Averaging the average: Morphology transitions in spin precession of black-hole binaries

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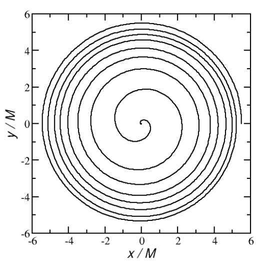
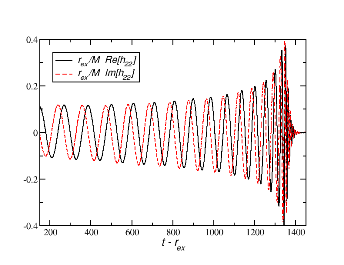
Overview

- Introduction
- The model
- The morphologies
- Precession averaged inspiral
- Conclusions

The 2-body problem

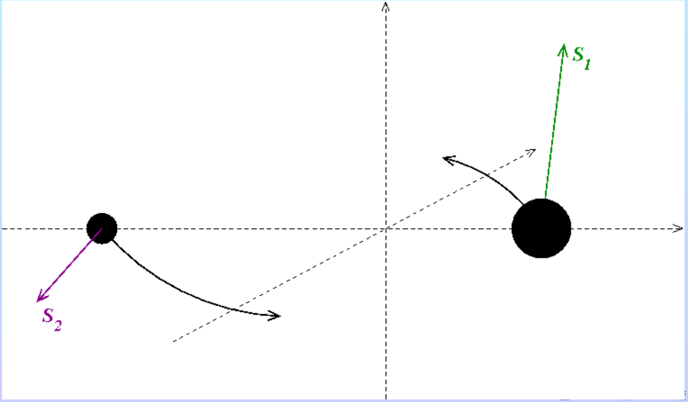
- Newtonian
  - Point masses
  - Kepler orbits
- General Relativity
  - Dissipative ⇒ GWs
  - Black holes
  - Spins ⇒ Additional parameters
- GW observations: LIGO, VIRGO,...





Spin precessing BH binaries

- Inspiral due to GW emission
- Precession of spins $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, orbital angular momentum $\boldsymbol{L}$
- Orbital motion



Timescale

- Adiabatic inspiral at 3.5PN; spin-spin coupling at 2PN order
- Zero eccentricity
- Timescales
  - Radiation reaction: $t_{RR} \sim r^4$
  - Precession: $t_{pre} \sim r^{5/2}$
  - Orbital motion: $t_{orb} \sim r^{3/2}$

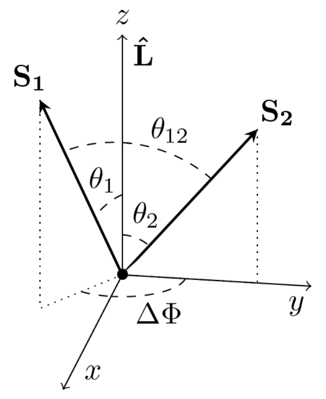
  $t_{orb} \ll t_{pre} \ll t_{RR}$
- Usual PN dynamics: orbit averaged $t_{orb} \ll t \ll t_{pre}$
- Here: precession averaged $t_{pre} \ll t \ll t_{RR}$

Parametrizing BBHs

- Zero eccentricity $\Rightarrow$ 9 parameters: $\boldsymbol{S}_1,\ \boldsymbol{S}_2,\ \boldsymbol{L}$
- Align z axis (e.g. with $\boldsymbol{L}$ or $\boldsymbol{J}$) $\rightarrow$ 7
- Rotation about z axis $\rightarrow$ 6
- S_1, S_2 conserved $\rightarrow$ 4
- $L \sim r^{1/2}$ is a measure for the separation, i.e. time $\rightarrow$ 3

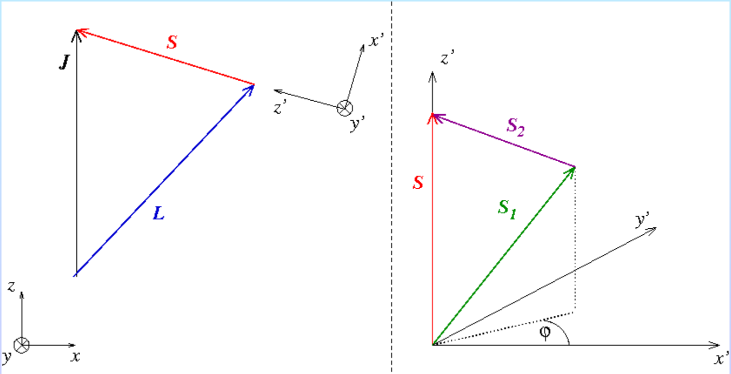
Option 1: Orientation of spin vectors

- $\theta_{1,2} = \angle(\boldsymbol{S}_{1,2}, \boldsymbol{L})$
- $\Delta\phi = \angle(\boldsymbol{S}_{1,\perp}, \boldsymbol{S}_{2,\perp})$
- Advantage: Easy to visualize
- Drawback: All vary on t_{pre}
- Conserved or averaged variables better!



Option 2: Variables adapted to timescales

- $\xi \equiv M^{-2}\left[(1+q)\boldsymbol{S}_1 + (1+q^{-1})\boldsymbol{S}_2\right] \cdot \hat{\boldsymbol{L}}$ conserved!
- J conserved on t_{pre}
- $S \leftrightarrow \varphi$ varies on t_{pre}



The precession cycle

- $S_{min} = \max\{|J - L|,\ |S_1 - S_2|\}$

  $S_{max} = \min\{J + L,\ S_1 + S_2\}$

- Constraint on S, φ:

  $$\xi(S,\varphi) = \{(J^2 - L^2 - S^2)\left[S^2(1+q)^2 - (S_1^2 - S_2^2)(1-q^2)\right] - (1-q^2)A\cos\varphi\}/(4qM^2S^2L)$$

  $A = A(J, L, S_1, S_2, S) \geq 0$

- $\cos\varphi = \mp 1 \quad \Rightarrow$ effective potentials $\xi_\pm(S)$: $\quad \xi_- \leq \xi \leq \xi_+$

- $A = 0$ if $S = S_{min}$ or $S = S_{max}$

  $\xi_\pm(S)$ form closed loop

The precession cycle for $L > S_1 + S_2$

- $\Delta\phi = 0$ or π on $\xi_\pm$
- $\Delta\phi$ jumps $0 \leftrightarrow \pi$

  if $\cos\theta_i = \pm 1$
- $\xi = \xi_{max}$:

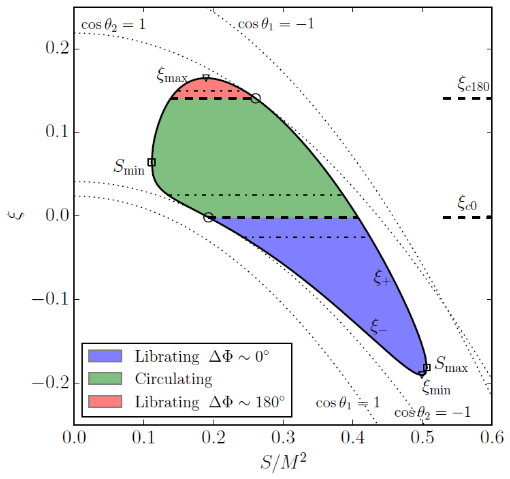
  $\Delta\phi = \pi$ resonance

  Schnittman ’04
- $\xi = \xi_{min}$:

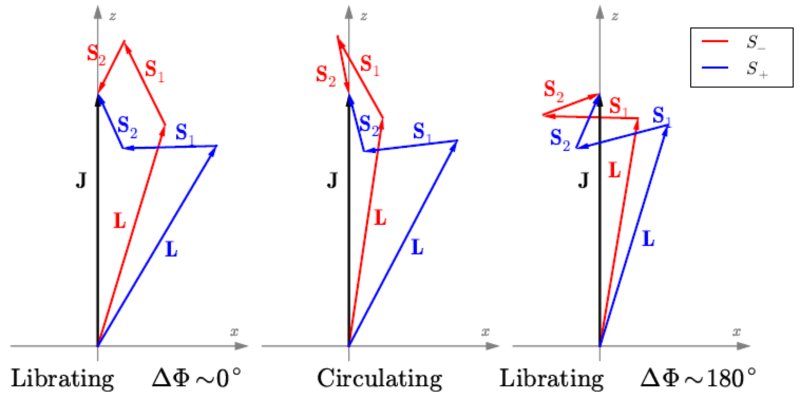
  $\Delta\phi = 0$ resonance
- Librating $\Delta\phi \sim 0$

  Circulating

  Librating $\Delta\phi \sim \pi$



The morphologies



- Over t_{RR}, a binary can change morphology
- Empirically: Only from circulating to librating!

Evolution of J

- Note: S parametrizes the precession
- Spin precession eqs.

$$\Rightarrow \frac{dS}{dt} = -\frac{3(1-q^2)}{2q}\frac{S_1 S_2}{S}\frac{(\eta^2 M^3)^3}{L^5}\left(1-\frac{\eta M^2 \xi}{L}\right)\sin\theta_1\ \sin\theta_2\ \sin\Delta\phi$$

- Except for a new resonant case $\alpha = 2\pi n$, this gives

$$\left\langle \frac{dJ}{dL} \right\rangle = \frac{1}{2JL}\left[J^2 + L^2 - \frac{2}{\tau}\int_{S_-}^{S_+}\frac{S^2 dS}{dS/dt}\right]$$

with $\tau \equiv 2\int_{S_-}^{S_+}\frac{dS}{|dS/dt|}$

- Evolve J numerically but with $t_{pre} \ll \Delta t \lesssim t_{RR}$

Conclusions

- Use hierarchy of time scales: $t_{orb} \ll t_{pre} \ll t_{RR}$
- Use convenient variables: ξ, J, S
- BBHs precession represented in $S\,\xi$ diagram
- $\Delta\phi = 0,\ \pi$ on edge of allowed configurations
- 3 morphologies: circulating, librating about $\Delta\phi = 0,\ \pi$
- BBHs undergo phase transitions over t_{RR}
- Precession averaged inspiral $\Rightarrow$ faster algorithm for evolving J