Black-hole simulations on supercomputers

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Overview

- Introduction
- Numerical modeling of black holes
- Applications
  - Gravitational wave physics
  - Astrophysics
  - High-energy physics
  - AdS/CFT correspondence
  - Fundamental and mathematical studies
- Conclusions and outlook

The Schwarzschild solution

- Einstein 1915

  General relativity: geometric theory of gravity

- Schwarzschild 1916

  $$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

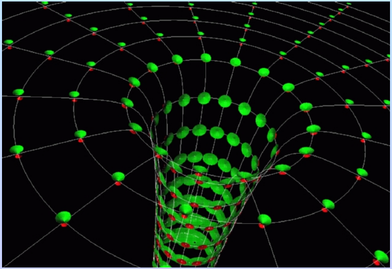
- Singularities:

  $r = 0$: physical

  $r = 2M$: coordinate

- Newtonian escape velocity

  $$v = \sqrt{\frac{2M}{r}}$$



Evidence for astrophysical black holes

- X-ray binaries

  e. g. Cygnus X-1 (1964)

  MS star + compact star

  $\Rightarrow$ Stellar Mass BHs

  $\sim 5 \ldots 50\ M_{\odot}$

- Stellar dynamics

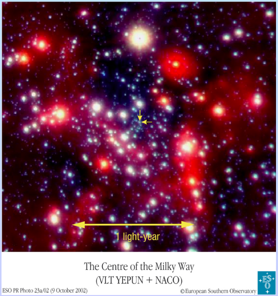
  near galactic centers,

  iron emission line profiles

  $\Rightarrow$ Supermassive BHs

  $\sim 10^6 \ldots 10^9\ M_{\odot}$

  AGN engines



Conjectured BHs

- Intermediate mass BHs

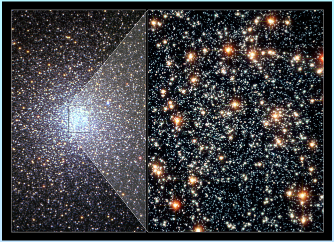
  $\sim 10^2 \dots 10^5\ M_\odot$

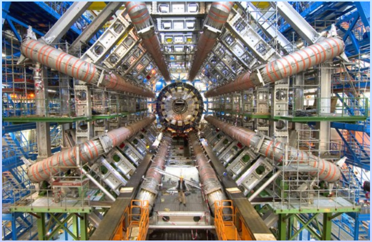
- Primordial BHs

  $\leq M_{Earth}$

- Mini BHs, LHC

  $\sim TeV$



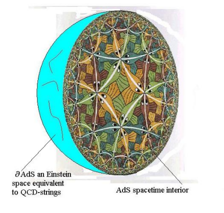


Note: BH solution is scale invariant!

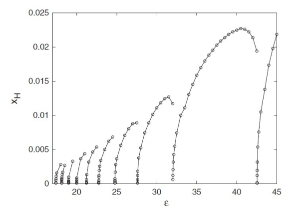
Research areas: Black holes have come a long way!

Astrophysics

Gauge-gravity duality

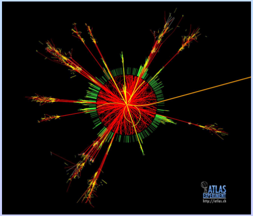


Fundamental studies

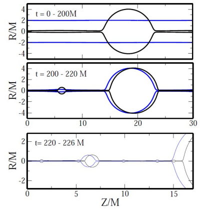


GW physics

High-energy physics



Fluid analogies



Modeling black holes in GR

General Relativity: Curvature

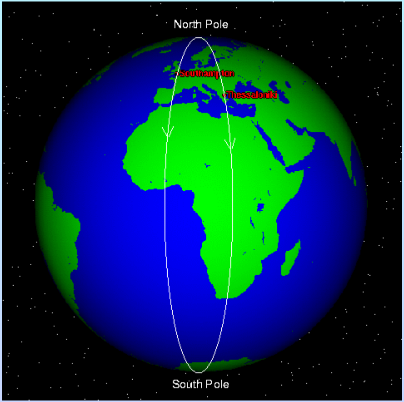
- Curvature generates acceleration

  "geodesic deviation"

  No "force"!!

- Description of geometry

Metric	$g_{\alpha\beta}$
Connection	$\Gamma^{\alpha}_{\beta\gamma}$
Riemann Tensor	$R^{\alpha}{}_{\beta\gamma\delta}$



How to get the metric?

Train cemetery
Uyuni, Bolivia

- Solve for the metric $g_{\alpha\beta}$

How to get the metric?

- The metric must obey the Einstein Equations
- Ricci-Tensor, Einstein Tensor, Matter Tensor

  $R_{\alpha\beta} \equiv R^{\mu}{}_{\alpha\mu\beta}$

  $G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\mu}{}_{\mu}$ "Trace reversed" Ricci

  $T_{\alpha\beta}$ "Matter"

- Einstein Equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$
- Solutions: Easy! Take metric

  $\Rightarrow$ Calculate $G_{\alpha\beta}$

  $\Rightarrow$ Use that as matter tensor

- Physically meaningful solutions: Difficult!

The Einstein Equations in vacuum

- "Spacetime tells matter how to move,
  matter tells spacetime how to curve"
- Field equations in vacuum: $R_{\alpha\beta} = 0$
  Second order PDEs for the metric components
  Invariant under coordinate (gauge) transformations
- System of equations extremely complex: Pile of paper!
  Analytic solutions: Minkowski, Schwarzschild, Kerr,
  Robertson-Walker, ...
- Numerical methods necessary for general scenarios!!!

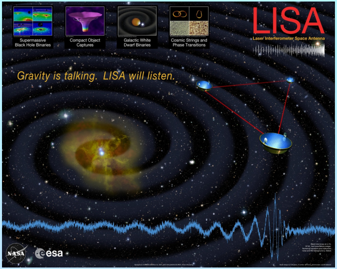
A list of tasks

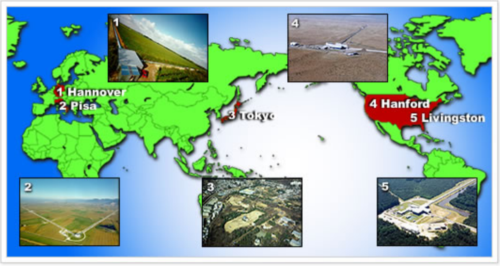
- Target: Predict time evolution of BBH in GR
- Einstein equations: 1) Cast as evolution system
  2) Choose specific formulation
  3) Discretize for computer
- Choose coordinate conditions: Gauge
- Fix technical aspects: 1) Mesh refinement / spectral domains
  2) Singularity handling / excision
  3) Parallelization
- Construct realistic initial data
- Start evolution and waaaaiiiiit...
- Extract physics from the data

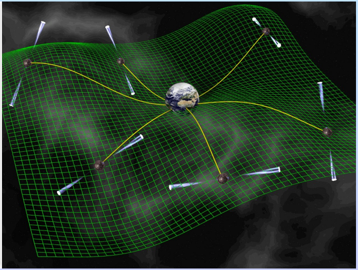
Gravitational Wave Physics

Gravitational wave detectors

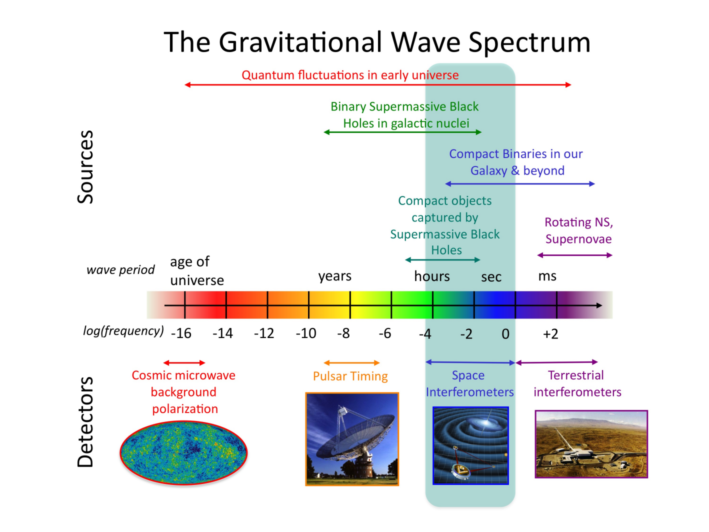
- Accelerated masses $\Rightarrow$ GWs
- Weak interaction!
- Laser interferometric detectors







The gravitational wave spectrum



Some targets of GW physics

- Confirmation of GR

  Hulse & Taylor 1993 Nobel Prize

- Parameter determination

  of BHs: M, $\vec{S}$

- Optical counter parts

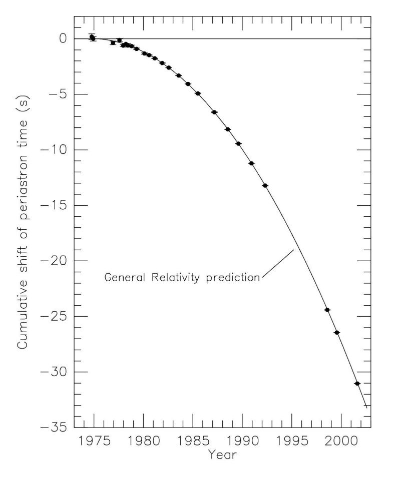
  Standard sirens (candles)

  Mass of graviton

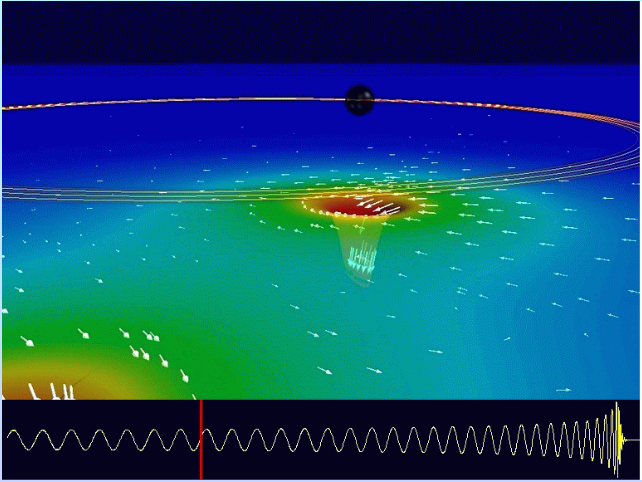
- Test Kerr Nature of BHs

- Cosmological sources

- Neutron stars: EOS



Morphology of a BBH inspiral

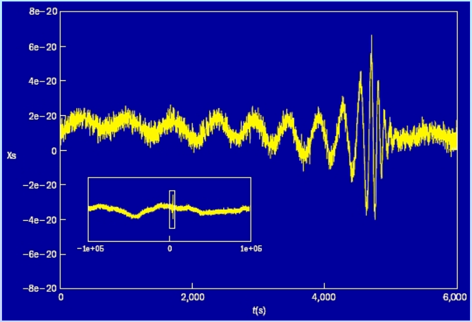


Thanks to Caltech, CITA, Cornell

Matched filtering

- BH binaries have 7 parameters: 1 mass ratio, 2×3 for spins
- Sample parameter space, generate waveform for each point

  - NR + PN
  - Effective one body
  - Ninja, NRAR Projects
  - GEO 600 noise
    chirp signal

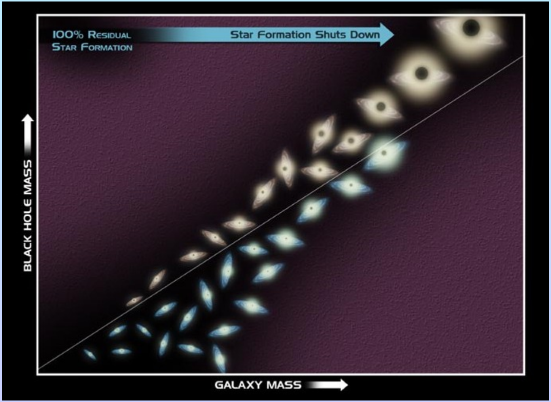


Astrophysics

Galaxies host SMBHs

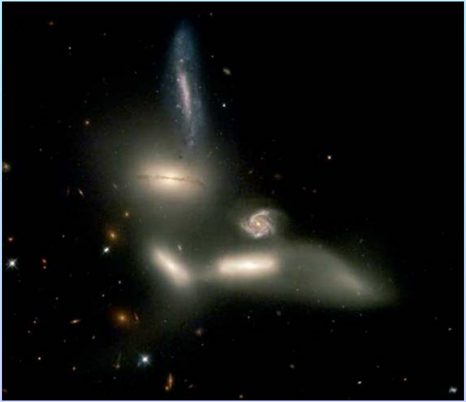
- Galaxies ubiquitously harbor BHs
- BH properties correlated with bulge properties

  e. g. J.Magorrian *et al.*, AJ 115, 2285 (1998)



SMBH formation

- Most widely accepted scenario for galaxy formation: hierarchical growth; “bottom-up”
- Galaxies undergo frequent mergers $\Rightarrow$ BH merger



Gravitational recoil

- Anisotropic GW emission $\Rightarrow$ recoil of remnant BH

  Bonnor & Rotenburg '61, Peres '62, Bekenstein '73

- Escape velocities:

Globular clusters	$30\ \mathrm{km/s}$
dSph	$20-100\ \mathrm{km/s}$
dE	$100-300\ \mathrm{km/s}$
Giant galaxies	$\sim 1000\ \mathrm{km/s}$

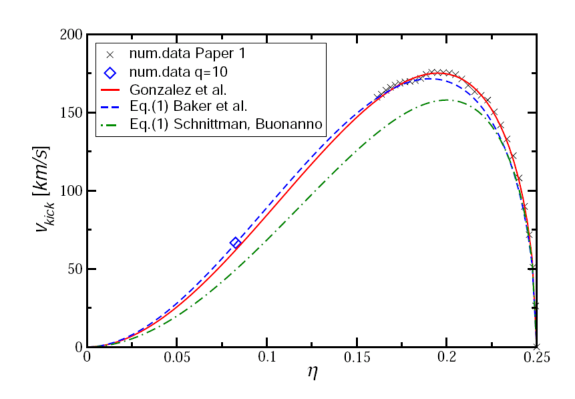
Ejection / displacement of BH $\Rightarrow$

- Growth history of SMBHs
- BH populations, IMBHs
- Structure of galaxies

Kicks from non-spinning BHs

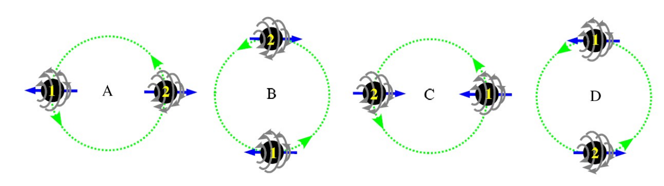
- Max. kick: $\sim 180\ km/s$, harmless!

  González et al., PRL 98, 091101 (2009)



Spinning BHs: Superkicks

- Kidder '95, UTB-RIT '07: maximum kick expected for



- Kicks up to $v_{\rm max} \approx 4\,000\ {\rm km/s}$

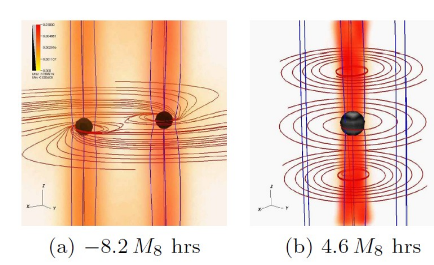
  González *et al.* '07, Campanelli *et al.* '07

- "Hang-up kicks" of up to $5\,000\ km/s$ Lousto & Zlochower '12
- Suppression via spin alignment and Resonance effects in inspiral
  Schnittman '04, Bogdanovicź et al. '07, Kesden, US & Berti '10, '10a, '12
- Dependence on mass ratio?

Double jets and spin flips

- BH binary with plasma
- Jets driven by **L**
- Optical signature: double jets

- Spin re-alignment
  - $\Rightarrow$ new + old jet
  - $\Rightarrow$ X-shaped radio sources



Palenzuela, Lehner & Liebling '10

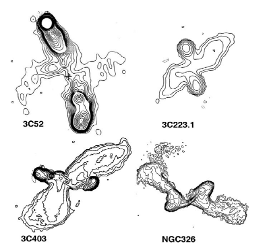


Fig. 2. A composite of four clear examples of the X-shape morphology. VLA radio observations of 3C52 (28), 3C223.1 (29), 3C403 (29), and NGC 326 (12).

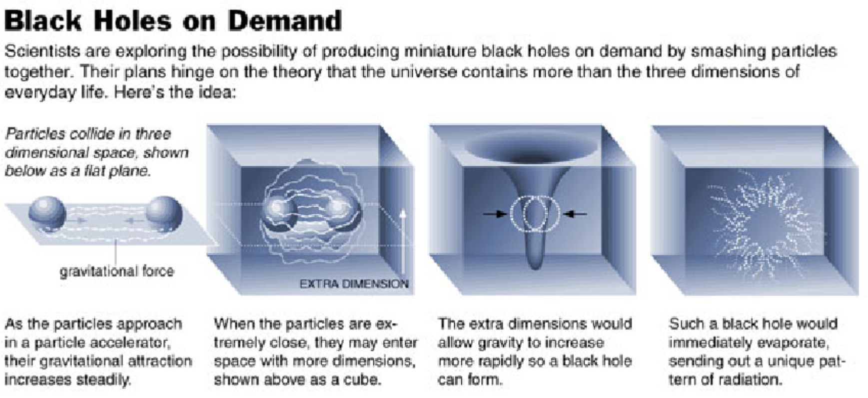
Campanelli et al. '06

High-energy collisions of BHs

The Hierarchy Problem of Physics

- Gravity $\approx 10^{-39} \times$ other forces
- Higgs field $\approx \mu_{obs} \approx 250\ GeV = \sqrt{\mu^2 - \Lambda^2}$
  where $\Lambda \approx 10^{16}\ GeV$ is the grand unification energy
- Requires enormous finetuning!!!
- Finetuning exist: $\frac{987654321}{123456789} = 8.0000000729$
- Or E_{Planck} much lower? Gravity strong at small r?
  $\Rightarrow$ BH formation in high-energy collisions at LHC
- Gravity not measured below 0.16 mm! Diluted due to...
  - Large extra dimensions   Arkani-Hamed, Dimopoulos & Dvali '98
  - Extra dimension with warp factor   Randall & Sundrum '99

Stages of BH formation



- Matter does not matter at energies well above the Planck scale

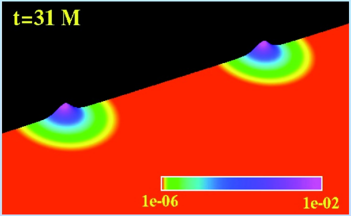
  $\Rightarrow$ Model particle collisions by black-hole collisions

  Banks & Fischler '99; Giddings & Thomas '01

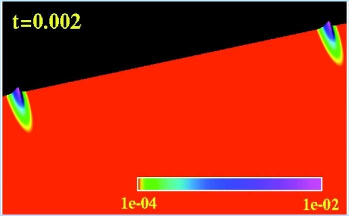
Does matter "matter"?

- Hoop conjecture ⇒ kinetic energy triggers BH formation
- Einstein plus minimally coupled, massive, complex scalar filed

  "Boson stars"   Pretorius & Choptuik '09



$\gamma = 1$



$\gamma = 4$

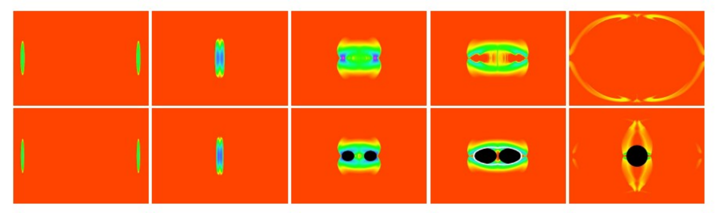
- BH formation threshold: $\gamma_{\rm thr} = 2.9 \pm 10\ \% \sim 1/3\ \gamma_{hoop}$
- Model particle collisions by BH collisions

Does matter "matter"?

- Perfect fluid "stars" model
- $\gamma = 8 \dots 12$; BH formation below Hoop prediction

  East & Pretorius '12

- Gravitational focussing $\Rightarrow$ Formation of individual horizons



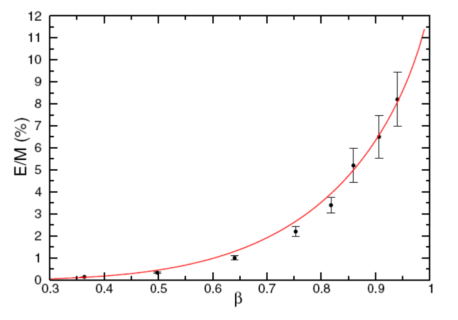
- Type-I critical behaviour
- Extrapolation by 60 orders would imply no BH formation at LHC

  Rezzolla & Tanaki '12

BH Head-on collision: $D = 4, \quad b = 0, \quad \vec{S} = 0$

- Total radiated energy: 14 ± 3 % for $v \to 1$ US *et al.* '08

  About half of Penrose '74



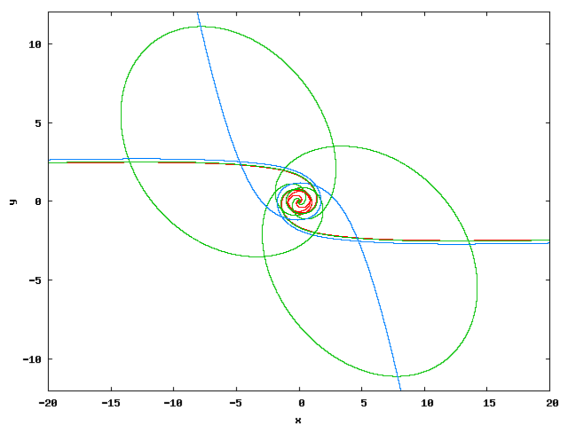
- Agreement with approximative methods

  Flat spectrum, multipolar GW structure Berti *et al.* '10

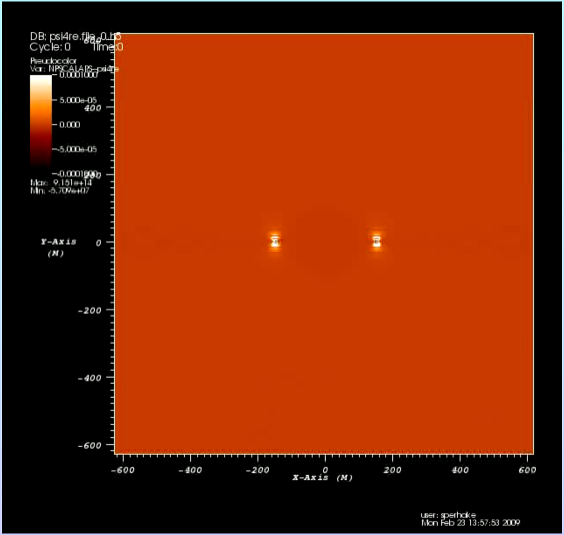
BH Grazing collisions: $D = 4, \quad b \neq 0, \quad \gamma = 1.52$

- Zoom-whirl orbits   Pretorius & Khurana '07
- Immediate vs. Delayed vs. No merger

  US, Cardoso, Pretorius, Berti, Hinderer & Yunes '09



Gravitational radiation: Delayed merger



Scattering threshold b_{scat} in $D = 4$

- $b < b_{\rm scat} \quad \Rightarrow \quad$ Merger

  $b > b_{\rm scat} \quad \Rightarrow \quad$ Scattering

- Numerical study: $\quad b_{\rm scat} = \frac{2.5 \pm 0.05}{v} M$

  Shibata, Okawa & Yamamoto '08

- Independent study by US, Pretorius, Cardoso, Berti *et al.* '09, '12

  $\gamma = 1.23 \ldots 2.93$:

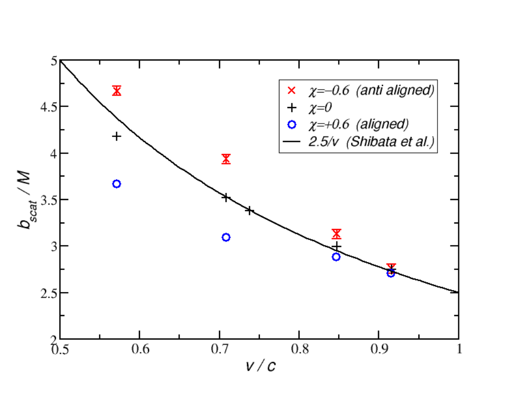
  $\chi = -0.6,\ 0,\ +0.6$ (anti-aligned, nonspinning, aligned)

- Limit from Penrose construction: $b_{\rm crit} = 1.685\ M$

  Yoshino & Rychkov '05

Diminishing impact of structure as $v \to 1$

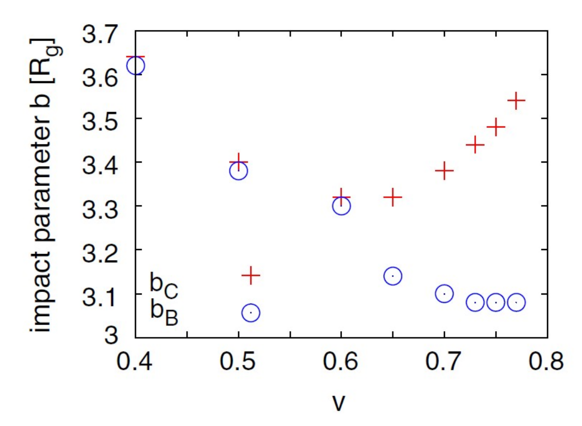
- Spin $\mathbf{S} \parallel \mathbf{L}$, $S = \pm 0.6,\ 0$ US, Berti, Cardoso & Pretorius, in prep.



- Effect of spin reduced for large γ
- $b_{\rm scat}$ for $v \to 1$ not quite certain

Scattering threshold in $D = 5$

Okawa, Nakao & Shibata '11

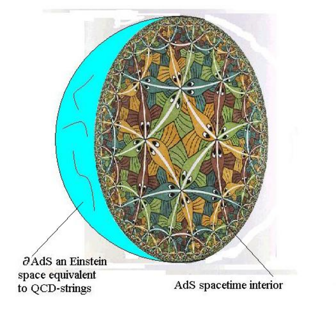


Numerical stability still an issue...

BH Holography

Large N and holography

- Holography
  - BH entropy $\propto A_{Hor}$
  - For a Local Field Theory
    entropy $\propto V$
  - Gravity in D dims
    $\Leftrightarrow$ local FT in $D-1$ dims



- Large N limit
  - Perturbative expansion of gauge theory in $g^2 N$
    $\sim$ loop expansion in string theory
  - N: # of "colors"
    $g^2 N$: t'Hooft coupling

The AdS/CFT conjecture

Maldacena '98

- "strong form": Type IIb string theory on $AdS_5 \times S^5$

  $\Leftrightarrow \mathcal{N} = 4$ super Yang-Mills in $D = 4$

  Hard to prove; non-perturbative Type IIb String Theory?

- "weak form": low-energy limit of string-theory side

  $\Rightarrow$ Type IIb Supergravity on $AdS_5 \times S^5$

- Some assumptions, factor out S^5

  $\Rightarrow$ General Relativity on AdS_5

- Corresponds to limit of large N, $g^2 N$ in the field theory

- E. g. Stationary AdS BH $\Leftrightarrow$ Thermal Equil. with T_{Haw} in dual FT

  Witten '98

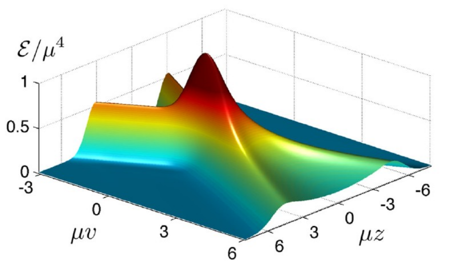
Collision of planar shockwaves in $\mathcal{N} = 4$ SYM

- Dual to colliding gravitational shock waves in AADS
- Characteristic study with translational invariance

  Chesler & Yaffe '10, '11

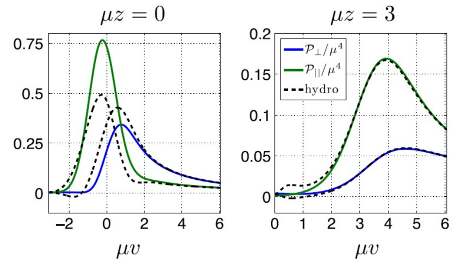
- Initial data: 2 superposed shockwaves

  $ds^2 = r^2[-dx_+ dx_- + d\mathbf{x}_\perp] + \frac{1}{r^2}[dr^2 + h(x_\pm)dx_\pm^2]$



Collision of planar shockwaves in $\mathcal{N} = 4$ SYM

- Initially system far from equilibrium
- Isotropization after $\Delta v \sim 4/\mu \sim 0.35\ fm/c$
- Confirms hydrodynamic simulations of QGP $\sim 1\ fm/c$ Heinz '04



- Non-linear vs. linear Einstein Eqs. agree within $\sim 20\ \%$

Heller et al. '12

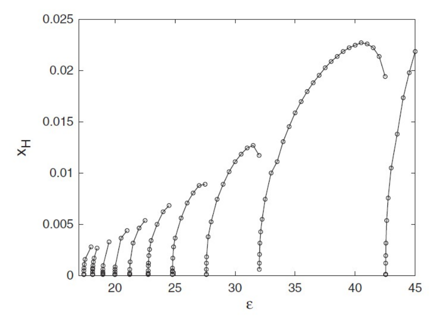
Fundamental properties of BHs

Stability of AdS

- $m = 0$ scalar field in as. flat spacetimes Choptuik '93

  $p > p^* \Rightarrow$ BH, $p < p^* \Rightarrow$ flat

- $m = 0$ scalar field in as. AdS Bizon & Rostworowski '11

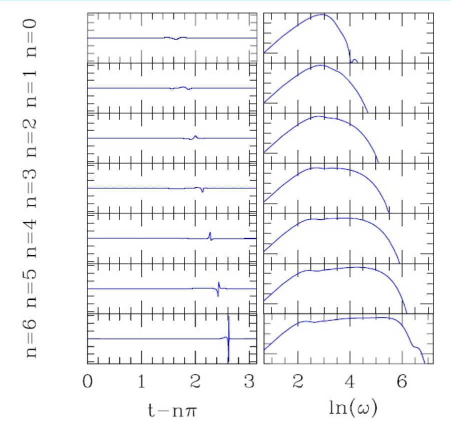


- Similar behaviour for "Geons" Dias, Horowitz & Santos '11

Stability of AdS

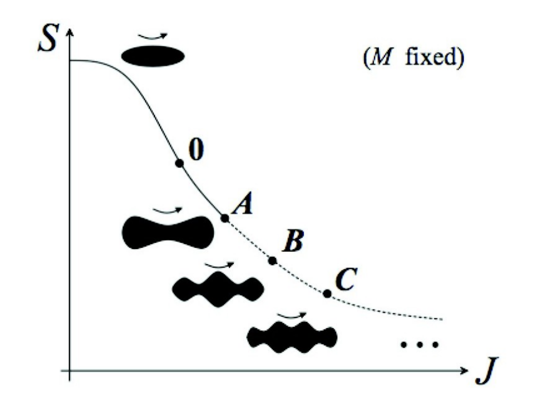
- Pulses narrow under successive reflections

  Buchel, Lehner & Liebling '12



Bar mode instability of Myers-Perry BH

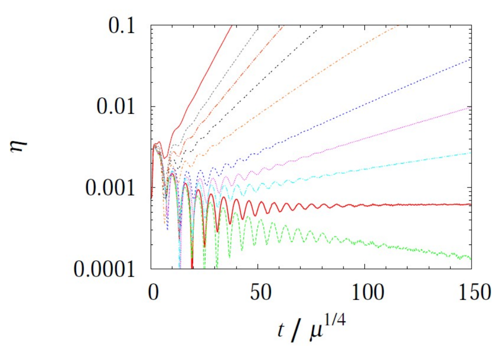
- MP BHs (with single ang.mom.) should be unstable.
- Linearized analysis Dias *et al.* '09



Non-linear analysis of MP instability

Shibata & Yoshino ’10

- Myers-Perry metric; transformed to Puncture like coordinate
- Add small bar-mode perturbation
- Deformation $\eta := \frac{2\sqrt{(l_0 - l_{\pi/2})^2 + (l_{\pi/4} - l_{3\pi/4})^2}}{l_0 + l_{\pi/2}}$



Cosmic Censorship in $D = 5$

Pretorius & Lehner '10

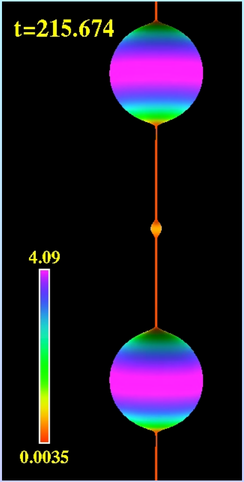
- Axisymmetric code
- Evolution of black string...
- Gregory-Laflamme instability

  cascades down

  in finite time

  until string has zero width

  $\Rightarrow$ naked singularity

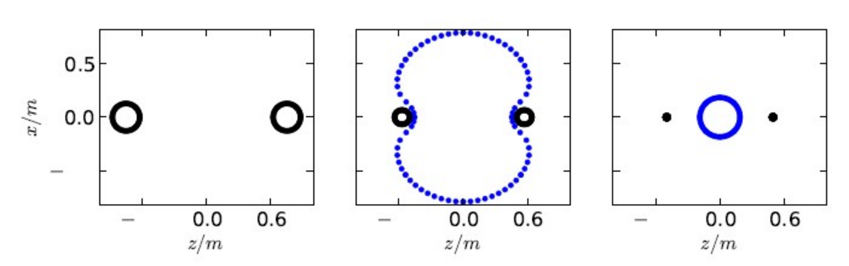


Cosmic Censorship in $D = 4$ de Sitter

Zilhão et al. '12

- Two parameters: MH, d
- Initial data: McVittie type binaries McVittie '33
- "Small BHs": $d < d_{crit} \Rightarrow$ merger

  $d > d_{crit} \Rightarrow$ no common AH
- "Large" holes at small d: Cosmic Censorship holds



Conclusions

Conclusions

- NR breakthroughs

  Pretorius '05, Brownsville, Goddard '05

- GW Template construction $\to$ Cover parameter space
- BH kicks $\to$ $m1/m2$dependence ofsuperkicks
- High-energy collisions $\to$ Extension to $D \geq 5$
- AdS/CFT $\to$ Generic NR framework, What studies?
- Fundamental properties $\to$ Cosmic censorship, BH Stability

BH have applications in many areas!