Collisions of black holes and gravitational wave emission in four and higher dimensional spacetimes

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24th May 2013

Overview

- Introduction
- Numerical modeling of black holes
- TeV gravity
- Collisions in $D = 4$
- Collisions in $D \geq 5$
- Non-asymptotically flat, non-vacuum spacetimes
- Conclusions and outlook

1. Introduction

The Schwarzschild solution

- Einstein 1915

  General relativity: geometric theory of gravity

- Schwarzschild 1916

  $$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

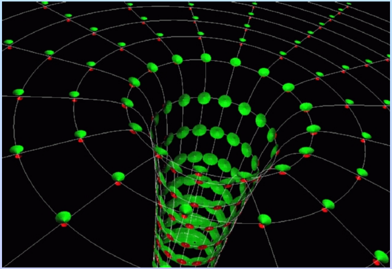
- Singularities:

  $r = 0$: physical

  $r = 2M$: coordinate

- Newtonian escape velocity

  $$v = \sqrt{\frac{2M}{r}}$$



Evidence for astrophysical black holes

- X-ray binaries

  e. g. Cygnus X-1 (1964)

  MS star + compact star

  $\Rightarrow$ Stellar Mass BHs
  $\sim 5 \ldots 50\ M_{\odot}$

- Stellar dynamics

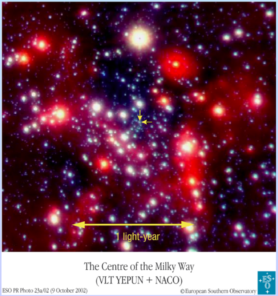
  near galactic centers,

  iron emission line profiles

  $\Rightarrow$ Supermassive BHs

  $\sim 10^6 \ldots 10^9\ M_{\odot}$

  AGN engines



Conjectured BHs

- Intermediate mass BHs

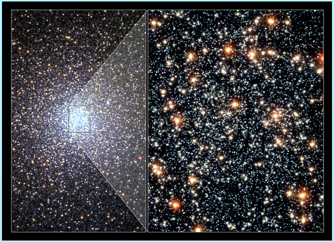
  $\sim 10^2 \dots 10^5\ M_\odot$

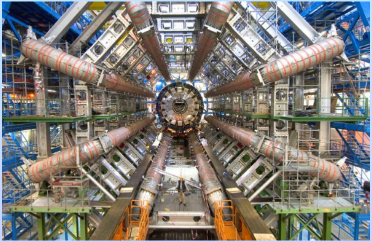
- Primordial BHs

  $\leq M_{Earth}$

- Mini BHs, LHC

  $\sim TeV$



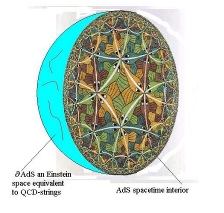


Note: BH solution is scale invariant!

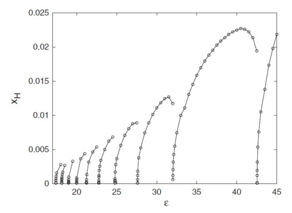
Research areas: Black holes have come a long way!

Astrophysics

Gauge-gravity duality

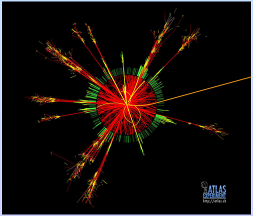


Fundamental studies

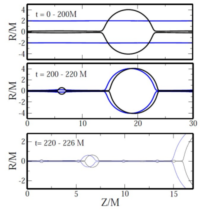


GW physics

High-energy physics



Fluid analogies



General Relativity: Curvature

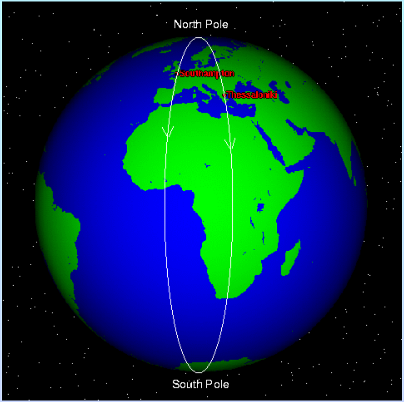
- Curvature generates acceleration

  "geodesic deviation"

  No "force"!!

- Description of geometry

Metric	$g_{\alpha\beta}$
Connection	$\Gamma^{\alpha}_{\beta\gamma}$
Riemann Tensor	$R^{\alpha}{}_{\beta\gamma\delta}$



How to get the metric?

Train cemetery
Uyuni, Bolivia

- Solve for the metric $g_{\alpha\beta}$

How to get the metric?

- The metric must obey the Einstein Equations

- Ricci-Tensor, Einstein Tensor, Matter Tensor

  $R_{\alpha\beta} \equiv R^{\mu}{}_{\alpha\mu\beta}$

  $G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\mu}{}_{\mu}$ "Trace reversed" Ricci

  $T_{\alpha\beta}$ "Matter"

- Einstein Equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$

- Solutions: Easy! Take metric

  $\Rightarrow$ Calculate $G_{\alpha\beta}$

  $\Rightarrow$ Use that as matter tensor

- Physically meaningful solutions: Difficult!

Solving Einstein's equations: Different methods

- Analytic solutions
  - Symmetry assumptions
  - Schwarzschild, Kerr, FLRW, Myers-Perry, Emparan-Reall,...
- Perturbation theory
  - Assume solution is close to known solution $g_{\alpha\beta}$
  - Expand $\hat{g}_{\alpha\beta} = g_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{\alpha\beta} + \ldots \quad \Rightarrow$ linear system
  - Regge-Wheeler-Zerilli-Moncrief, Teukolsky, QNMs, EOB,...
- Post-Newtonian Theory
  - Assume small velocities $\Rightarrow$ expansion in $\frac{v}{c}$
  - N^{th} order expressions for GWs, momenta, orbits,...
  - Blanchet, Buonanno, Damour, Kidder, Will,...
- Numerical Relativity

2. Numerical modeling of BHs in GR

A list of tasks

- Target: Predict time evolution of BBH in GR
- Einstein equations: 1) Cast as evolution system

  2) Choose specific formulation

  3) Discretize for computer
- Choose coordinate conditions: Gauge
- Fix technical aspects: 1) Mesh refinement / spectral domains

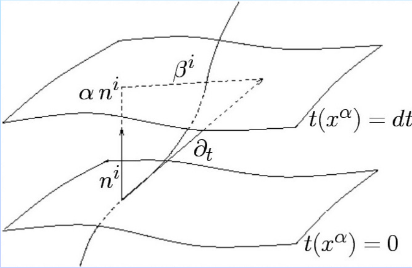
  2) Singularity handling / excision

  3) Parallelization
- Construct realistic initial data
- Start evolution and waaaaiiiiit...
- Extract physics from the data

3+1 Decomposition

- GR: "Space and time exist as a unity: Spacetime"
- NR: ADM 3+1 split   Arnowitt, Deser & Misner '62
  York '79, Choquet-Bruhat & York '80

$$g_{\alpha\beta} = \left(\begin{array}{c|c} -\alpha^2 + \beta_m \beta^m & \beta_j \\ \hline \beta_i & \gamma_{ij} \end{array}\right)$$



- 3-Metric γ_{ij}
  Lapse α
  Shift β^i
- lapse, shift $\Rightarrow$ Gauge

ADM Equations

The Einstein equations $R_{\alpha\beta} = 0$ become

- 6 Evolution equations

  $$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

  $$(\partial_t - \mathcal{L}_\beta)K_{ij} = -D_i D_j \alpha + \alpha[R_{ij} - 2K_{im}K^m{}_j + K_{ij}K]$$

- 4 Constraints

  $$R + K^2 - K_{ij}K^{ij} = 0$$

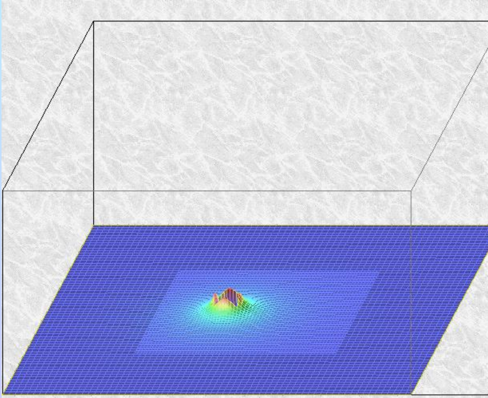
  $$-D_j K^{ij} + D^i K = 0$$

  preserved under evolution!

- Evolution

  1) Solve constraints

  2) Evolve data



Formulations I: BSSN

- One can easily change variables. E. g. wave equation

$$\partial_{tt} u - c\partial_{xx} u = 0 \qquad \Leftrightarrow \qquad \begin{aligned} \partial_t F - c\partial_x G &= 0 \\ \partial_x F - \partial_t G &= 0 \end{aligned}$$

- BSSN: rearrange degrees of freedom

$$\chi = (\det\gamma)^{-1/3} \qquad \tilde{\gamma}_{ij} = \chi\gamma_{ij}$$

$$K = \gamma_{ij}K^{ij} \qquad \tilde{A}_{ij} = \chi\left(K_{ij} - \tfrac{1}{3}\gamma_{ij}K\right)$$

$$\tilde{\Gamma}^i = \tilde{\gamma}^{mn}\tilde{\Gamma}^i_{mn} = -\partial_m\tilde{\gamma}^{im}$$

Shibata & Nakamura '95, Baumgarte & Shapiro '98

- BSSN strongly hyperbolic, but depends on details...

Sarbach *et al.*'02, Gundlach & Martín-García '06

Formulations I: BSSN

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\boxed{\begin{array}{ll} \phi = \frac{1}{12}\ln\gamma & \hat{\gamma}_{ij} = e^{-4\phi}\gamma_{ij} \\ K = \gamma_{ij}K^{ij} & \hat{A}_{ij} = e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij}K\right) \\ \hat{\Gamma}^i = \gamma^{ij}\hat{\Gamma}^i_{jk} = -\partial_j\hat{\gamma}^{ij} & \end{array}}$$

$$(\partial_t - \mathcal{L}_\beta)\hat{\gamma}_{ij} = -2\alpha\hat{A}_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\frac{1}{6}\alpha K$$

$$(\partial_t - \mathcal{L}_\beta)\hat{A}_{ij} = e^{-4\phi}(-D_iD_j\alpha + \alpha R_{ij})^{\mathrm{TF}} + \alpha(K\hat{A}_{ij} - 2\hat{A}_{ik}\hat{A}^k{}_j)$$

$$(\partial_t - \mathcal{L}_\beta)K = -D^iD_i\alpha + \alpha\left(\hat{A}_{ij}\hat{A}^{ij} + \frac{1}{3}K^2\right)$$

$$\begin{aligned}\partial_t\hat{\Gamma}^i = {} & 2\alpha\left(\hat{\Gamma}^i_{jk}\hat{A}^{jk} + 6\hat{A}^{ij}\partial_j\phi - \frac{2}{3}\hat{\gamma}^{ij}\partial_jK\right) - 2\hat{A}^{ij}\partial_j\alpha + \hat{\gamma}^{jk}\partial_j\partial_k\beta^i \\ & + \frac{1}{3}\hat{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\hat{\Gamma}^i + \frac{2}{3}\hat{\Gamma}^i\partial_j\beta^j \underbrace{- \left(\chi + \frac{2}{3}\right)\left(\hat{\Gamma}^i - \hat{\gamma}^{jk}\Gamma^i_{jk}\right)\partial_l\beta^l}_{\text{Yo et al. (2002)}}\end{aligned}$$

Formulations II: Generalized harmonic (GHG)

- Harmonic gauge: choose coordinates such that

  $\nabla_\mu \nabla^\mu x^\alpha = 0$

- 4-dim. version of Einstein equations

  $R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots$

  Principal part of wave equation

- Generalized harmonic gauge: $H_\alpha \equiv g_{\alpha\nu} \nabla_\mu \nabla^\mu x^\nu$

  $\Rightarrow R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots - \frac{1}{2} (\partial_\alpha H_\beta + \partial_\beta H_\alpha)$

  Still principal part of wave equation !!! Manifestly hyperbolic

  Friedrich '85, Garfinkle '02, Pretorius '05

- Constraint preservation; constraint satisfying BCs

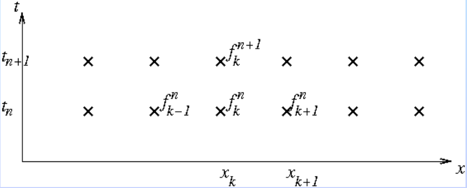
  Gundlach et al. '05, Lindblom et al. '06

Discretization of the time evolution

- Finite differencing (FD)

  Pretorius, RIT, Goddard, Georgia Tech, LEAN, BAM, UIUC,...

- Spectral Caltech-Cornell-CITA
- Parallelization with MPI, $\sim$ 128 cores, $\sim$ 256 *Gb* RAM
- Example: advection equation $\partial_t f = \partial_x f$, FD
- Array f_k^n for fixed n



$$f_k^{n+1} = f_k^n + \Delta t \frac{f_{k+1}^n - f_{k-1}^n}{2\Delta x}$$

Initial data

Two problems: Constraints, realistic data

- Rearrange degrees of freedom

  York-Lichnerowicz split: $\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$

  $$K_{ij} = A_{ij} + \frac{1}{3}\gamma_{ij} K$$

  York & Lichnerozwicz, O'Murchadha & York,

  Wilson & Mathews, York

- Make simplifying assumptions

  Conformal flatness: $\tilde{\gamma}_{ij} = \delta_{ij}$, and $K = 0$

- Find good elliptic solvers, e. g. Ansorg et al. '04

Mesh refinement

3 Length scales :	BH	$\sim 1\ M$
	Wavelength	$\sim 10...100\ M$
	Wave zone	$\sim 100...1000\ M$

- Critical phenomena

  Choptuik '93

- First used for BBHs

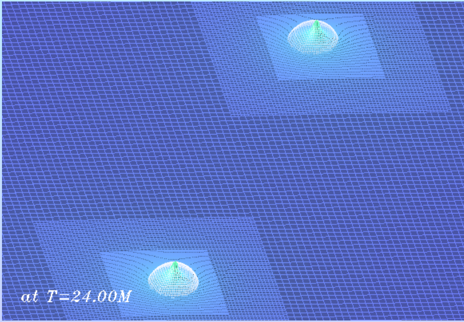
  Brügmann '96

- Available Packages:

  Paramesh MacNeice *et al.* '00

  Carpet Schnetter *et al.* '03

  SAMRAI MacNeice *et al.* '00



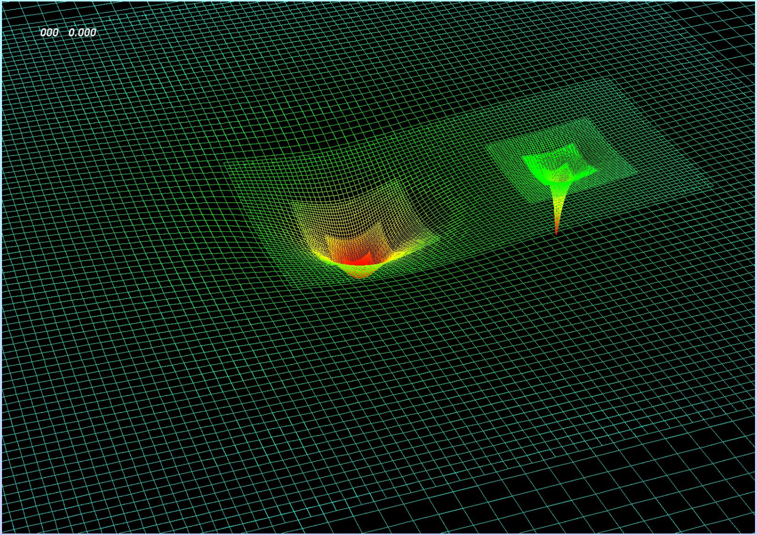
The gauge freedom

- Remember: Einstein equations say nothing about $\alpha, \quad \beta^i$
- Any choice of lapse and shift gives a solution
- This represents the coordinate freedom of GR
- Physics do not depend on $\alpha, \quad \beta^i$

  So why bother?
- The performance of the numerics DO depend strongly on the gauge!
- How do we get good gauge?

  Singularity avoidance, avoid coordinate stretching, well posedness

What goes wrong with bad gauge?



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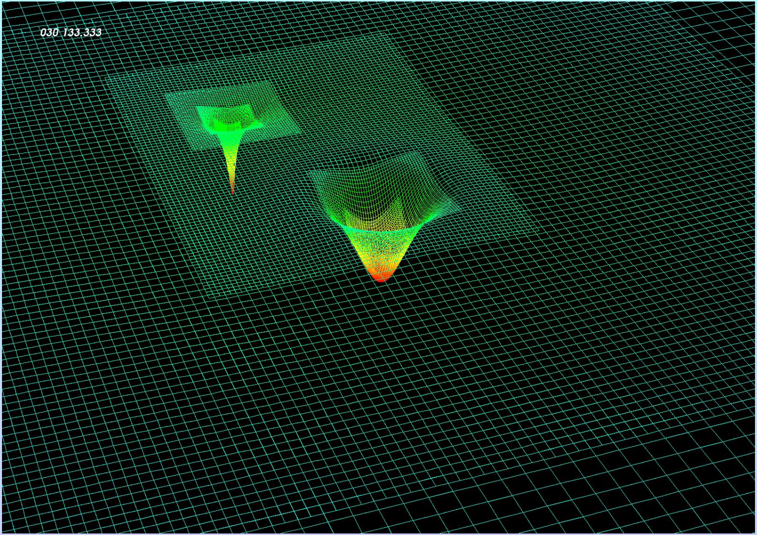

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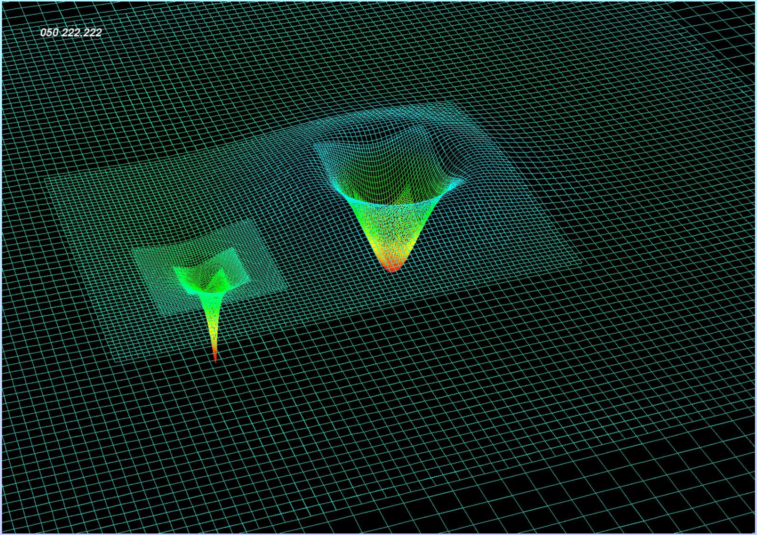

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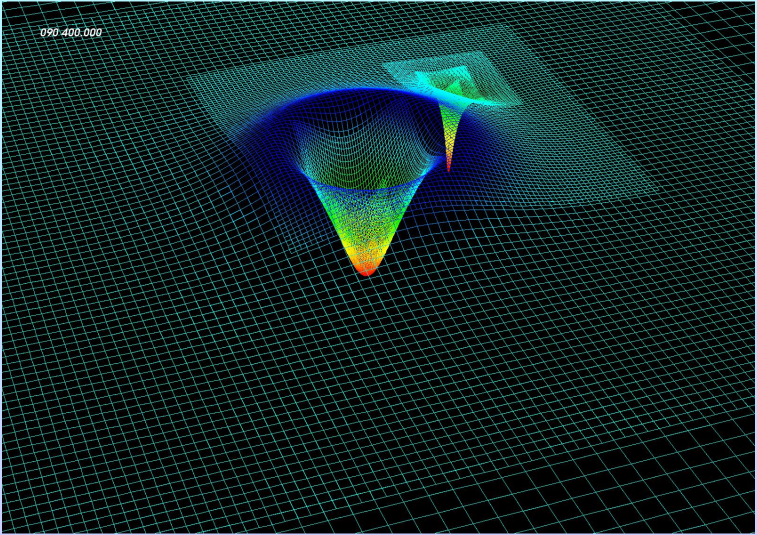


A brief history of BH simulations

- Pioneers: Hahn & Lindquist '60s, Eppley, Smarr *et al.* '70s
- Grand Challenge: First $3D$ Code Anninos *et al.* '90s
- Further attempts: Bona & Massó, Pitt-PSU-Texas

  AEI-Potsdam, Alcubierre *et al.*

  PSU: first orbit Brügmann *et al.* '04

  Codes unstable!

- Breakthrough: Pretorius '05 — GHG

  UTB, Goddard'05 — Moving Punctures

- Currently about 10 codes world wide

3. TeV Gravity and BH formation

The Hierarchy Problem of Physics

- Gravity $\approx 10^{-39} \times$ other forces
- Higgs field $\approx \mu_{obs} \approx 250\ GeV = \sqrt{\mu^2 - \Lambda^2}$

  where $\Lambda \approx 10^{16}\ GeV$ is the grand unification energy
- Requires enormous finetuning!!!
- Finetuning exist: $\frac{987654321}{123456789} = 8.0000000729$
- Or E_{Planck} much lower? Gravity strong at small r?

  $\Rightarrow$ BH formation in high-energy collisions at LHC
- Gravity not measured below $0.16\ mm$! Diluted due to...
  - Large extra dimensions Arkani-Hamed, Dimopoulos & Dvali '98
  - Extra dimension with warp factor Randall & Sundrum '99

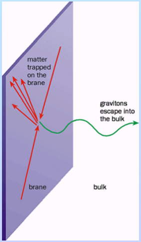
The Hierarchy proble in physics: TeV Gravity

Large extra dimensions

Arkani-Hamed, Dimopoulos & Dvali '98

- SM confined to "3+1" brane
- Gravity lives in bulk

  $\Rightarrow$ Gravity diluted



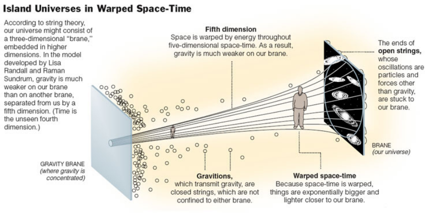
Warped geometry

Randall & Sundrum '99

- 5D AdS Universe with 2 branes:

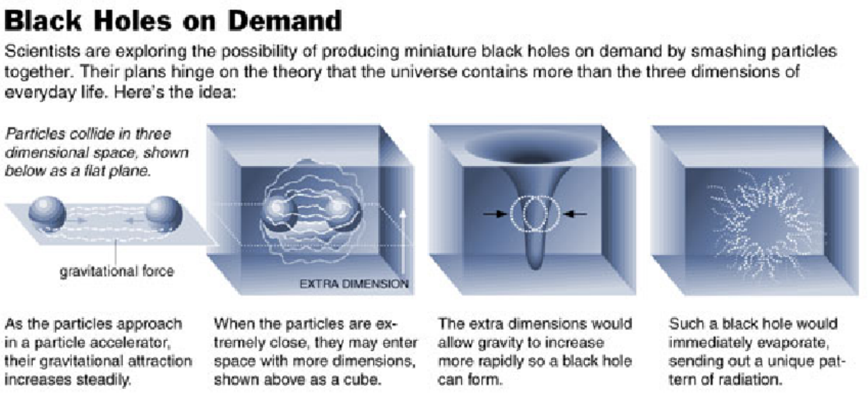
  "our" 3+1 world, gravity brane
- 5^{th} dimension warped

  $\Rightarrow$ Gravity weakened



Either way: Gravity strong at $\gtrsim$ TeV

Motivation (High-energy physics)



- Matter does not matter at energies well above the Planck scale

$\Rightarrow$ Model particle collisions by black-hole collisions

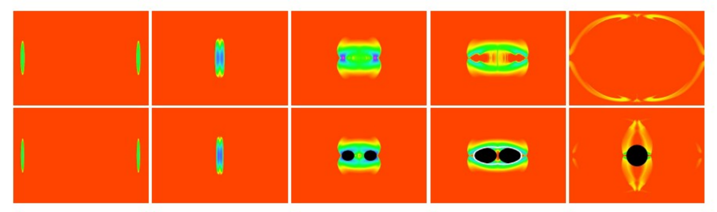
Banks & Fischler '99; Giddings & Thomas '01

Does matter "matter"?

- Perfect fluid "stars" model
- $\gamma = 8 \ldots 12$; BH formation below Hoop prediction

  East & Pretorius '12

- Gravitational focussing $\Rightarrow$ Formation of individual horizons



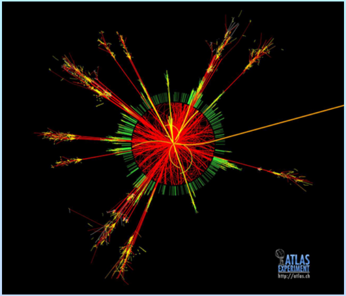
- Type-I critical behaviour
- Extrapolation by 60 orders would imply no BH formation at LHC

  Rezzolla & Tanaki '12

Experimental signature at the LHC

Black hole formation at the LHC could be detected by the properties of the jets resulting from Hawking radiation. `BlackMax`, `Charybdis`

- Multiplicity of partons: Number of jets and leptons
- Large transverse energy
- Black-hole mass and spin are important for this!



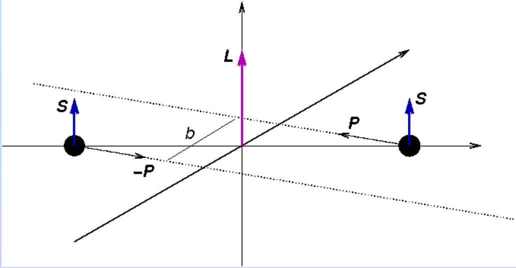
ToDo:

- Exact cross section for BH formation
- Determine loss of energy in gravitational waves
- Determine spin of merged black hole

4. High-energy collisions in $D = 4$

Initial setup: 1) Aligned spins

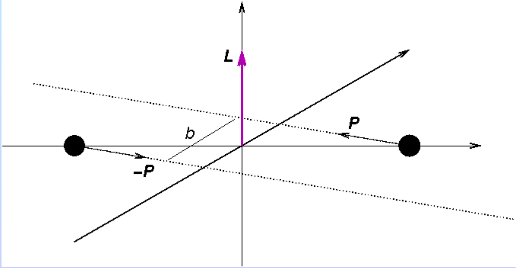
- Orbital hang-up Campanelli et al. '06
- 2 BHs: Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$
  Boost: $\gamma = 1/\sqrt{1-v^2}$, $M = \gamma M_0$
- Impact parameter: $b \equiv \frac{L}{P}$



Initial setup: 2) No spins

- Orbital hang-up Campanelli et al. '06
- 2 BHs: Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

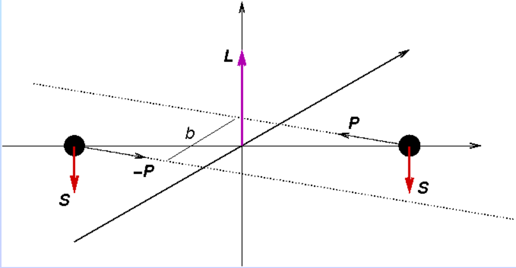
  Boost: $\gamma = 1/\sqrt{1-v^2}$, $M = \gamma M_0$
- Impact parameter: $b \equiv \frac{L}{P}$



Initial setup: 3) Anti-aligned spins

- Orbital hang-up   Campanelli et al. '06
- 2 BHs: Total rest mass: $M_0 = M_{\mathrm{A},\,0} + M_{\mathrm{B},\,0}$

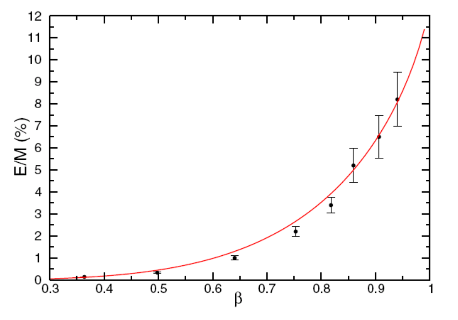
  Boost: $\gamma = 1/\sqrt{1-v^2}$, $M = \gamma M_0$
- Impact parameter: $b \equiv \frac{L}{P}$



Head-on: $b = 0, \quad \vec{S} = 0$

- Total radiated energy: 14 ± 3 % for $v \to 1$ US *et al.* '08

  About half of Penrose '74



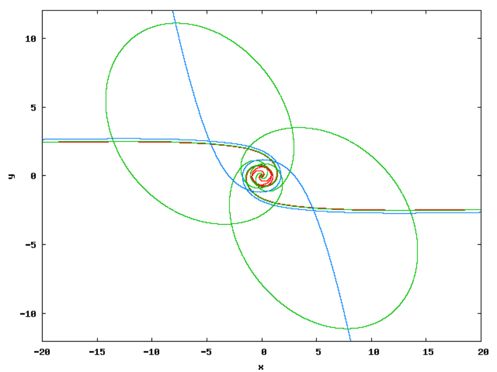
- Agreement with approximative methods

  Flat spectrum, multipolar GW structure Berti *et al.* '10

Grazing: $b \neq 0, \quad \vec{S} = 0, \quad \gamma = 1.52$

- Radiated energy up to at least 35 % M
- Immediate vs. Delayed vs. No merger

US, Cardoso, Pretorius, Berti, Hinderer & Yunes '09



Scattering threshold b_{scat} for $D = 4$, $\vec{S} = 0$

- $b < b_{\rm scat}$ $\Rightarrow$ Merger

  $b > b_{\rm scat}$ $\Rightarrow$ Scattering

- Numerical study: $b_{\rm scat} = \frac{2.5 \pm 0.05}{v} M$

  Shibata, Okawa & Yamamoto '08

- Independent study by US, Pretorius, Cardoso, Berti *et al.* '09, '12

  $\gamma = 1.23 \ldots 2.93$:

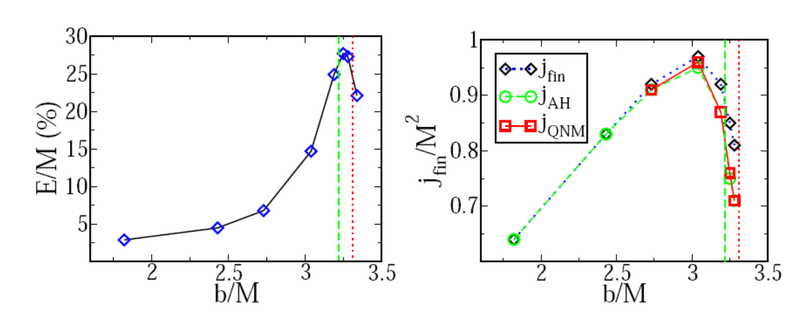
  $\chi = -0.6,\ 0,\ +0.6$ (anti-aligned, nonspinning, aligned)

- Limit from Penrose construction: $b_{\rm crit} = 1.685\ M$

  Yoshino & Rychkov '05

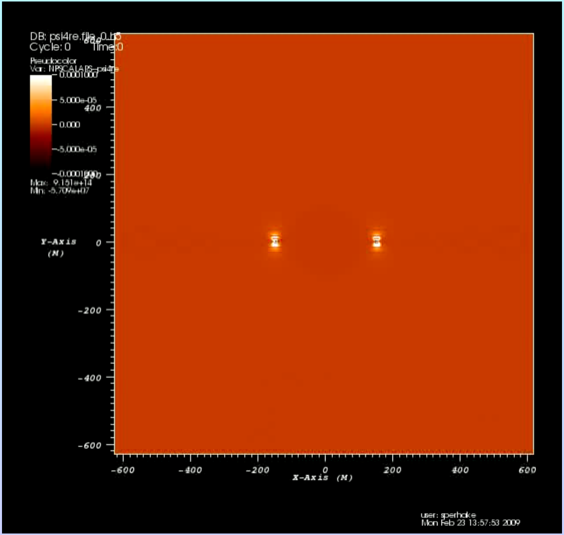
Radiated quantities $b \neq 0, \quad \vec{S} = 0$

- $b-$sequence with $\gamma = 1.52$
- Threshold of immediate merger Pretorius & Khurana '07
- $E_{\rm rad} \sim 35$ % for $\gamma = 2.93$; about 10 % of Dyson luminosity

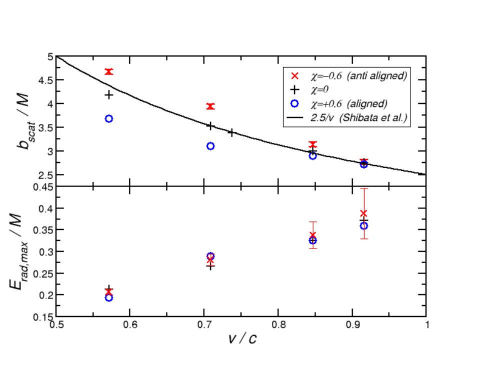


Sperhake *et al.* '09

Gravitational radiation: Delayed merger



Scattering threshold and radiated energy $\vec{S} \neq 0$

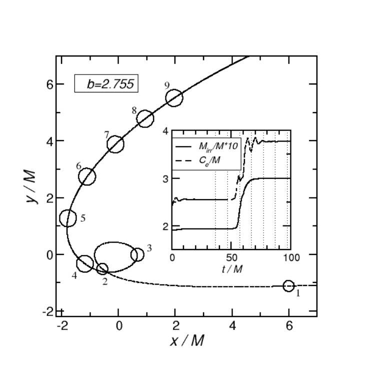


US, Berti, Cardoso & Pretorius ’12

- At speeds $v \gtrsim 0.9$ spin effects washed out
- E_{rad} always below $\lesssim 50\ \%\ M$

Absorption

- For large γ: $E_{kin} \approx M$
- If E_{kin} is not radiated, where does it go?
- Answer: ~ 50 % into E_{rad}, ~ 50 % is absorbed



US, Berti, Cardoso & Pretorius '12

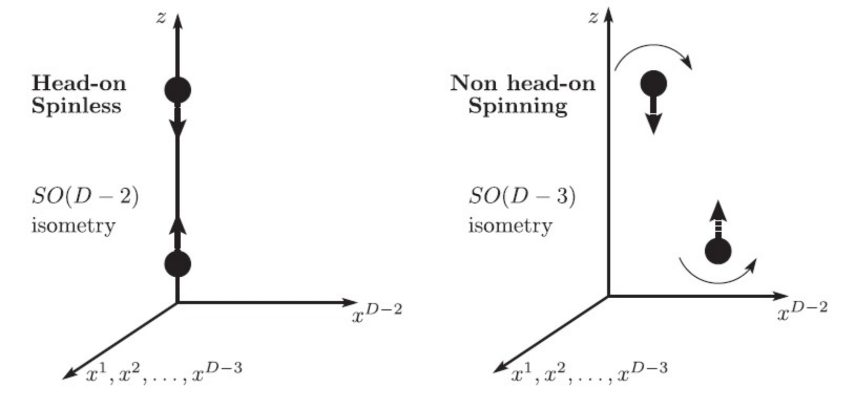
5. Collisions in $D > 4$

Moving to $D > 4$

- Symmetries allow dimensional reduction

  Geroch '70

- Reduces to "3+1" plus quasi-matter terms: scalar field



BSSN formulation with quasi matter

$$\partial_t \tilde{\gamma}_{ij} = [\mathrm{BSSN}],$$

$$\partial_t \chi = [\mathrm{BSSN}],$$

$$\partial_t K = [\mathrm{BSSN}] + 4\pi\alpha(E + S),$$

$$\partial_t \tilde{A}_{ij} = [\mathrm{BSSN}] - 8\pi\alpha\left(\chi S_{ij} - \tfrac{1}{3} S \tilde{\gamma}_{ij}\right),$$

$$\partial_t \tilde{\Gamma}^i = [\mathrm{BSSN}] - 16\pi\alpha\chi^{-1} j^i,$$

$$\partial_t \zeta = -2\alpha K_\zeta + \beta^m \partial_m \zeta - \tfrac{2}{3}\zeta \partial_m \beta^m + 2\zeta \frac{\beta^y}{y},$$

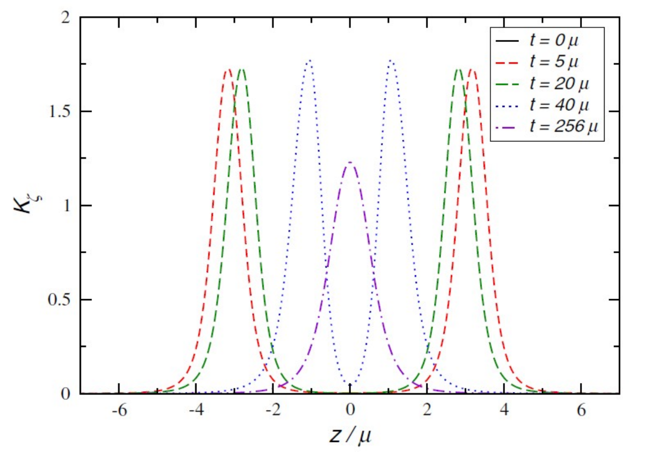
$$\partial_t K_\zeta = \ldots \quad ,$$

$$E,\ j^i,\ S_{ij} = f(\mathrm{BSSN},\ \zeta,\ K_\zeta).$$

Zilhão *et al.* '10

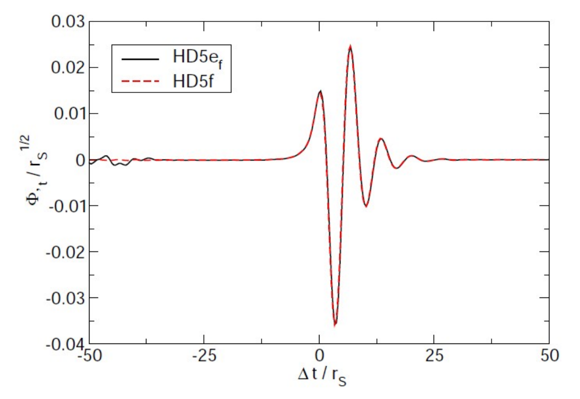
Head-on in $D = 5$

Initial data: $D = 5$ analogue of Brill-Lindquist data



GWs from head-on in $D = 5$

Wave extraction based on Kodama & Ishibashi '03

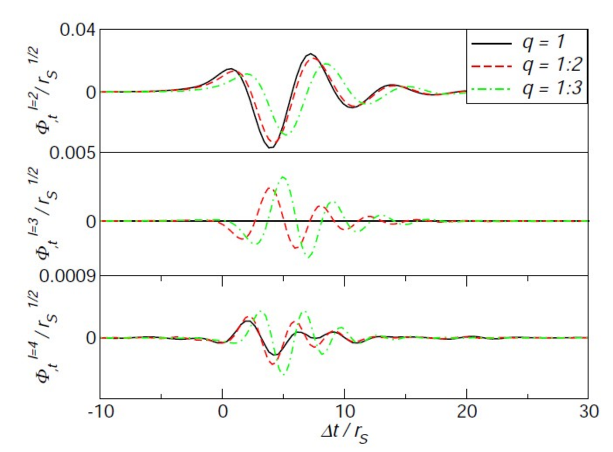


$E_{\rm rad} = 0.089\ \%M$ cf. $0.055\ \%M$ in $D = 4$

Witek *et al.* '10a

Unequal-mass head-on in $D = 5$

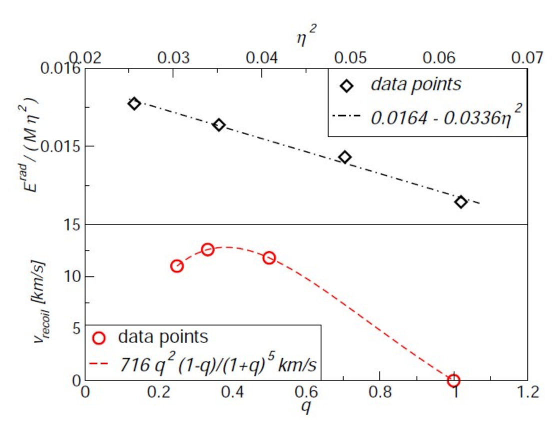
Kodama-Ishibashi multipoles



Witek *et al.* '10b

Unequal-mass head-on in $D = 5$

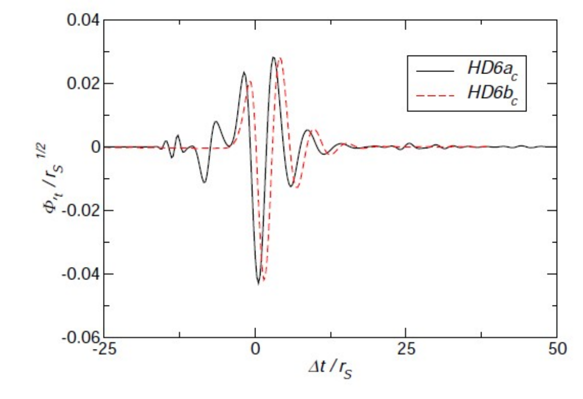
Radiated energy and momentum



Agreement within < 5 % with extrapolated point particle calculations

First black-hole collisions in $D = 6$

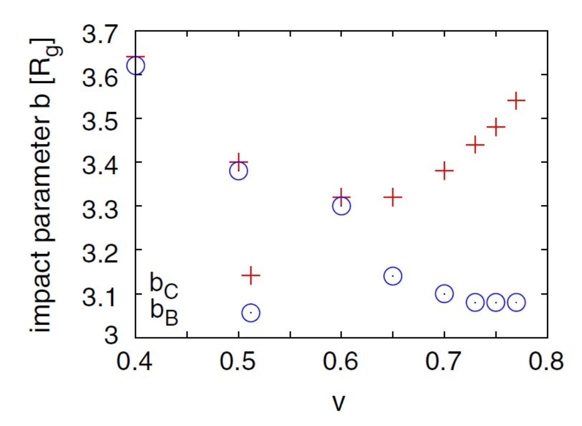
Witek *et al.* ’10



- Adjust shift parameters
- Use LaSh system Witek, Hilditch & US ’10

Scattering threshold in $D = 5$

Okawa, Nakao & Shibata ’11



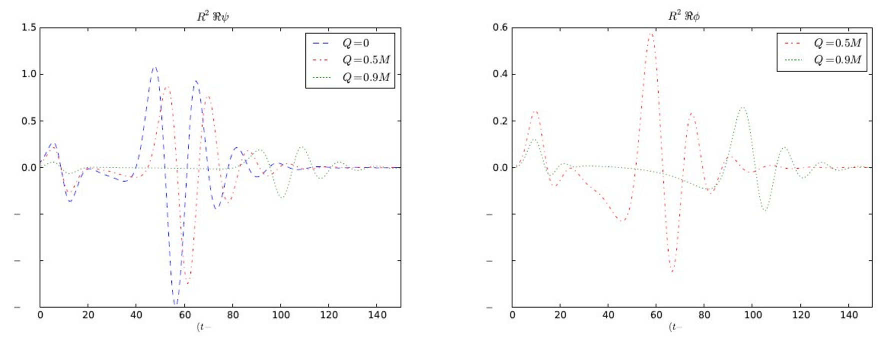
Numerical stability still an issue...

6. Non-asymptotically flat, non-vacuum spacetimes

Collisions of charged BHs in $D = 4$

Zilhão, Cardoso, Herdeiro, Lehner & US

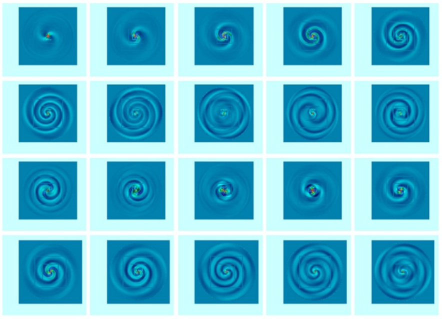
- Electro-vacuum Einstein-Maxwell Eqs.; Moesta et al. '10
- Brill-Lindquist construction for equal mass, charge BHs
- Wave extraction $\Phi_2 := F_{\mu\nu}\bar{m}^{\mu}k^{\nu}$

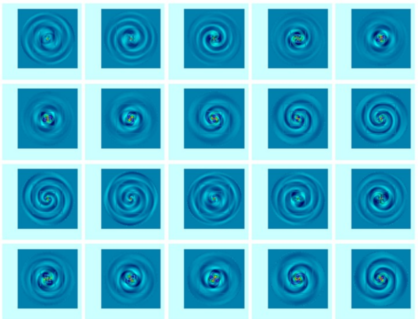


GW superradiance model

Witek *et al.* '10

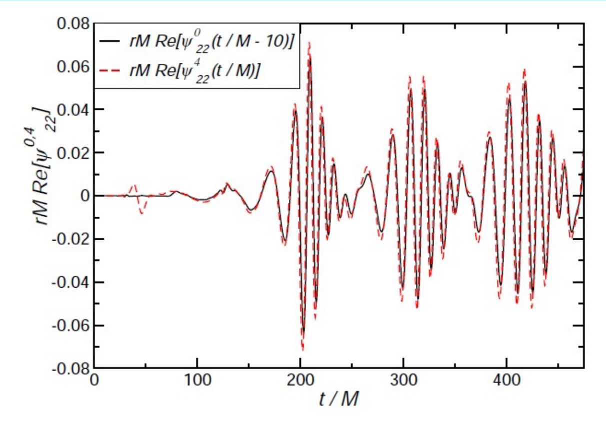
- BH binary inside lego sphere with reflective BC
- Extract Ψ_4, Ψ_0



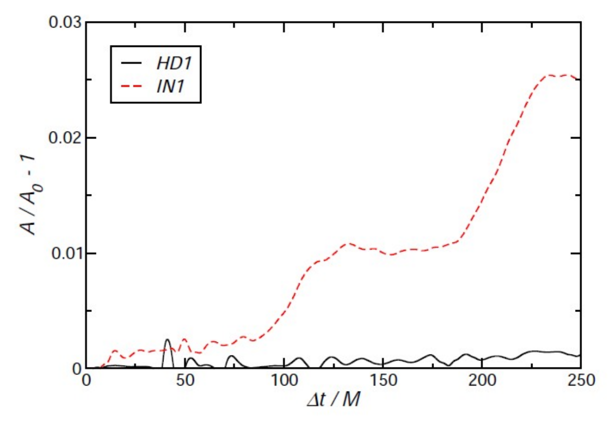


Quadrupole mode

Gravitational radiation (out going and ingoing)



Horizon area



Superradiance: high frequency absorbed, low frequency amplified

No conclusive evidence... More superradiance: Helvi's talk

Cosmic Censorship in $D = 5$

Pretorius & Lehner '10

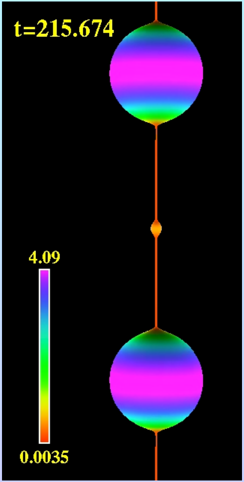
- Axisymmetric code
- Evolution of black string...
- Gregory-Laflamme instability

  cascades down

  in finite time

  until string has zero width

  $\Rightarrow$ naked singularity

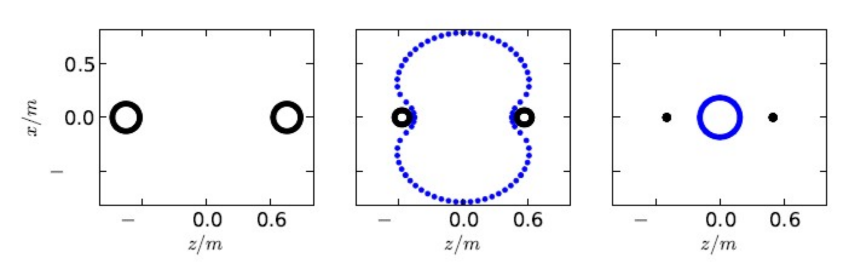


Cosmic Censorship in $D = 4$ de Sitter

Zilhão et al. '12

- Two parameters: MH, d
- Initial data: McVittie type binaries McVittie '33
- "Small BHs": $d < d_{crit} \Rightarrow$ merger

  $d > d_{crit} \Rightarrow$ no common AH
- "Large" holes at small d: Cosmic Censorship holds



7. Conclusions

Conlcusions and outlook

- High-energy collisions in $D = 4$
  - Scattering threshold $\sim 2.5\ M/v$
  - Maximal radiation $\sim 50\ \%\ M$
  - Rest of E_{kin} absorbed
  - Spin effects washed out at large boosts
- Collisions in higher D
  - Numerical stability still an issue
  - Head-on from rest: good agreement with PP calcs.
- Collisions of charged BHs; equal Q/m
- Collisions in de Sitter: Cosmic censorship holds
- ToDo: higher dimensions, charged BHs, AdS