Strong gravity on supercomputers

Ulrich Sperhake

DAMTP, University of Cambridge

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Gravitational Waves: Ripples in spacetime

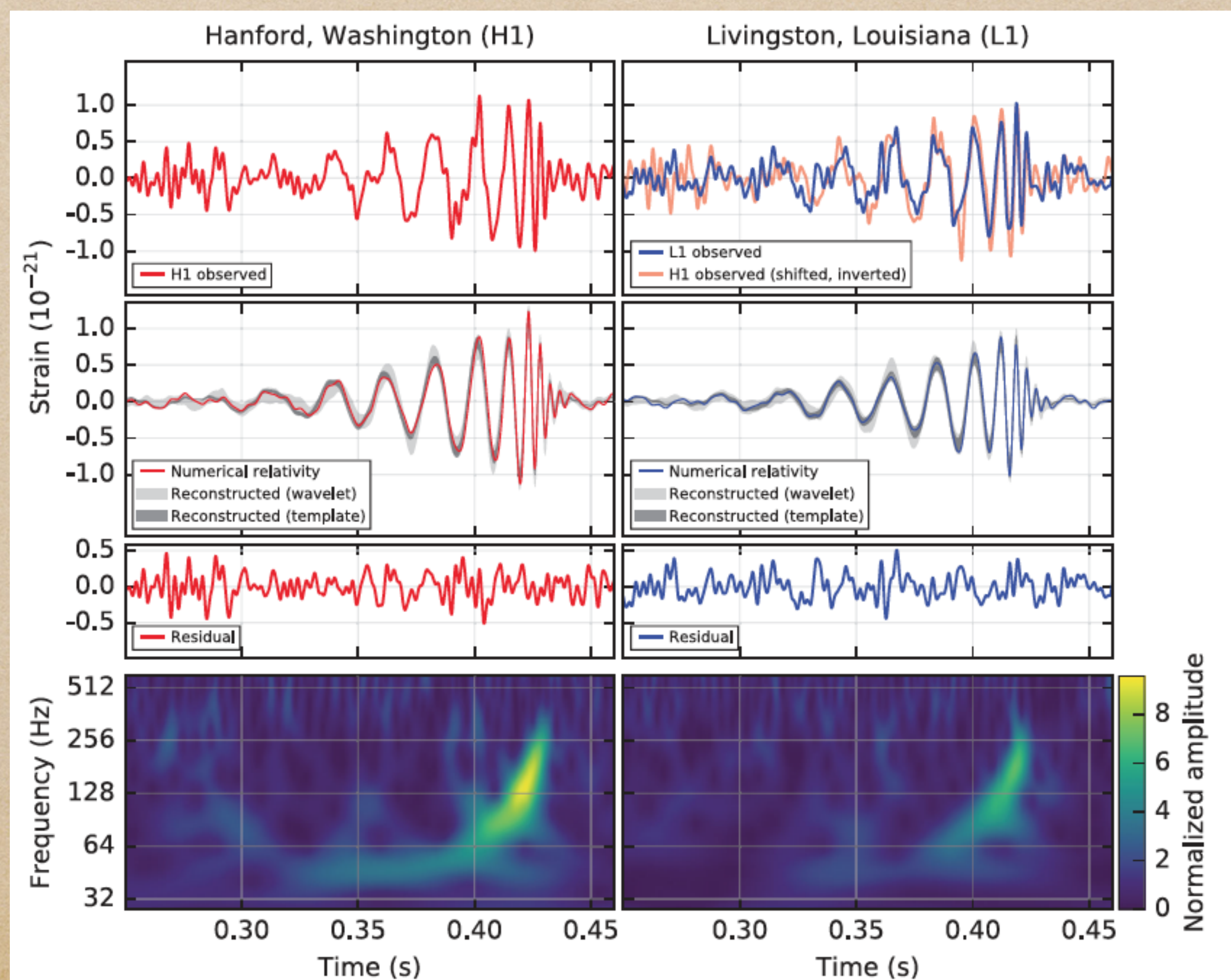
- Unusual news headlines on 11/12 February 2016
- First direct detection of gravitational waves: GW150914

So, what happened?

- Sep 14, 2015 at 09:50:45 UTC: SNR ~ 24

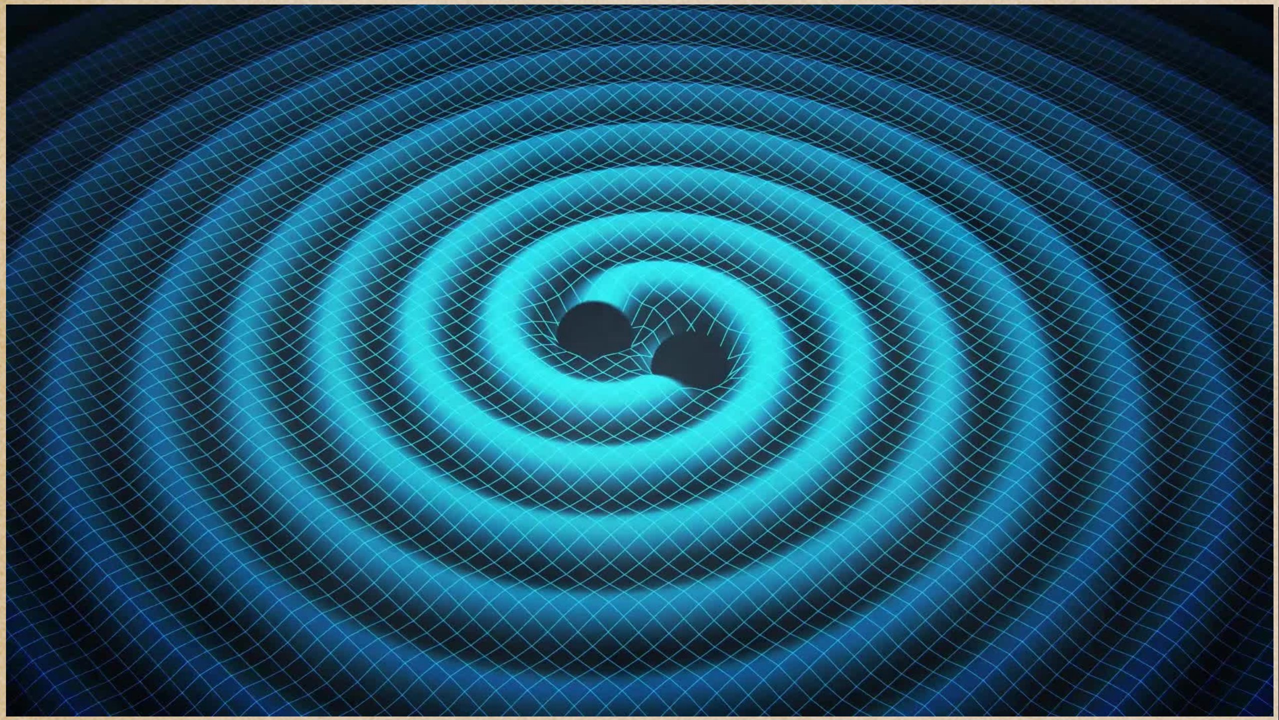
  Abbott et al. PRL 1602.03837, Abbott et al. 1606.01210

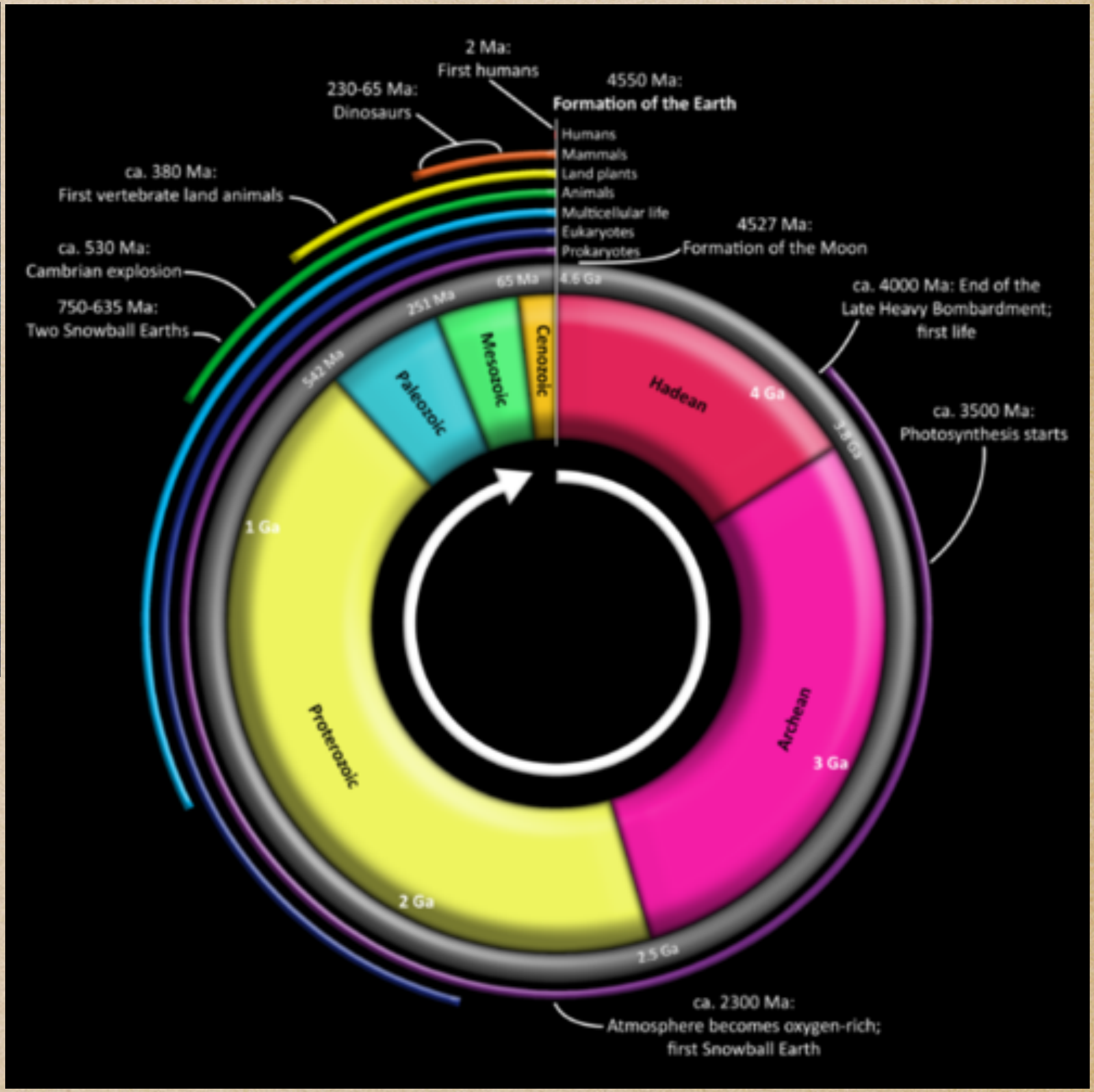
- BBH inspiral, merger and ringdown: $m_1 = 35^{+5}_{-3}\, m_\odot\,, \quad m_2 = 30^{+3}_{-4}\, M_\odot$



What really happened...

- Once upon a time: $1.34^{+0.52}_{-0.59}\ \mathrm{Gyr}$ ago, somewhere in the universe





- Deep Precambrian

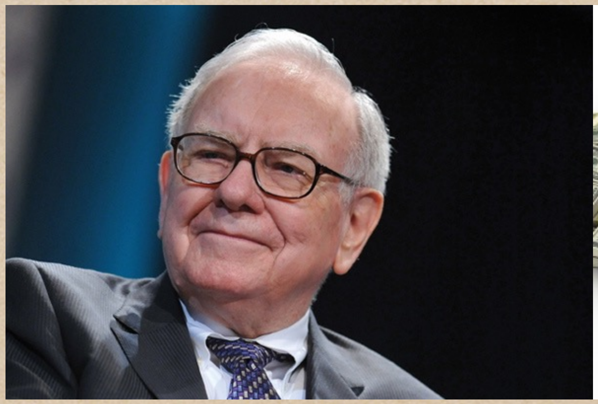
Overview

- Introduction, Motivation
- Foundations of numerical relativity
  - Formulations of Einstein's Eqs.: 3+1, BSSN, GHG
  - Initial data, gauge
  - Technical ingredients: Discretization, AMR, boundaries…
  - Diagnostics: Horizons, momenta, GWs,…
- Applications and selected results
  - Gravitational wave physics
  - Fundamental properties of gravity

1. Introduction

Strong gravity = non linearity

- What is non-linearity?   Think of the stock market


$\Rightarrow$ linear

$\Rightarrow$ NON-LINEAR!

Strongest possible gravity: Black holes

- Einstein 1915: General Relativity; geometric theory of gravity
- Schwarzschild 1916: Solution to Einstein's equations

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 d\phi^2$$

- Singularities

  $r = 0$: physical

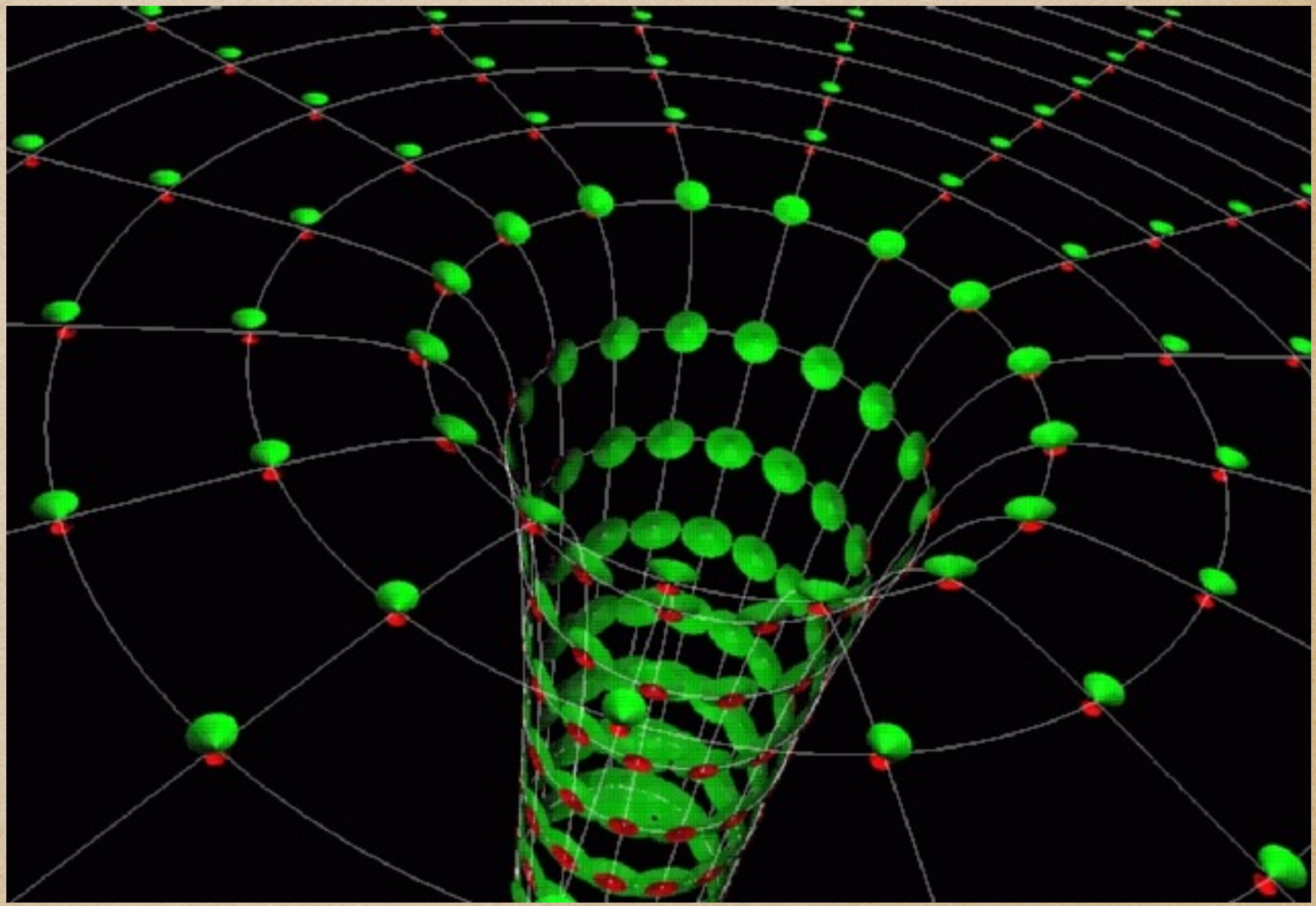
  $r = 2M$: singularity

- Horizon at $r = 2M$

  Light cones tilt over

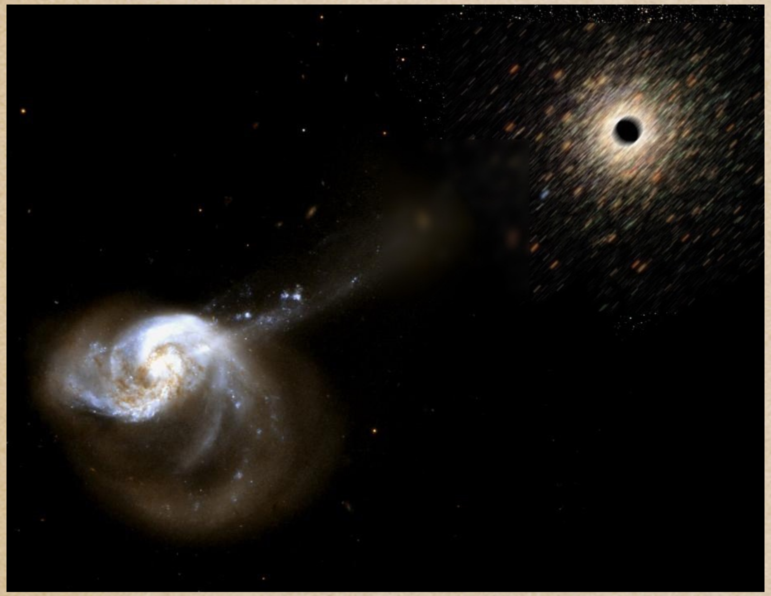
- Newtonian escape velocity

$$v = \sqrt{\frac{2M}{r}}$$

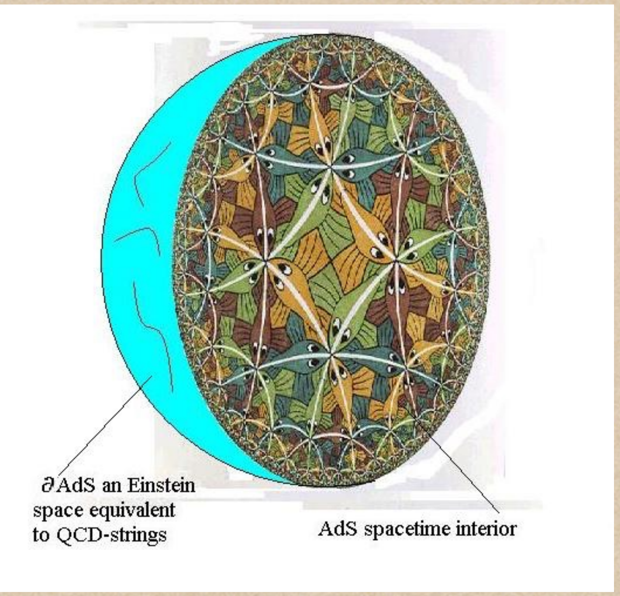


Research areas

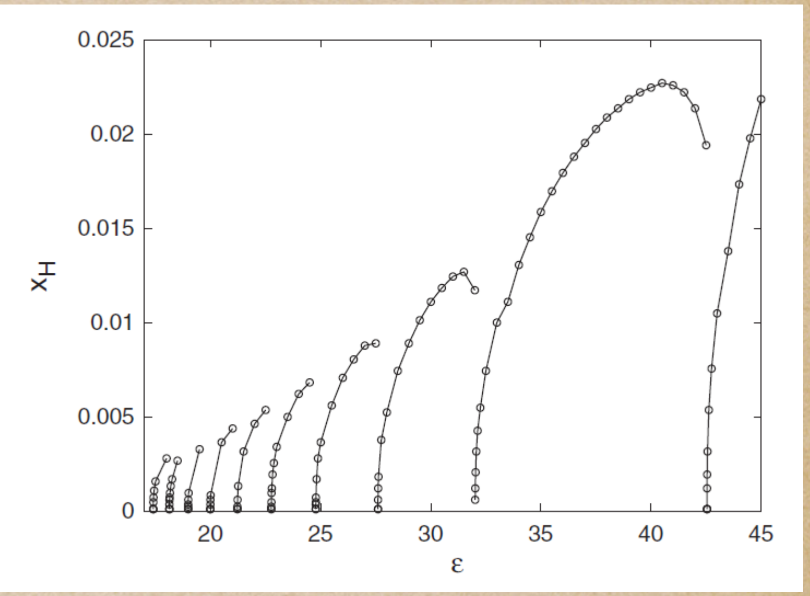
Astrophysics



Gauge gravity duality

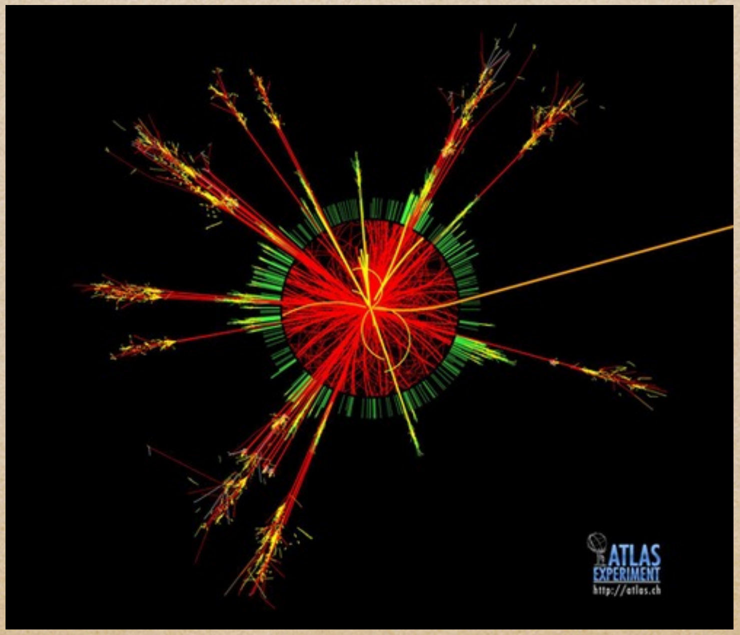


Fundamental studies

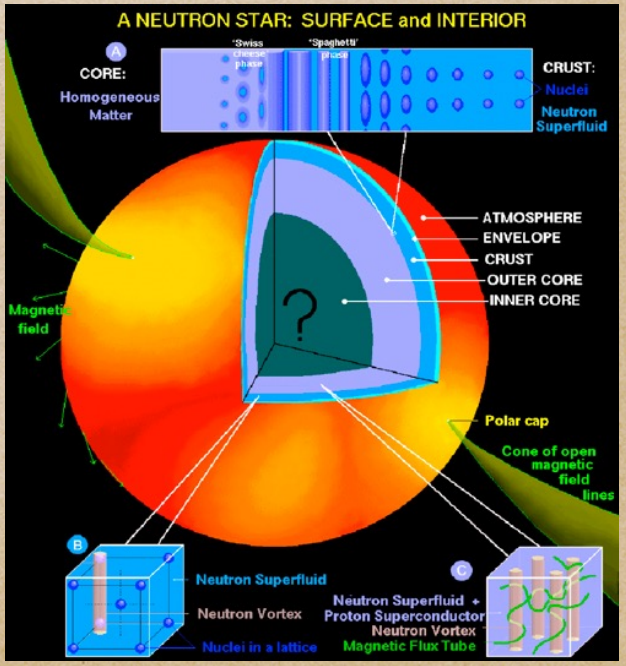


GW physics

High-energy physics



Equation of state



General Relativity in 30 seconds

- Spacetime as a curved manifold
- Key quantity: spacetime metric $g_{\alpha\beta}$
- Curvature, geodesics etc. all follow
- Einstein equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + \Lambda g_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

10 non-linear PDEs for $g_{\alpha\beta}$

$T_{\alpha\beta} =$ Matter fields

- Conceptually simple,

  hard in practice
- E.g. Schwarzschild

$$g_{\mu\nu} = \begin{pmatrix} \left(1 - \frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & -\left(1 - \frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{pmatrix}$$

$$ds^2 = c^2 dt^2 \left(1 - \frac{2GM}{rc^2}\right) - \frac{dr^2}{1 - 2GM/rc^2} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

How do we get the metric?

- The metric must obey $R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + \Lambda g_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$
- Ricci tensor, Einstein tensor, matter tensor

$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$

$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\mu}{}_{\mu}$ "Trace reverse Ricci"

$T_{\alpha\beta}$ "Matter"; see Talk by Luciano Rezzolla

Λ "Cosmological constant"

- Solutions: Easy!

Take metric $g_{\alpha\beta}$

$\Rightarrow$ Calculate $G_{\alpha\beta}$

$\Rightarrow$ Use that for $T_{\alpha\beta}$

- Physically meaningful solutions: That's the hard part!

Solving Einstein's Eqs.: The toolbox

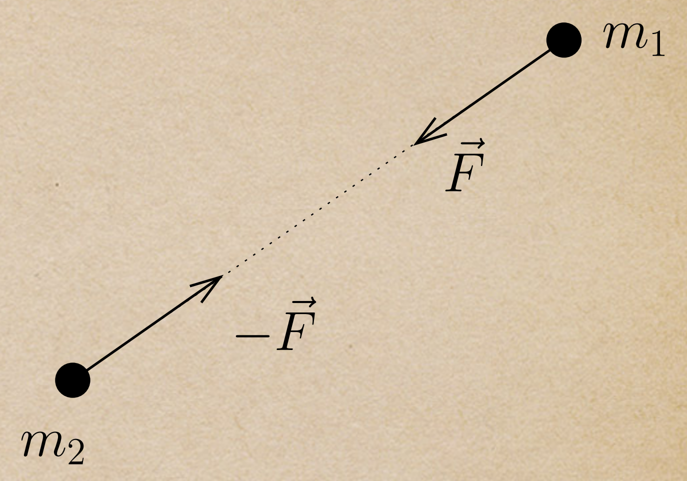
- **Analytic solutions**
  - Symmetry assumptions
  - Schwarzschild, Kerr, FLRW, Vaidya, Tangherlini, Myers-Perry, ...
- **Perturbation theory**
  - Assume solution is close to a known "background" $g^{(0)}_{\alpha\beta}$
  - Regge-Wheeler-Zerilli-Moncrief, Teukolsky, QNMs, EOB, ...
- **Post-Newtonian theory**
  - Assume small velocities $\Rightarrow$ Expansion in $\frac{v}{c}$
  - Blanchet, Buonanno, Damour, Will,...
- **Numerical Relativity**

2. Foundations of Numerical Relativity

The Newtonian 2-body problem

- Eqs. of motion

$$m_1 \frac{d^2\vec{r}_1}{dt^2} = \vec{F} = -G\frac{m_1 m_2}{r^2}\hat{\vec{r}} = -m_2\frac{d^2\vec{r}_2}{dt^2}$$



- Solution: Keppler ellipses, parabolic, hyperolic

$$r = \frac{r_0}{1+\epsilon\cos\theta}$$

e.g. Sperhake CQG 1411.3997

- What's different in GR?
  - No point particles in GR!
  - GR is non-linear
  - No "background" time and space
  - Systems typically are dissipative $\Rightarrow$ Gravitational waves
  - No obvious formulation as time evolution problem

A list of tasks in NR

- **Target:** Predict time evolution of a physical system in GR
- **Einstein eqs.:** 1) Cast as evolution system

  2) Choose a “good” formulation

  3) Discretize for a computer
- **Gauge:** Choose “good” coordinates
- **Technical aspects:** 1) Mesh refinement / spectral domains

  2) Singularity handling (excision)

  3) Parallelization
- **Initial data:** 1) Solve constraints

  2) Get “realistic” initial data
- **Diagnostics:** 1) GW extraction, kicks, …

  2) Horizon data, ADM mass,…

2.1 Formulations of Einstein's equations

The Einstein equations

- Recall: $$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + \Lambda g_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

- In this form, the mathematical character is unclear!

  hyperbolic, elliptic, parabolic?

- Coordinates x^α are on equal footing.

  Time singled out only through signature of the metric!

- Well-posedness of the equations? Suitable for numerics?

- There are various ways to address these questions

  $\rightarrow$ Formulations of the equations

2.1.1 ADM type 3+1 formulations

The 3+1 decomposition

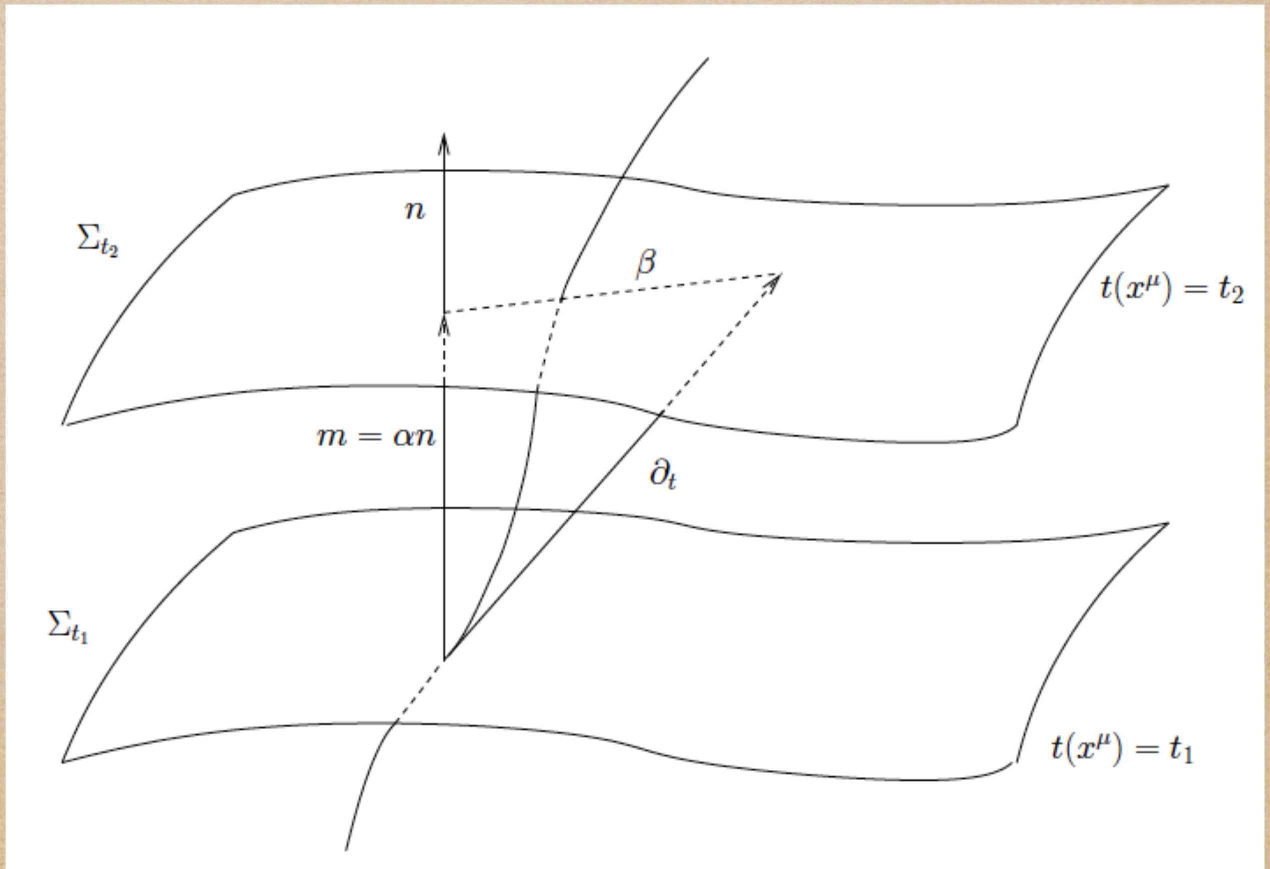
- ADM 3+1 split: Arnowitt, Deser & Misner 1962

  York 1979, Choquet-Bruhat & York 1980

Def.: Spacetime := $(\mathcal{M}, g)$

= Manifold with metric of signature $-+++$

Def.: Cauchy surface := A spacelike hypersurface Σ in $\mathcal{M}$ such that each timelike or null curve without endpoint intersects Σ exactly once.



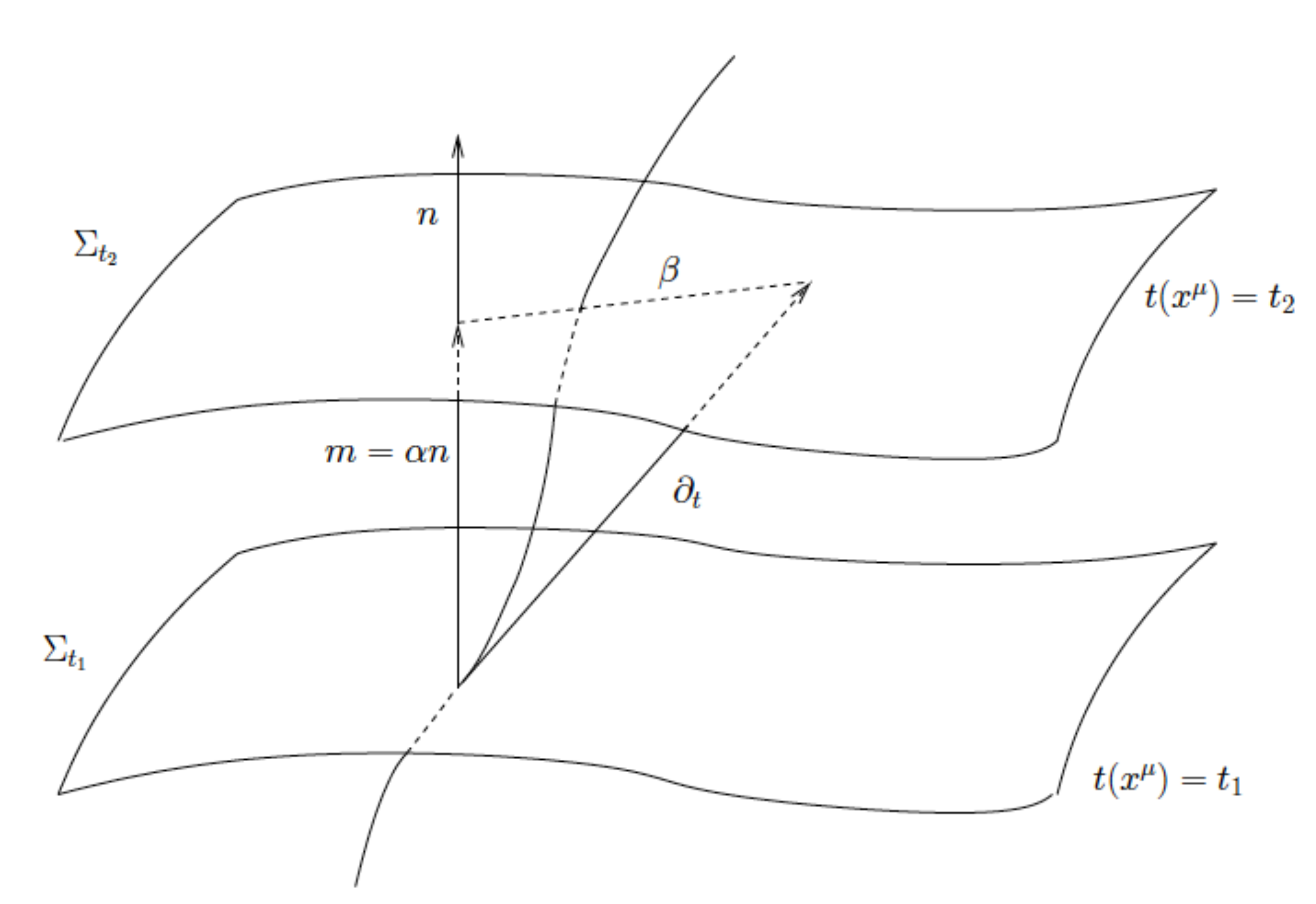
The (D-1)+1 decomposition

Def.: A spacetime is globally hyperbolic

$:\Leftrightarrow$ it admits a Cauchy surface

- From now on:

  Let $(\mathcal{M}, g)$ be glob.hyp.



Then one can show:

$\exists$ smooth $t : \mathcal{M} \mapsto \mathbb{R}$

such that

1) The gradient $\mathbf{d}t \neq 0$

   everywhere

2) level surfaces $t = \mathrm{const}$ are hypersurfaces:

$$\forall_{t_1 \in \mathbb{R}} \ \ \Sigma_{t_1} = \{p \in \mathcal{M} \ \ : \ \ t(p) = t_1\} \ , \ \ \Sigma_{t_1} \cap \Sigma_{t_2} = \emptyset \ \Leftrightarrow \ t_1 \neq t_2$$

The 3+1 decomposition

- 1-Form: $\mathbf{d}t$; vector: $\frac{\partial}{\partial t} =: \partial_t \quad \Rightarrow \quad \langle \mathbf{d}t, \partial_t \rangle = 1$

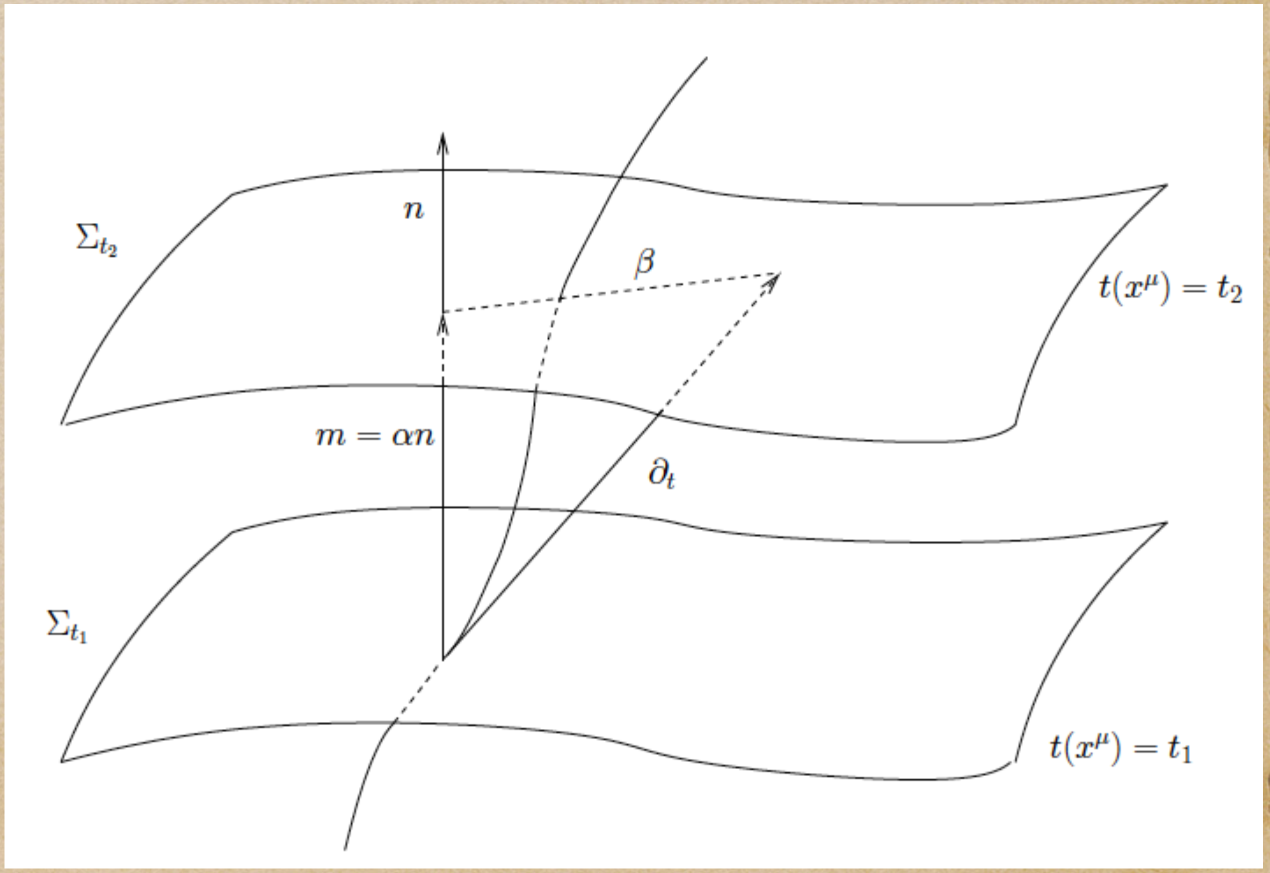
Def.: Time like unit field: $n_\mu := -\alpha(\mathbf{d}t)_\mu$

Lapse function: $\alpha := \frac{1}{||\mathbf{d}t||}$ Shift vector: $\beta^\mu := (\partial_t)^\mu - \alpha n^\mu$

Adapted coordinates: $(t,\, x^i)\,,$ x^i label points in Σ_t

Adapted coordinate basis:

$$\partial_t = \alpha n + \beta\,, \quad \partial_i := \frac{\partial}{\partial x^i}$$



The 3+1 decomposition

Def.: A vector $\boldsymbol{v}^\alpha$ is tangent to Σ_t $:\Leftrightarrow$ $\langle \mathbf{d}t, \boldsymbol{v} \rangle = (\mathbf{d}t)_\mu v^\mu = 0$

Def.: Projector $\perp^\alpha{}_\mu := \delta^\alpha{}_\mu + n^\alpha n_\mu$

- Projection of the metric

$$\gamma_{\alpha\beta} := \perp^\mu{}_\alpha \perp^\nu{}_\beta g_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \quad \Rightarrow \quad \gamma_{\alpha\beta} = \perp_{\alpha\beta}$$

- In adapted coordinates (t, x^i) : Ignore t component in $\gamma_{\alpha\beta} \to \gamma_{ij}$

  "spatial metric" or "First fundamental form"

- Spacetime metric:

$$g_{\alpha\beta} = \left(\begin{array}{c|c} -\alpha^2 + \beta_m \beta^m & \beta_j \\ \hline \beta_i & \gamma_{ij} \end{array} \right)$$

$$\Leftrightarrow \qquad ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

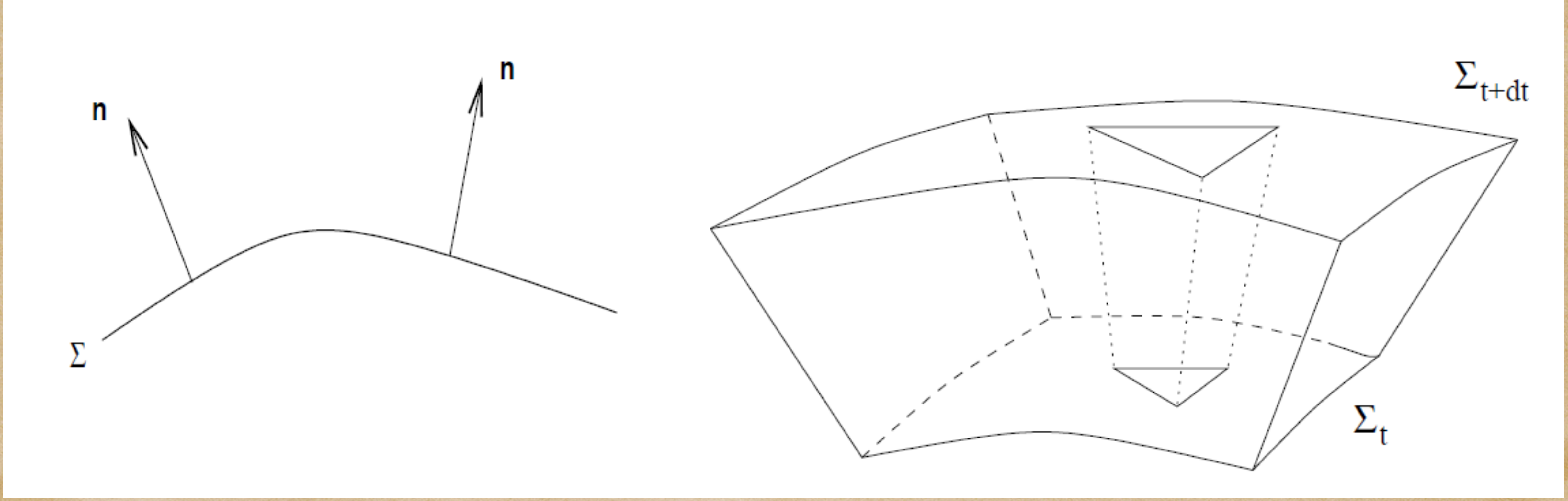
Extrinsic curvature

Def.: Extrinsic curvature: $K_{\alpha\beta} := -\perp \nabla_\beta n_\alpha$

- $\nabla_\beta n_\alpha$ is not symmetric, but $\perp \nabla_\beta n_\alpha$ is!
- The minus sign is a non-universal convention
- One can show that

$$\mathcal{L}_n \gamma_{\alpha\beta} = n^\mu \nabla_\mu \gamma_{\alpha\beta} + \gamma_{\mu\beta} \nabla_\alpha n^\mu + \gamma_{\alpha\mu} \nabla_\beta n^\mu = -2K_{\alpha\beta}$$

- Interpretation of $K_{\alpha\beta}$ $\rightarrow$ Embedding of Σ_t in $\mathcal{M}$



Decomposition of the Einstein eqs.

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + \Lambda g_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

$$\Leftrightarrow R_{\alpha\beta} = 8\pi \left(T_{\alpha\beta} - \frac{1}{D-2} g_{\alpha\beta} T \right) + \frac{2}{D-2} \Lambda g_{\alpha\beta}$$

- Energy momentum tensor

$$\rho := T_{\mu\nu} n^\mu n^\nu \,,$$

$$j_\alpha := -\perp^\mu{}_\alpha T_{\mu\nu} n^\nu \,,$$

$$S_{\alpha\beta} := \perp T_{\alpha\beta} \,, \quad S = \gamma^{\mu\nu} S_{\mu\nu} \,,$$

$$T_{\alpha\beta} = S_{\alpha\beta} + n_\alpha j_\beta + n_\beta j_\alpha + \rho n_\alpha n_\beta \,, \quad T = S - \rho \,.$$

- Lie derivative

$$\mathcal{L}_{\boldsymbol{m}} K_{ij} = \partial_t K_{ij} - \beta^m \partial_m K_{ij} - K_{mj} \partial_i \beta^m - K_{im} \partial_j \beta^m$$

$$\mathcal{L}_{\boldsymbol{m}} \gamma_{ij} = \partial_t \gamma_{ij} - \beta^m \partial_m \gamma_{ij} - \gamma_{mj} \partial_i \beta^m - \gamma_{im} \partial_j \beta^m$$

The ADM version of the Einstein eqs.

- Introduction of the extrinsic curvature:

$$\boxed{\mathcal{L}_{\boldsymbol{m}}\gamma_{ij} = -2\alpha K_{ij}}$$

- $\perp^{\mu}{}_{\alpha}\perp^{\nu}_{\beta}$ projection

$$\boxed{\mathcal{L}_{\boldsymbol{m}}K_{ij} = -D_iD_j\alpha + \alpha(\mathcal{R}_{ij} + KK_{ij} - 2K_{im}K^{m}{}_{j}) + 8\pi\alpha\left(\frac{S-\rho}{D-2}\gamma_{ij} - S_{ij}\right) - \frac{2}{D-2}\Lambda\gamma_{ij}}$$

"Evolution equations"

- $n^{\mu}n^{\nu}$ projection

$$\boxed{\mathcal{R} + K^2 - K^{mn}K_{mn} = 2\Lambda + 16\pi\rho}$$

"Hamiltonian constraint"

- $\perp^{\mu}{}_{\alpha}n^{\nu}$ projection

$$\boxed{D_iK - D_mK^m_i = -8\pi j_i}$$

"Momentum constraints"

- Problem: Doesn't work! ADM eqs. not "Strongly hyperbolic"

The BSSN system

- Goal: Modify ADM eqs. to get a strongly hyperbolic system

Shibata & Nakamura PRD 52 (1995), Baumgarte & Shapiro PRD gr-qc/9810065

- Use (i) conformal desomposition, (ii) trace split, (iii) aux. variables

$$\gamma := \det \gamma_{ij}\,, \quad \chi = \gamma^{-1/(D-1)}\,, \quad K = \gamma^{mn} K_{mn}\,,$$

$$\tilde{\gamma}_{ij} = \chi \gamma_{ii} \qquad \Leftrightarrow \qquad \tilde{\gamma}^{ij} = \frac{1}{\chi}\gamma^{ij}$$

$$\tilde{A}_{ij} = \chi \left(K_{ij} - \frac{1}{D-1}\gamma_{ij} K \right) \qquad \Leftrightarrow \qquad K_{ij} = \frac{1}{\chi}\left(\tilde{A}_{ij} + \frac{1}{D-1}\tilde{\gamma}_{ij} K \right)$$

$$\tilde{\Gamma}^i = \tilde{\gamma}^{mn} \tilde{\Gamma}^i_{mn}$$

- Auxiliary constraints

$$\tilde{\gamma} = 1, \qquad \tilde{\gamma}^{mn} \tilde{A}_{mn} = 0, \qquad \mathcal{G}^i \equiv \tilde{\Gamma}^i - \tilde{\gamma}^{mn} \tilde{\Gamma}^i_{mn} = 0\,.$$

2.1.2 Generalized harmonic formulation

The generalized harmonic gauge (GHG)

- Harmonic gauge: Choose coordinates such that

$$\Box x^\alpha = \nabla^\mu \nabla_\mu x^\alpha = -g^{\mu\nu}\Gamma^\alpha_{\mu\nu} = 0$$

- D dimensional Einstein eqs. in harmonic gauge:

$$R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu}\partial_\mu \partial_\nu g_{\alpha\beta} + \ldots$$

principle part of wave equation $\Rightarrow$ Manifestly hyperbolic!

- Problem: Start with a hypersurface $t = \text{const}$

Does t remain timelike?

- Goal: Generalize the harmonic gauge

Garfinkle PRD gr-qc/0110013; Pretorius CQG gr-qc/0407110;
Lindblom et al CQG gr-qc/0512093

$\rightarrow$ Source function $H^\alpha = \nabla^\mu \nabla_\mu x^\alpha = -g^{\mu\nu}\Gamma^\alpha_{\mu\nu}$

The generalized harmonic equations

- Any spacetime in any coordinates can formulated in GH form!

  Problem: find the corresponding H^α

- Promote the H^α to evolution variables

- Einstein equations in GH form:

$$
\begin{aligned}
\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} = & - \partial_\nu g_{\mu(\alpha} \partial_{\beta)} g^{\mu\nu} - \partial_{(\alpha} H_{\beta)} + H_\mu \Gamma^\mu_{\alpha\beta} - \Gamma^\mu_{\nu\alpha} \Gamma^\nu_{\mu\beta} \\
& - \frac{2}{D-2} \Lambda\, g_{\alpha\beta} - 8\pi \left(T_{\mu\nu} - \frac{1}{D-2} T\, g_{\alpha\beta} \right) .
\end{aligned}
$$

with constraints $\mathcal{C}^\alpha = H^\alpha - \Box x^\alpha = 0$

- Still has principle part of the wave equation!!! Manifestly hyperbolic

Friedrich Comm.Math.Phys. 1985; Garfinkle PRD gr-qc/0110013;
Pretorius CQG gr-qc/0407110

2.2 Initial data, gauge

Analytic initial data

- Schwarzschild, Kerr, Tangherlini, Myers-Perry,...

  e.g. Schwarzschild in isotropic coordinates

$$ds^2 = -\left(\frac{2r-M}{2r+M}\right)^2 dt^2 + \left(1+\frac{M}{2r}\right)^4 \left[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

- Time symmetric initial data with n BHs:

  Brill & Lindquist PR 131 (1963) 471, Misner PR 118 (1960) 1110

- Problem: Find initial data for dynamic systems

- Goals: 1) Solve constraints

  2) Realistic snapshot of physical system

- This is mostly done using the ADM 3+1 split

The York-Lichnerowicz split

- We work in $D=4$; generalization to $D>4$ possible
- Conformal metric $\gamma_{ij} = \psi^4 \bar{\gamma}_{ij}$

Lichnerowicz J.Math.Pures Appl. 23 (1944) 37

York PRL 26 (1971) 1656, PRL 28 (1972) 1082

- Note: In contrast to BSSN, we do not require $\det \bar{\gamma}_{ij} = 1$
- Conformal traceless split of the extrinsic curvature

$$K_{ij} = A_{ij} + \frac{1}{3} K \gamma_{ij} \,,$$

$$A^{ij} = \psi^{-10} \bar{A}^{ij} \quad \Leftrightarrow \quad A_{ij} = \psi^{-2} \bar{A}_{ij}$$

- Further assume: γ_{ij} Conformally flat; $K=0$

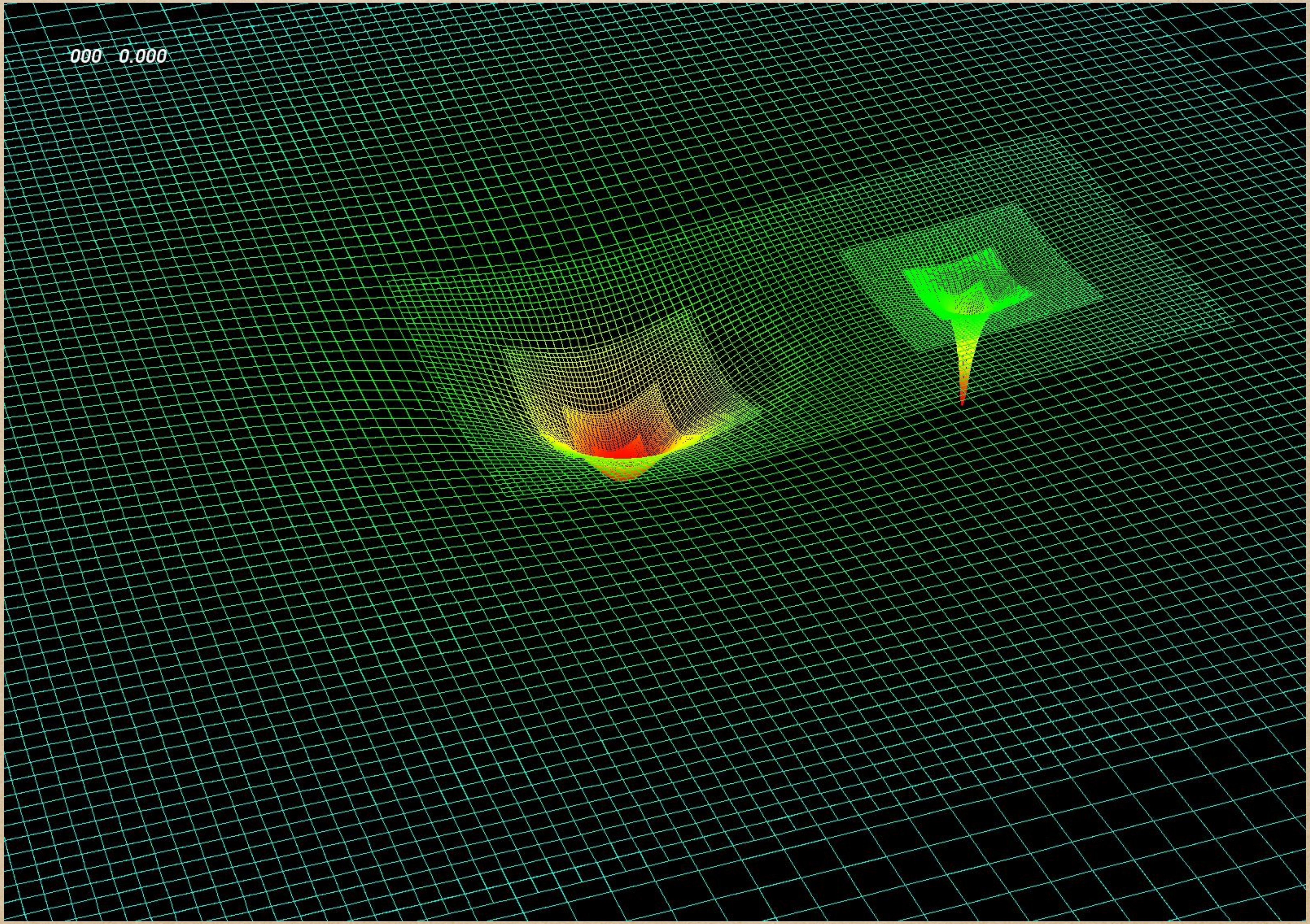
$\Rightarrow$ Mom.Constr.: Analytic solution; Ham.constr.: Numerics for ψ

Bowen & York PRD (1980); Brandt & Bügmann PRL gr-qc/9703066

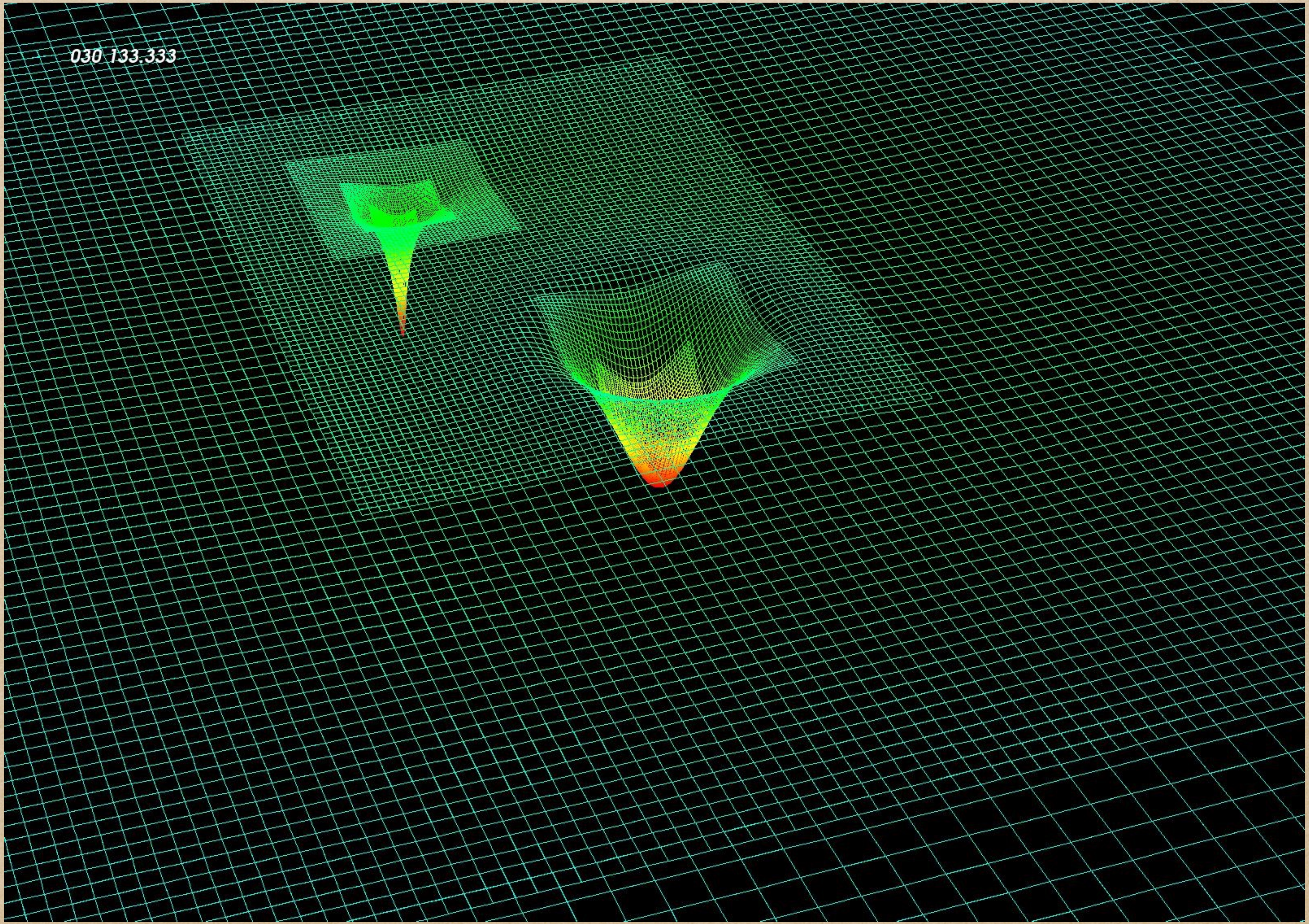
The gauge freedom

- Recall: Einstein's equations say nothing about $\alpha\,,\ \ \beta^i$
- Any choice of lapse and shift gives a solution to Einstein's eqs.
- This is the coordinate or gauge freedom of GR
- If the physics do not depend on $\alpha\,,\ \ \beta^i$, then why bother?
- Answer: The performance of the numerics DO depend very sensitively on the gauge!

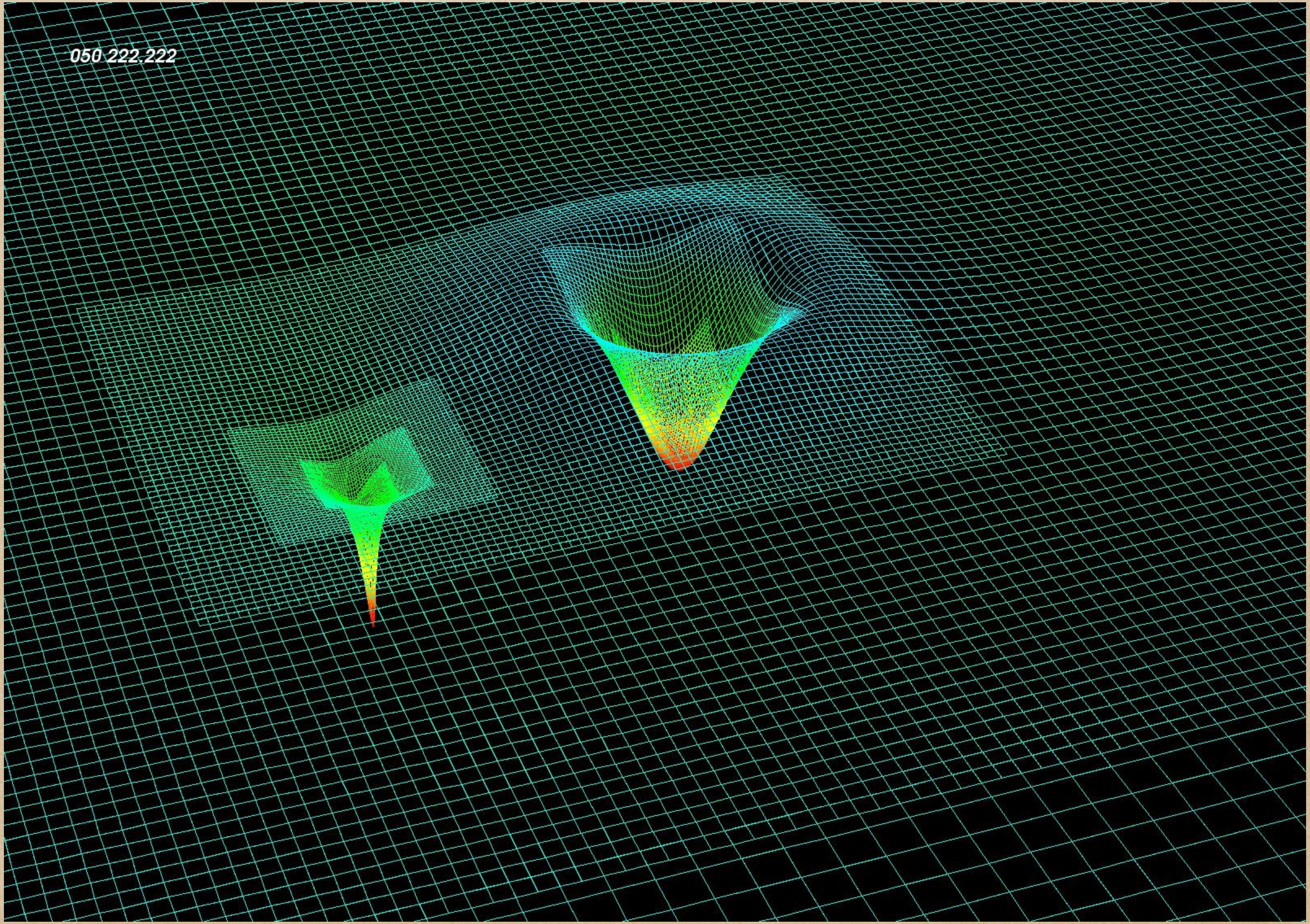
What goes wrong with bad gauge?



What goes wrong with bad gauge?



What goes wrong with bad gauge?



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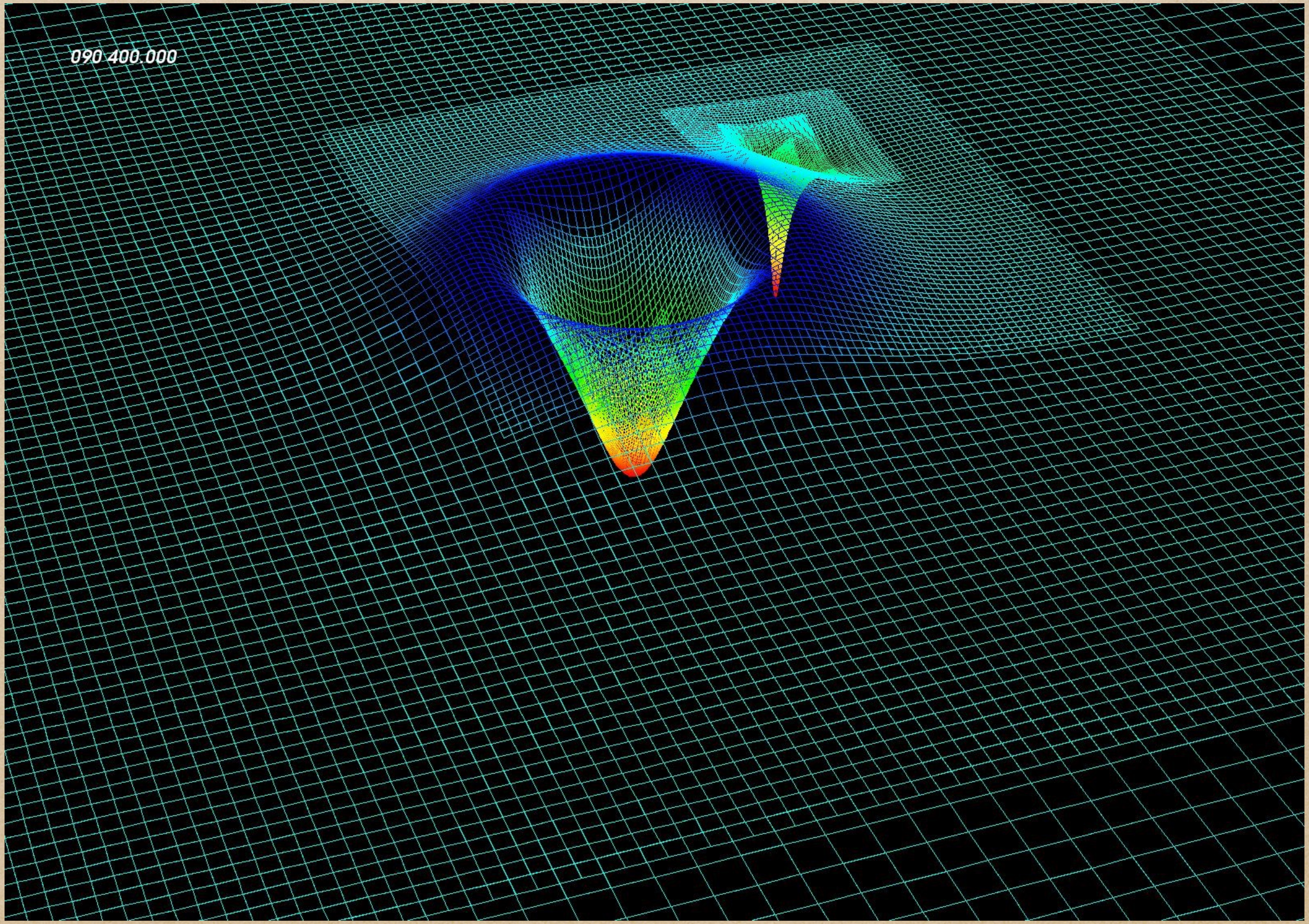


Ingredients for good gauge

- Singularity avoidance
- Avoid slice stretching
- Aim for stationarity in a co-moving frame
- Well-posedness of the system of PDEs
- Generalize "good" gauge, e.g. harmonic
- Lots of good luck!

Bona et al PRL (1995)

Alcubierre et al PRD gr-qc/0206072

Alcubierre CQG gr-qc/0210050

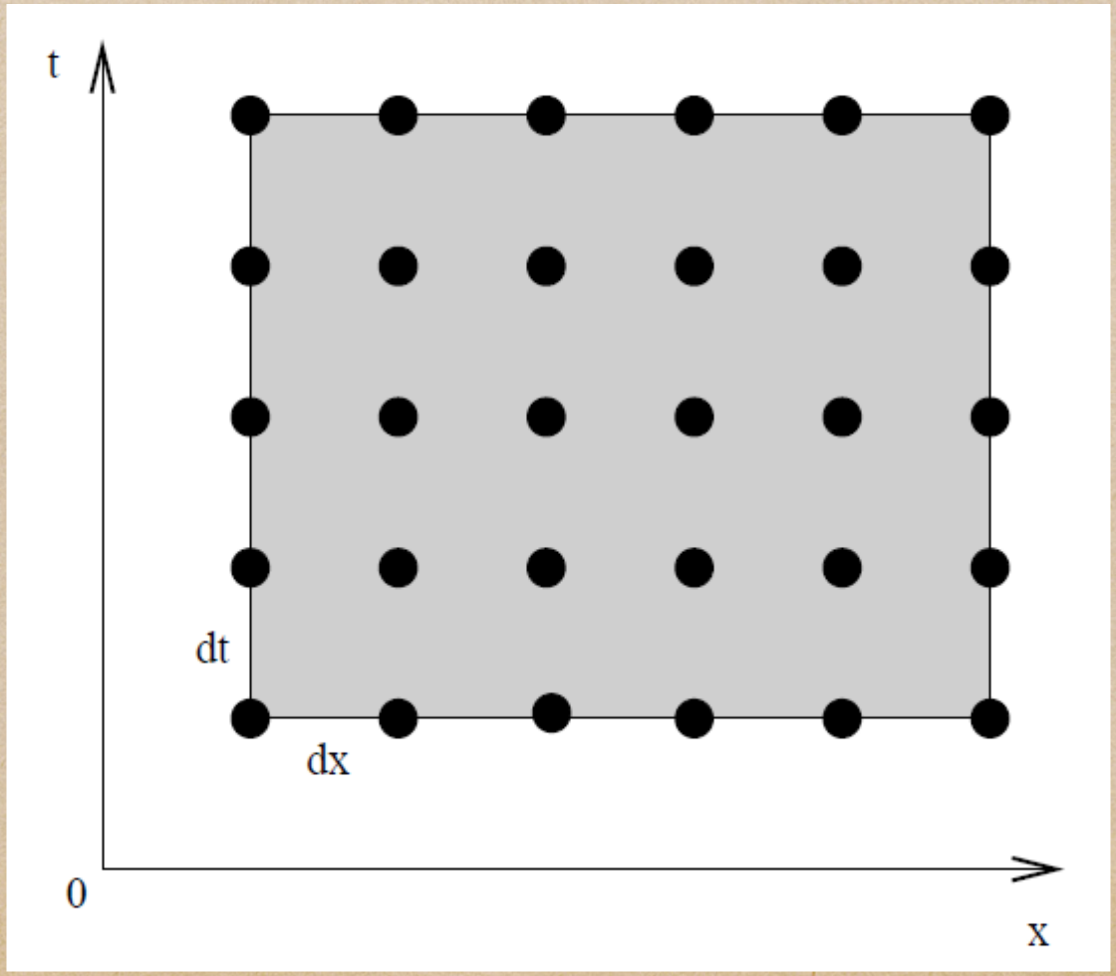
Garfinkle PRD gr-qc/0110013

2.3 Discretization of the equations

Finite differencing

- Consider one spatial and one time dimension: $t, \quad x$
- Replace computational domain by discrete points

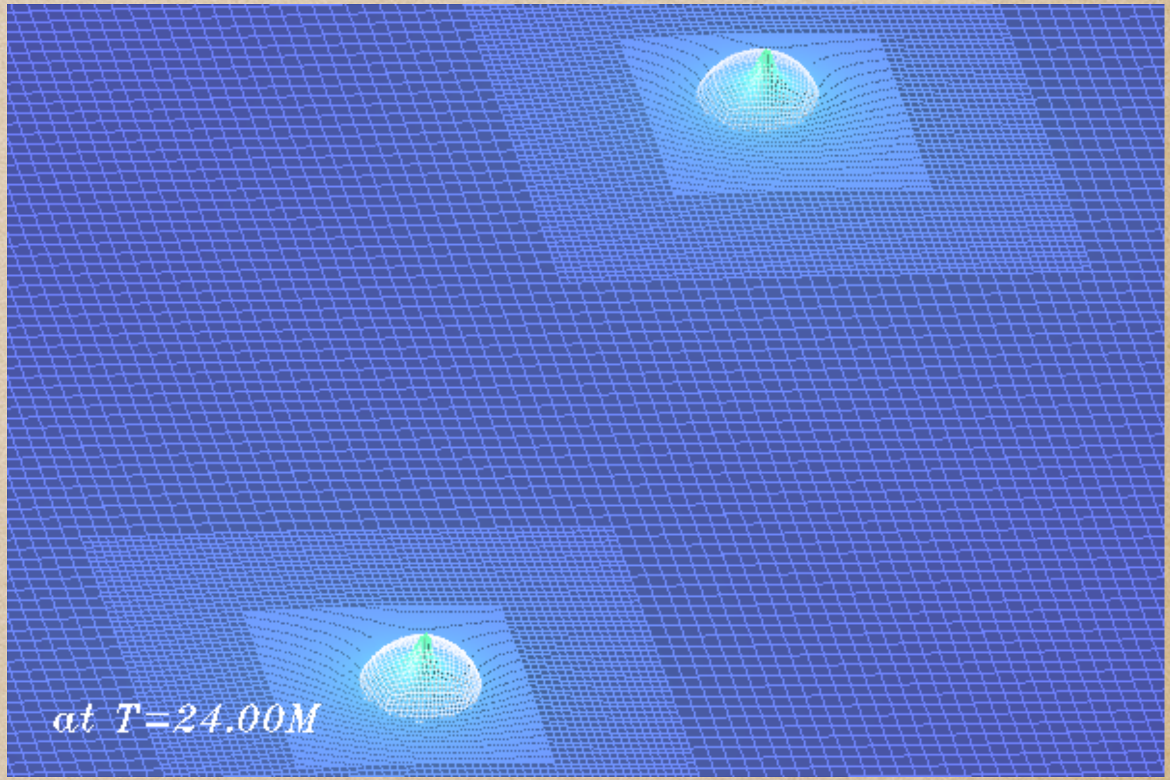
  $x_i = x_0 + i\,dx\,, \quad t_n = t_0 + n\,dt$
- Approximate function: $f(t_n, x_i) \approx f_{n,i}$



Mesh refinement

- 3 length scales: BH $\sim 1\ M$
  Wavelength $\sim 10 \ldots 100\ M$
  Wave zone $\sim 100 \ldots 1000\ M$
- First mesh refinement in GR: Critical phenomena

  Choptuik PRL **70** 9-12



- First use for BBHs

  Brügmann PRD gr-qc/9608050

- Available packages

  SAMRAI

  Paramesh MacNeice et al Comp.Phys.Comm. 136 (2000) 330

  Carpet Schnetter et al gr-qc/0310042

  Chombo Clough et al 1503.03436

Alternative discretization schemes

- Spectral methods: high accuracy, efficiency, complexity

  e.g. Caltech-Cornell-CITA code; `http://www.black-holes.org/SpEC.html`

  Application to moving punctures hard

  e.g. Tichy PRD 0911.0973

  Also used in symmetric asymptotically AdS spacetimes

  e.g. Chesler & Yaffe PRL 1011.3562; Santos & Sopuerta PRL 1511.04344

- Finite volume methods

- Finite element methods

  e.g. Arnold, Mukherjee & Pouly gr-qc/9709038
  Sopuerta et al CQG gr-qc/0507112
  Sopuerta & Laguna PRD gr-qc/0512028

2.4 Excision of the singularity

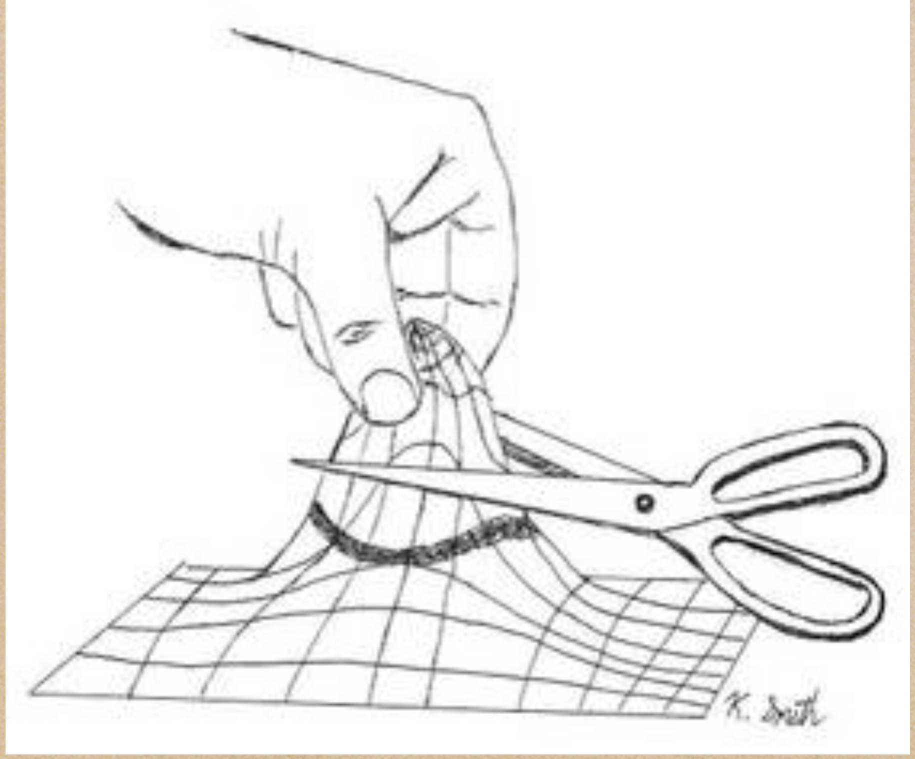
Inner boundary: Singularity treatment

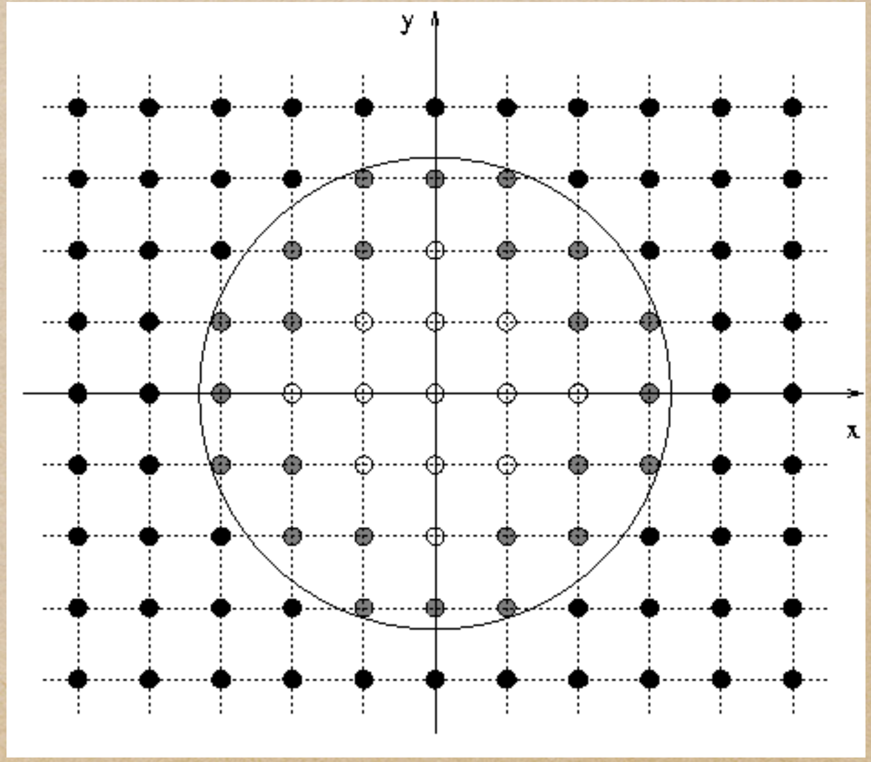
- Cosmic censorship $\Rightarrow$ horizon protects outside from singularity
- Moving puncture method: "we get away with it…"

  Baker et al PRL gr-qc/0511103; Campanelli et al PRL gr-qc/0511048

- Excision: Cut out region around the singularity

  Caltech-Cornell-CITA code, Pretorius' code





2.5 Diagnostics

The subtleties of diagnostics in GR

- Successful NR simulation $\rightarrow$ Tons of numbers for grid functions
- Typically: Spacetime metric $g_{\alpha\beta}$ and time derivative $\partial_t g_{\alpha\beta}$, or

  ADM variables $\gamma_{ij}\,,\quad K_{ij}\,,\quad \alpha\,,\quad \beta^i$
- Challenges
  - Coordinate dependence of numbers $\Rightarrow$ Gauge invariants
  - Global quantities at ∞, domain finite $\Rightarrow$ Extrapolation
  - Complexity of variables, e.g. GWs $\Rightarrow$ Spherical harmonics
  - Local quantities meaningful? $\Rightarrow$ Horizons
- AdS/CFT correspondence: Dictionary

Global quantities vs. local quantities

- Global mass, momentum, angular momentum
  - Well defined through asymptotics of metric

    Arnowitt, Deser & Misner gr-qc/0405109; Gourgoulhon gr-qc/0703035
  - These are spacetime properties and constant by construction

- At null infinity: Bondi mass $\rightarrow$ Gravitational wave energy

  Bondi et al; Sachs Proc.Roy.Soc.A 1962

- Local mass, energy, ... : No rigorous definition in GR!

  Except: Black holes $\rightarrow$ Apparent, isolated, dynamic Horizons

  E.g.: Ashtekar & Krishnan gr-qc/0308033; Thornburg gr-qc/9508014

Alternative extraction methods

- Newman-Penrose scalars: Convenient projections of Weyl tensor

  $\rightarrow$ Newman Penrose formalism   Newman & Penrose J.Math.Phys. '61

  Outgoing GWs: $\Psi_4 = -C_{\alpha\beta\gamma\delta} k^\alpha \bar{m}^\beta k^\gamma \bar{m}^\delta$

- Landau-Lifshitz pseudo tensor: simple but gauge dependent

  see e.g. Lovelace et al PRD 0907.0869

- Regge-Wheeler-Zerilli-Moncrief perturbation formalism:

  perturbations on Schwarzschild $\rightarrow$ gauge invariant master function

  Regge & Wheeler PR '57; Zerilli PRL '70; Moncrief Ann.Phys. '74

- Cauchy-characteristic extraction at $\mathcal{I}^+$ using a compactified exterior vacuum patch with characteristic coordinates: very accurate

  Reisswig et al PRL 0907.2637; Babiuc et al PRD 1011.4223

2.6 History of Numerical Relativity

A brief history of NR

- 1952: Cauchy problem has locally unique soln. Choquet-Bruhat
- 1964: First documented num.study Hahn & Lindquist Ann.Phys. '64
- 1977: First coordinated NR effort De Witt, Smarr, Eppley, Cadez
- 1990s: "Binary black hole Grand Challenge" Project

  First BH mergers (head-on), GWs, 3D code

  Matzner, Anninos, Price, Seidel, Smarr + many others
- 1998: BSSN: Shibata & Nakamura PRD '95, Baumgarte & Shapiro PRD '98
- 1999 : First 3+1 Binary BH merger Brügmann gr-qc/9912009
- 2000-2004: Much progress in gauge, formulations, excision
- 2005: Breakthrough Pretorius (GHG) Brownsville, Goddard (Mov.Puncs.)
- 2006 - ...: Gold rush Years

3. Results from BH simulations

3.1 BHs in GW physics

Detection and parameter estimation

Generic transient search

- No specific waveform model
- Identify excess power in detector strain data
- Use multi detector maximum likelihood   Klimenko et al. 1511.05999

Binary coalescence search

- "Matched Filtering" e.g. Allen et al. PRD 2012
- Compare data stream with GW templates
  ("Finger print search")
- Bayesian analysis: Prior $\rightarrow$ Posterior

Black-hole binaries: parameters

- 8+2 Intrinsic parameters

  Masses m_1 , m_2

  Spins $\boldsymbol{S}_1 , \boldsymbol{S}_2$

  Eccentricity (often ignored; GW emission circularizes orbit)

- 7 Extrinsic parameters

  Location: Luminosity distance D_L , Right ascension α , Declination δ

  Orientation: Inclination ι , Polarization ψ

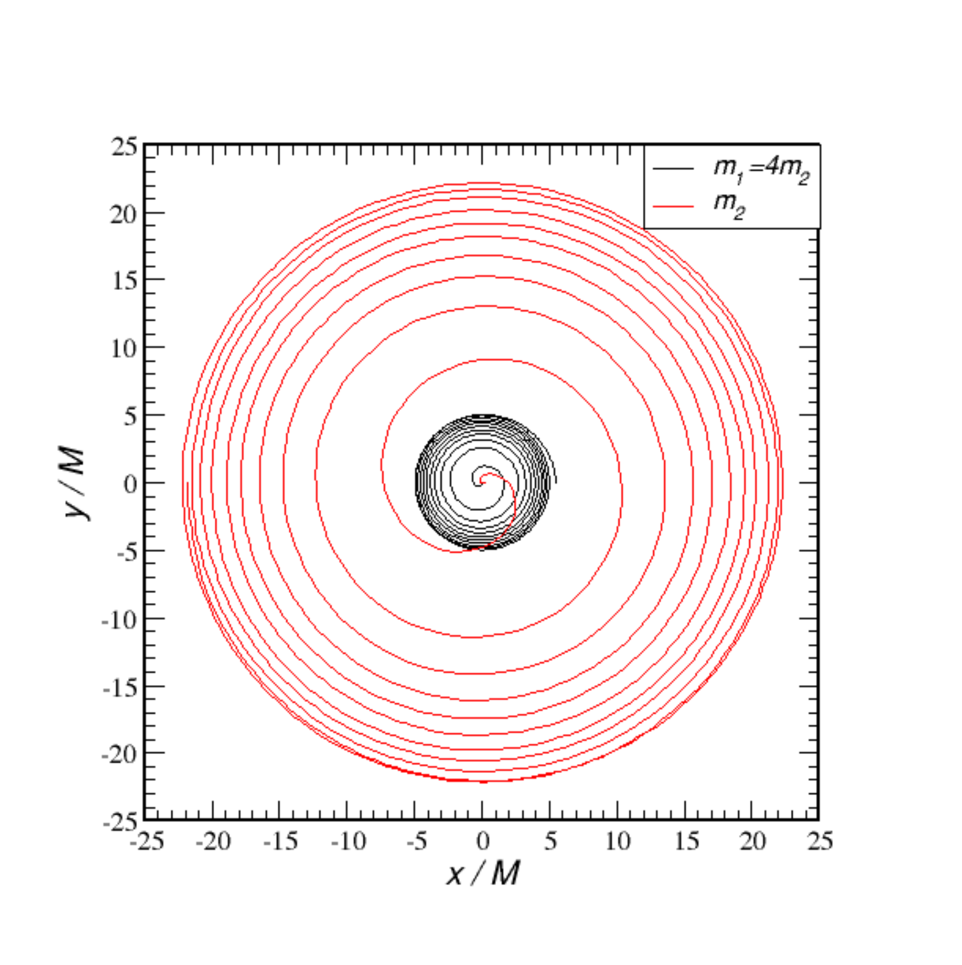
  Time t_c and Phase ϕ_c of coalescence

Binary BH trajectory and waveform

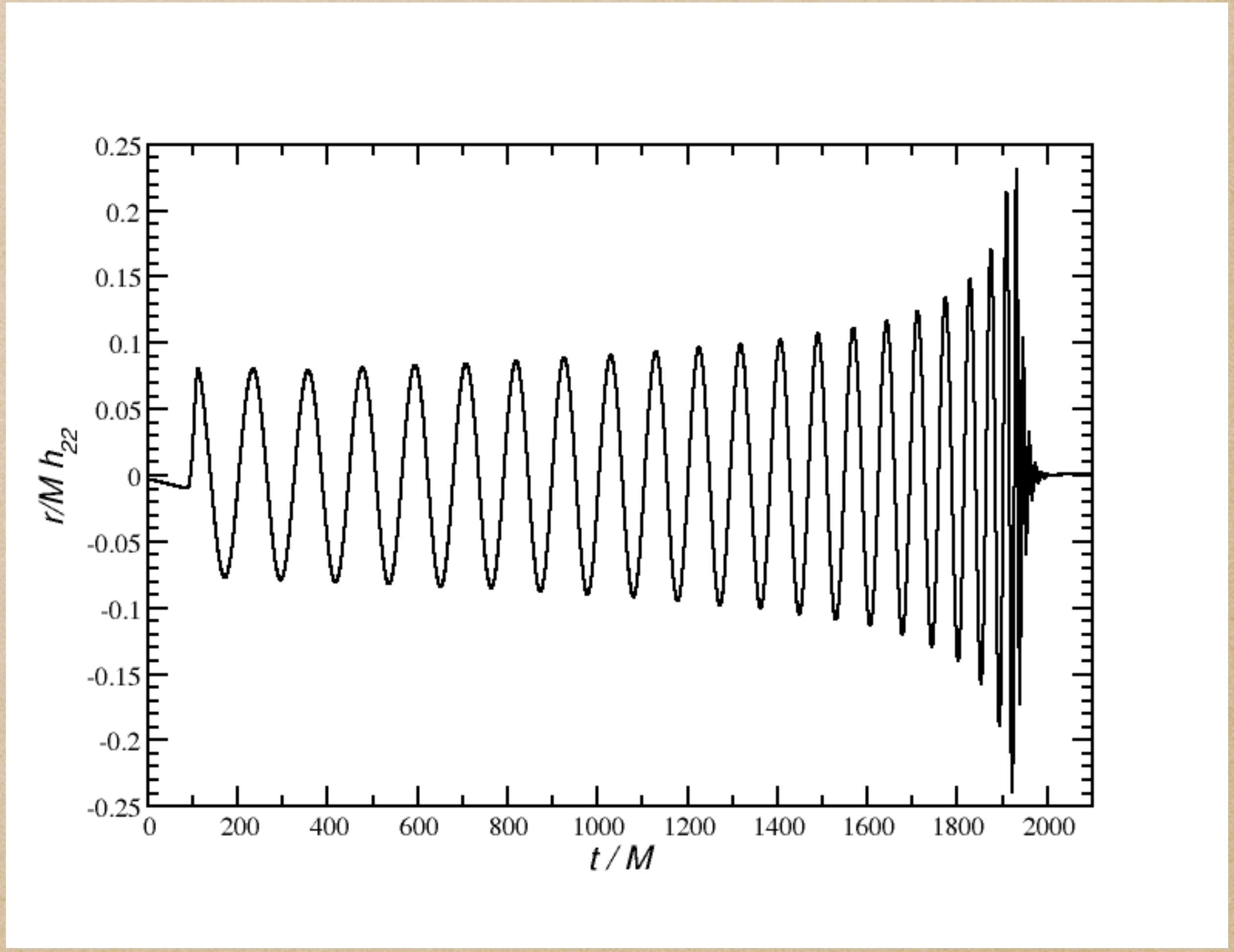
- $\frac{m_1}{m_2} = 4$ non-spinning binary; ≈ 11 orbits

Sperhake et al CQG 1012.3173

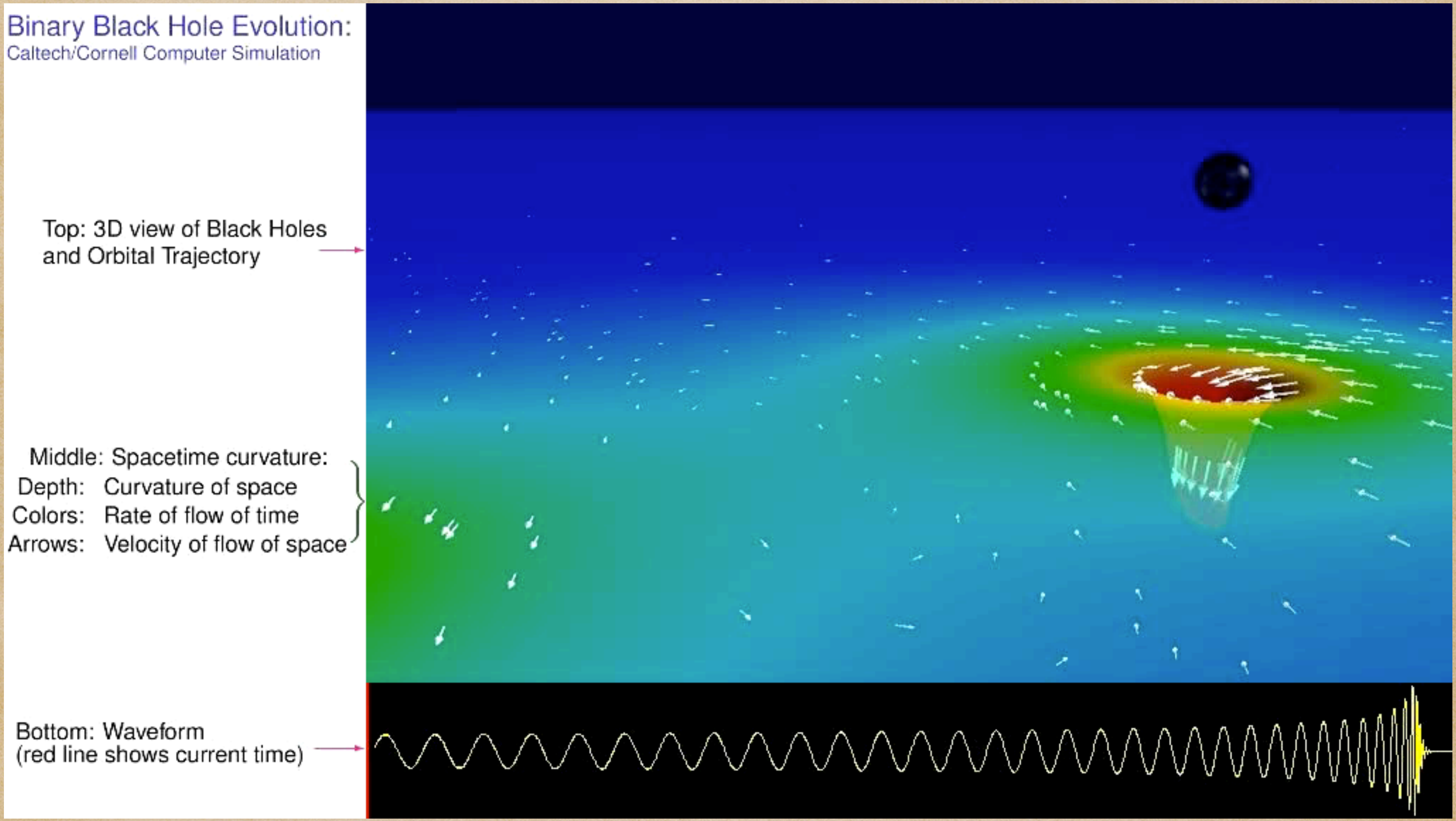
Trajectory



Quadrupole mode



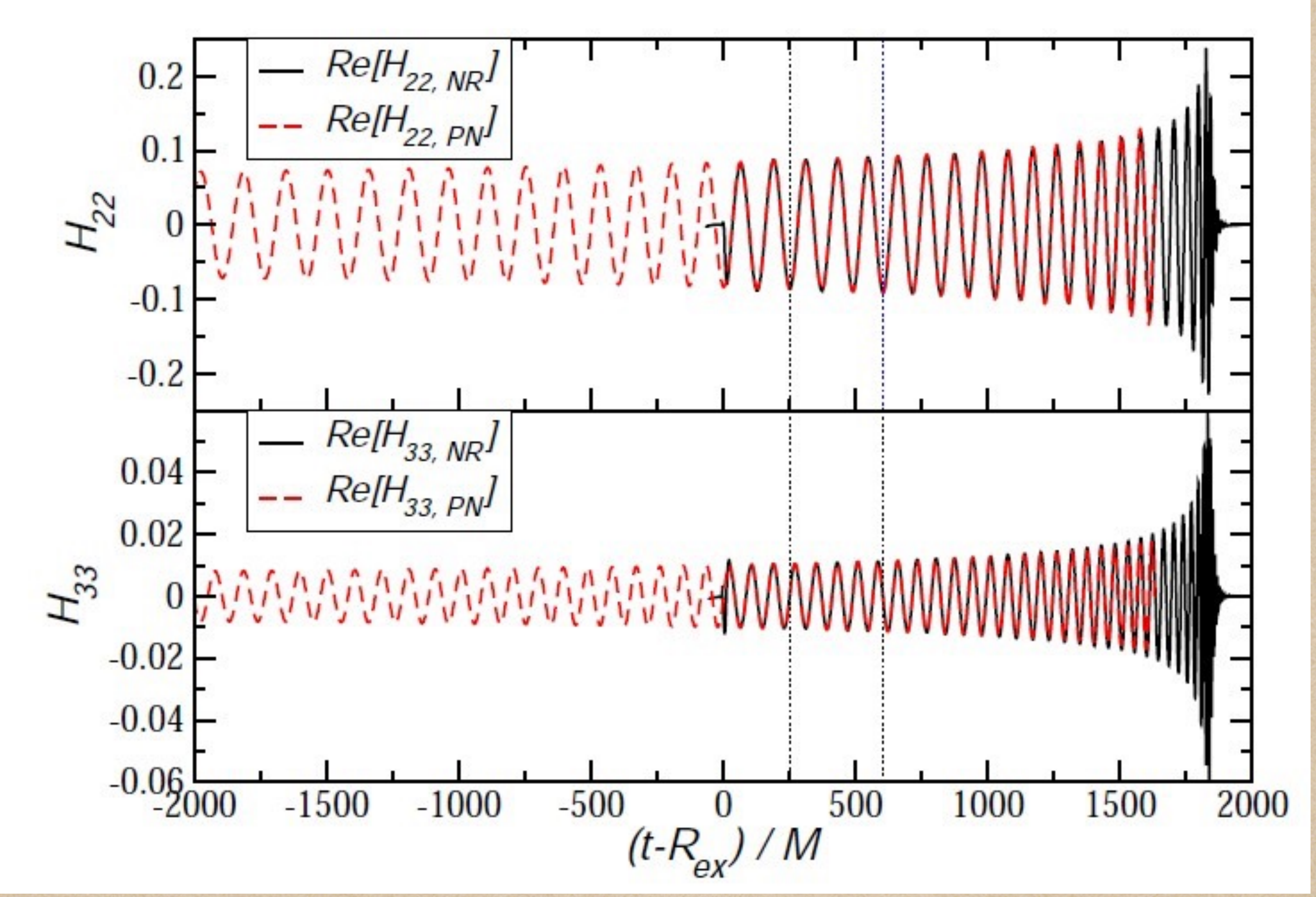
Anatomy of a BHB coalescence



Thanks to Caltech-Cornell groups

Hybrid waveforms and catalogs

- Stitch together PN and NR waveforms



Sperhake et al CQG 2011

- Mass produce waveforms; Hinder et al CQG 1307.5307;
  Mroué et al PRL 1004.4697

GW source modeling

- Key requirement for matched filtering: GW template catalog
- Model black holes in general relativity
  - Post Newtonian theory $\rightarrow$ Inspiral Blanchet LRR-2006-4
  - Numerical relativity $\rightarrow$ final orbits, merger
  - Perturbation theory $\rightarrow$ Ringdown
- Combine "NR" with "Post-Newtonian", "Effective one body" methods
- 2 families in use: Phenomenological, Effective one body
- Use reduced bases or similar to cover parameter space
- Multipolar decomposition

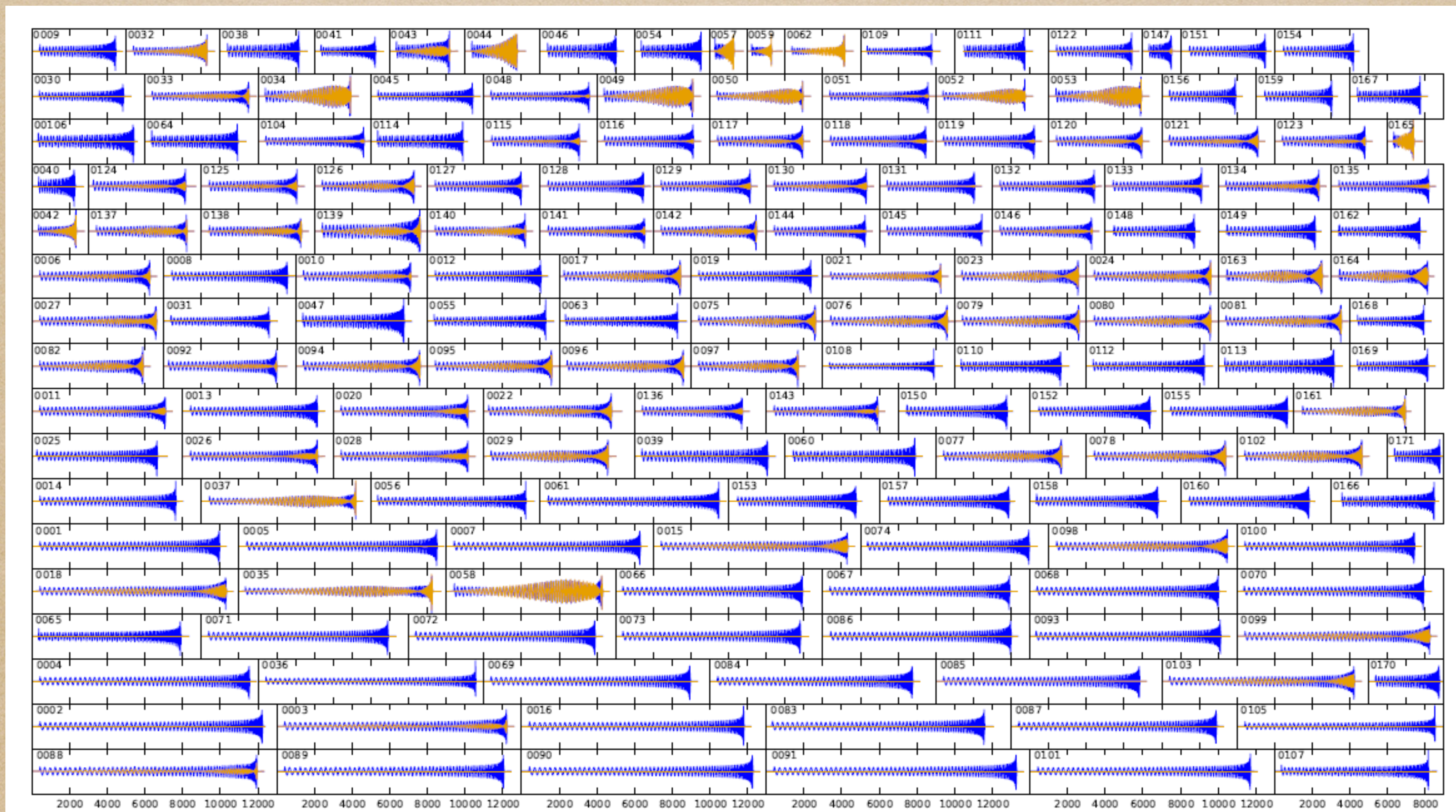
$$h_+ - ih_\times = \sum_{\ell m} {}_{-2}Y_{\ell m}(\theta,\phi) h_{\ell m}(t)$$

Tools of mass production

- Explore seven-dim. parameter space. E.g. SpEC catalogue:

  171 waveforms: $m_1/m_2 \leq 8$ up to 34 orbits

Mroué et al PRL 1304.6077



3.2 Fundamental properties of BHs

Why 4 dimensions? Cosmic censorship in D=5

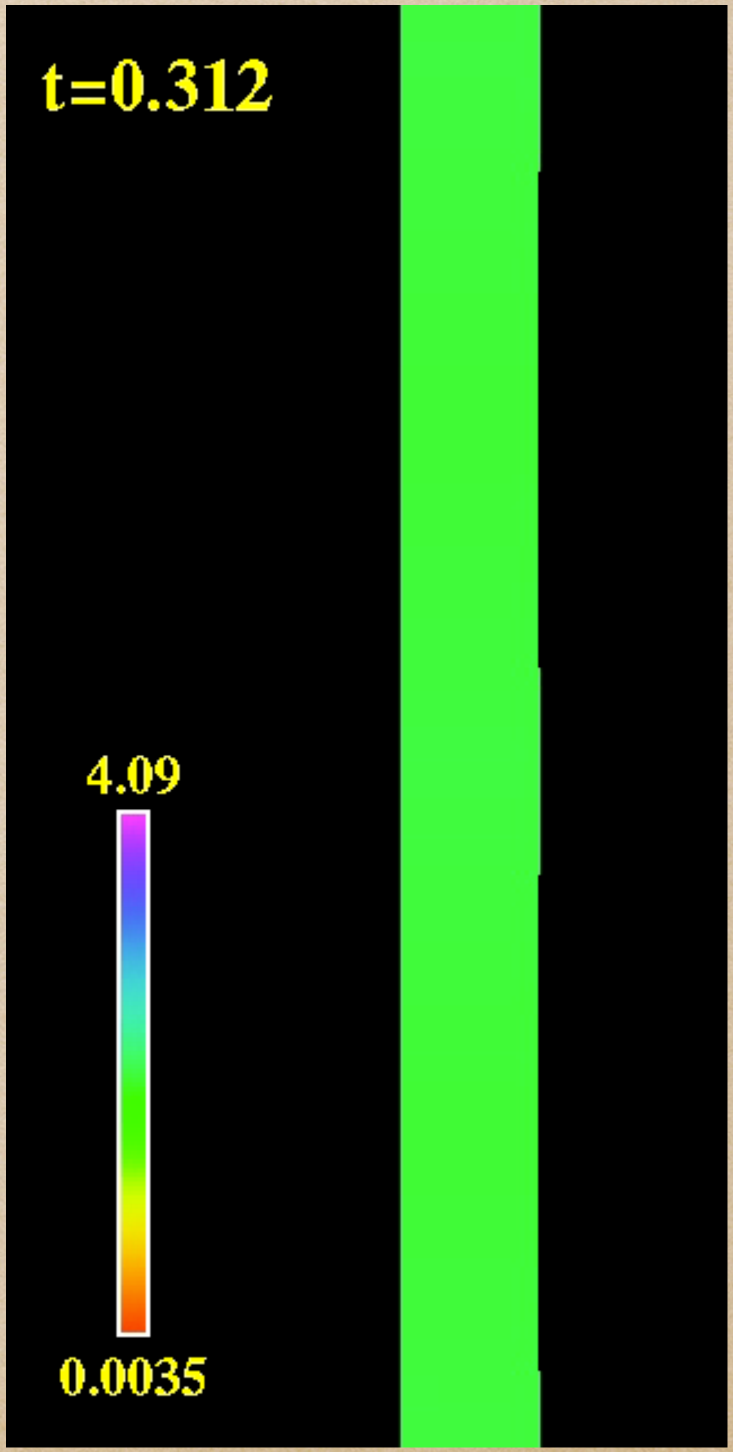
Lehner & Pretorius PRL 1006.5960

- Axisymmetric code
- Evolution of black string...
- Gregory-Laflamme instability;

  cascades down in finite time

  until string has zero width

  ⇒ Naked singularity
- Note: spacetime not asympt.flat!



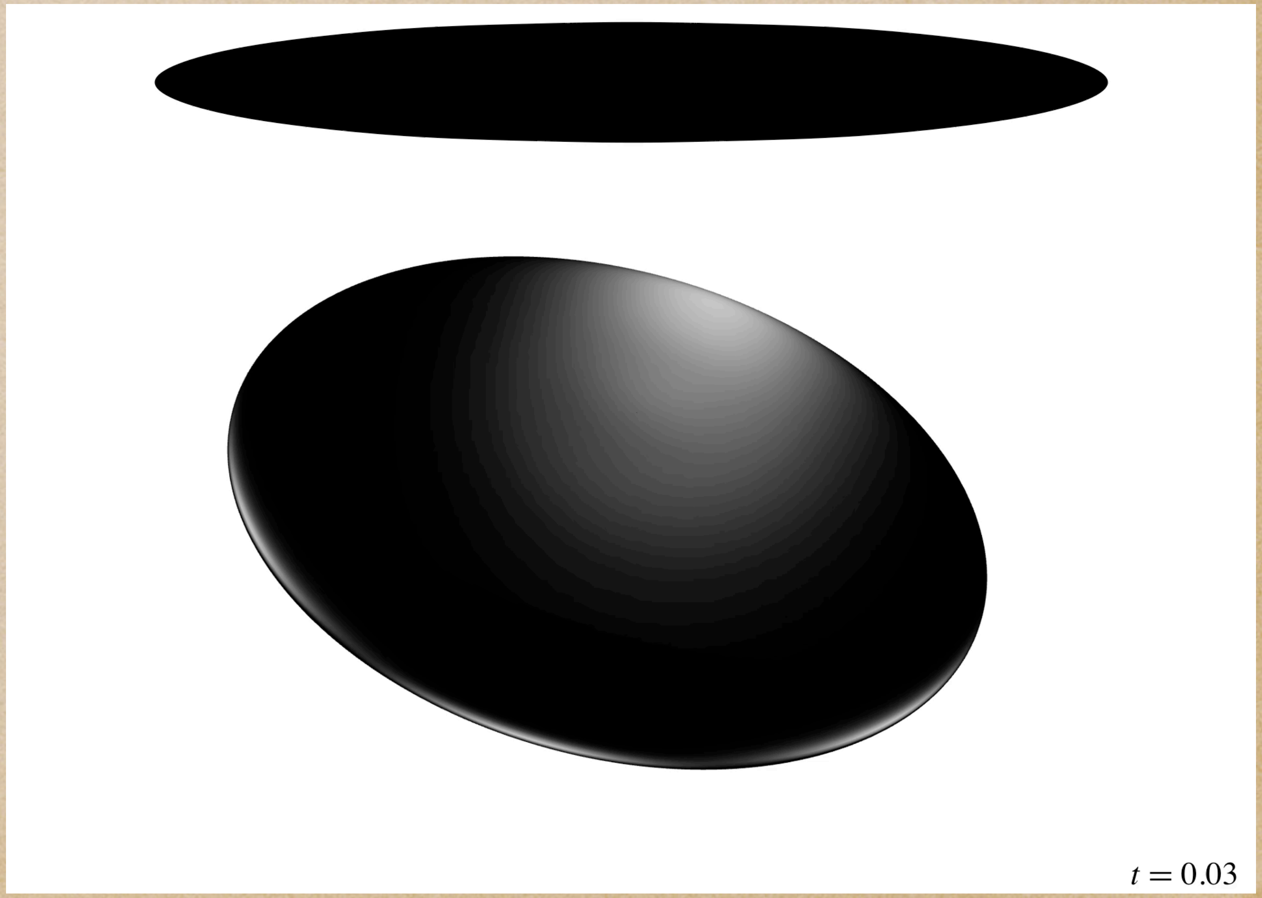
Cosmic censorship in D=5

Figueras, Kunesch & Tunyasuvunakool PRL 1512.04532

- 3+1 code with modified cartoon for 5th dimension
- Conformal Z4 system
- Fast rotating black hole: assympt.flat!
- Gregory-Laflamme instability

  develops for thin ring

  $\Rightarrow$ Violation of CC!



Conclusions

- GW150914 marks the dawn of GW astronomy
- "We" measured the change in length by a fraction of an atomic nucleus caused by sth. 1 Gyr away!
- Numerical relativity accurately models this!
- First surprise: BHs heavier than expected
- Parameter estimation requires a lot more GW modeling
- NR Applications: Astrophysics, High-energy phys., Fundamentals
- A new window to the universe reveals interesting things…