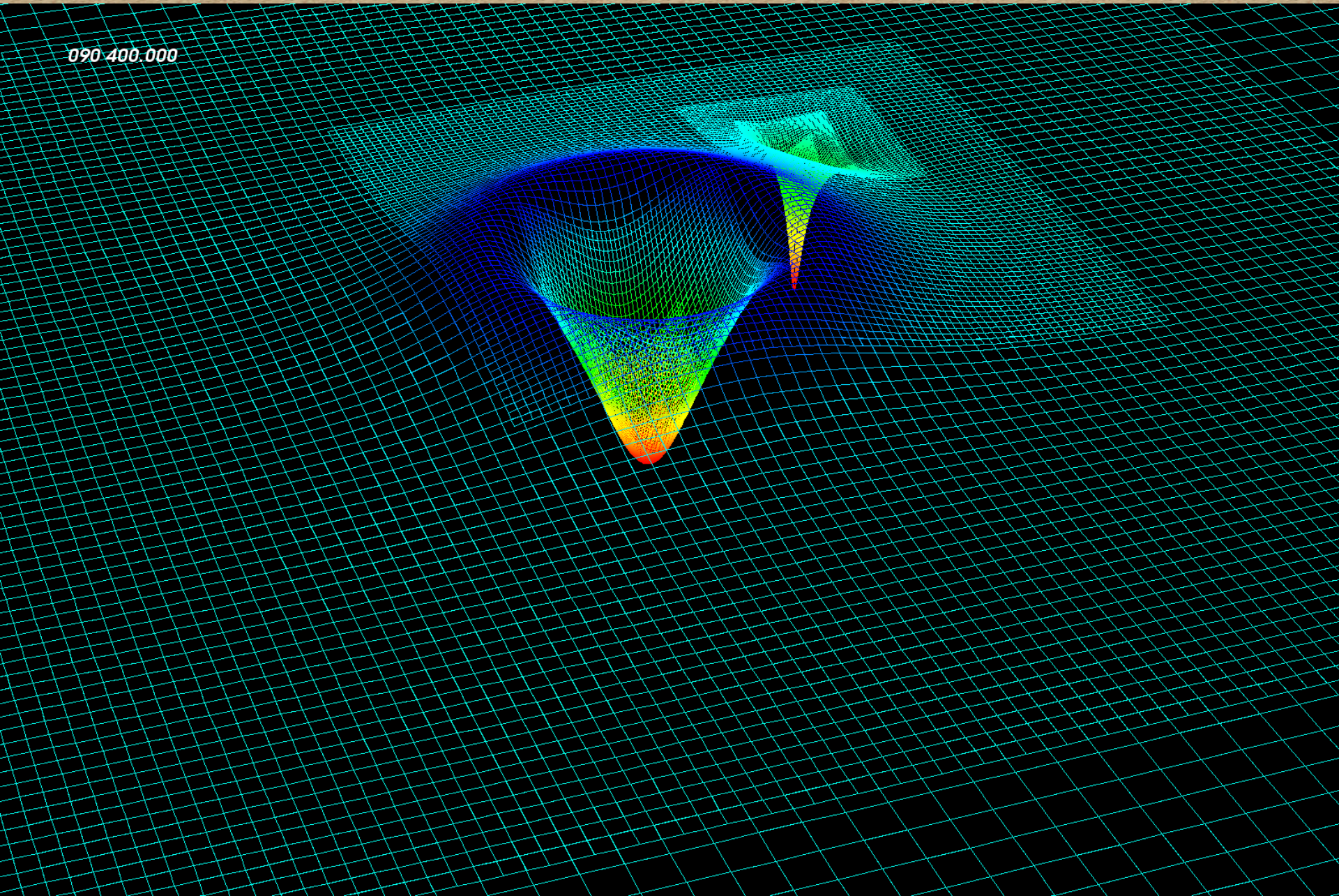
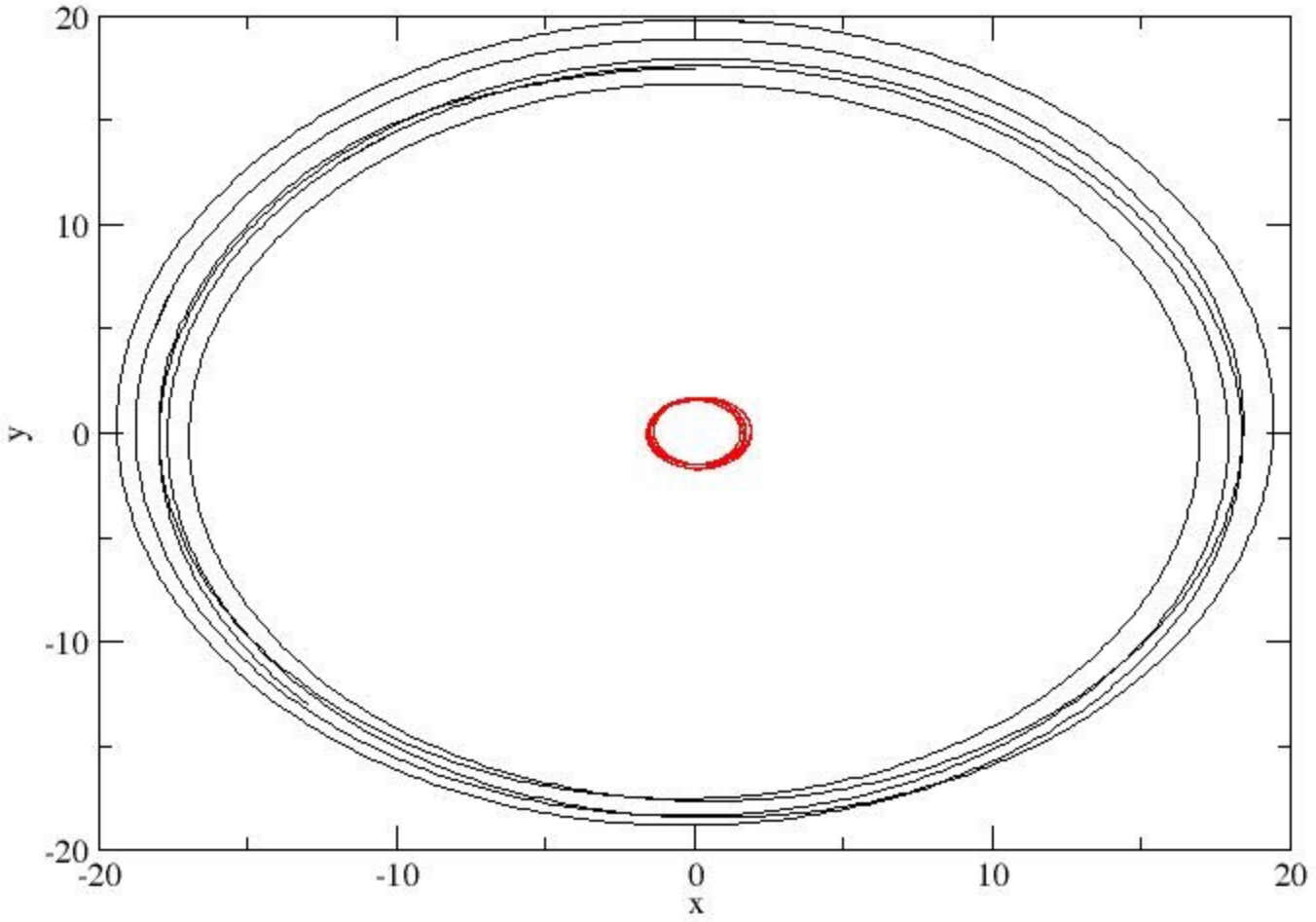


Numerical difficulties: Resolution

- 6th order Convergence obtained for $h = \frac{M}{165}, \frac{M}{187}, \frac{M}{209}$
- Insufficient resolution
  - Loss of convergence
  - No inspiral…
- $q = 10$ more difficult than earlier study $q = 1...4$

González et al. '07

What went wrong?



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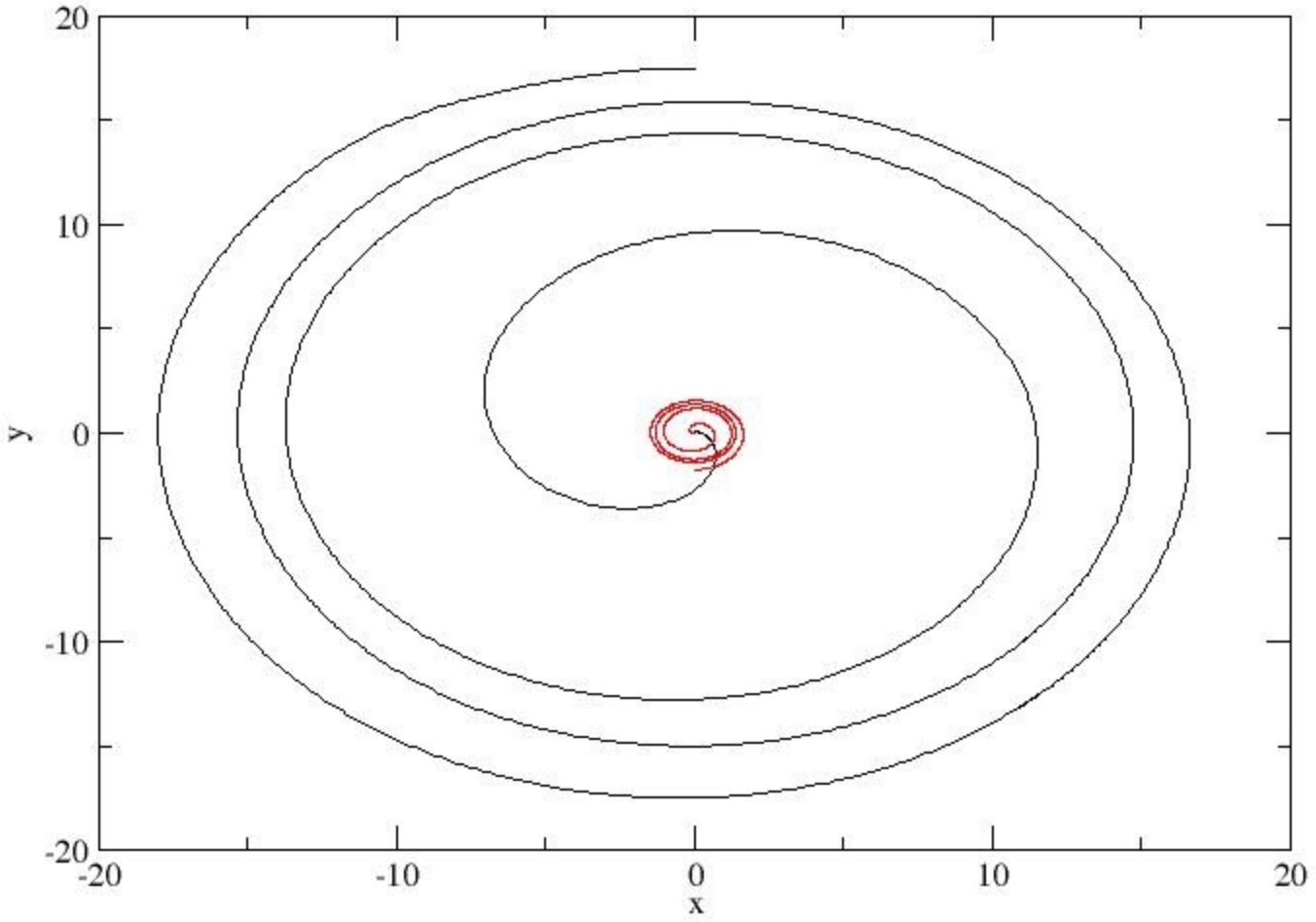
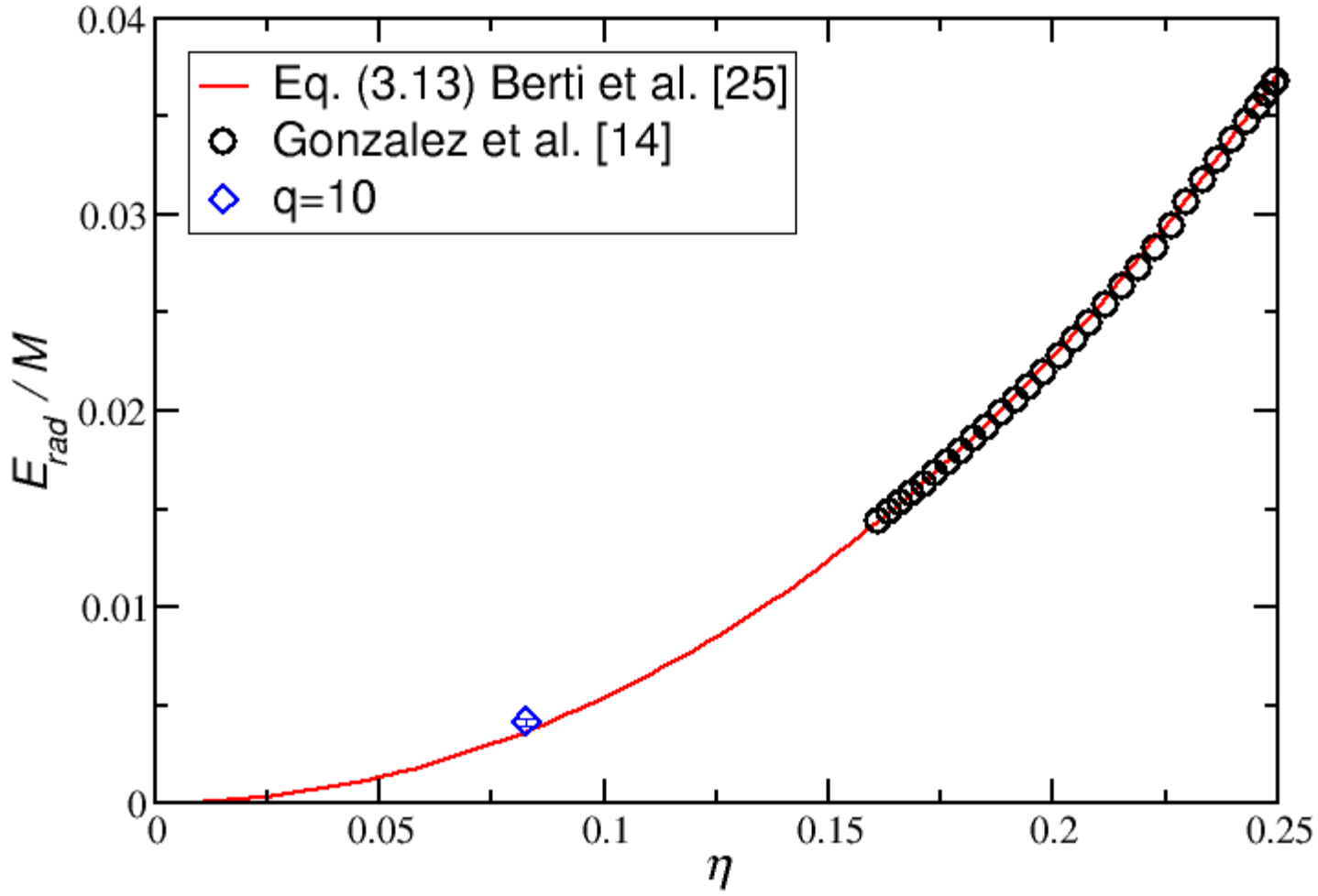


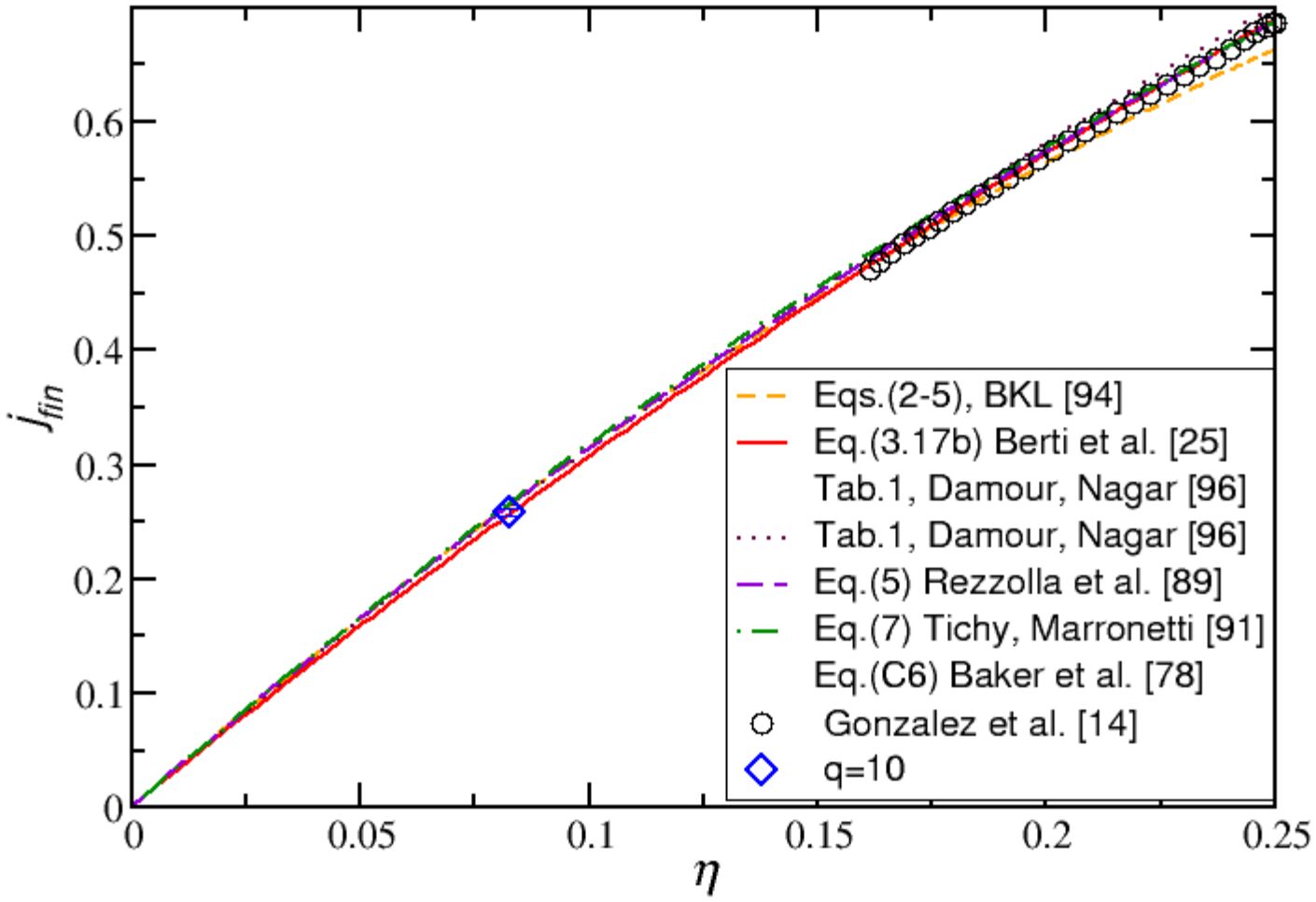
Radiated energy

$E_{\rm rad} = (0.415 \pm 0.017)\,\%\ M_{\rm ADM}$ for $\Delta\phi \approx 40$ prior to $A_{\rm max}$



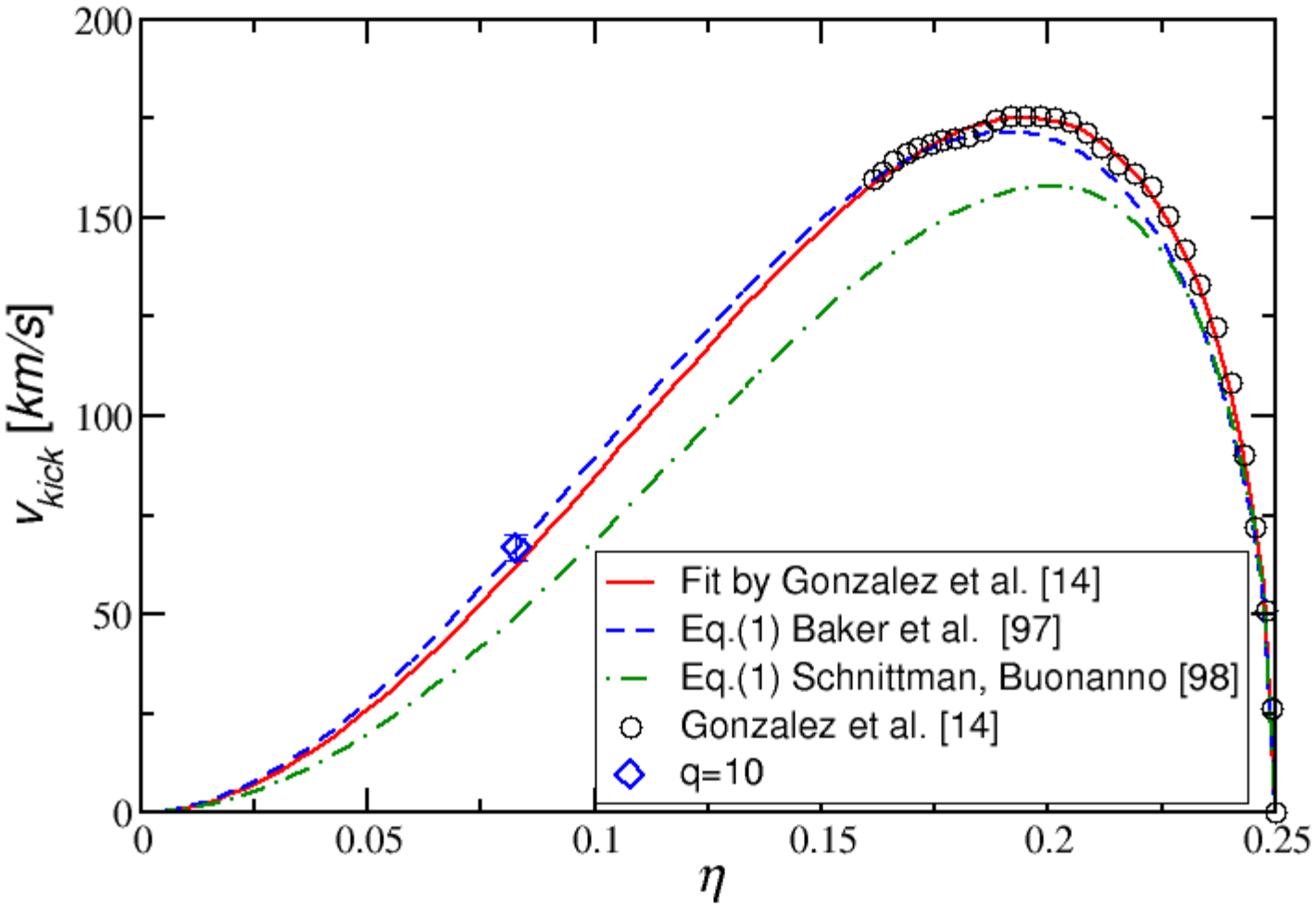
Final spin

$$j_{\rm fin} = 0.259 \pm 0.003$$



Recoil

$$v_{\rm kick} = (66.7 \pm 3.3) \ \ {\rm km/s}$$



Summary

- $E_{\rm rad} = (0.415 \pm 0.017)\,\%\ M_{\rm ADM}$

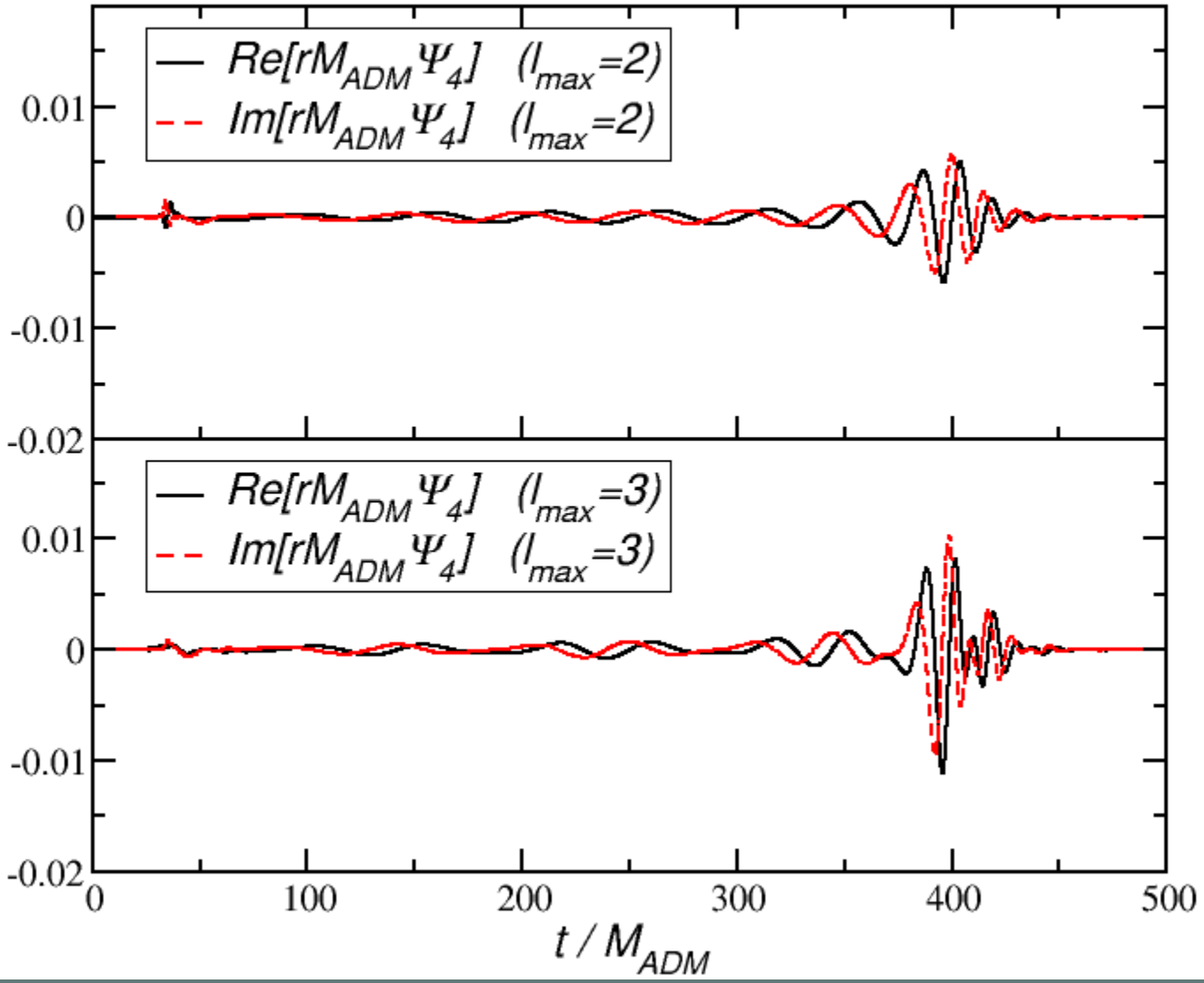
  $j_{\rm fin} = 0.259 \pm 0.003$

  $v_{\rm kick} = (66.7 \pm 3.3)\ \ {\rm km/s}$

- Good: Excellent agreement with various fitting formulae.
- Bad: Does not discriminate between different formulae.
- Needed: $q = 10$ simulations with spin!

Wave signal

Inclination angle: $\theta = 45.5^{\circ}$



Wave signal

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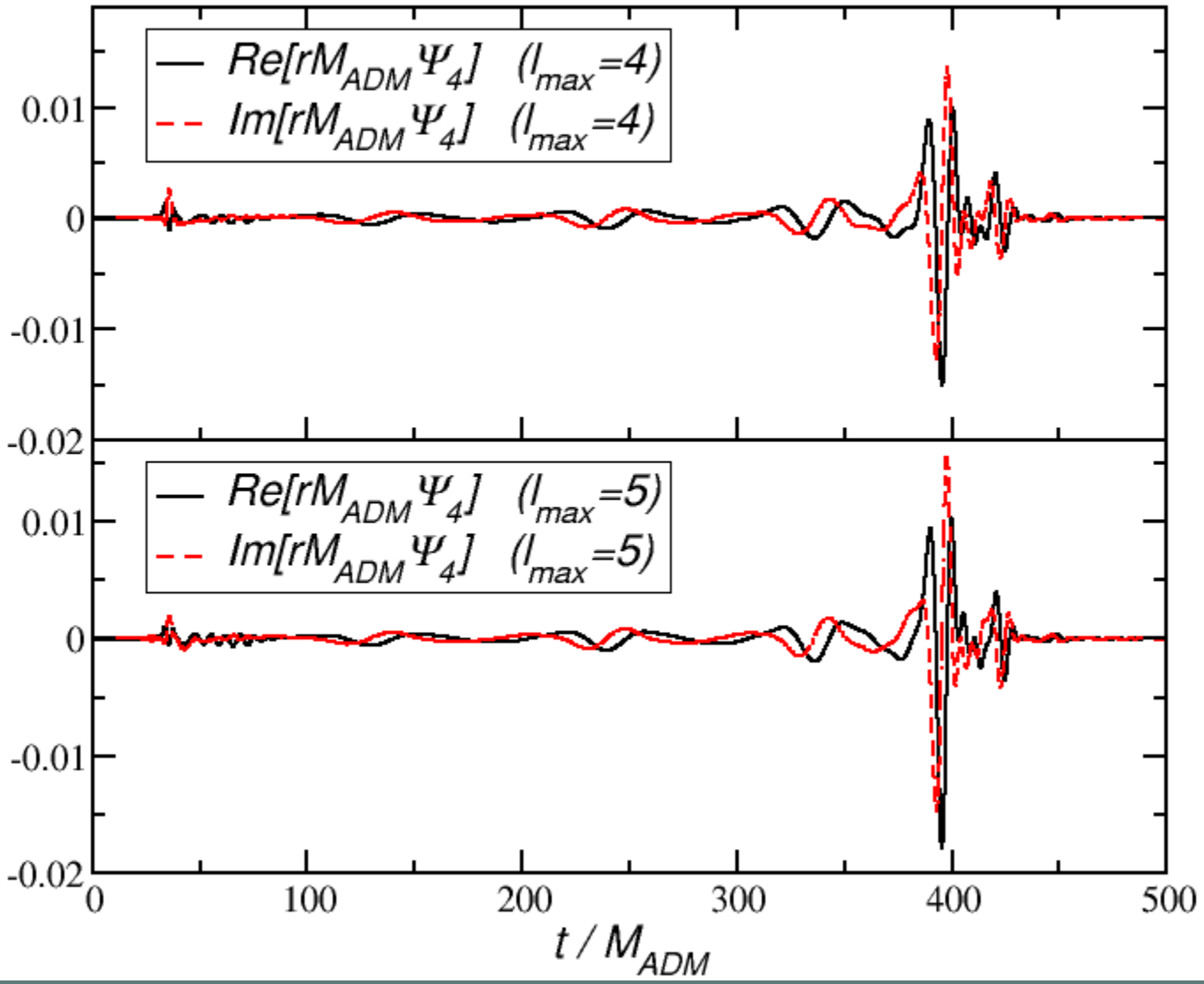
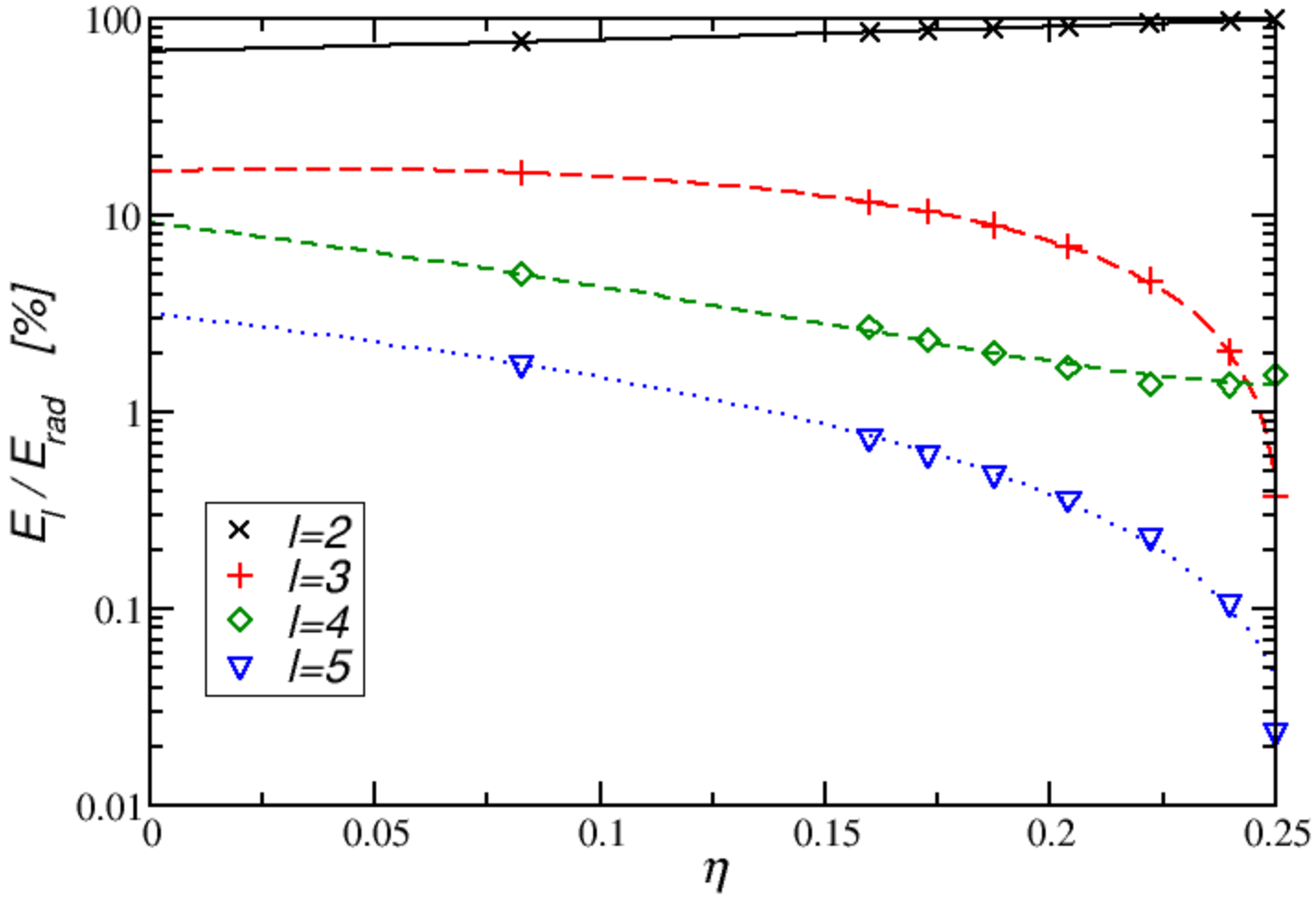


Multipolar distribution of radiated energy

Quadratic polynomial fits incl. data from Berti et al. ‘07



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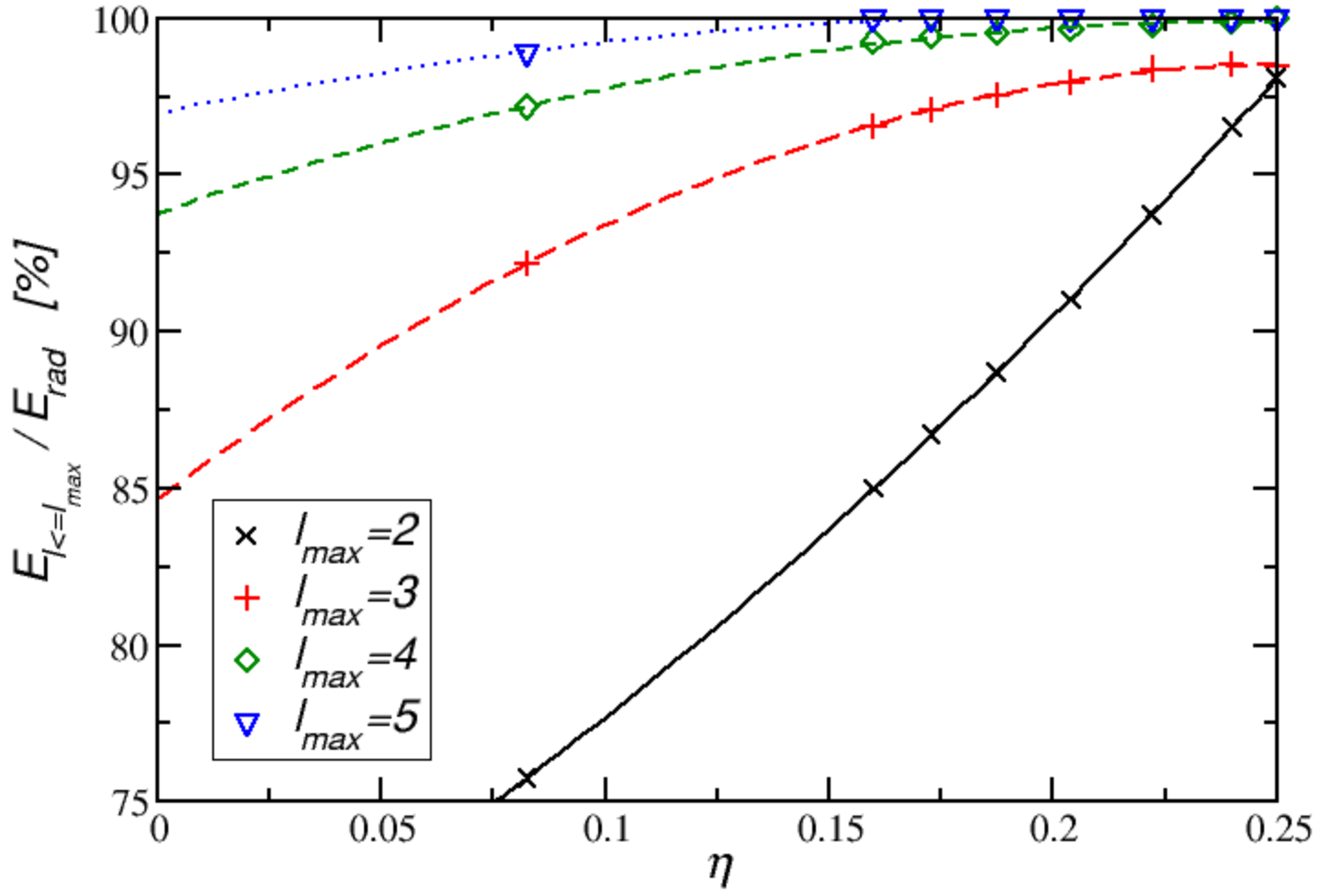


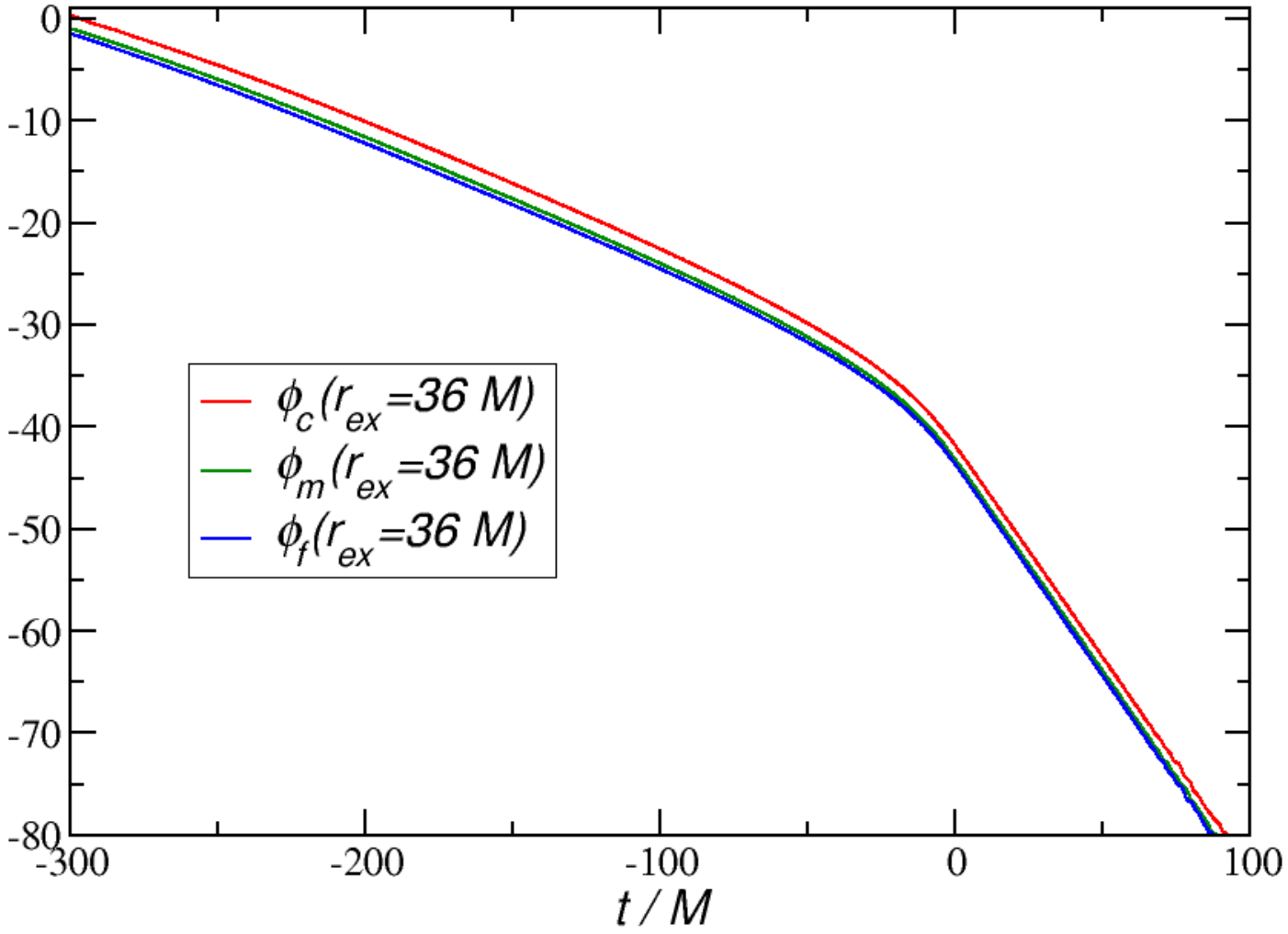
Accuracy of waveform

- Resolution: 6th order convergence
- Extraction radius: $r_{\rm ex} = 18\,M,\ \ 27\,M,\ \ 36\,M$
- Procedure: Quadrupole only! $\ell = 2,\ m = 2$
  - Decompose $\psi_{\ell m} = A_{\ell m} e^{im\phi}$
  - Align waveforms at $\max\left(A_{22}\right)$
  - Richardson extrapolation of A, ϕ using 6th and 4th order
  - Fit power law for dependence on extraction radius

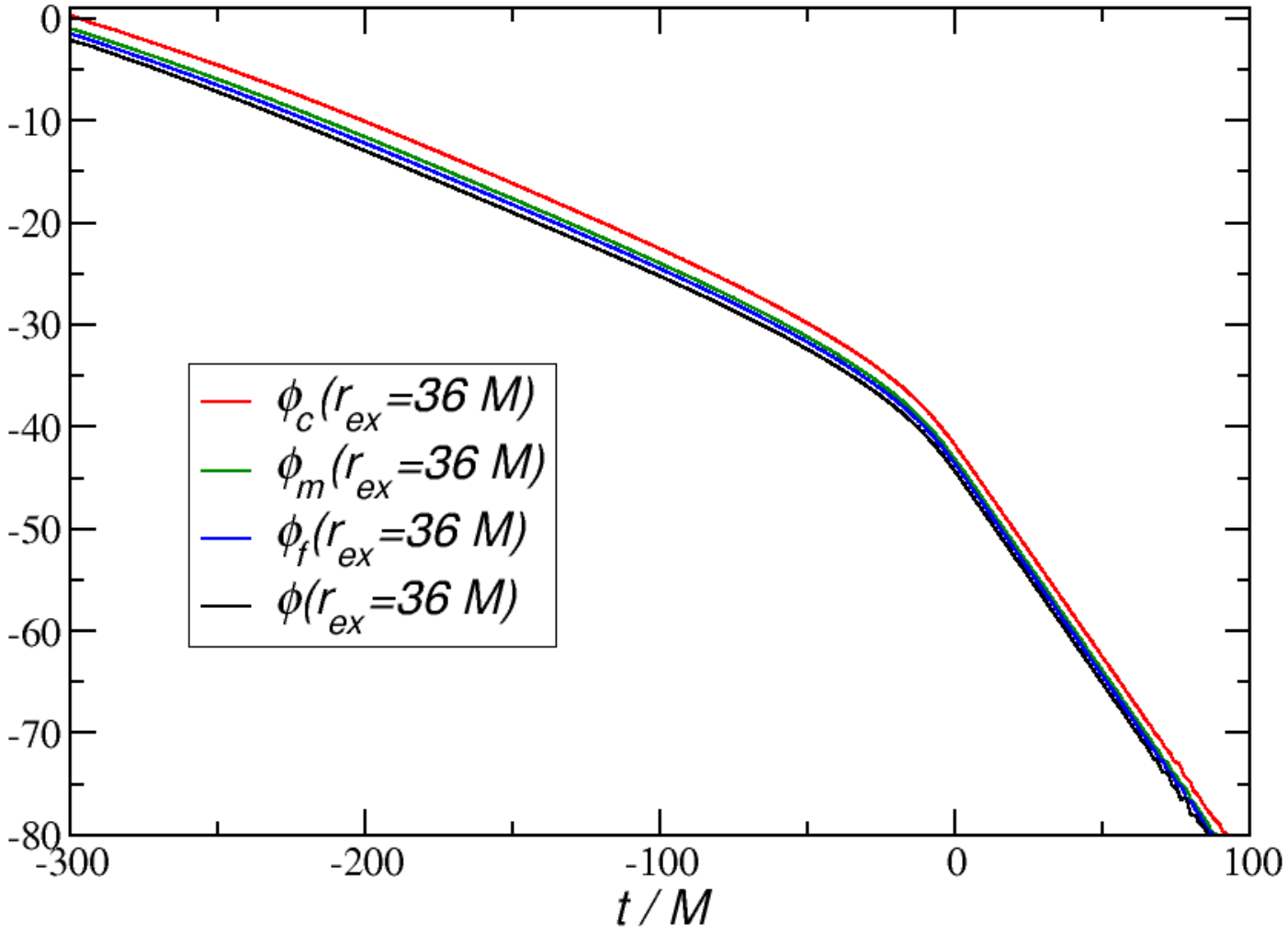
$$\phi = \phi_0 + \phi_1\, r_{\rm ex}^{-1} \left(+ \phi_2\, r_{\rm ex}^{-2} \right)$$

$$A = A_0 + A_1\, r_{\rm ex}^{-1} \left(+ A_2\, r_{\rm ex}^{-2} \right)$$

Accuracy of waveform: Phase



Accuracy of waveform: Phase



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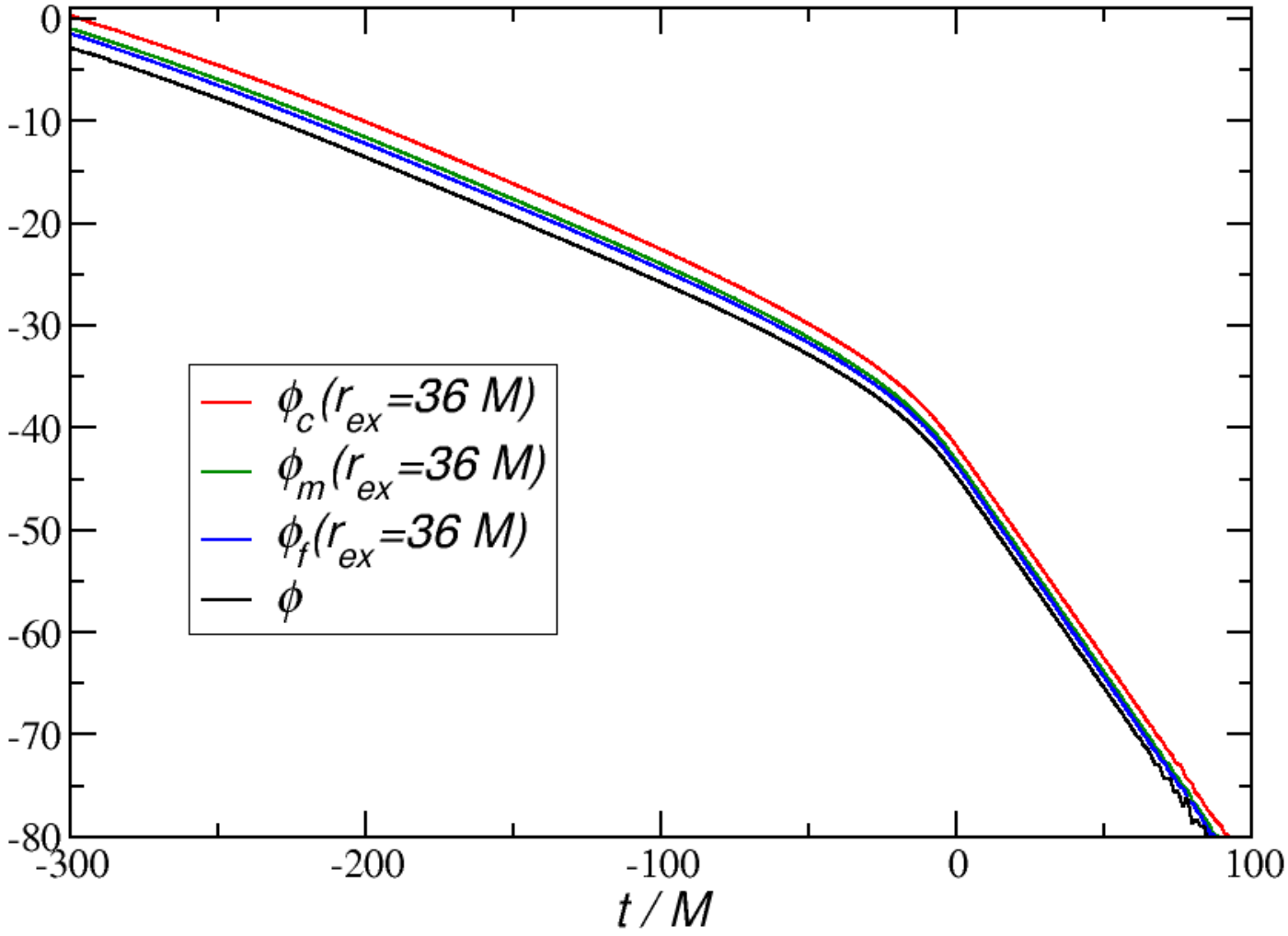
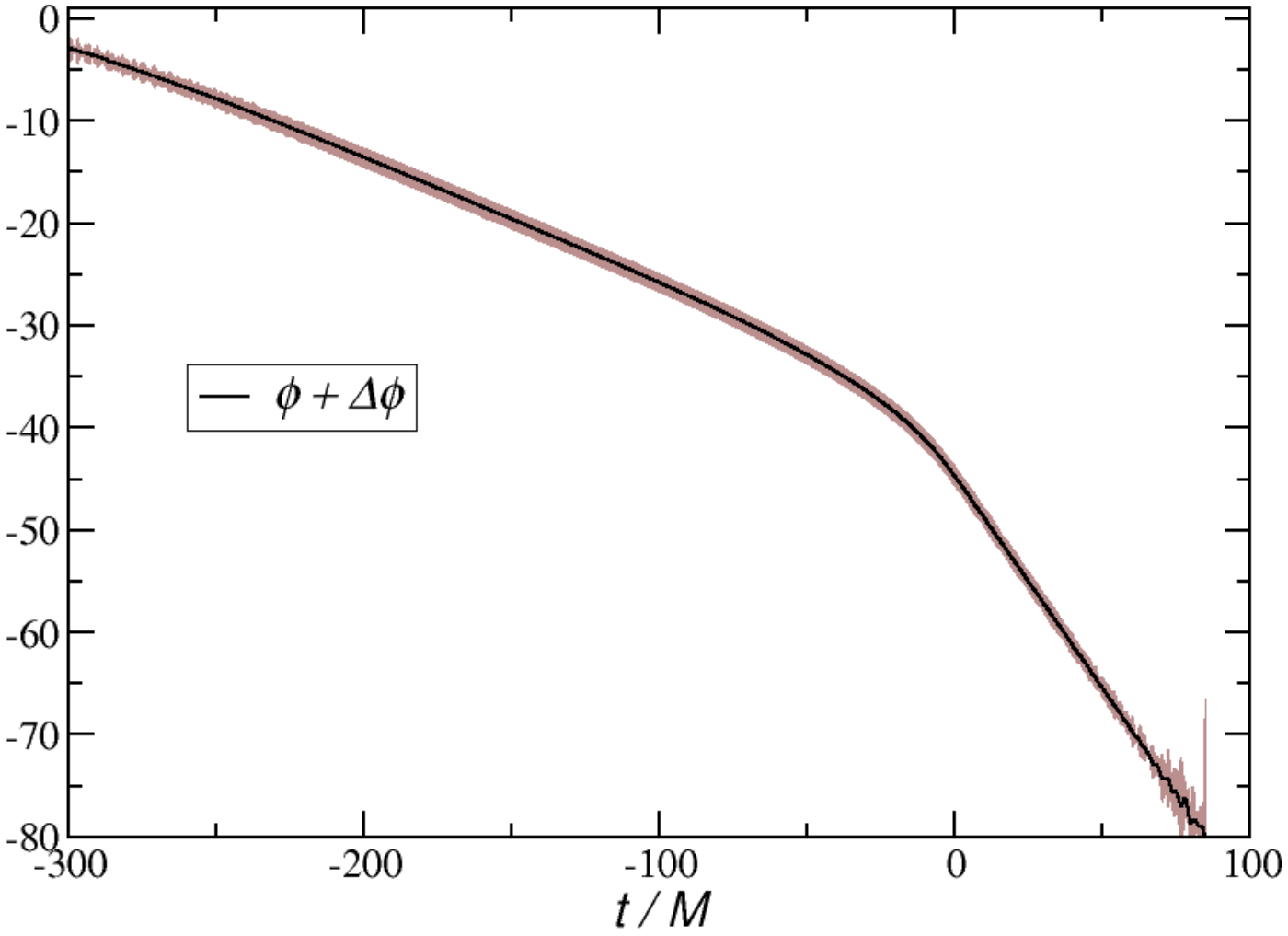
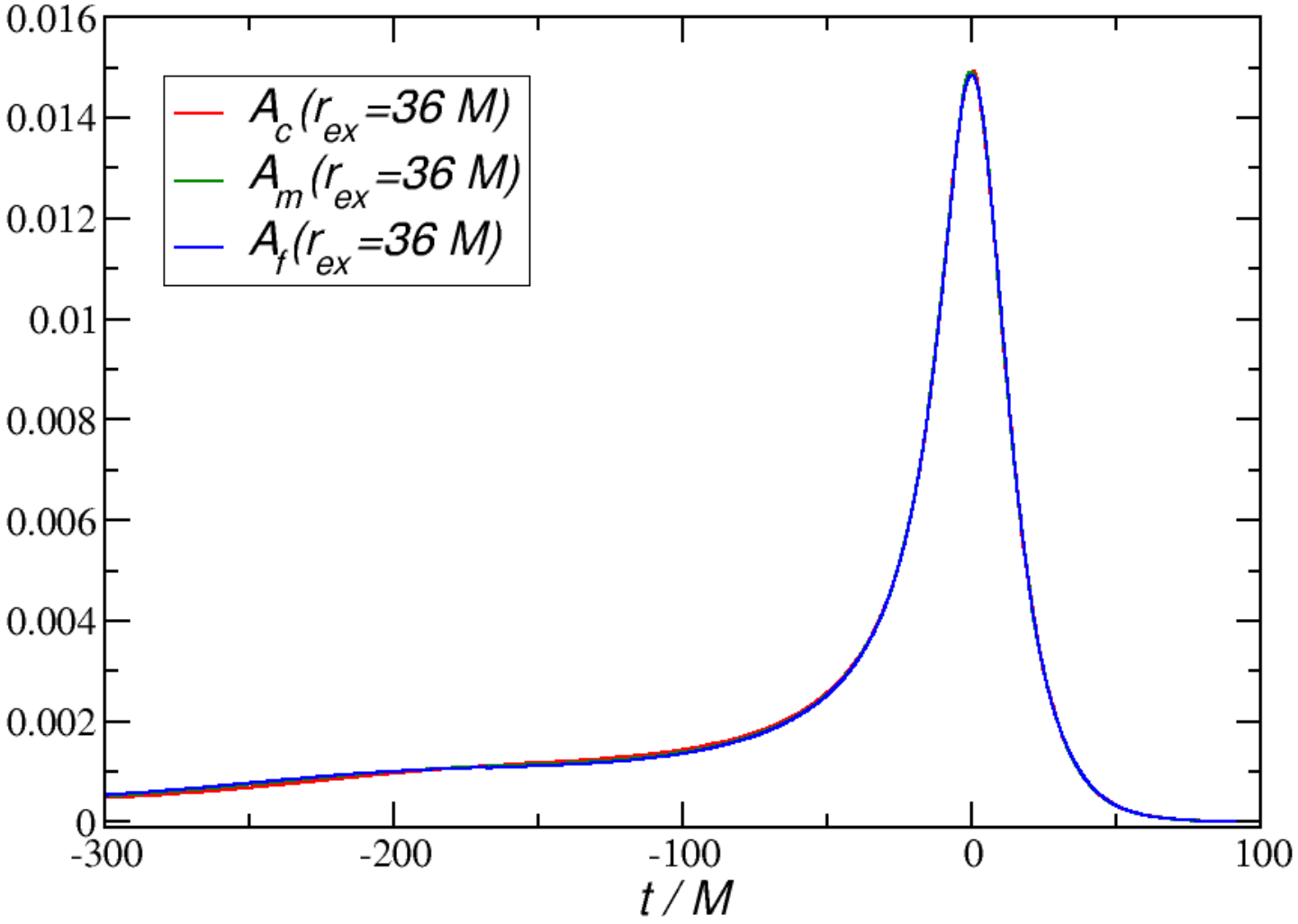


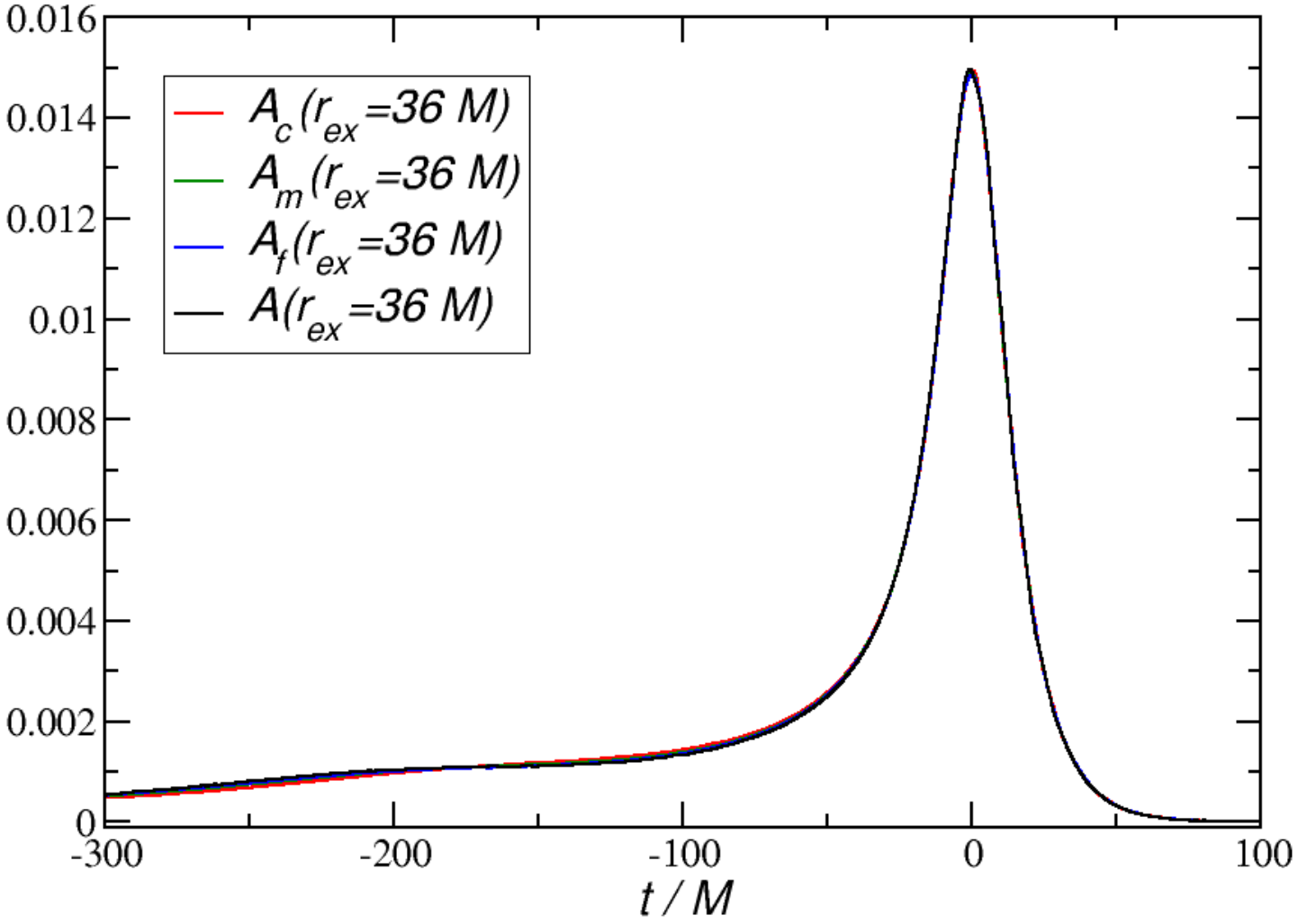
Accuracy of waveform: Phase



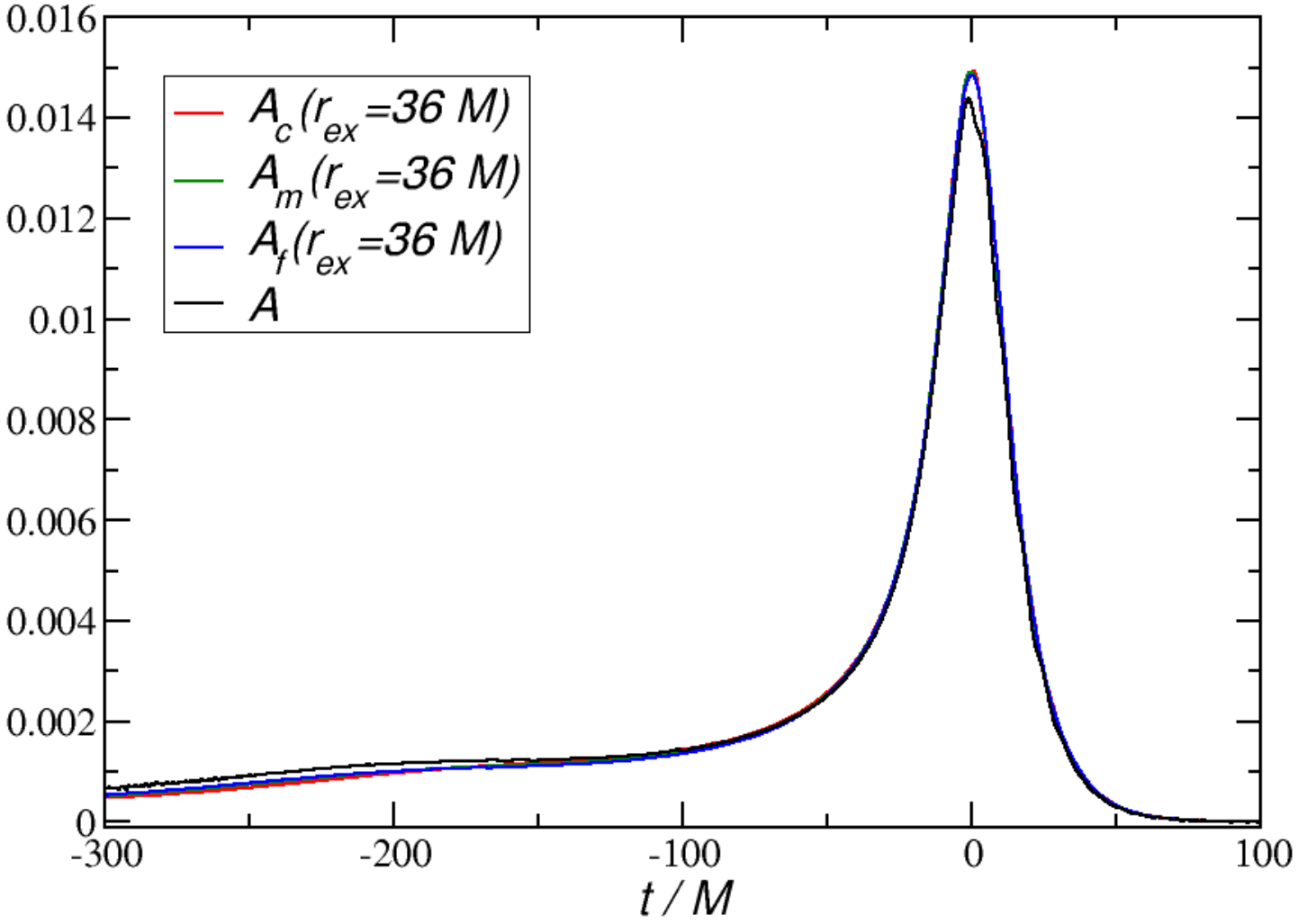
Accuracy of waveform: Amplitude



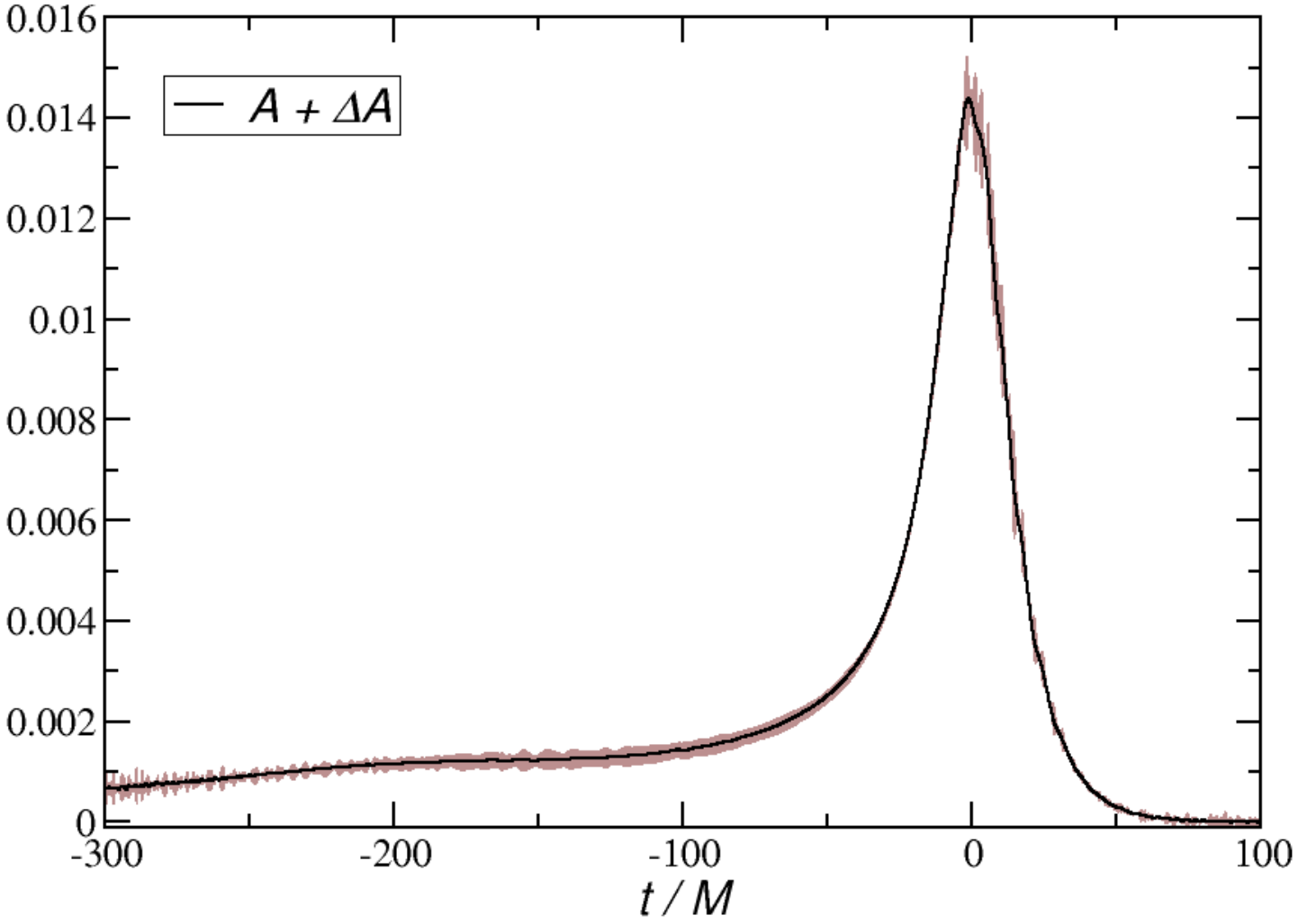
Accuracy of waveform: Amplitude



Accuracy of waveform: Amplitude



Accuracy of waveform: Amplitude



Summary

- Higher multipoles significant in energy and wave signal
- Accuracy limited by small and few r_{ex}
- Local maximum in A difficulties in extrapolating
- Phase accuracy: $\Delta\phi \approx 0.6$ (optimistic)
  $\Delta\phi \approx 0.8$ (conservative)
- Amplitude accuracy: $\frac{\Delta A}{A} \approx 6\,\%$ (optimistic)
  $\frac{\Delta A}{A} \approx 10\,\%$ (conservative)

Numerical framework: q=4 (long)

- Lean code Sperhake '07
  - Puncture initial data
  - BSSN evolution
  - Moving puncture gauge
  - FD 6th order in space, 4th order in time
- Parameters (about 20 cycles)

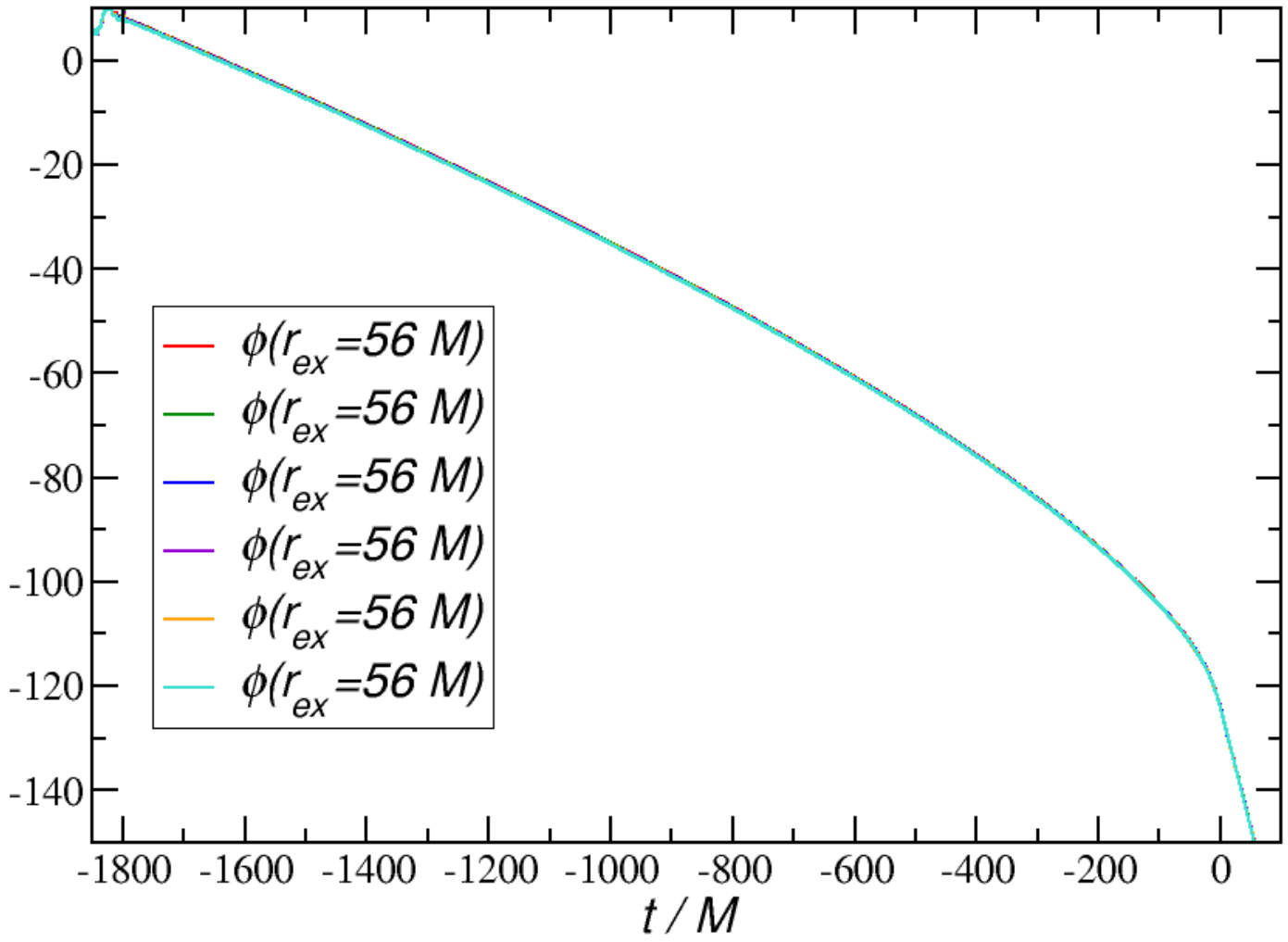
$$M := M_1 + M_2 \qquad q := \frac{M_1}{M_2} = 4$$

$$D = 10.9\, M \qquad \eta := \frac{M_1 M_2}{(M_1 + M_2)^2} = 0.16$$

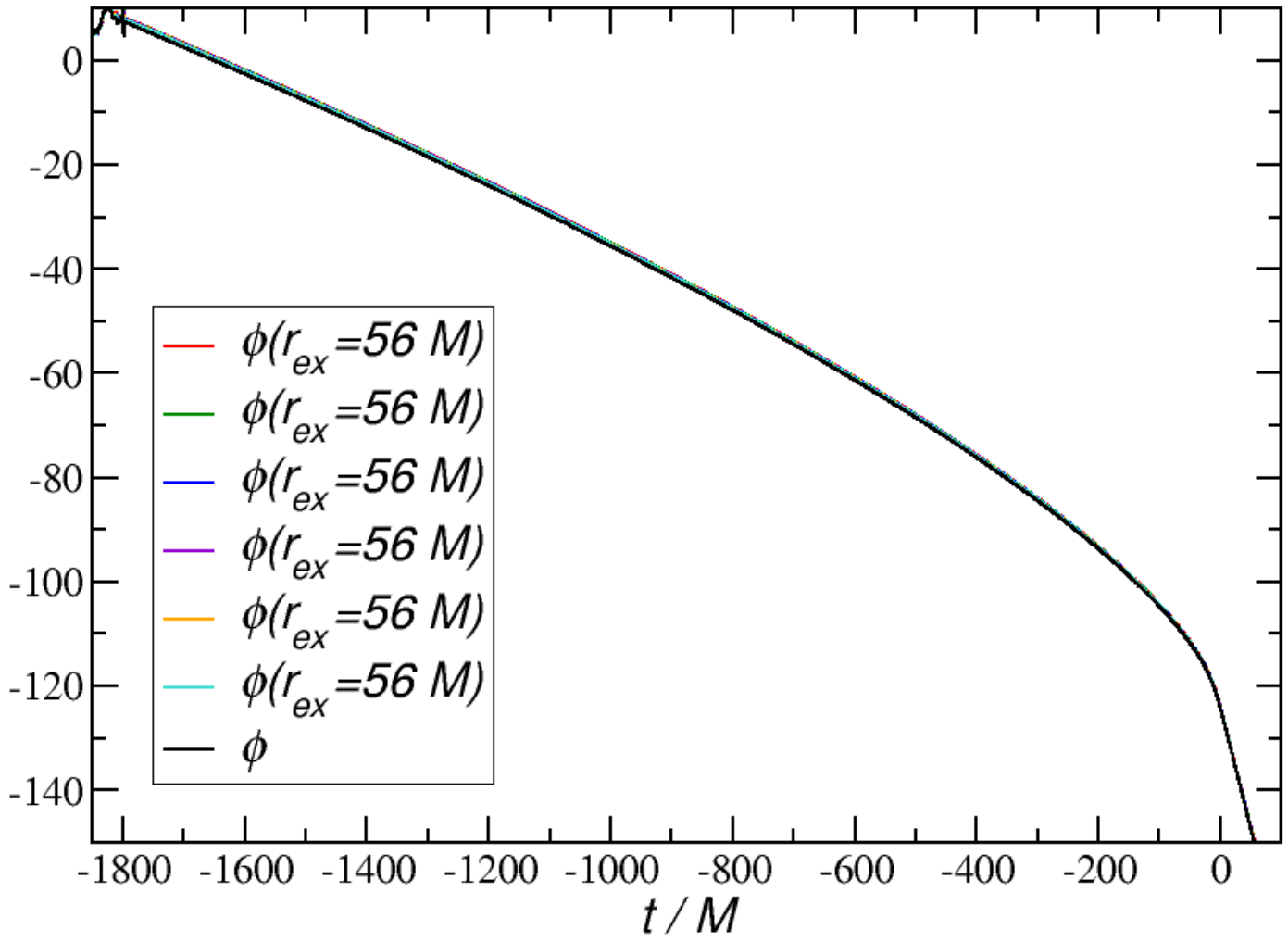
$$\boldsymbol{S}_1 = \boldsymbol{S}_2 = 0$$

- Reduced eccentricity Caltech/Cornell '07
- Work in progress: Only extraction radius so far…

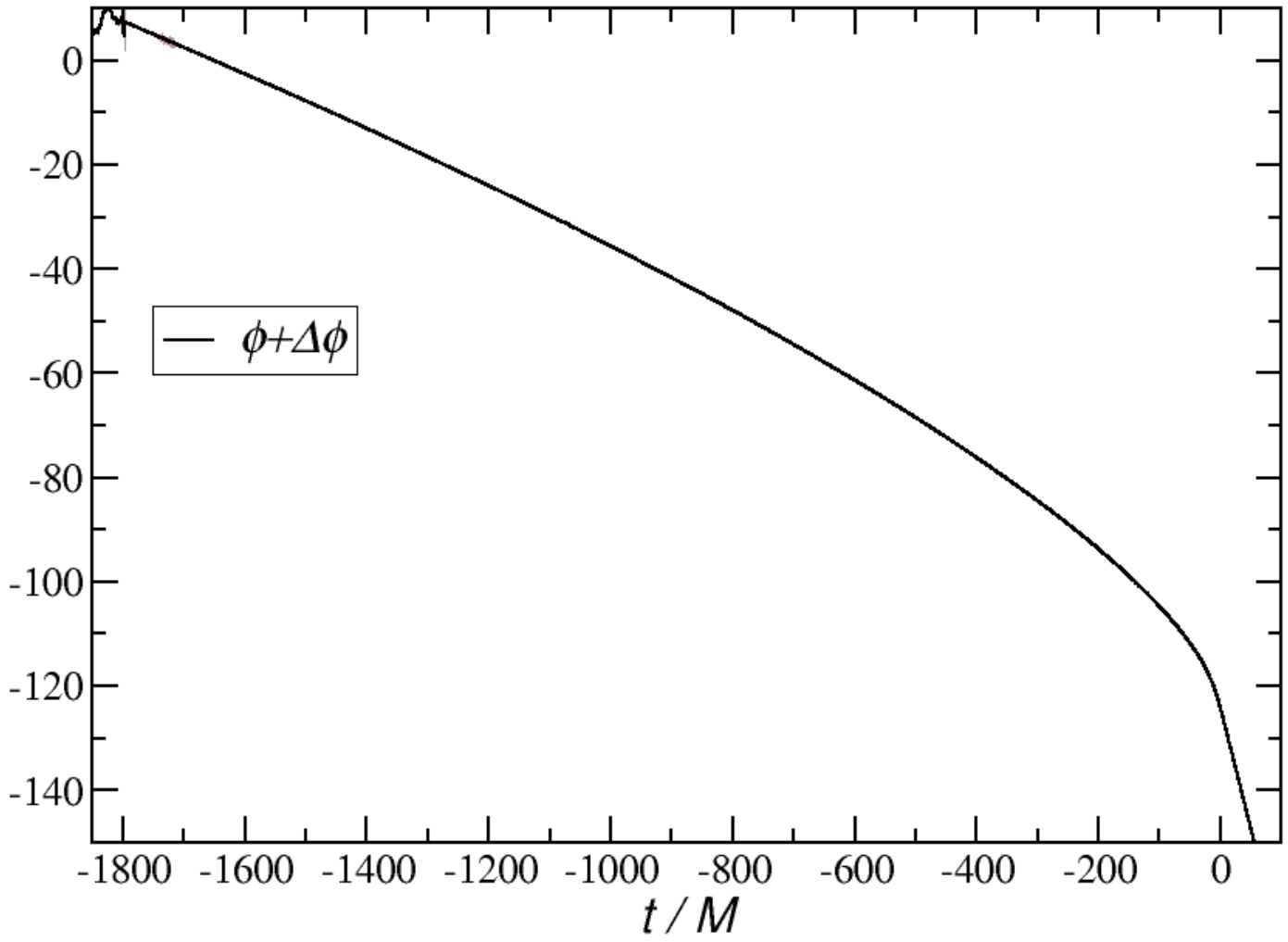
Accuracy of waveform: Phase



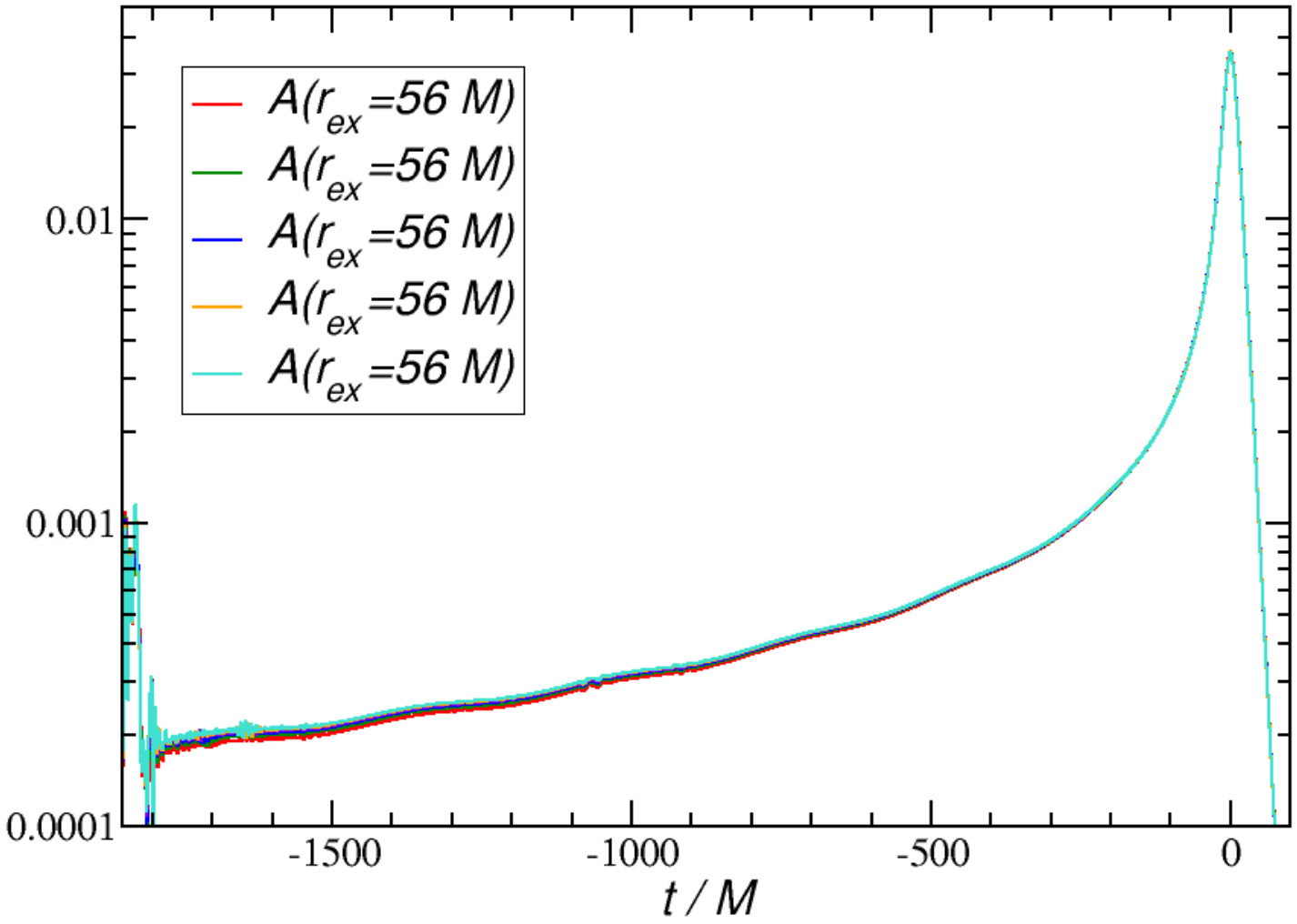
Accuracy of waveform: Phase



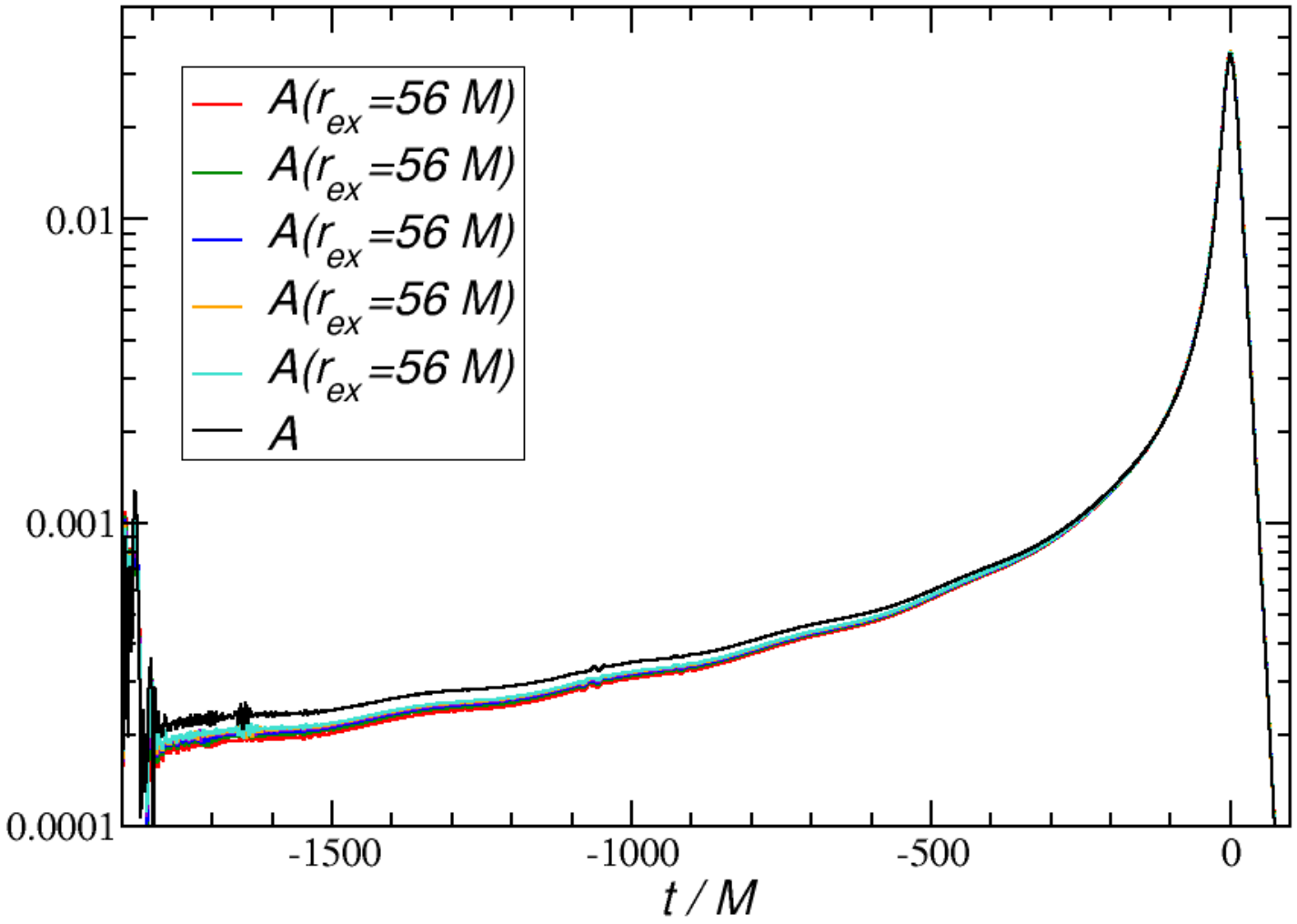
Accuracy of waveform: Phase



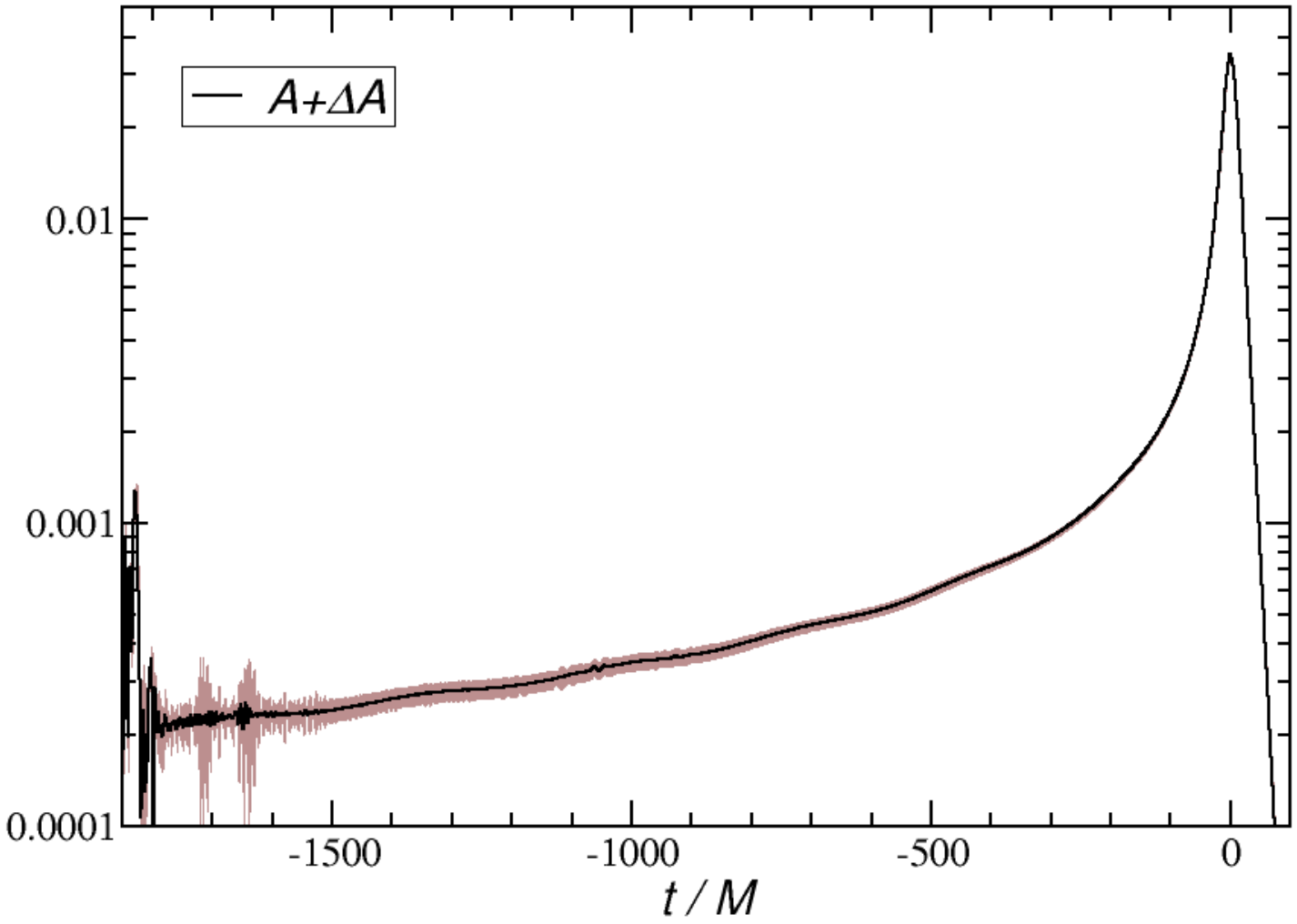
Accuracy of waveform: Amplitude



Accuracy of waveform: Amplitude



Accuracy of waveform: Amplitude



Summary

- Resolution $M/160$ already appears to give reaonable accuracy for long simulations
- Phase accuracy: $\Delta\phi \approx 0.01...0.02$ (inspiral)

  $\Delta\phi \approx 0.02...0.09$ (plunge, merger, ringdown)
- Amplitude accuracy: $\frac{\Delta A}{A} \approx 7\,\%$ (early)

  $\frac{\Delta A}{A} \approx 2\,\%$ (merger)
- Expect improvement by fitting higher order power laws!
- Warning: Low resolution dissipates GWs; Don't xpol!!!

Discussion

- Kick, final spin and radiated energy for $q = 10$ agree well with formulae
- Simulations of spinning BHBs needed to check formulae
- Higher order multipoles important!
- Accuracy limited by extraction radii; better than expected

$$\Delta\phi \approx 0.6, \qquad \frac{\Delta A}{A} \approx 6\,\%$$

- Long runs with $q = 4$ "easier"

$$\Delta\phi \approx 0.02...0.09, \qquad \frac{\Delta A}{A} \approx 2...7\,\%$$

- Convergence study to be completed and included in uncertainties