Black hole collisions and gravitational waves

U. Sperhake

CSIC-IEEC Barcelona
California Institute of Technology
University of Mississippi

University of Southampton General Relativity Seminar
31$^{\text{th}}$ March 2011

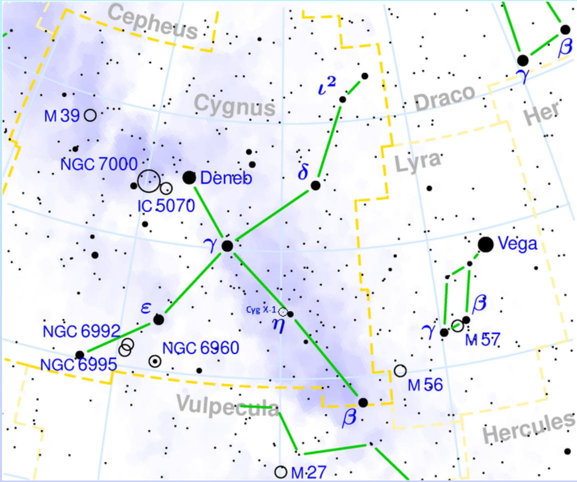
Overview

- Motivation
- Modeling black holes in GR
- Black holes in astrophysics
- Black holes in fundamental physics
  - Trans Planckian scattering
  - Non-assymptotically flat boundaries: AdS/CFT
- Other topics in $D \geq 5$
  - Instabilities of Myers-Perry BHs
  - Cosmic censorship in $D \geq 5$
- Summary

1. Motivation

Black holes are out there: Stellar BHs

- high-mass X-ray binaries: Cygnus X-1 (1964)

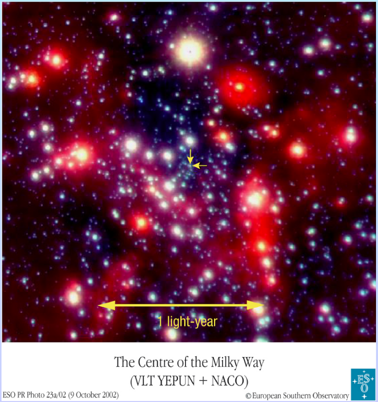


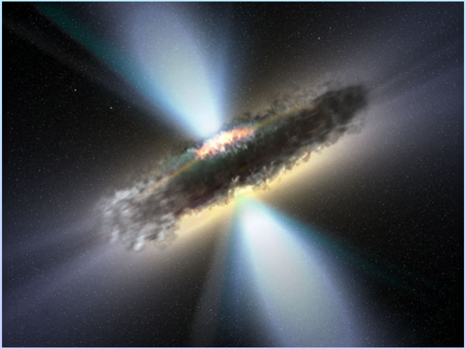
Black holes are out there: Stellar BHs

- One member is very compact and massive ⇒ Black Hole

Black holes are out there: galactic BHs

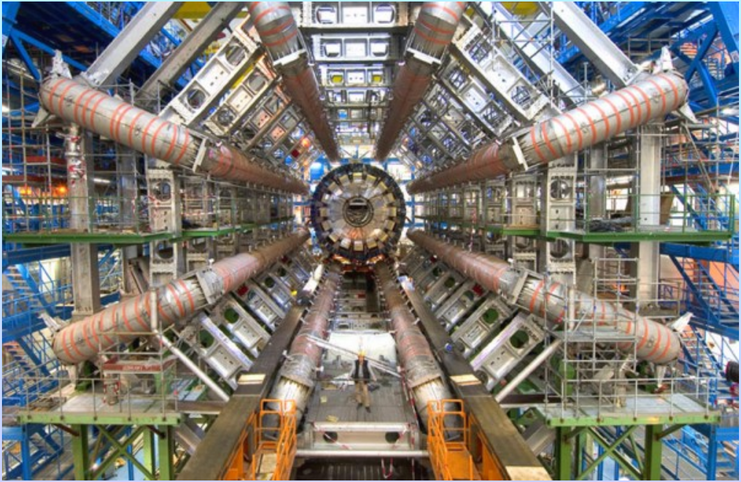
- Supermassive BHs found at center of virtually all galaxies
- SMBHs conjectured to be responsible for quasars starting in the 1980s





Black holes might be in here: LHC

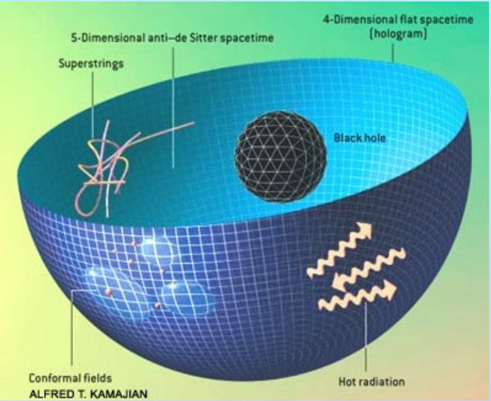
- LHC CERN



Motivation (AdS/CFT correspondence)

- BH spacetimes “know” about physics without BHs

  AdS/CFT correspondence   Maldacena ’97



2. Modeling black holes in GR

How to get the metric?

Train cemetery
Uyuni, Bolivia

- Solve for the metric $g_{\alpha\beta}$

How to get the metric?

- The metric must obey the Einstein Equations
- Ricci-Tensor, Einstein Tensor, Matter Tensor

  $R_{\alpha\beta} \equiv R^{\mu}{}_{\alpha\mu\beta}$

  $G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\mu}{}_{\mu}$ “Trace reversed” Ricci

  $T_{\alpha\beta}$ “Matter”
- Einstein Equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$
- Solutions: Easy! Take metric

  $\Rightarrow$ Calculate $G_{\alpha\beta}$

  $\Rightarrow$ Use that as matter tensor
- Physically meaningful solutions: Difficult! $\Rightarrow$ Numerics!

A list of tasks

- Target: Predict time evolution of BBH in GR
- Einstein equations: 1) Cast as evolution system
  2) Choose specific formulation
  3) Discretize for computer
- Choose coordinate conditions: Gauge
- Fix technical aspects: 1) Mesh refinement / spectral domains
  2) Singularity handling / excision
  3) Parallelization
- Construct realistic initial data
- Start evolution and waaaaiiiiit...
- Extract physics from the data

3. Black holes in astrophysics

Free parameters of BH binaries

- Total mass M

  Relevant for GW detection: Frequencies scale with M

  Not relevant for source modeling: trivial rescaling

- Mass ratio $q \equiv \frac{M_1}{M_2}$, $\eta \equiv \frac{M_1 M_2}{(M_1+M_2)^2}$

- Spin: $\vec{S}_1$, $\vec{S}_2$ (6 parameters)

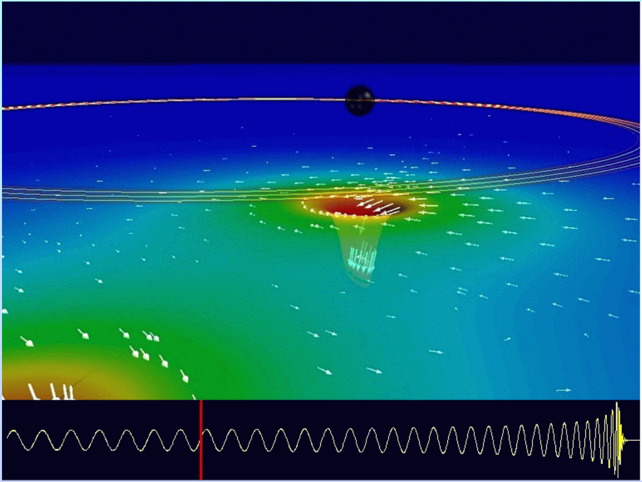
- Initial parameters

  Binding energy E_b — Separation

  Orbital ang. momentum L — Eccentricity

  Alternatively: frequency, eccentricity

Morphology of a BBH inspiral



Thanks to Caltech, CITA, Cornell

Gravitational recoil

- Anisotropic GW emission $\Rightarrow$ recoil of remnant BH

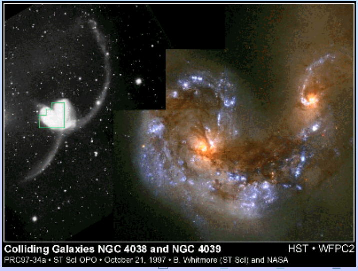
  Bonnor & Rotenburg '61, Peres '62, Bekenstein '73

- Escape velocities:

Globular clusters	30 $\mathrm{km/s}$
dSph	$20 - 100$ $\mathrm{km/s}$
dE	$100 - 300$ $\mathrm{km/s}$
Giant galaxies	~ 1000 $\mathrm{km/s}$

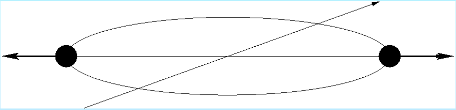
Ejection / displacement of BH $\Rightarrow$

- Growth history of SMBHs
- BH populations, IMBHs
- Structure of galaxies



Superkicks

- Kidder '95, UTB-RIT '07: maximum kick expected for



- Measured kicks $v \approx 2500\ \mathrm{km/s}$ for spin $a \approx 0.75$

  Extrapolated to maximal spins: $v_{\max} \approx 4000\ \mathrm{km/s}$

  González *et al.* '07, Campanelli *et al.* '07

- Unlikely configuration!

  Bogdanović *et al.* '07, Kesden, US & Berti '10, '10a

- Hyperbolic encounters: v up to $10000\ \mathrm{km/s}$

  Healy *et al.* '08

Spin precession and flip

- X-shaped radio sources

  Merrit & Ekers '07

- Jet along spin axis

- Spin re-alignment

  $\Rightarrow$ new + old jet

- Spin precession 98°

  Spin flip 71°

  UTB-RIT '06

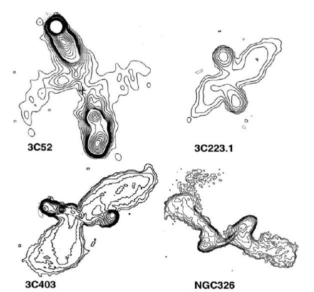
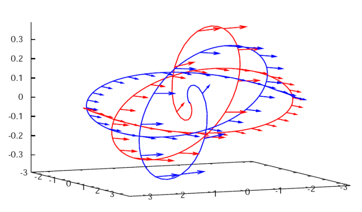


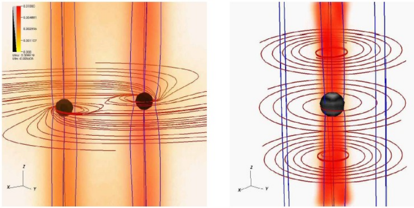
Fig. 2. A composite of four clear examples of the X-shape morphology. VLA radio observations of 3C52 (28), 3C223.1 (29), 3C403 (29), and NGC 326 (12).



Jets generated by binary BHs

Palenzuela, Lehner & Liebling ’10

- Non-spinning BH binary
- Einstein-Maxwell equtions with “force free” plasma
- Electromagnetic field extracts energy from **L** $\Rightarrow$ jets
- Optical signature: double jets

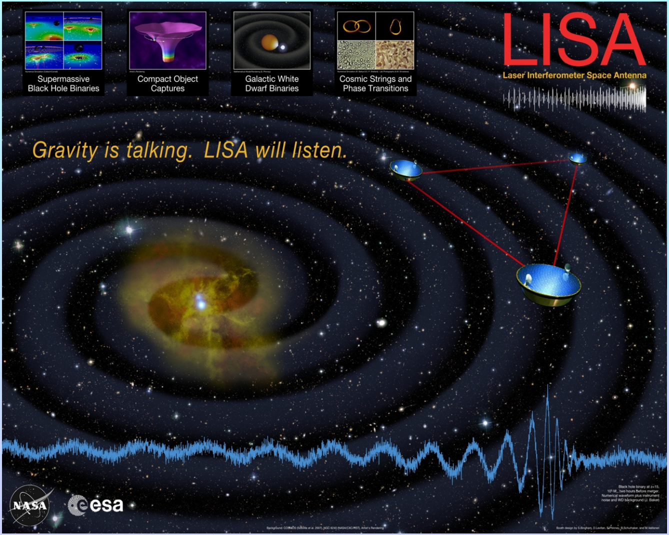


(a) $-8.2\,M_8$ hrs (b) $4.6\,M_8$ hrs

Gravitational Wave observations

- Accelerated masses generate GWs
- Interaction with matter very weak!
- Earth bound detectors: LIGO, VIRGO, GEO600, LCGT

Space interferometer LISA



Some targets of GW physics

- Confirmation of GR

  Hulse & Taylor 1993 Nobel Prize

- Parameter determination

  of BHs: M, $\vec{S}$

- Optical counter parts

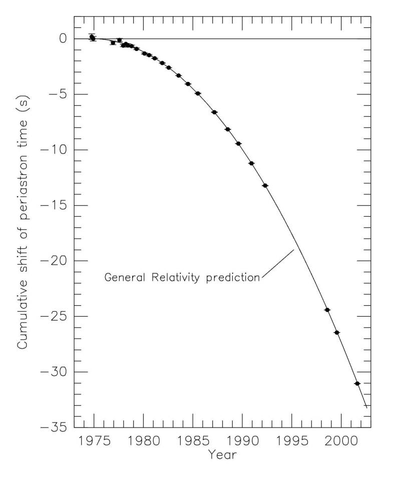
  Standard sirens (candles)

  Mass of graviton

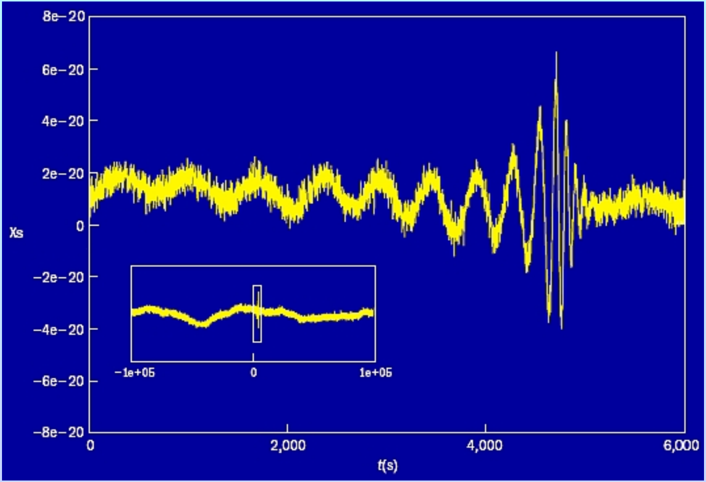
- Test Kerr Nature of BHs

- Cosmological sources

- Neutron stars: EOS



Matched filtering



Long, accurate waveforms required

⇒ combine NR with PN, perturbation theory

3. Black holes in fundamental physics

So what other interesting physics can we do with NR?

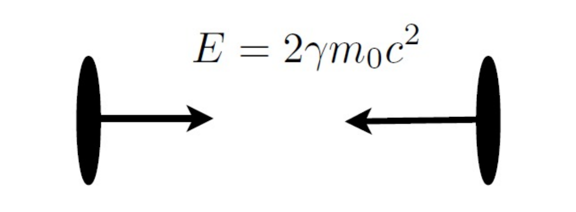
- High-energy physics
  - Trans-Planckian scattering
  - AdS/CFT duality
- Mathematical physics and theoretical physics
  - Cosmic censorship
  - Critical phenomena
  - BH instabilities (Myers-Perry)

3.1. Transplanckian scattering

BH formation and hoop conjecture

- Hoop conjecture

  Thorne '72



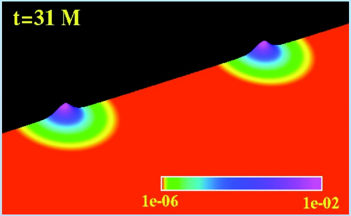
- de Broglie wavelength: $\lambda = \frac{hc}{E}$
- Schwarzschild radius: $r = \frac{2GE}{c^4}$
- BH will form if $\lambda < r \quad \Leftrightarrow \quad E \gtrsim \sqrt{\frac{hc^5}{G}} \equiv E_{\mathrm{Planck}}$

BH formation in boson field collisions

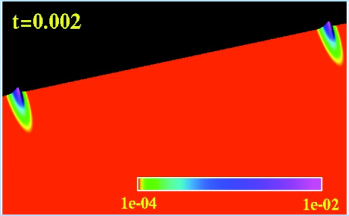
Pretorius & Choptuik '09

- Einstein plus minimally coupled, massive, complex scalar filed

  "Boson stars"



$\gamma = 1$



$\gamma = 4$

- BH formation threshold: $\gamma_{\text{thr}} = 2.9 \pm 10\ \%$
- About $1/3$ of hoop conjecture prediction

Motivation (High-energy physics)

- Matter does not matter at energies well above the Planck scale

  ⇒ Model particle collisions by black-hole collisions

  Banks & Fischler '99; Giddings & Thomas '01

- TeV-gravity scenarios

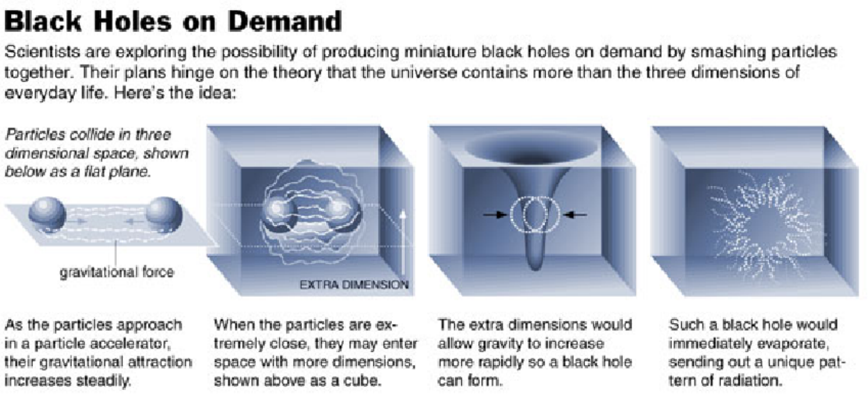
  ⇒ The Planck scale might be as low as TeVs due to extra dimensions

  Arkani-Hamed, Dimopulos & Dvali '98, Randall & Sundrum '99

  ⇒ Black holes could be produced in colliders

  Eardley & Giddings '02, Dimopoulos & Landsberg '01,...

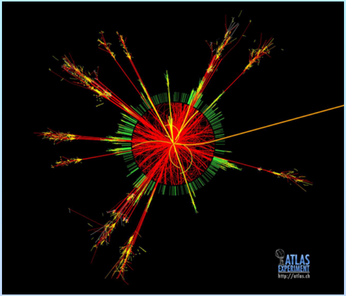
Motivation (High-energy physics)



Experimental signature at the LHC

Black hole formation at the LHC could be detected by the properties of the jets resulting from Hawking radiation.

- Multiplicity of partons: Number of jets and leptons
- Large transverse energy
- Black-hole mass and spin are important for this!



ToDo:

- Exact cross section for BH formation
- Determine loss of energy in gravitational waves
- Determine spin of merged black hole

Black-hole collisions in $D = 4$

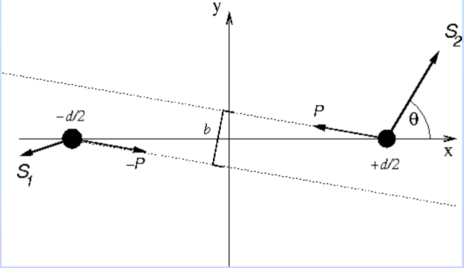
- Take two black holes

  Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

  Initial position: $\pm x_0$

  Linear momentum: $\mp P[\cos\alpha,\ \sin\alpha,\ 0]$

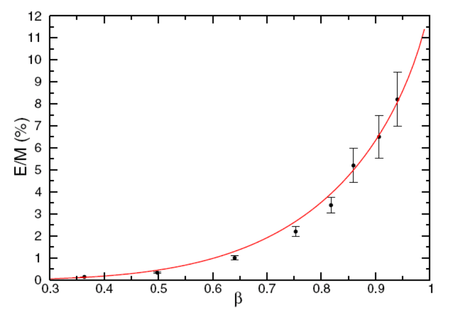
- Impact parameter: $b \equiv \frac{L}{P}$



Head-on collisions: $b = 0, \quad \vec{S} = 0$

- Total radiated energy: 14 ± 3 % for $v \to 1$ Sperhake *et al.* '08

  About half of Penrose '74

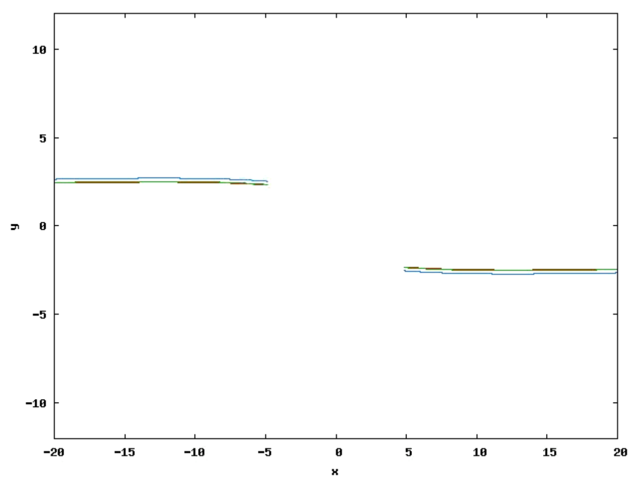


- Agreement with approximative methods

  Flat spectrum, multipolar GW structure Berti *et al.* '10

Grazing collisions: $b \neq 0$, $\vec{S} = 0$, $\gamma = 1.52$

Immediate vs. Delayed vs. No merger   Sperhake *et al.* '09



Critical impact parameter

- $b < b_{\text{crit}} \quad \Rightarrow \quad$ Merger

  $b > b_{\text{crit}} \quad \Rightarrow \quad$ Scattering

- Numerical study: $\quad b_{\text{crit}} = \frac{2.5 \pm 0.05}{v} M$

  Shibata *et al.* '08

- Independent study by Sperhake *et al.* '09

  $\gamma = 1.52$: $\quad 3.39 < b_{\text{crit}}/M < 3.4$

  $\gamma = 2.93$: $\quad 2.3 < b_{\text{crit}}/M < 2.4$

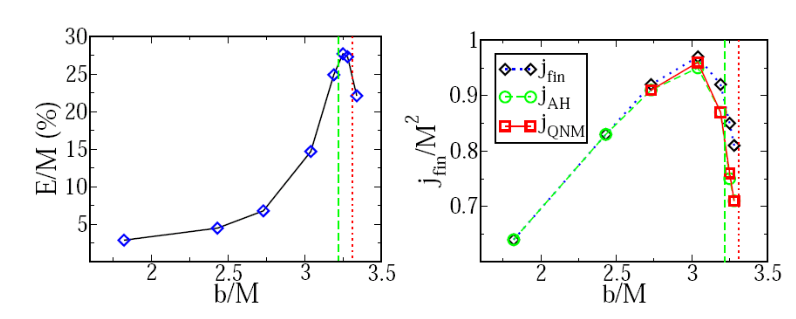
  $v \to 1$ limit still needs to be determined

- Limit from Penrose construction: $b_{\text{crit}} = 1.685\ M$

  Yoshino & Rychkov '05

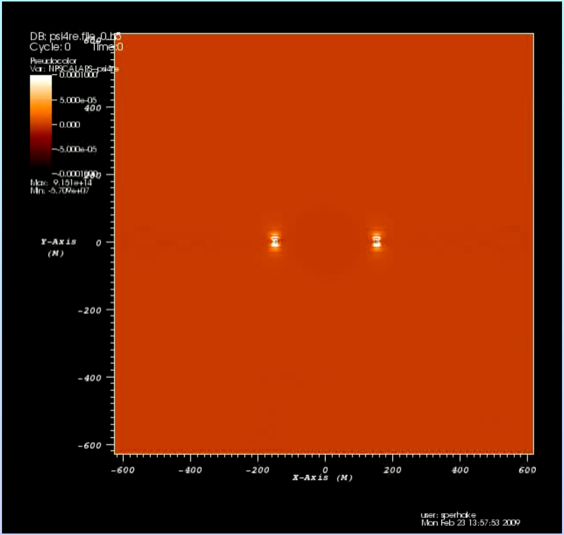
Radiated quantities

- $b-$sequence with $\gamma = 1.52$
- Final spin close to Kerr limit
- $E_{\rm rad} \sim 35$ % for $\gamma = 2.93$; about 10 % of Dyson luminosity

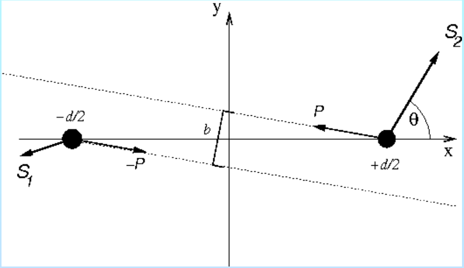


Sperhake *et al.* '09

Gravitational radiation: Delayed merger



Recoil in grazing collisions

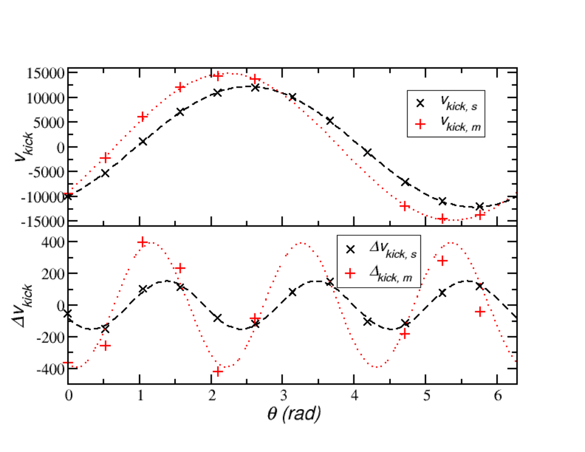


- equal-mass, superkick, $\chi = 0.621$
- $\gamma = 1.52$
- 2 sequences

  merging: $b = 3.34\ M$

  scattering: $b = 3.25\ M$

Recoil in grazing collisions



Expansion in θ according to Boyle, Kesden & Nissanke '08

Recoil in grazing collisions

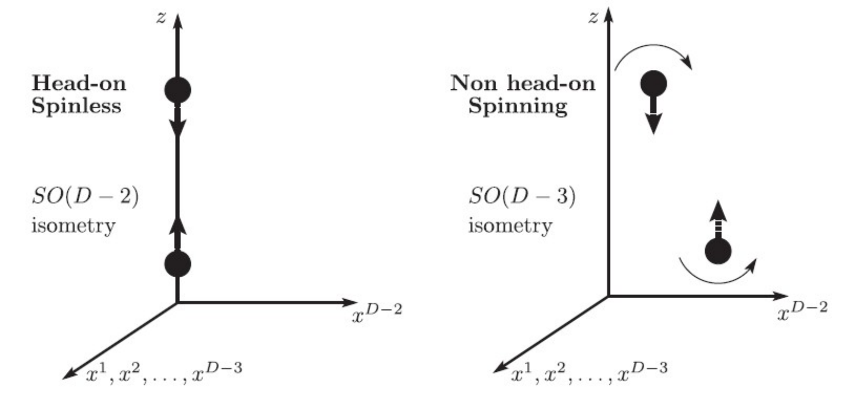
- $v_{\mathrm{max,s}} = 12\,200\ \mathrm{km/s}$
  $v_{\mathrm{max,m}} = 14\,900\ \mathrm{km/s}$
- Large recoils for merger and scattering!
- $v_{\mathrm{max}} \propto E_{\mathrm{rad}}$
- Antikicks can occur in both $\Rightarrow$ not a merger-only feature!
- Ultimate kick
  $v_{\mathrm{max}} \propto E_{\mathrm{rad}} \quad \Rightarrow \quad \sim 45\,000\ \mathrm{km/s}$
  spin insignificant for large $\gamma \quad \Rightarrow \quad \sim 25\,000\ \mathrm{km/s}$
  no simple picture $\Rightarrow$ more data needed...

Moving to $D > 4$

- Symmetries allow dimensional reduction

  Geroch '70

- Reduces to "3+1" plus quasi-matter terms: scalar field



BSSN formulation with quasi matter

$$\partial_t \tilde{\gamma}_{ij} = [\text{BSSN}],$$

$$\partial_t \chi = [\text{BSSN}],$$

$$\partial_t K = [\text{BSSN}] + 4\pi\alpha(E + S),$$

$$\partial_t \tilde{A}_{ij} = [\text{BSSN}] - 8\pi\alpha\left(\chi S_{ij} - \frac{1}{3} S \tilde{\gamma}_{ij}\right),$$

$$\partial_t \tilde{\Gamma}^i = [\text{BSSN}] - 16\pi\alpha\chi^{-1} j^i,$$

$$\partial_t \zeta = -2\alpha K_\zeta + \beta^m \partial_m \zeta - \frac{2}{3}\zeta\partial_m\beta^m + 2\zeta\frac{\beta^y}{y},$$

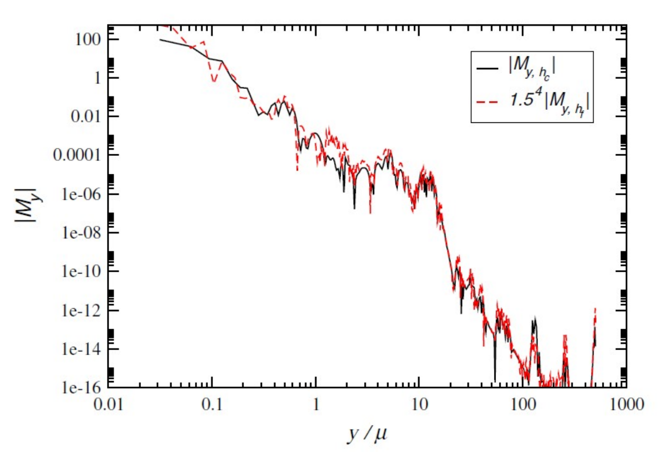
$$\partial_t K_\zeta = \ldots \ ,$$

$$E,\ j^i,\ S_{ij} = f(\text{BSSN},\ \zeta,\ K_\zeta).$$

Zilhão *et al.* '10

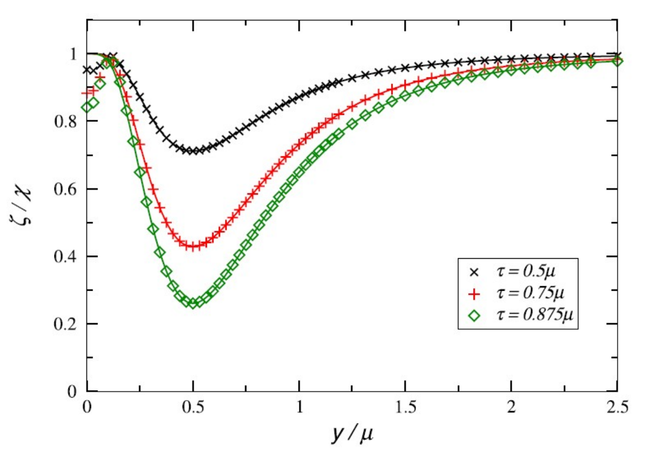
Single black hole in $D = 5$

Initial data: Tangherlini '63



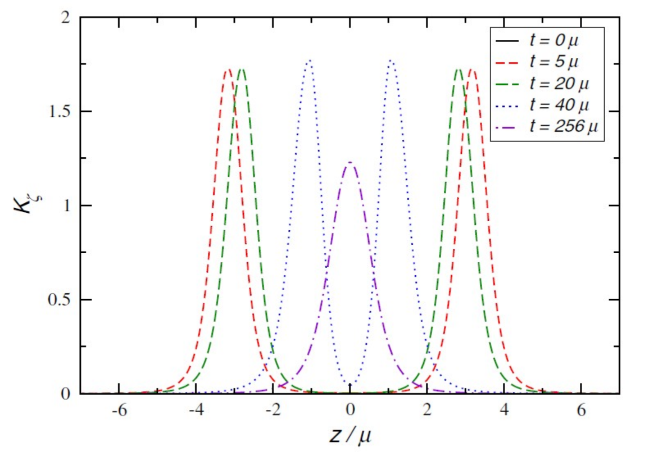
Single black hole in $D = 5$

In geodesic slicing

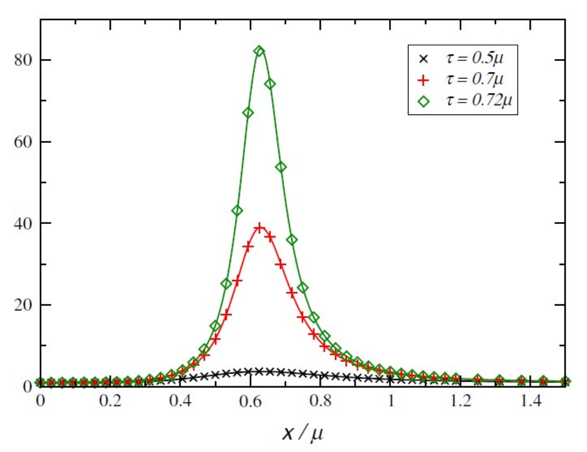


Head-on in $D = 5$

Initial data: $D = 5$ analogue of Brill-Lindquist data

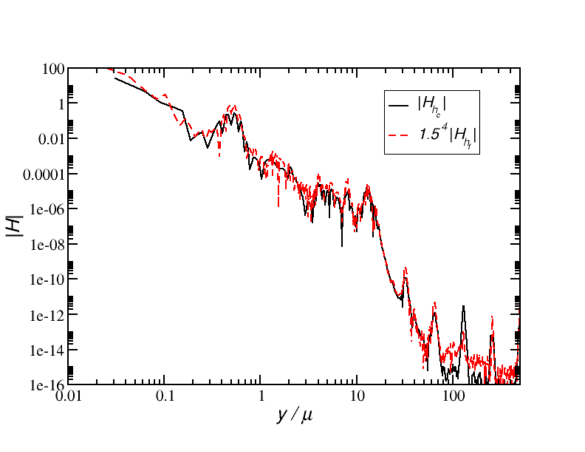


Single black hole in $D = 6$



Geoesic slicing, zero shift

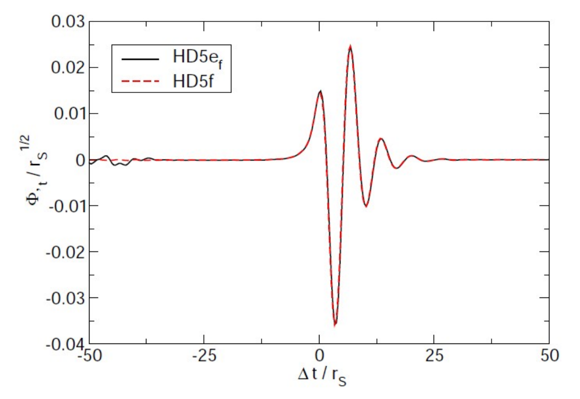
Single black hole in $D = 6$



ToDo: long term stable evolutions

GWs from head-on in $D = 5$

Wave extraction based on Kodama & Ishibashi '03

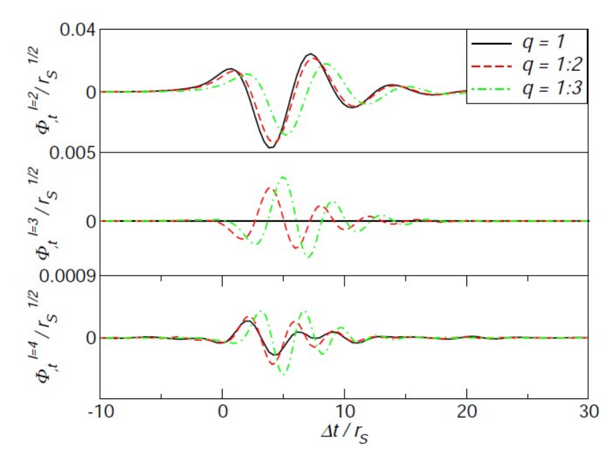


$E_{\text{rad}} = 0.089\ \%M$ cf. $0.055\ \%M$ in $D = 4$

Witek *et al.* '10a

Unequal-mass head-on in $D = 5$

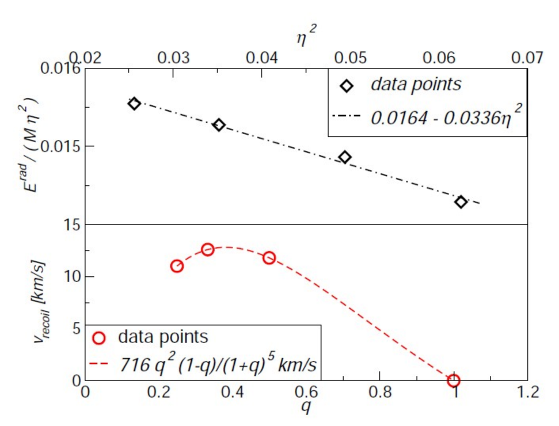
Kodama-Ishibashi multipoles



Witek *et al.* '10b

Unequal-mass head-on in $D = 5$

Radiated energy and momentum

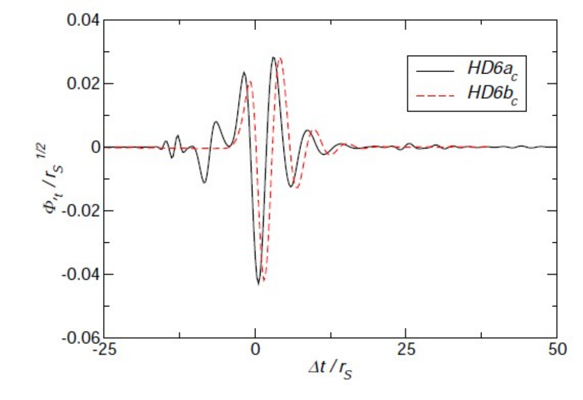


Agreement within $< 5\ \%$ with extrapolated point particle calculations

Breaking news!

First black-hole collisions in $D = 6$

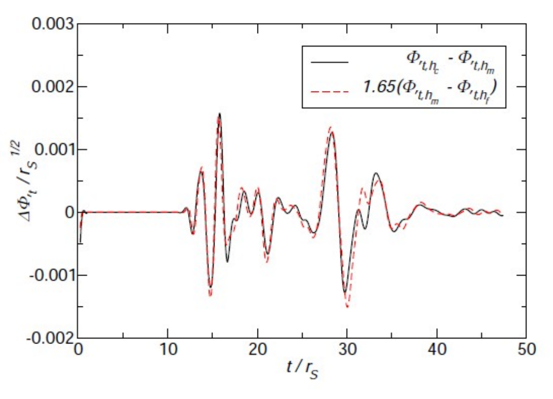
Witek *et al.* '10



- Adjust shift parameters
- Use LaSh system Witek, Hilditch & US '10

First black-hole collisions in $D = 6$

Witek *et al.* '10

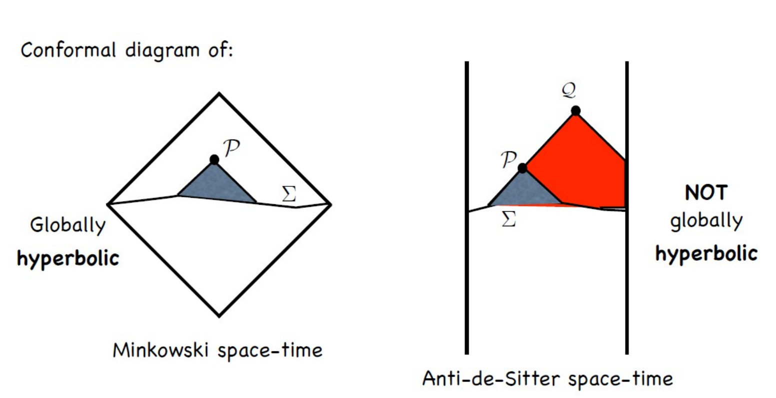


Second order convergence

3.2. Non-assymptotically flat boundaries: AdS/CFT

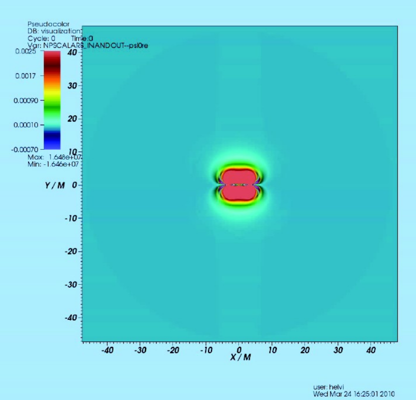
AdS/CFT correspondence

Challenge: Model the active role of the boundary!



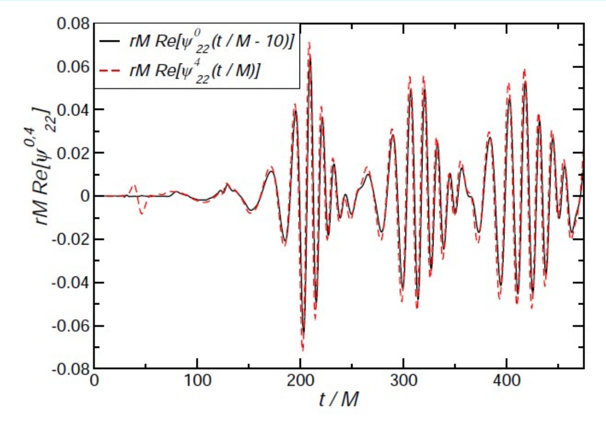
Toy model: Black hole inspiral in a lego sphere

- Lego sphere with reflective boundary
- Goddard R1 run Baker *et al.* ’06
- Calculate Ψ_4 and Ψ_0

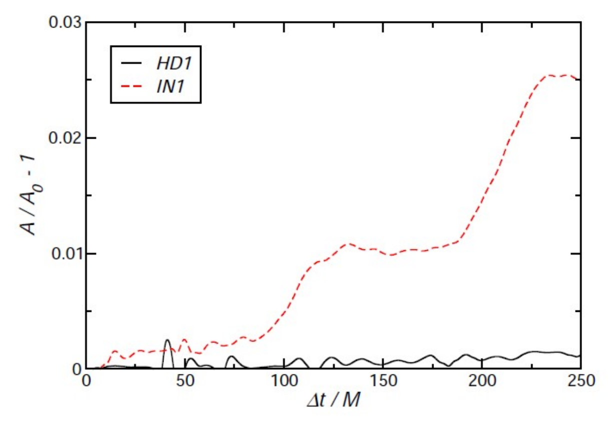


Quadrupole mode

Gravitational radiation (out going and ingoing)



Horizon area



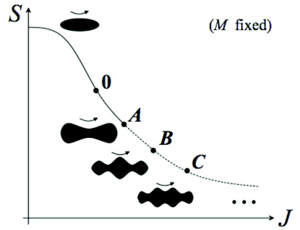
Superradiance: high frequency absorbed, low frequency amplified
No conclusive evidence yet...

4. Other topics in $D \geq 5$

Other topics: Instabilities of Myers-Perry

- Ultra-spinning Myers-Perry black holes (with single angular momentum parameter) should be unstable.
- Confirmed by linearized analysis of axisymmetric perturbations

  Dias *et al.* '09



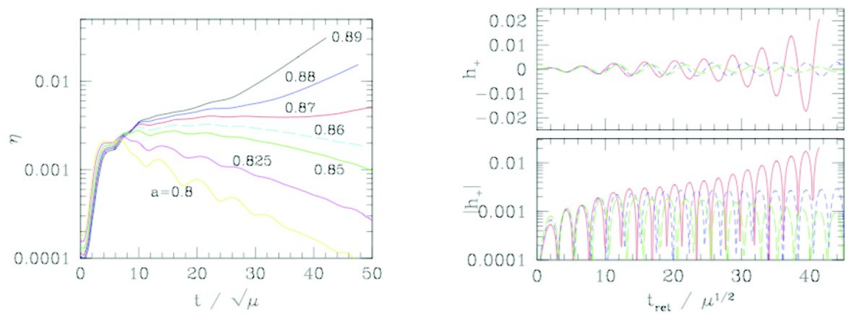
d	$(a/r_m)\vert_{\ell=1}$	$(a/r_m)\vert_{\ell=2}$	$(a/r_m)\vert_{\ell=3}$
7	1.075	1.714	2.141
8	1.061	1.770	2.275
9	1.051	1.792	2.337

Other topics: Instabilities of Myers-Perry

- Numerical study of non-axisymmetric instabilities of $D = 5$ Myers-Perry BH with single ang. momentum parameter.

  Shibata & Yoshino '09

- Found onset of instabilities at spin $a/\sqrt{\mu} \approx 0.87$



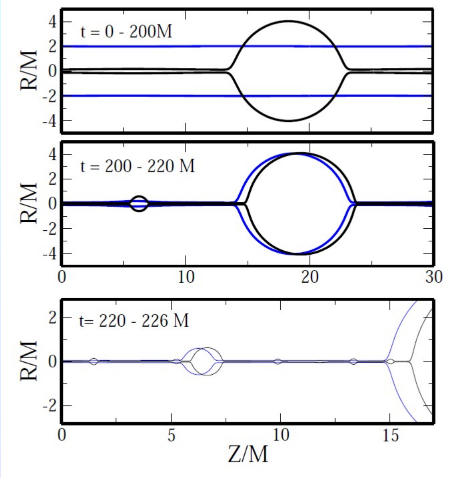
Other topics: Cosmic censorship in $D = 5$

Pretorius & Lehner '10

- Axisymmetric code
- Study evolution of black string...
- Gregory-Laflamme instability cascades down

  until string reaches zero radius

  $\Rightarrow$ naked singularity



5. Summary

Summary

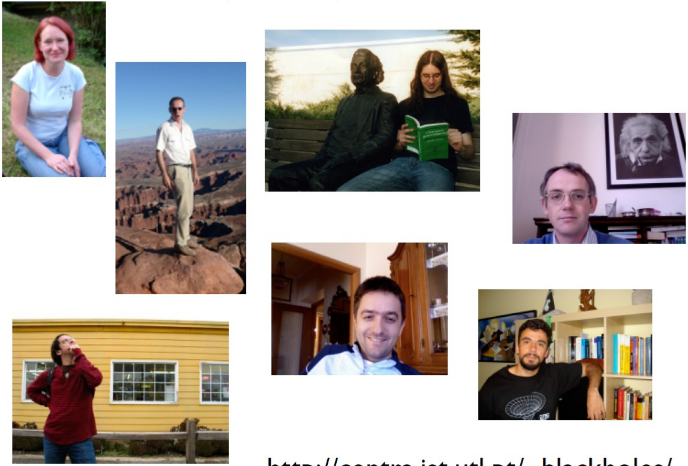
- Black holes are real objects in many areas of physics!
- Astrophysics: Recoil, Spin flips, jets
- Gravitational wave physics: template banks needed
- High-energy collisions in $D = 4$:

  largest kicks $\sim 15\,000\ \mathrm{km/s}$

  largest radiation $\sim 30\ \%$

  largest post-merger spin $a \lesssim 1$
- Formalism for arbitrary spatial dimension D
- Head-on collisions from rest
- Test non-assymptotically flat OBCs
- Signs of cosmic censorship violation in $D = 5$

The team



http://centra.ist.utl.pt/~blackholes/