Morphologies and phase transitions in precessing black-hole binaries

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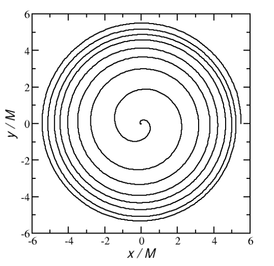
Overview

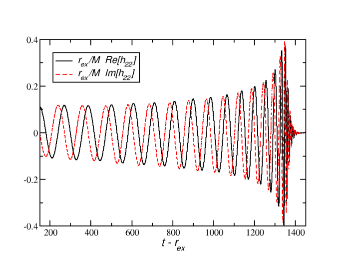
- Introduction
- Time scales
- The precessional time scale
- Evolutions on t_{RR}
- Conclusions

1. Introduction

The 2-body problem

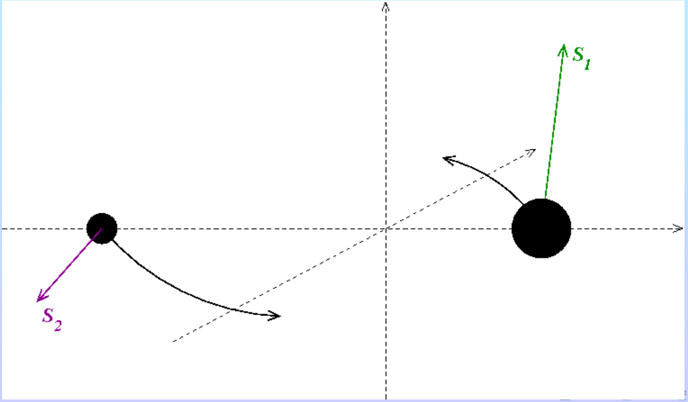
- Newtonian
  - Point masses
  - Kepler orbits
- General Relativity
  - Dissipative ⇒ GWs
  - Black holes
  - Spins ⇒ Additional parameters
- GW observations: LIGO, VIRGO,...





Spin precessing BH binaries

- Inspiral due to GW emission
- Precession of spins $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, orbital angular momentum $\boldsymbol{L}$
- Orbital motion



2. Time scales

Time scales

- Adiabatic inspiral at 2.5 PN; spin-spin coupling at 2 PN order
- Zero eccentricity
- Timescales
  - Radiation reaction: $t_{RR} \sim r^4$
  - Precession: $t_{pre} \sim r^{5/2}$
  - Orbital motion: $t_{orb} \sim r^{3/2}$

  $t_{orb} \ll t_{pre} \ll t_{RR}$
- Usual PN dynamics: orbit averaged $t_{orb} \ll t \ll t_{pre}$
- Here: precession averaged $t_{pre} \ll t \ll t_{RR}$

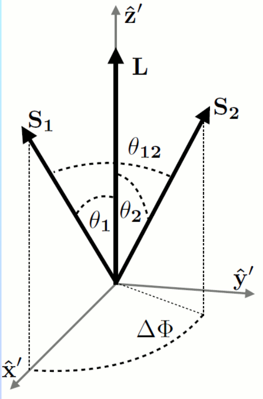
3. The precessional time scale

Parametrizing BBHs

- Zero eccentricity $\Rightarrow$ 9 parameters: $\boldsymbol{S}_1,\ \boldsymbol{S}_2,\ \boldsymbol{L}$
- Align z axis (e.g. with $\boldsymbol{L}$ or $\boldsymbol{J}$) $\rightarrow$ 7
- Rotation about z axis $\rightarrow$ 6
- S_1, S_2 conserved $\rightarrow$ 4
- $L \sim r^{1/2}$ is a measure for the separation, i.e. time $\rightarrow$ 3

Option 1: Orientation of spin vectors

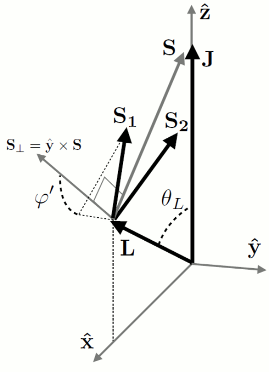
- $\theta_{1,2} = \angle(\boldsymbol{S}_{1,2}, \boldsymbol{L})$
- $\Delta\phi = \angle(\boldsymbol{S}_{1,\perp}, \boldsymbol{S}_{2,\perp})$
- Advantage: Easy to visualize
- Drawback: All vary on t_{pre}
- Conserved or averaged variables better!



Option 2: Variables adapted to timescales

- $J = |\boldsymbol{J}|$
- $\theta_L = \angle(\boldsymbol{L}, \boldsymbol{J})$
- $\varphi' =$ rotation of $\boldsymbol{S}_1$, $\boldsymbol{S}_2$ around $\boldsymbol{S}$
- Replace θ_L with S
- J const on $t_{\rm pre}$

  S, φ' vary on $t_{\rm pre}$



Constant of motion

- Note: S, φ', J completely determine the binary evolution on $t_{\rm pre}$:

$$\boldsymbol{L} = \boldsymbol{L}(S, \varphi'; \; J, S_1, S_2, L)\,, \qquad \boldsymbol{S}_{1,2} = \boldsymbol{S}_{1,2}(S, \varphi'; \; J, S_1, S_2, L)$$

- At 2 PN spin-precession, 2.5 PN Radiation Reaction:

$$\begin{aligned} \xi(S, \varphi') &= \{(J^2 - L^2 - S^2)[S^2(1+q^2) - (S_1^2 - S_2^2)(1-q^2)] \\ &\quad -(1-q^2)\, A_1\, A_2\, A_3\, A_4 \cos\varphi'\} \;/\; (4qM^2 S^2 L) \\ A_i &= A_i(J, L, S, S_1, S_2) \geq 0 \end{aligned}$$

conserved: *projected effective spin*

- Constraint on (S, φ') $\Rightarrow$ trade φ' for ξ

Summary on t_{pre}

- Choose binary parameters $S_1,\ S_2,\ \xi,\ q,\ M$
- Fix J of the binary
- Fix its separation r by specifying L

$\Rightarrow$ On the precession timescale t_{pre},

its evolution is a 1-parameter evolution in S

Physically allowed parameter ranges I

- $|\cos\varphi'| \le 1$
- Recall

$$\xi(S,\varphi') = \{(J^2-L^2-S^2)[S^2(1+q^2)-(S_1^2-S_2^2)(1-q^2)] \\ -(1-q^2)\,A_1\,A_2\,A_3\,A_4\,\cos\varphi'\}\;/\;(4qM^2S^2L)$$

- Then $\xi_- \le \xi \le \xi_+$ with

$$\xi_\pm(S) = \{(J^2-L^2-S^2)[S^2(1+q^2)-(S_1^2-S_2^2)(1-q^2)] \\ \pm(1-q^2)\,A_1\,A_2\,A_3\,A_4\}\;/\;(4qM^2S^2L)$$

- Note: $\xi(S,\varphi') = \xi_\pm(S) \;\Rightarrow\; \varphi' = 0$ or $\varphi' = \pi$

$\Rightarrow$ $\boldsymbol{L}$, $\boldsymbol{S}_1$, $\boldsymbol{S}_2$ co-planar

Phycially allowed parameter ranges II

- $S_{\min} \leq S \leq S_{\max}$, where

  $S_{\min} = \max\left(|J - L|,\ |S_1 - S_2|\right)$,

  $S_{\max} = \min\left(J + L,\ S_1 + S_2\right)$

- $S = S_{\min}$ or $S = S_{\max}$ $\Rightarrow \ldots \Rightarrow$ one $A_i = 0$

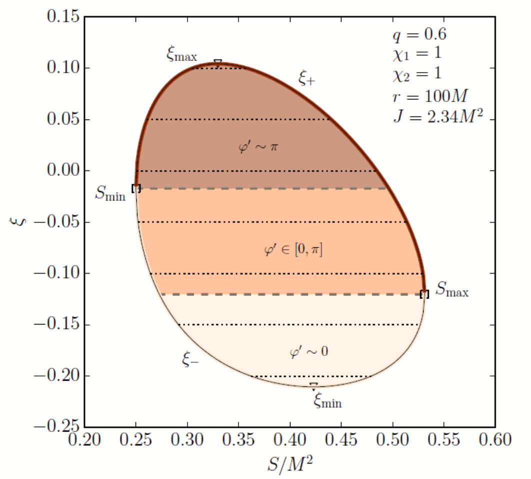
  $\Rightarrow \xi = \xi_- = \xi_+$

- For any other value of S: all $A_i > 0$

  $\Rightarrow$ $\xi_-(S) \leq \xi(S, \varphi') \leq \xi_+(S)$ as $\cos\varphi'$ varies from $+1$ to -1

$\Rightarrow$ Closed loop in (S, ξ) plane: allowed configs. inside

Effective potential diagram



Note: $\varphi' = 0$ on ξ_- ; $\varphi' = \pi$ on ξ_+

The precession cycle

Now consider a binary with fixed ξ:

- As S varies, binary moves on horizontal line in (S, ξ) plane
- Turning points: $\xi(S) = \xi_\pm$

  $\Rightarrow \ldots \Rightarrow$ there are 2 solutions for S : S_- , S_+
- Binary precession quantified by evolution of $S \in [S_-, S_+]$

The precession cycle

What about φ' during a precession cycle?

1) Both turning points on $\xi_+ \;\Rightarrow\; \cos\varphi' = -1 \;\Rightarrow\; \varphi' = \pi$

   $\Rightarrow\;\; \varphi'$ oscillates around π but never reaches 0

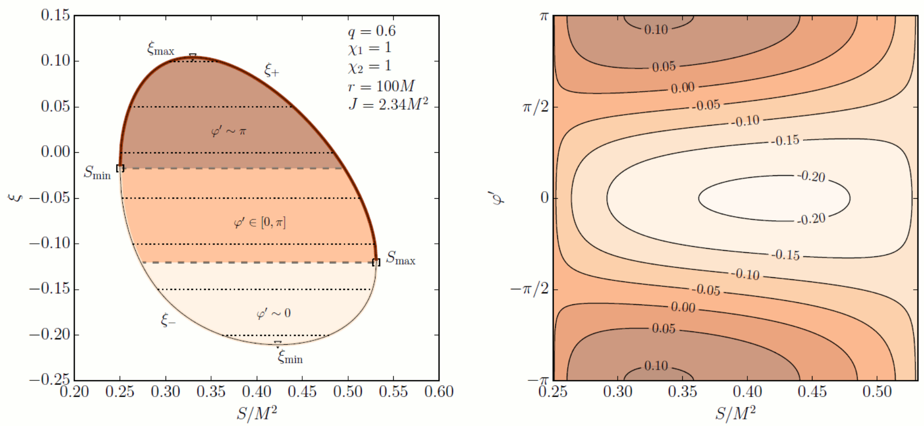
2) Both turning points on $\xi_- \;\Rightarrow\; \cos\varphi' = +1 \;\Rightarrow\; \varphi' = 0$

   $\Rightarrow\;\; \varphi'$ oscillates around 0 but never reaches π

3) One turning point on ξ_-, the other on ξ_+

   $\Rightarrow\;\; \varphi'$ circulates all the way from 0 to π and back

The precession cycle



Summary

- We now have 3 morphologies!
- Where do they meet?

  Answer: When $S_\pm = S_{\rm min}$ or $S_{\rm max}$

- Note: At $\xi_{\rm max}$, $\xi_{\rm min}$ we have $S = {\rm const}$

  Schnittmann's (2004) spin-orbit resonances

- Note: What about $q \to 1$?

  Then $\xi_+(S) = \xi_-(S) \quad \Rightarrow \quad S$ fixed by prescribing ξ

  "Egg" squashed to "line"

Switching coordinates: $(J,\, S,\, \xi) \leftrightarrow (\theta_1,\, \theta_2,\, \Delta\Phi)$

One straightforwardly shows (cosine theorem):

$$
\begin{aligned}
\cos\theta_1 &= \frac{1}{2(1-q)S_1}\left[\frac{J^2-L^2-S^2}{L} - 2\frac{2qM^2\xi}{1+q}\right]\,, \\
\cos\theta_2 &= \frac{q}{2(1-q)S_2}\left[-\frac{J^2-L^2-S^2}{L} + 2\frac{2M^2\xi}{1+q}\right]\,, \\
\cos\theta_{12} &= \frac{S^2-S_1^2-S_2^2}{2S_1S_2}\,, \\
\cos\Delta\Phi &= \frac{\cos\theta_{12}-\cos\theta_1\;\cos\theta_2}{\sin\theta_1\;\sin\theta_2}\,,
\end{aligned}
$$

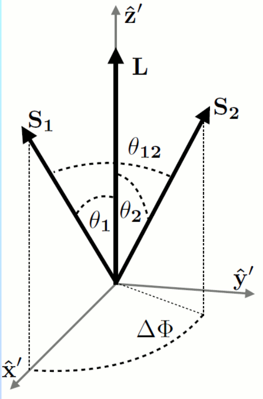
3 morphologies for $\Delta\Phi$

- Recall:

  $\varphi' = 0$ or $\varphi' = \pi \quad \Rightarrow \quad \sin\varphi' = 0$

  $\Rightarrow \quad \boldsymbol{L},\ \boldsymbol{S}_1,\ \boldsymbol{S}_2$ co-planar

  $\Rightarrow \quad \sin\Delta\Phi' = 0 \quad \Rightarrow \quad \Delta\Phi = 0$ or $\Delta\Phi = \pi$



- 3 morphologies in $\Delta\Phi$:

  1) $\Delta\Phi$ librates around π

  2) $\Delta\Phi$ librates around 0

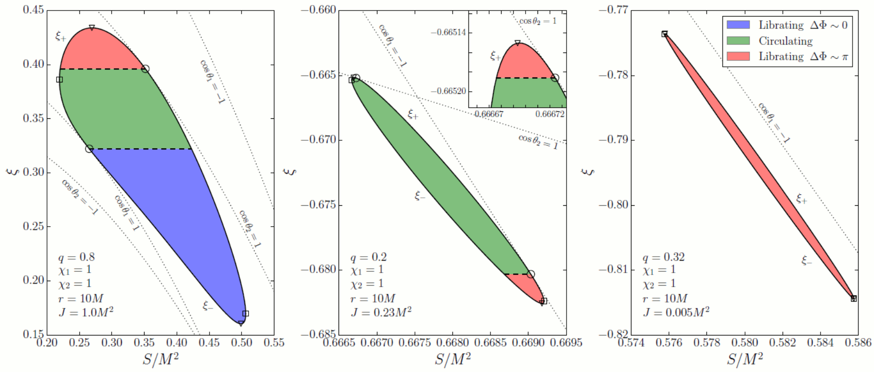
  3) $\Delta\Phi$ circulates in $[0, \pi]$

- Warning: Although $\sin\varphi' = 0 \quad \Leftrightarrow \quad \sin\Delta\Phi = 0$,

  $\varphi' = 0$ or π and $\Delta\Phi = 0$ or π is NOT determined!!

Morphology diagrams

Depending on BH parameters, not all morphologies may be available



When do morphologies change?

- Clearly, $\Delta\Phi$ must change on the $\xi_{\pm}$ loop!
- We know: $\varphi' = 0$ or π on loop

  $\Rightarrow \quad \Delta\Phi = 0$ or π on loop

  $$\Rightarrow \quad \cos\Delta\Phi = \frac{\cos\theta_{12} - \cos\theta_1 \; \cos\theta_2}{\sin\theta_1 \; \sin\theta_2} = \pm 1 \;\; \text{on loop}$$
- Discontinuous change only possible if $\sin\theta_1 = 0$ or $\sin\theta_2 = 0$

  $\Rightarrow$ one BH spin aligned with $\boldsymbol{L}$

4. Evolutions on t_{RR}

Traditional orbit averaged PN evolutions

- Valid for times $t \approx t_{orb} \ll t_{pre}$
- Ignore precession: $\boldsymbol{L}(t),\ \boldsymbol{S}_1(t),\ \boldsymbol{S}_2(t)$ held fixed
- Then we have: $\left\langle \frac{d\boldsymbol{J}}{dt} \right\rangle = \frac{1}{T} \int_0^{2\pi} \frac{d\boldsymbol{J}}{dt} \frac{d\psi}{d\psi/dt}$
- Here: $T =$ orbital period

  $\psi =$ e.g. Kepler's true anomaly
- Then handle precession as a quasi-adiabatic process...

Averaging the average

- Let X be some quantity
- Precession average $\langle X \rangle_{pre} := \frac{2}{\tau} \int_{S_-}^{S_+} \langle X \rangle_{orb} \frac{dS}{|dS/dt|}$
- $\ldots \Rightarrow$ 1.5 PN angular momentum flux:

$$\frac{dJ}{dL} = \frac{1}{2LJ}\left(J^2 + L^2 - \langle S \rangle_{pre}\right)$$

with

$$\frac{dS}{dt} = -\frac{3(1-q^2)}{2q} \frac{S_1 S_2}{S} \frac{(\eta^2 M^3)^3}{L^5} \left(1 - \frac{\eta M^2 \xi}{L}\right) \sin\theta_1 \; \sin\theta_2 \; \sin\Delta\Phi$$

Comments

- ξ conserved on t_{RR} $\Rightarrow$ only L, J vary on $t > t_{pre}$
- One exception: Special resonance configurations.

  Set of measure zero in parameter space.

  But: Possibly interesting physics... (under study)
- "Only" $1.5PN$, but:

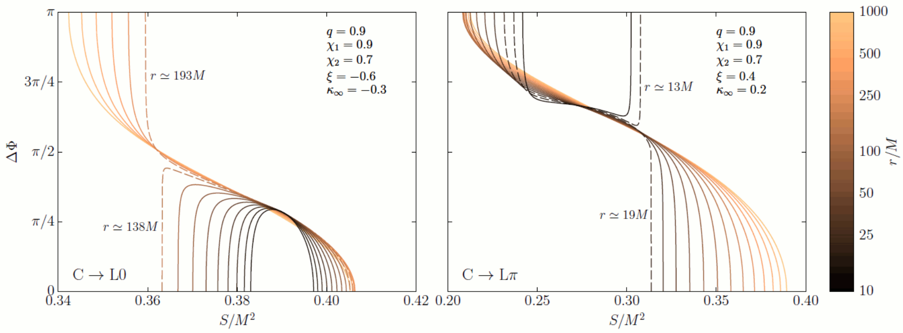
  Comparison with full PN orbit averaged evolutions

  $\Rightarrow$ excellent agreement down to at least $50\ M$ separation
- Precession averaged: Huge computational speed-up!

Example applications I: Phase transition

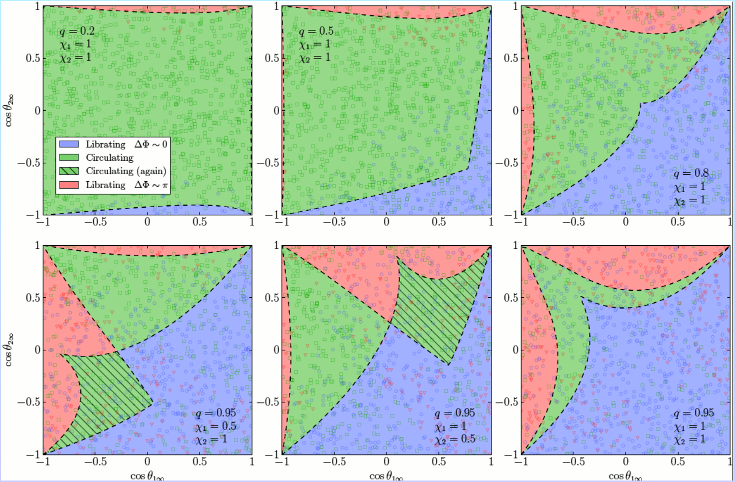
Evolution of the $\Delta\Phi$ range during inspiral:

circulating $\rightarrow$ librating



Example applications II: BH binary memory effect

- Determine morphology with LIGO/Virgo measurements
- Color shows $\theta_1,\ \theta_2$ of BHs at large distances



5. Conclusions

Conclusions

- Use hierarchy of time scales: $t_{orb} \ll t_{pre} \ll t_{RR}$
- Use convenient variables: ξ, J, S
- BBHs precession represented in $S\,\xi$ diagram
- $\Delta\phi = 0,\ \pi$ on edge of allowed configurations
- 3 morphologies: circulating, librating about $\Delta\phi = 0,\ \pi$
- BBHs undergo phase transitions over t_{RR}
- Precession averaged inspiral $\Rightarrow$ faster algorithm for evolving J