Black-hole binary inspiral and merger in scalar-tensor theory of gravity

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General Relativity Seminar, DAMTP, University of Cambridge
24^{th} January 2014

Overview

Joined work with

E. Berti, V. Cardoso, L. Gualtieri, M. Horbatsch

Berti et al. 2013 (PRD **87**)

- Introduction, motivation
- Analytic results
- Numerical framework
- Numerical results
- Conclusions and outlook

1. Introduction, motivation

Motivation

- Goal: BHs in ST theory with non-trivial dynamics
- Time varying BCs (e.g. Cosmology)

  $\Rightarrow$ induce scalar charge of BHs
- Non-uniform scalar field due to galactic matter

  $\approx$ non-asymptotically flat BCs
- Super massive boson stars

  $\Rightarrow$ scalar field gradients
- Scalar field modifications of GR
  - Brans-Dicke
  - Bergmann-Wagoner $\omega(\phi),\ V(\phi)$
  - Multiple scalar fields
- Here: single scalar field, vacuum

Theoretical framework

Jordan frame: Physical metric $g^J_{\alpha\beta}$

- Action $S = \int d^4x \frac{\sqrt{-g^J}}{16\pi G}\left[F(\phi)R^J - 8\pi G Z(\phi) g^{\mu\nu}_J \partial_\mu\phi\,\partial_\nu\phi - U(\phi)\right]$

- GWs $\rightarrow$ 3 degs. of freedom
- Matter couples to $g^J_{\alpha\beta}$

Einstein frame: Conformal metric $g_{\alpha\beta} = F(\phi) g^J_{\alpha\beta}$

- $\varphi(\phi) = \int d\phi \left[\frac{3}{2}\frac{F'(\phi)^2}{F(\phi)^2} + \frac{8\pi G Z(\phi)}{F(\phi)}\right]^{1/2}$
- Action $S = \frac{1}{16\pi G}\int \left[R - g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - W(\varphi)\right]\sqrt{-g}\,d^4x$

Einstein vs. Jordan frame

Pro Einstein

- Minimally coupled scalar field $\Rightarrow$ numerics straightforward
- F, Z not explicitly present in evolutions

  $\Rightarrow$ Evolve whole class of theories at once

Pro Jordan

- Strongly hyperbolic formulation also available

  Salgado 2005 (CQG **23**), Salgado et al. 2008 (PRD **77**)
- Matter couples to evolved metric $g^J_{\alpha\beta}$

Here: Einstein frame more suitable

GWs in the Einstein and Jordan frames

Einstein frame evolution eqs. $G_{\alpha\beta} = \partial_\alpha \varphi \, \partial_\beta \varphi - \frac{1}{2} g_{\alpha\beta} g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi$

$$\Box \varphi = 0$$

Perturbations

$$g^J_{\alpha\beta} = \bar{g}^J_{\alpha\beta} + \delta g^J_{\alpha\beta} \qquad g_{\alpha\beta} = \bar{g}_{\alpha\beta} + \delta g_{\alpha\beta}$$

$$\phi = \bar{\phi} + \delta\phi \qquad \varphi = \bar{\varphi} + \delta\varphi$$

$$\delta g^J_{\alpha\beta} = \frac{1}{F(\bar{\phi})} \left[\delta g_{\alpha\beta} - \bar{g}^J_{\alpha\beta} F'(\bar{\phi}) \delta\phi \right]$$

$$\delta\phi = \left[\frac{3}{2} \frac{F'(\bar{\phi})^2}{F(\bar{\phi})^2} + \frac{8\pi G Z(\bar{\phi})}{F(\bar{\phi})} \right]^{-1/2} \delta\varphi$$

Newman-Penrose scalar: $\Psi_4 = \ddot{h}_+ - i\ddot{h}_\times$

Jordan version Ψ_4^J from Ψ_4, φ: see Barausse et al. 2012 (PRD **87**)

2. Analytic solutions

Single BH solutions to the linearized equations

- Equations: $R_{\alpha\beta} = 0\,, \qquad \Box\varphi = 0$

  i.e. solve Laplace eq. on BH background

- Schwarzschild in isotropic coordinates

  $$ds^2 = \frac{(2\tilde{r}-M)^2}{(2\tilde{r}+M)^2} dt^2 + \left(1 + \frac{M}{2\tilde{r}}\right)^4 [d\tilde{r}^2 + \tilde{r}^2 d\Omega^2]$$

  $$\Rightarrow \ldots \Rightarrow \varphi = 2\pi\sigma \left(1 + \frac{M^2}{4\tilde{r}^2}\right) \tilde{r}\cos\theta \approx 2\pi\sigma z$$

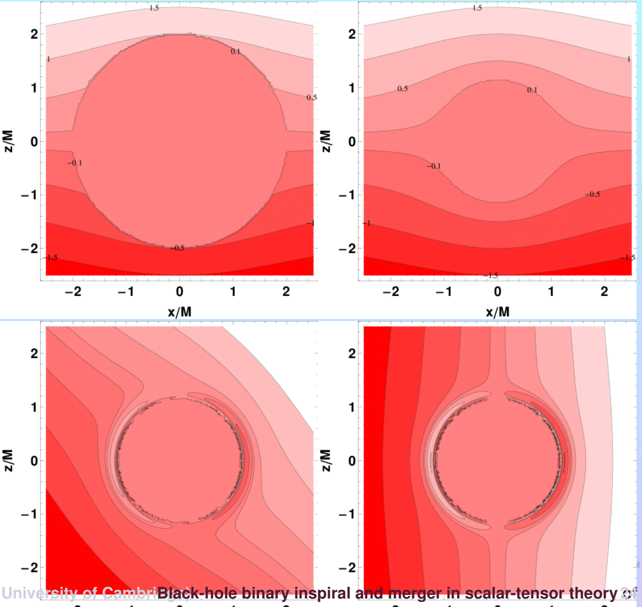
  asymptotically: constant gradient in z dir.

- Kerr BH; cf. Press 1972 (ApJ **175**)

  $$\varphi = 2\pi\sigma(r-M)\left[\frac{z}{r}\cos\gamma + \frac{x}{r} f_a \sin\gamma\right]\,, \qquad f_a = f_a(M,a,r)$$

  $\gamma =$ angle between BH spin and z axis

Contour plots of φ



Boundary conditions and multipolar expansion of φ

- Outgoing radiation condition at large r

$$\varphi = \varphi_{\text{ext}} + \frac{\Phi(t-r,\theta,\phi)}{r}$$

$$\Rightarrow \quad \partial_r(r\varphi) + \partial_t(r\varphi) = 4\pi\sigma r\cos\theta$$

- Multipolar expansion of Φ

$$\Phi(t-r,\theta,\phi) = \mathcal{M} + n^i\dot{\mathcal{D}}_i + \frac{1}{2}n^i n^j \ddot{\mathcal{Q}}_{ij} + \ldots$$

$$\vec{n} \equiv \vec{r}/r$$

$\mathcal{M}$ Monopole

$\mathcal{D}_i$ Dipole

$\mathcal{Q}_{ij}$ Quadrupole

Scalar radiation from BH binaries

- Scalar field background: $\varphi_{\rm ext} = 2\pi\sigma r \sin\theta \, \sin\phi$

  Orbital plane $yz \quad \Rightarrow \quad \theta$ relative to x axis

- Consider rotating source with frequency Ω

  $\Rightarrow$ Modulation in $\quad \varphi = \varphi_{\rm ext}[1 + f(\phi - \Omega t)]$

  $$\Rightarrow \varphi = 2\pi\sigma r \sin\theta \, \sin\phi \left[1 + \sum_m f_m e^{im(\phi - \Omega t)} \right]$$

  $$\Rightarrow \varphi_{lm} \sim \left[e^{-i(m+1)\Omega t} + e^{-i(m-1)\Omega t} \right]$$

- Monopole: Oscillation with Ω

  Dipole: Oscillation with 2Ω

- Confirmed by more elaborate calculation

3. Numerical framework

Evolution system

- "3+1" formalism with BSSN

  Baumgarte & Shapiro 1998 (PRD **59**) ,   Shibata & Nakamura 1995 (PRD **52**)

- Matter variables: $\varphi\,, \qquad (\partial_t - \mathcal{L}_\beta)\varphi = -2\alpha K_\varphi$

- "3+1" Matter sources

  $8\pi G\,\rho = 2K_\varphi^2 + \frac{1}{2}\partial_i\varphi\,\partial^i\varphi$

  $8\pi G\,j^i = 2K_\varphi\partial^i\varphi$

  $8\pi G\,S_{ij} = \partial_i\varphi\,\partial_j\varphi - \frac{1}{2}\gamma_{ij}\partial^m\varphi\,\partial_m\varphi + 2\gamma_{ij}K_\varphi^2$

  $8\pi G\,S = -\frac{1}{2}\partial^m\varphi\,\partial_m\varphi + 6K_\varphi^2$

- Straightforward to add to *Lean* code

  Moving punctures Campanelli et al.2005, Baker et al. 2005

  Cactus, Carpet, AHFinder   Schnetter et al. 2003, Thornburg 1995, 2003

Initial data

- Scalar field: Initialize as $\varphi = 2\pi\sigma z$

  Error: σ^2, $M^2/4\tilde{r}^2$

  $\Rightarrow$ Brief transient at early times

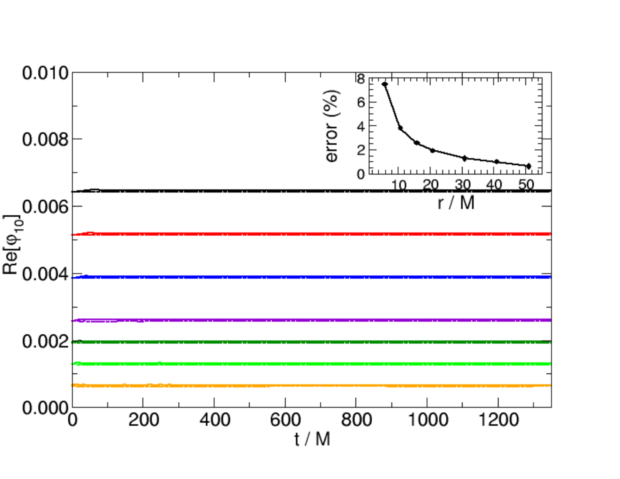
- BHs: Spectral solver Ansorg et al. 2004 (PRD **70)**

- Limits on σ
  - Scalar field energy $\sim (\nabla\varphi)^2 \sim \sigma^2 \sim \mathrm{const}$
  - Total scalar energy $M \sim \sigma^2 R^3$
  - Horizon if $M/R \sim \sigma^2 R^2 \sim 1$

    $\Rightarrow \quad \sigma < R^{-1} = \mathcal{O}(10^{-3}\, M_{\mathrm{BH}}^{-1})$
  - Conservative choice: $M_{\mathrm{BH}}\sigma = 10^{-7} \ldots 10^{-4}$

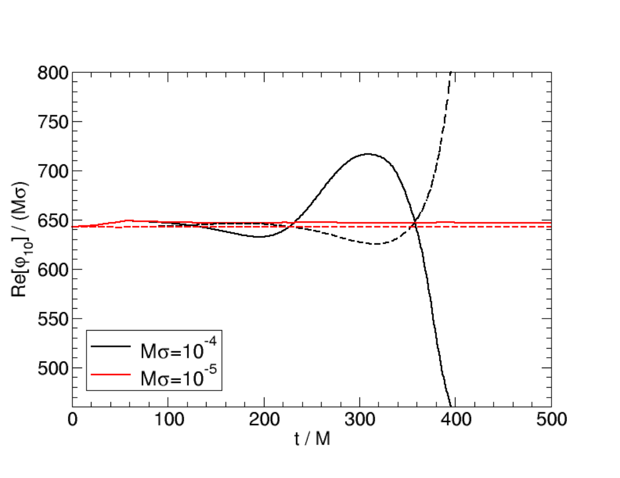
4. Numerical results

Schwarzschild BH: Num. vs. lin. solution $M\sigma = 10^{-5}$



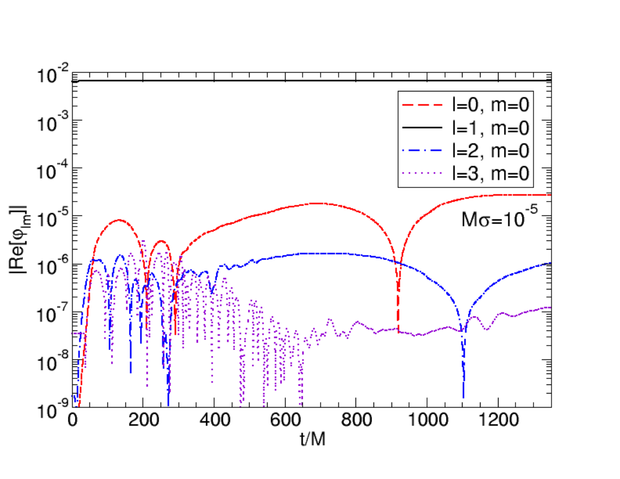
$\varphi_{10,\text{lin}} = \sqrt{\frac{4\pi}{3}} 2\pi\sigma(r - M)$ $\qquad r_{\text{ex}} = 5,\ 10,\ 15,\ 20,\ 30,\ 40,\ 50\ M$

Schwarzschild BH: σ dependence

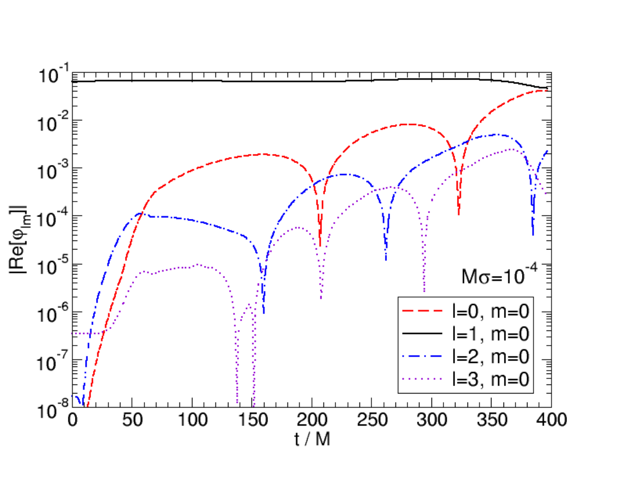


$r_{\text{ex}} = 50\ M$: Signs of collapse of scalar field for $M\sigma = 10^{-4}$

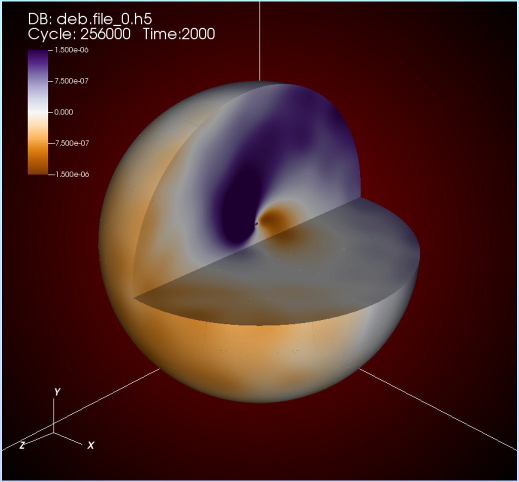
Schwarzschild BH: Scalar multipoles, $M\sigma = 10^{-5}$



Schwarzschild BH: Scalar multipoles, $M\sigma = 10^{-4}$

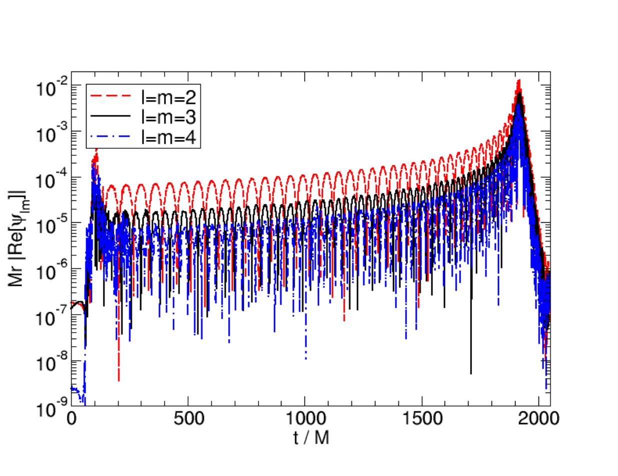


BH binary: Animation of $r\partial_t\varphi$



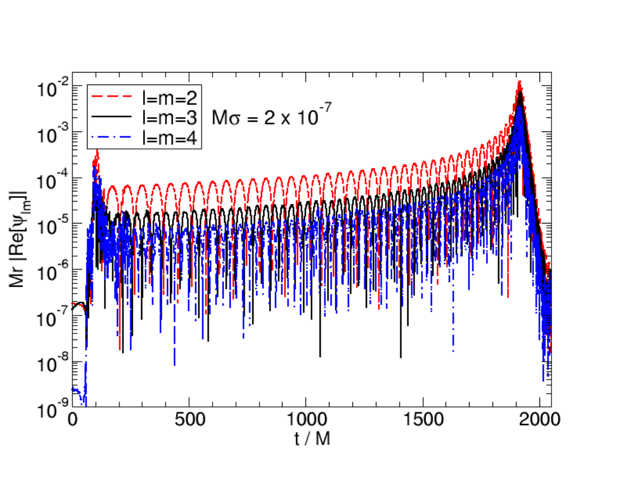
BH binary: Gravitational waves, $M\sigma = 0$

$q = 1/3$, $\boldsymbol{S} = 0$, *yz* plane: Multipoles of Ψ_4

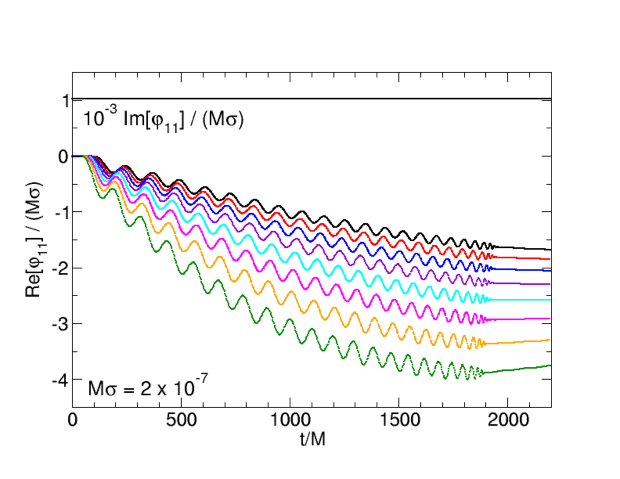


BH binary: Gravitational waves, $M\sigma = 2 \times 10^{-7}$

$q = 1/3$, $\boldsymbol{S} = 0$, yz plane: Multipoles of Ψ_4

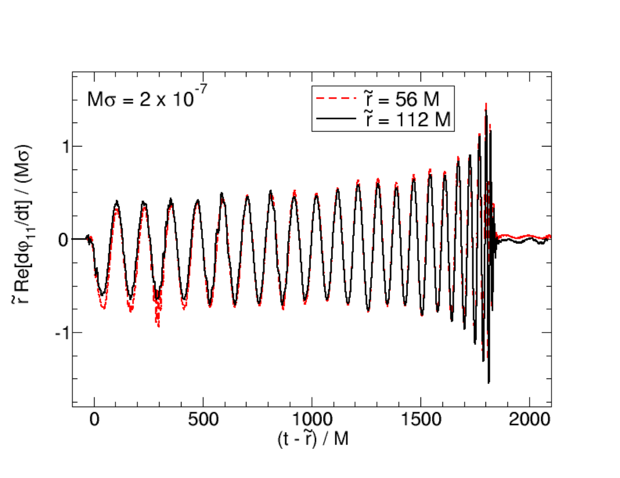


BH binary: Scalar dipole radiation, $M\sigma = 2 \times 10^{-7}$



$$r_{\rm ex} = 56 \ldots 112\, M$$

BH binary: Scalar dipole radiation, $M\sigma = 2 \times 10^{-7}$



Dipole oscillates at $2\Omega_{\rm orb}$ as expected

Features of the radiation

- Ringdown of $a/M = 0.543$ BH
  - GWs: $M\omega_{11\ \mathrm{lin}} = 0.476 - 0.0849i\,, \quad M\omega_{11\ \mathrm{num}} = 0.48 - 0.081i$
  - Scal.: $M\omega_{11\ \mathrm{lin}} = 0.351 - 0.0936i\,, \quad M\omega_{11\ \mathrm{num}} = 0.36 - 0.070i$
- Drift in φ_{11}
  - EFT calculation predicts some drift
  - Contribution from BH kick expected but not large enough
  - Frame dragging: order of magnitude ok, but r dependence not
  - Injection of scalar field energy through BCs

Probably: Combination of all effects

Conclusions and outlook

- Numerical simulations of BHs in ST Theory work very well!
- Einstein frame $\Rightarrow$ Simulate whole class of theories at once
- Single BHs: Excellent agreement with linearized calculations
- Large $M\sigma$ induces collapse of scalar field
- Our $M\sigma \gg$ values expected for dark matter models
- Large $M\sigma$ may still be possible: e.g. boson stars...
- Scalar radiation:
  - Monopole oscillates at $\Omega_{\rm orb}$
  - Dipole oscillates at $2\Omega_{\rm orb}$
- Scalar gradients circumvent the no-hair theorem