Searching for smoking gun effects of modified gravity in the gravitational-wave era

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Conclusions

- Introduction and motivation
- BH binaries
  - Introduction
  - Analytic predictions
  - Numerics and results
- Supernova core collapse
  - Introduction
  - Formalism and modeling techniques
  - Code tests and results
- Conclusions

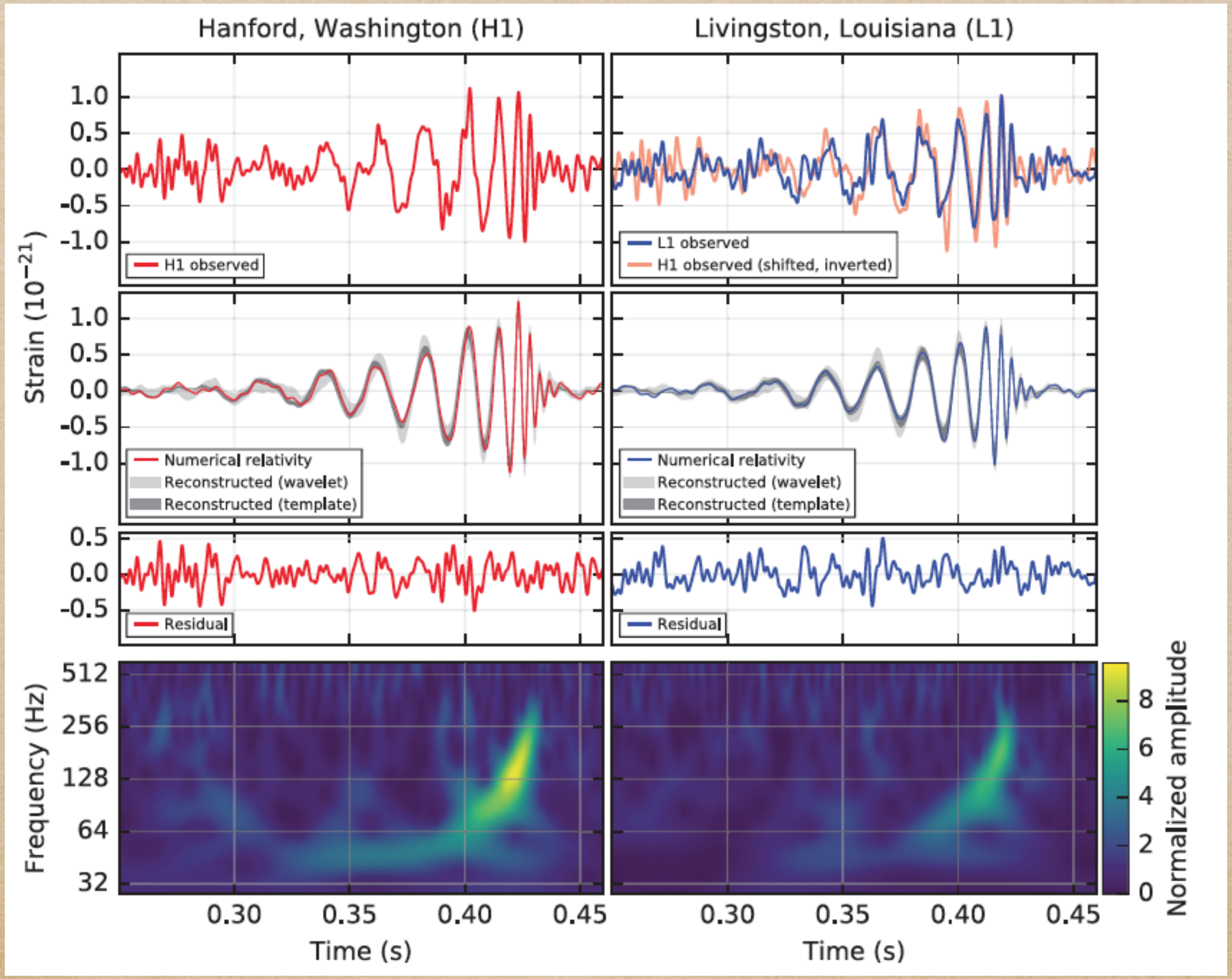
Introduction and motivation

Gravitational waves have arrived

- Sep 14, 2015 at 09:50:45 UTC: SNR ~ 24

  Abbott et al. PRL 1602.03837, Abbott et al. PRL 1606.01210

- BBH inspiral, merger and ringdown: $m_1 = 35^{+5}_{-3}\, m_\odot\,, \quad m_2 = 30^{+3}_{-4}\, M_\odot$



GW150914 compatible with GR

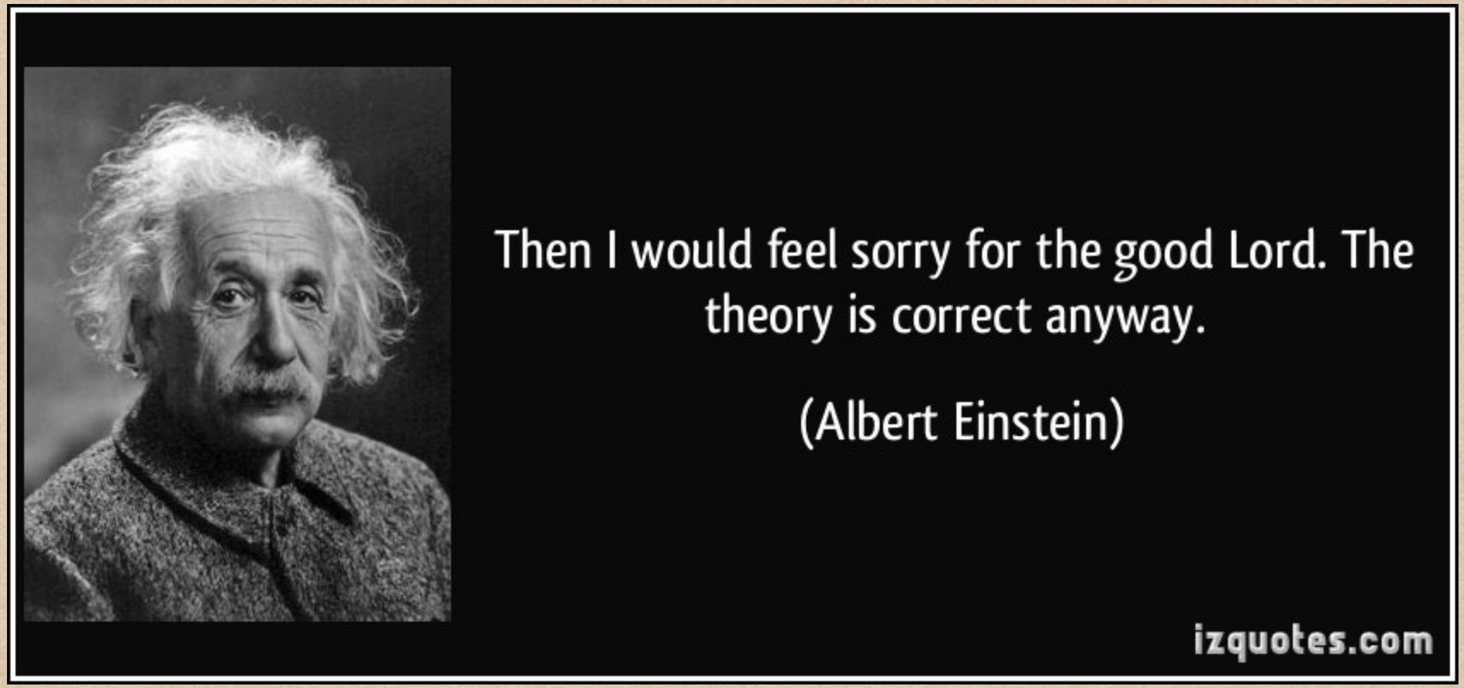
Here we perform several studies of GW150914, aimed at detecting deviations from the predictions of GR. Within the limits set by LIGO's sensitivity and by the nature of GW150914, we find no statistically significant evidence against the hypothesis that, indeed, GW150914 was emitted by a binary system composed of two black holes (i.e., by the Schwarzschild [17] or Kerr [18] GR solutions), that the binary evolved dynamically toward merger, and that it formed a merged rotating black hole consistent with the GR solution.

Taken from Abbott et al. PRL 2016 1602.03841 "Testing GR"

- GR waveform templates work stunningly well!
- So why bother looking for something other than GR?

Do we need a theory beyond GR?

- When asked what he would do if Eddington's mission failed…



- But we have reasons to search for "beyond GR"
  - Renormalization: Requires, e.g., higher curvature terms.

    $\rightarrow$ GR is low-energy limit of more fundamental theory
  - Dark energy: Why is Λ so small and why $\rho_{\rm dark} \sim \rho_{\rm mat}$
  - Dark matter: "Neptune" or "Vulcan" ?

A hard task

- (Weak) Gravity well tested on length scales $1\ \mu\mathrm{m} \lesssim \ell \lesssim 10^{11}\ \mathrm{m}$

  A modified theory needs to be consistent with these tests!

- How to interpret constraints?

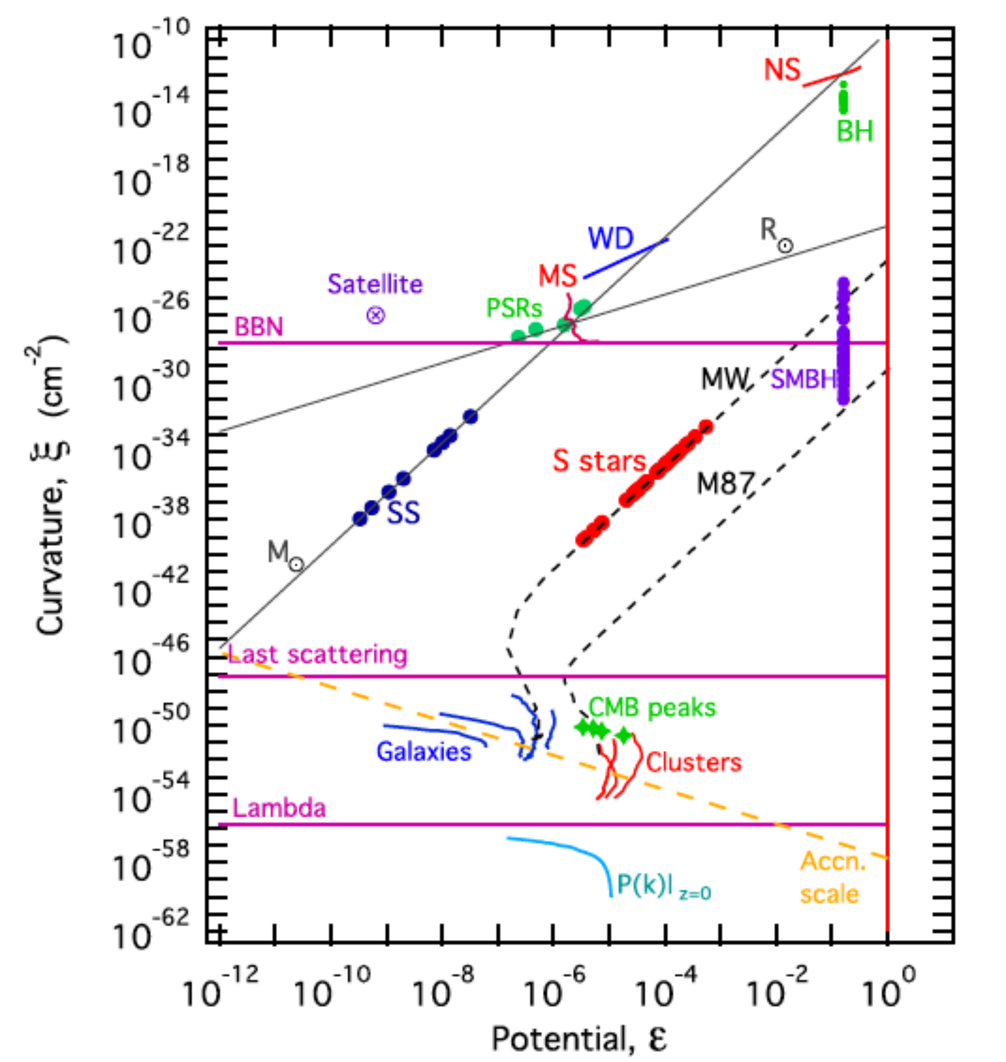
  Strong vs. weak field tests

- There's a zoo of theories!

  Berti et al CQG 1501.07274

- Full modeling requires a well-posed formulation

  e.g. Delsate et al PRD 1407.6727



Baker et al. ApJ 1412.3455

Predictions for GW tests: 2 approaches

- Parameterized post Newtonian / Einsteinium (PPN, PPE)
  - Introduce "phenomenological" extra terms to the theory and quantify these through extra parameters
  - No specific theory in mind
  - Very general

  e.g. Will ApJ 1971, Yunes & Pretorius PRD 0909.3328

- Smoking gun effects
  - Choose specific theory
  - Model physical systems; look for deviations from GR
  - Very concrete

  Topic of this talk

No-hair theorems

- Stationary BHs are the same in ST theory as in GR
  - For Brans-Dicke: Hawking '72, Thorne & Dykla '71, Chase '70
  - For Bergmann-Wagoner, $f(R)$: Sotirou & Faraoni 1109.6324
  - Supported by numerics of grav.collapse: e.g. Scheel et al '95
- How about BH binaries is ST theory?
  - In Brans-Dicke to leading PN order: Will & Zaglauer '89
  - No dipolar radiation in EMR limit: Yunes et al 1112.3351
- Generalized no-hair theorems rely on 4 assumptions
  - No matter
  - Vanishing scalar potential
  - Action truncated at second derivatives
  - Metric is asymptotically flat, scalar field assympt. constant

Part A: Black hole binaries

Berti, Cardoso, Gualtieri, Horbatsch & Sperhake PRD 1304.2836

A1. Introduction

Circumventing the no-hair theorems

Berti, Cardoso, Gualtieri, Horbatsch & Sperhake PRD 1304.2836

- Here focus on 4th item: asymptotic flatness, constancy
- E.g. time varying BCs as in cosmological expansion

  $\rightarrow$ Induces scalar charge. Binary may emit dipole radiation

  Jacobson 9905303; Horbatsch & Burgess 1111.4009

- Non-uniform scalar fields: $\approx$ non-asymptotically flat BCs

  E.g. through

  - Scalar fields that are anchored on galactic matter
  - Supermassive boson stars

  Length scale of scalar profile $\gg$ size of BH binary

Theoretical framework

Jordan frame: Physical metric $\tilde{g}_{\alpha\beta}$

- Action $\boxed{S = \int dx^4 \frac{\sqrt{-\tilde{g}}}{16\pi G}\left[F(\phi)\tilde{R} - 8\pi G Z(\phi)\tilde{g}^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - U(\phi)\right]}$
- GWs: 3 degrees of freedom
- Matter couples to $\tilde{g}_{\alpha\beta}$

Einstein frame: Conformal metric $g_{\alpha\beta} = F(\phi)\tilde{g}_{\alpha\beta}$

- Scalar field: $\varphi(\phi) = \int d\phi \left[\frac{3}{2}\frac{F'(\phi)^2}{F(\phi)^2} + \frac{8\pi G Z(\phi)}{F(\phi)}\right]^{1/2}$
- Action $\boxed{S = \int d^4x \frac{\sqrt{-g}}{16\pi G}\left[R - g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - W(\varphi)\right]}$
- Price: Matter couples to $g_{\alpha\beta}/F$

From now on: vanishing potential $U(\phi) = W(\varphi) = 0\,.$

A choice of frames

Pro Einstein

- Minimally coupled scalar field $\Rightarrow$ Numerics straightforward
- $F,\ Z$ not explicitly present in evolutions

  $\Rightarrow$ Evolve entire class of theories at once

Pro Jordan

- Strongly hyperbolic formulations also available

  Salgado gr-qc/0509001; Salgado et al 0801.2372

- Matter couples to the evolved metric $\tilde{g}_{\alpha\beta}$

Here: Einstein frame more suitable.

Gravitational waves in the 2 frames

- Evolution eqs. (Einstein): $G_{\alpha\beta} = \partial_\alpha\varphi\,\partial_\beta\varphi - \frac{1}{2}g_{\alpha\beta}g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi$

$$\Box\varphi = 0$$

- Perturbations: $\tilde{g}_{\alpha\beta} = \tilde{g}^{(0)}_{\alpha\beta} + \delta\tilde{g}_{\alpha\beta}$ $\qquad g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \delta g_{\alpha\beta}$

$$\phi = \phi^{(0)} + \delta\phi \qquad \varphi = \varphi^{(0)} + \delta\varphi$$

$$\delta\tilde{g}_{\alpha\beta} = \frac{1}{F(\phi^{(0)})}\left[\delta g_{\alpha\beta} - \tilde{g}^{(0)}_{\alpha\beta}F'(\phi^{(0)})\,\delta\phi\right]$$

$$\delta\phi = \left[\frac{3}{2}\,\frac{F'(\phi^{(0)})^2}{F(\phi^{(0)})^2} + \frac{8\pi G Z(\phi^{(0)})}{F(\phi^{(0)})}\right]^{-1/2}\delta\varphi$$

- Newman-Penrose scalar: $\Psi_4 = \ddot{h}_+ - i\ddot{h}_\times$
- Jordan version $\tilde{\psi}_4$ from $\psi_4,\ \varphi$: e.g. Barausse et al 1212.5053

A2. Analytic solutions

Single black holes

- Linearize Eqs. in φ

$$\boxed{R_{\alpha\beta} = 0\,, \qquad \Box\varphi = 0}$$

Laplace Eq. on BH background

- Schwarzschild in isotropic coordinates

$$ds^2 = -\frac{(2\hat{r}-M)^2}{(2\hat{r}+M)^2}dt^2 + \left(1+\frac{M}{2\hat{r}}\right)^4 (d\hat{r}^2 + \hat{r}^2 d\Omega^2)$$

$$\Rightarrow \ldots \Rightarrow \varphi = 2\pi\,\sigma\left(1+\frac{M^2}{4\hat{r}^2}\right)\hat{r}\,\cos\theta = 2\pi\sigma(r-M)\cos\theta \approx 2\pi\sigma z$$

Asymptotically: constant gradient in z dir.

- Kerr BH; cf. Press '72

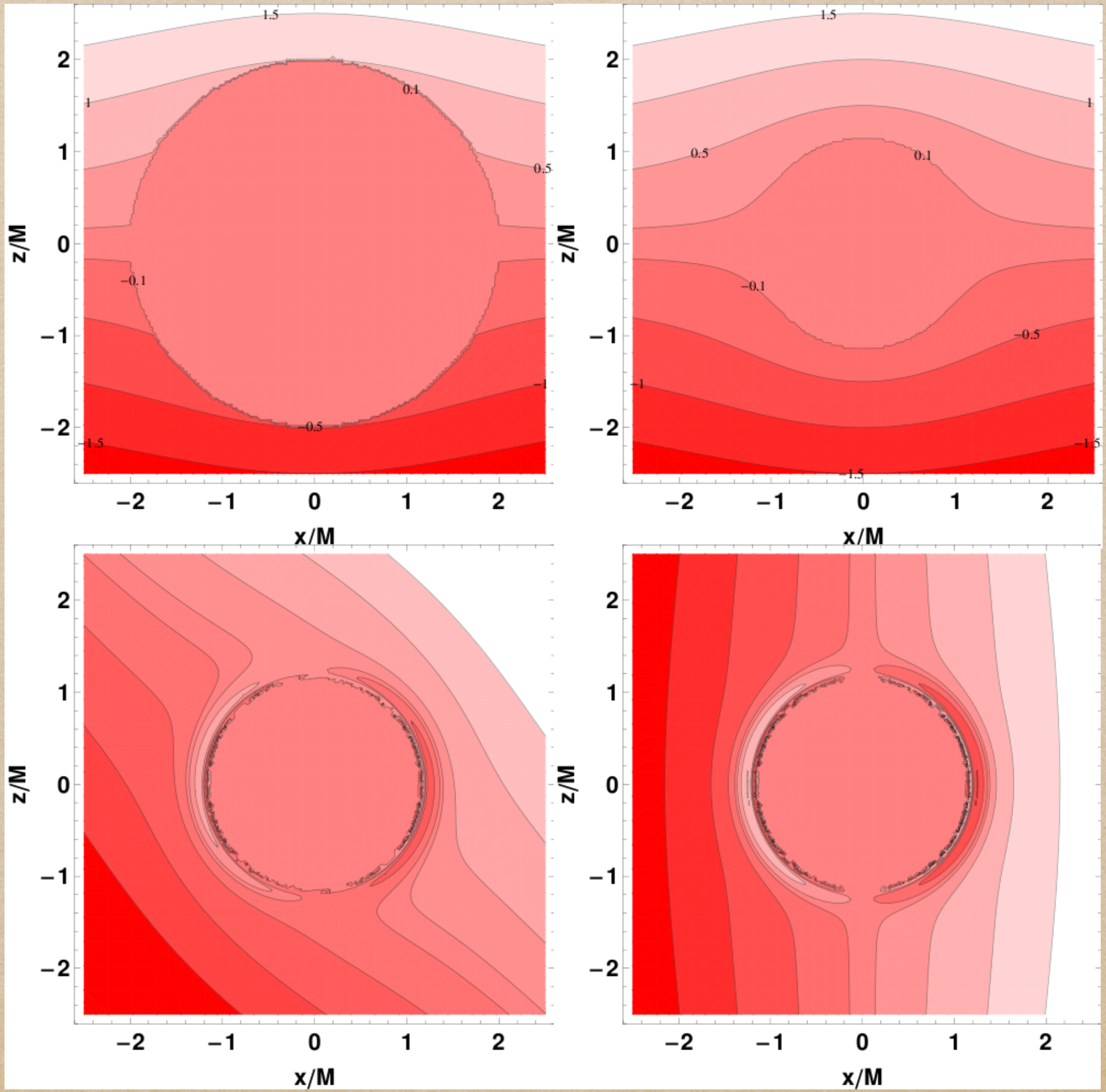
$$\varphi = 2\pi\sigma(r-M)\left[\frac{z}{r}\cos\gamma + \frac{x}{r}f_a\sin\gamma\right]\,, \qquad f_a = f_a(M,a,r)$$

$\gamma =$ Angle between BH spin and z axis

Contour plots of φ

$a = 0$
$\gamma = 0$

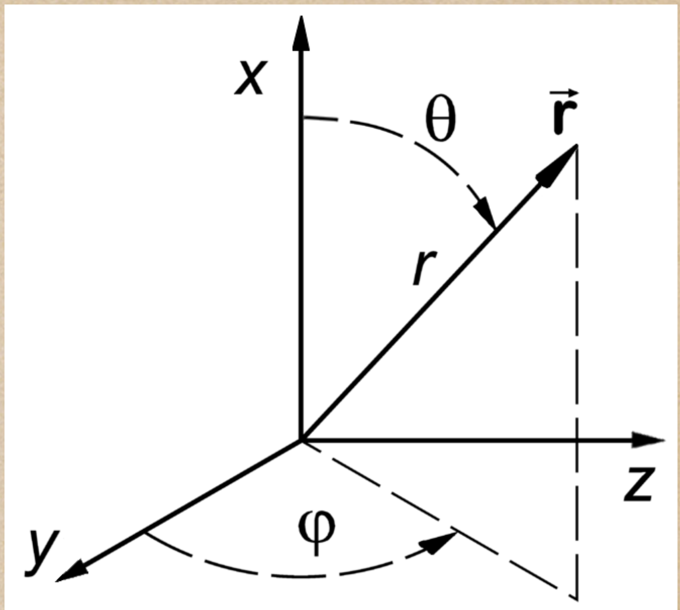
$a = 0.99$
$\gamma = 0$



$a = 0.99$
$\gamma = \frac{\pi}{4}$

$a = 0.99$
$\gamma = \frac{\pi}{2}$

BH Binaries: what would we expect?



- Coordinates: $y = r\sin\theta\,\cos\phi$

  $z = r\sin\theta\,\sin\phi$

  $x = r\cos\theta$

  Orbital plane: (y, z)

- Scalar background $\varphi_{\rm ext} = 2\pi\sigma\, r\sin\theta\,\sin\phi$

- Consider source rotating with frequency Ω

  $\Rightarrow$ Modulation in $\varphi = \varphi_{\rm ext}[1 + f(\phi - \Omega t)]$

  $$\Rightarrow \varphi = 2\pi\sigma\,\sin\theta\,\sin\phi\left[1 + \sum_m f_m e^{im(\phi-\Omega t)}\right]$$

  $$\Rightarrow \varphi_{lm} \sim \left[e^{-i(m+1)\Omega t} + e^{-i(m-1)\Omega t}\right]$$

- Monopole: Oscillation with Ω

  Dipole: Oscillation with 2Ω plus non-oscillating part

A3. Numerical framework

Evolution system

- "3+1" formalism using BSSN

Baumgarte & Shapiro gr-qc/9810065; Shibata & Nakamura PRD '95

- Matter variables $\varphi\,, \quad K_\varphi \equiv -\frac{1}{2\alpha}(\partial_t - \mathcal{L}_\beta)\varphi$

- Matter source terms

$$8\pi\rho = 2K_\varphi^2 + \frac{1}{2}\partial_i\varphi\,\partial^i\varphi\,,$$

$$8\pi j^i = 2K_\varphi\partial^i\varphi\,,$$

$$8\pi S_{ij} = \partial_i\varphi\,\partial_j\varphi - \frac{1}{2}\gamma_{ij}\partial^m\varphi\,\partial_m\varphi + 2\gamma_{ij}K_\varphi^2\,,$$

$$8\pi S = -\frac{1}{2}\partial^m\varphi\,\partial_m\varphi + 6K_\varphi^2\,.$$

- Straightforward to add to LEAN code Sperhake gr-qc/0606079

- Moving puncture technique for BHs

Campanelli et al gr-qc/0511048; Baker et al gr-qc/0511103

Initial data

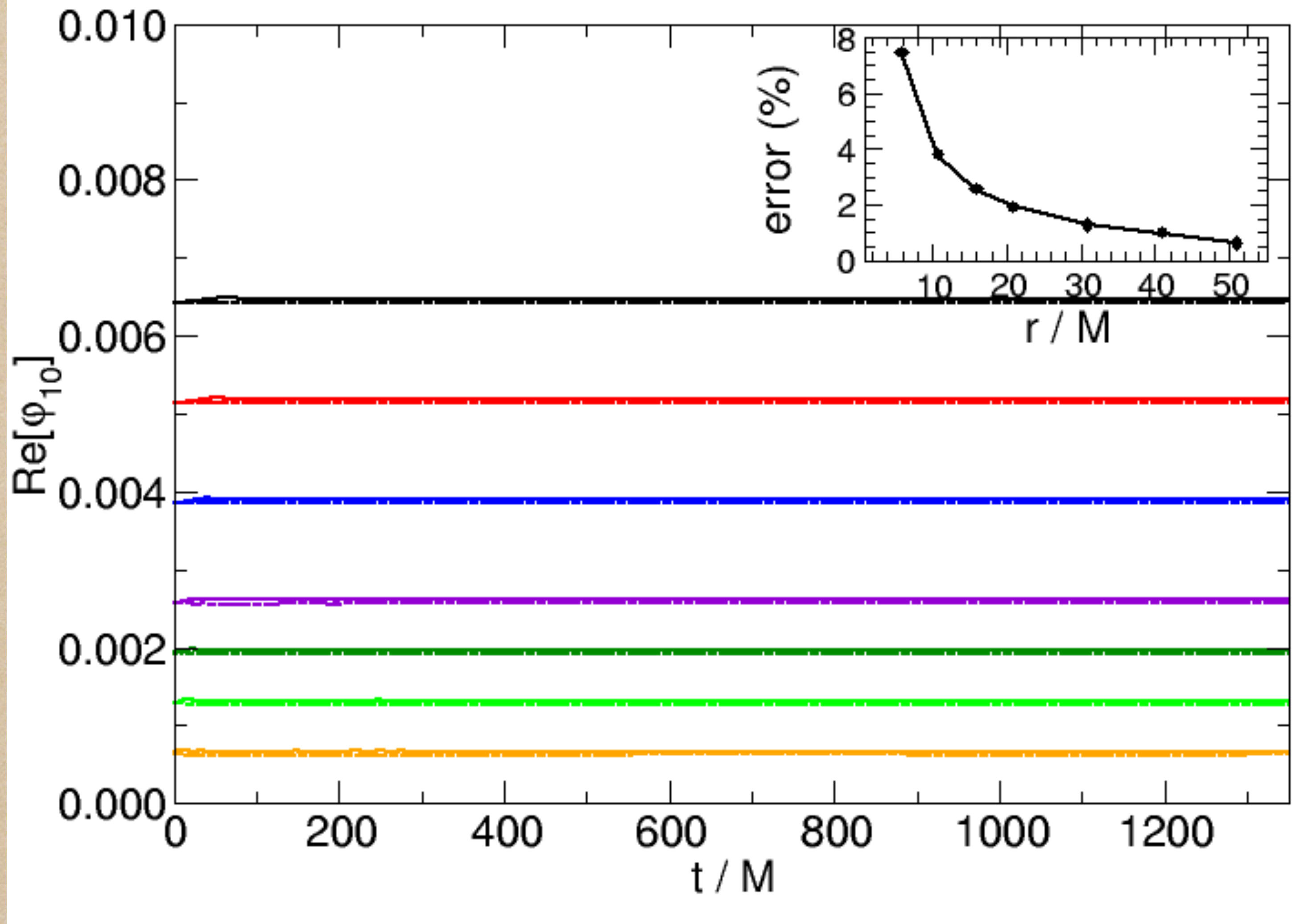
- Black holes: Puncture data from Ansorg et al gr-qc/0404056
- Scalar field: Initialize as $\varphi = 2\pi\sigma\, z$

  Error: $\mathcal{O}(\sigma^2,\ M^2/r^2)$

  $\Rightarrow$ Brief transient at early times
- Limits on σ
  - Scalar field energy $\sim (\nabla\varphi)^2 \sim \sigma^2 \sim \mathrm{const}$
  - Total scalar energy $M \sim \sigma^2 R^3$
  - Horizon if $M/R \sim \sigma^2 R^2 \sim 1 \quad \Rightarrow\ \sigma < R^{-1} = \mathcal{O}(10^{-3}\ M_{\mathrm{BH}}^{-1})$
  - Conservative choice: $M_{\mathrm{BH}}\sigma = 10^{-7}\ldots 10^{-4}$
  - Realistic values probably smaller $\sigma \sim \dfrac{10^{-15}}{10\ M_{\odot}}$

A4. Numerical results

Single Schwarzschild BH: num. vs. lin.



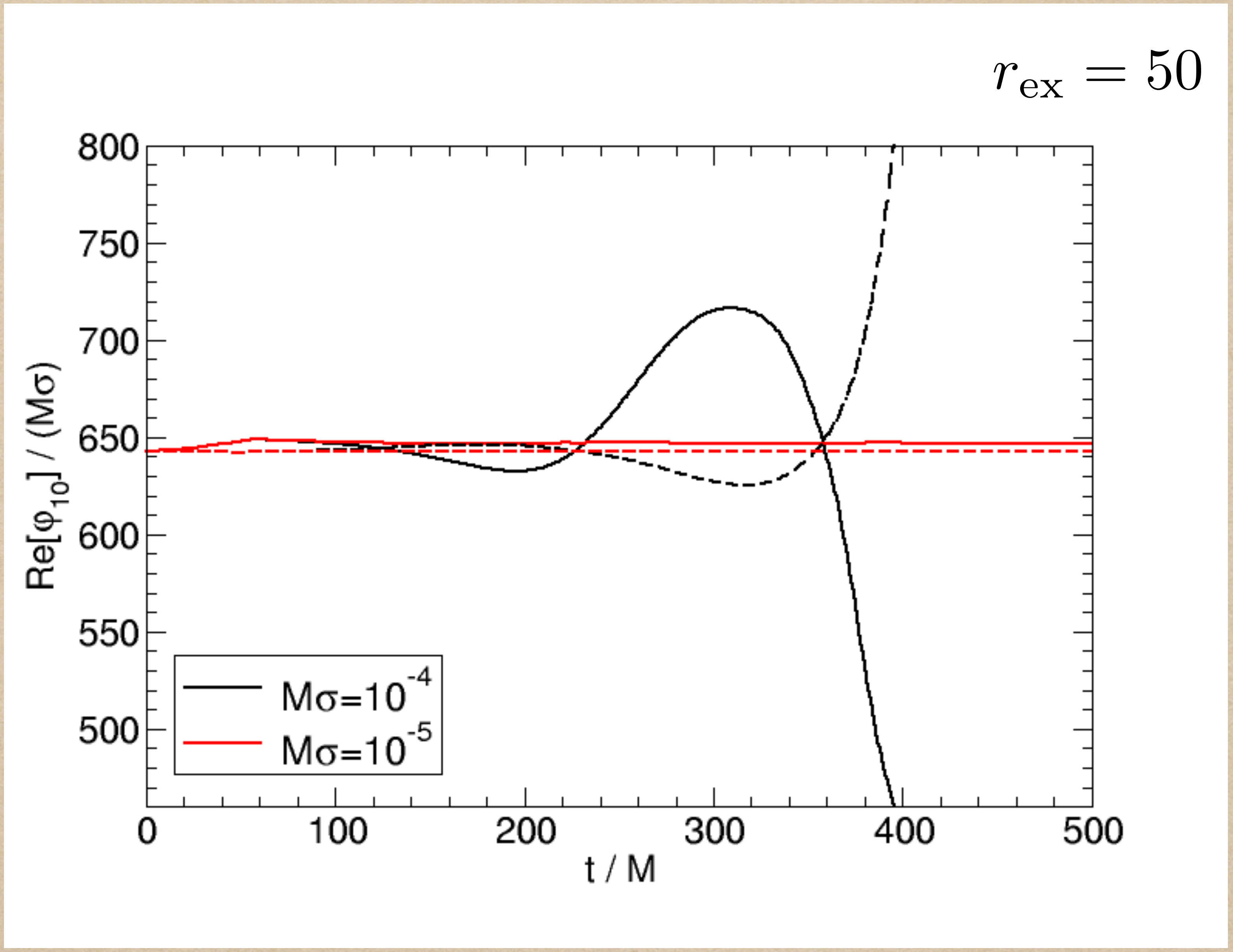
$\hat{r}_{\rm ex} = 50\ M$

$\vdots$

$5\ M$

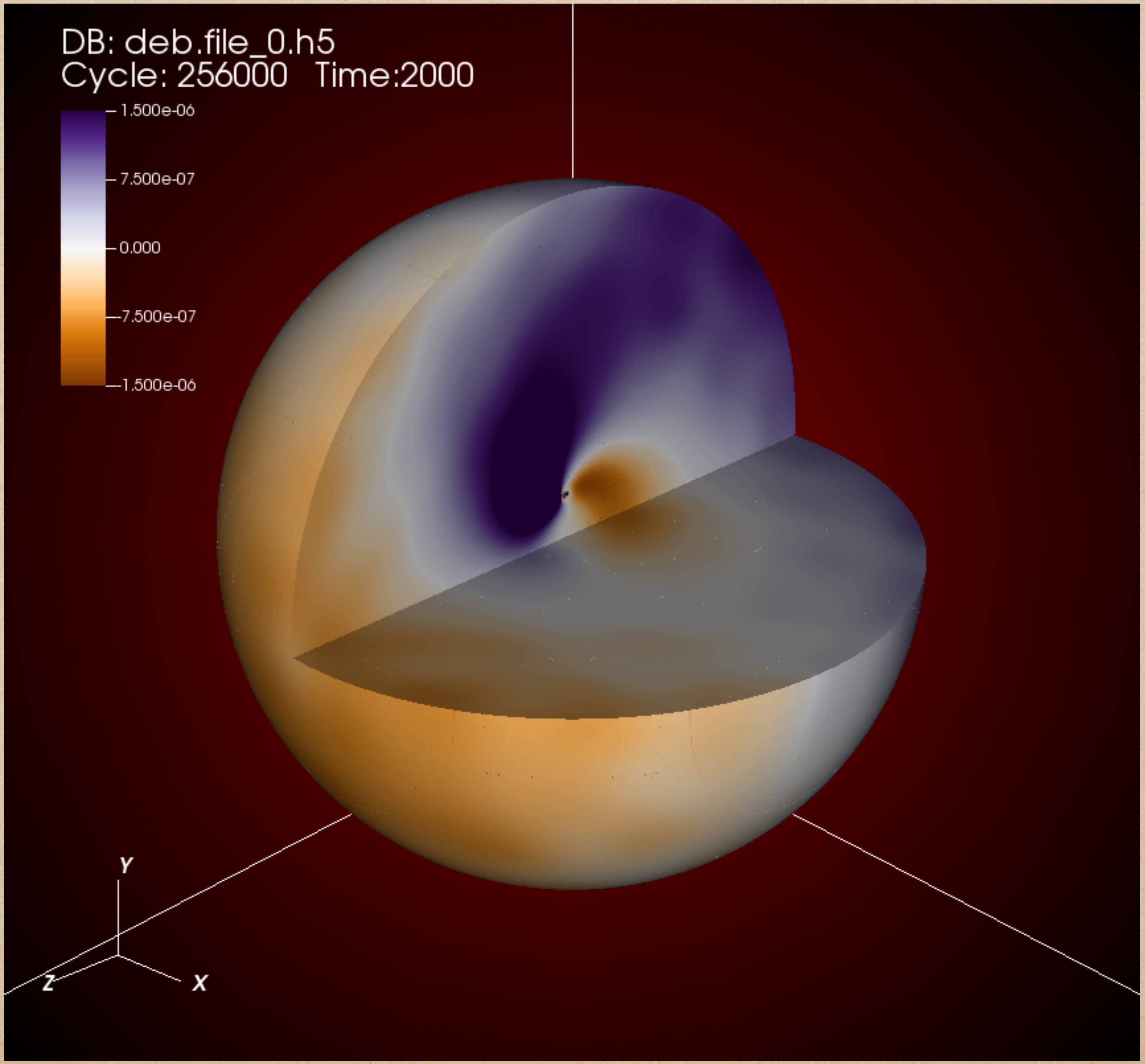
$$\varphi_{10,\rm lin} = \sqrt{\frac{4\pi}{3}} 2\pi\sigma(r - M)\,, \quad M\sigma = 10^{-5}$$

Large σ : Onset of collapse



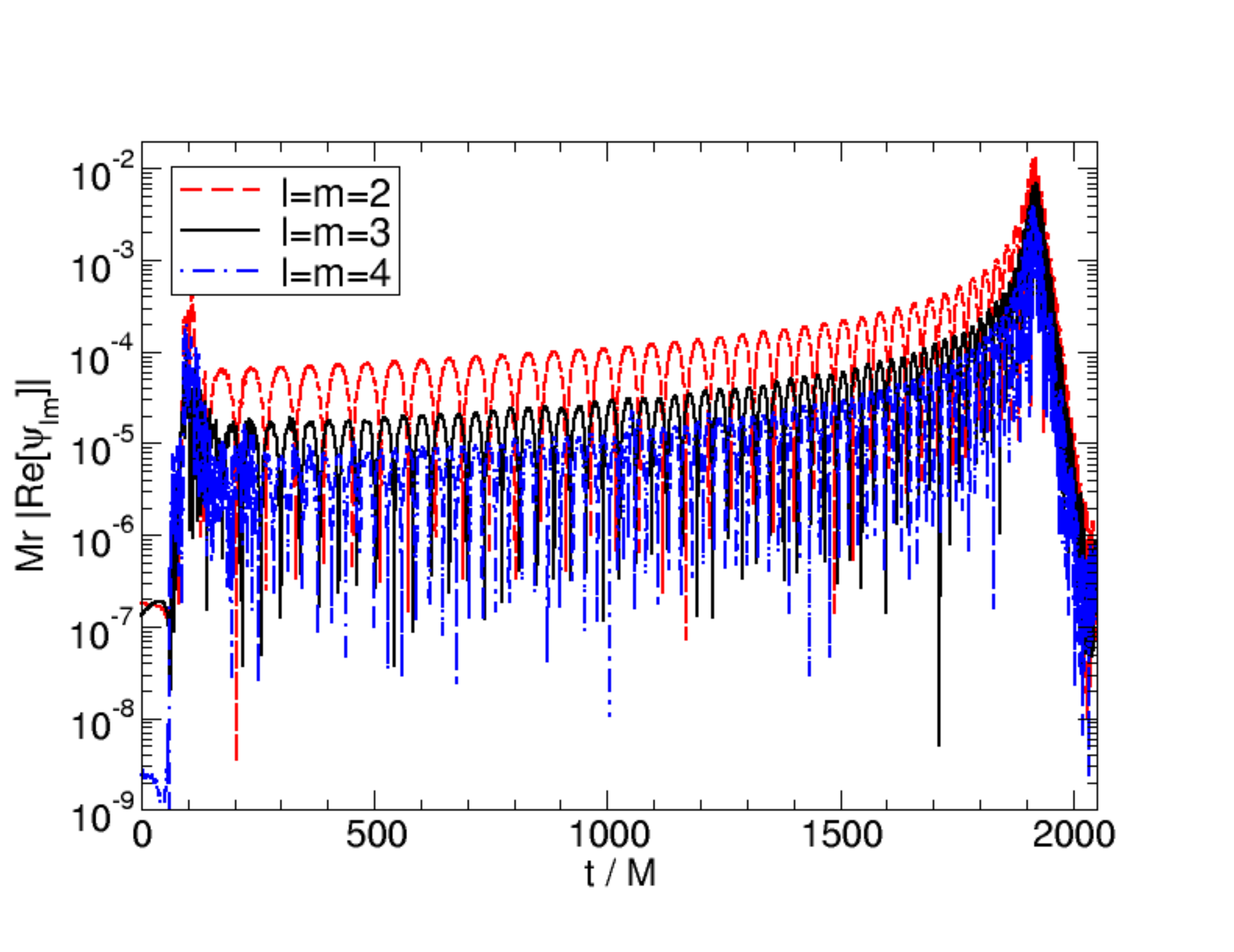
- For large σ , linearized (dashed) and numerical (solid) solution unstable!

BH binary: Animation of $r\partial_t\varphi$



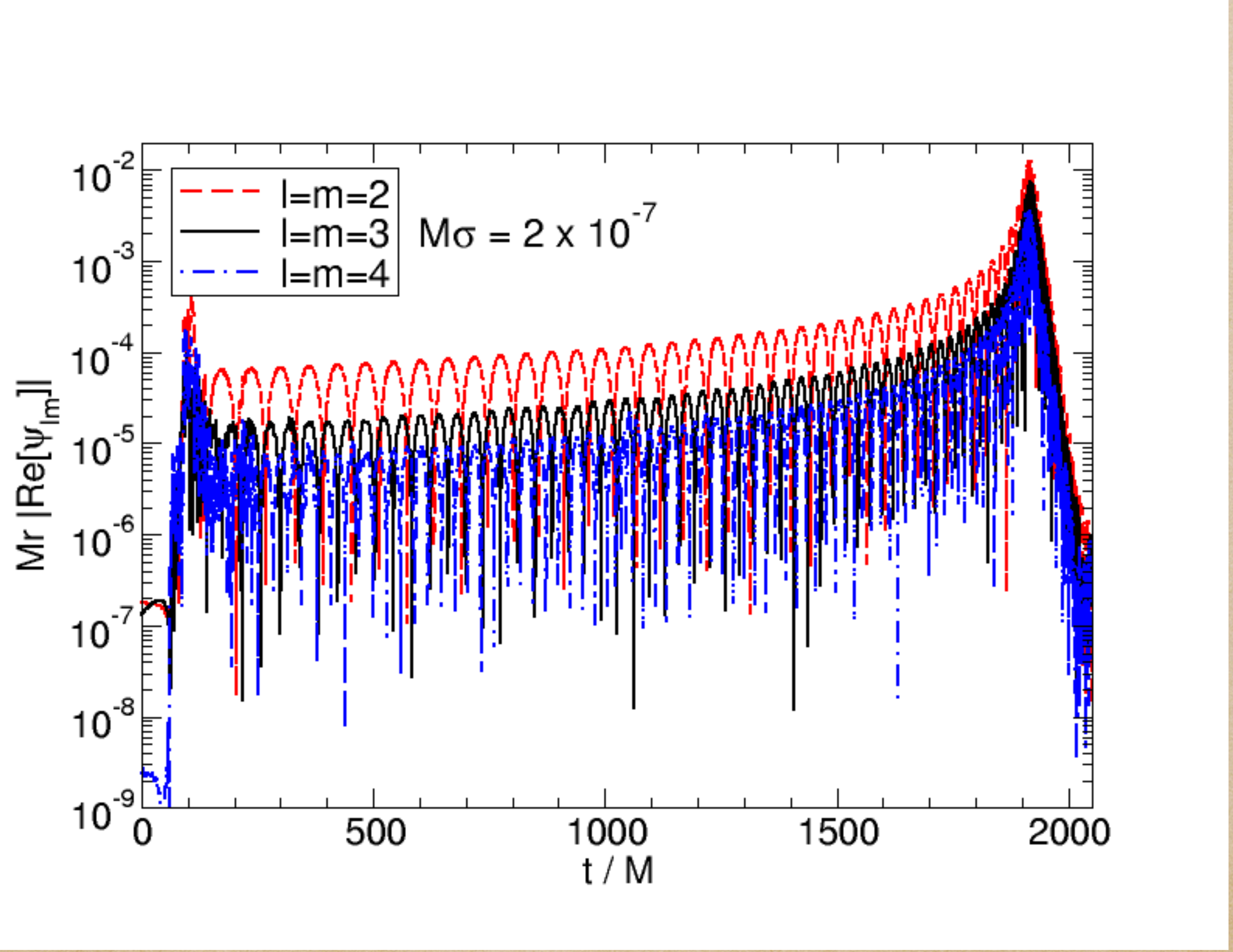
BH binary GW signal: $M\sigma = 0$

$q = 1/3\,,\quad \boldsymbol{S} = 0\,,\quad yz$ -plane; Multipoles of Ψ_4

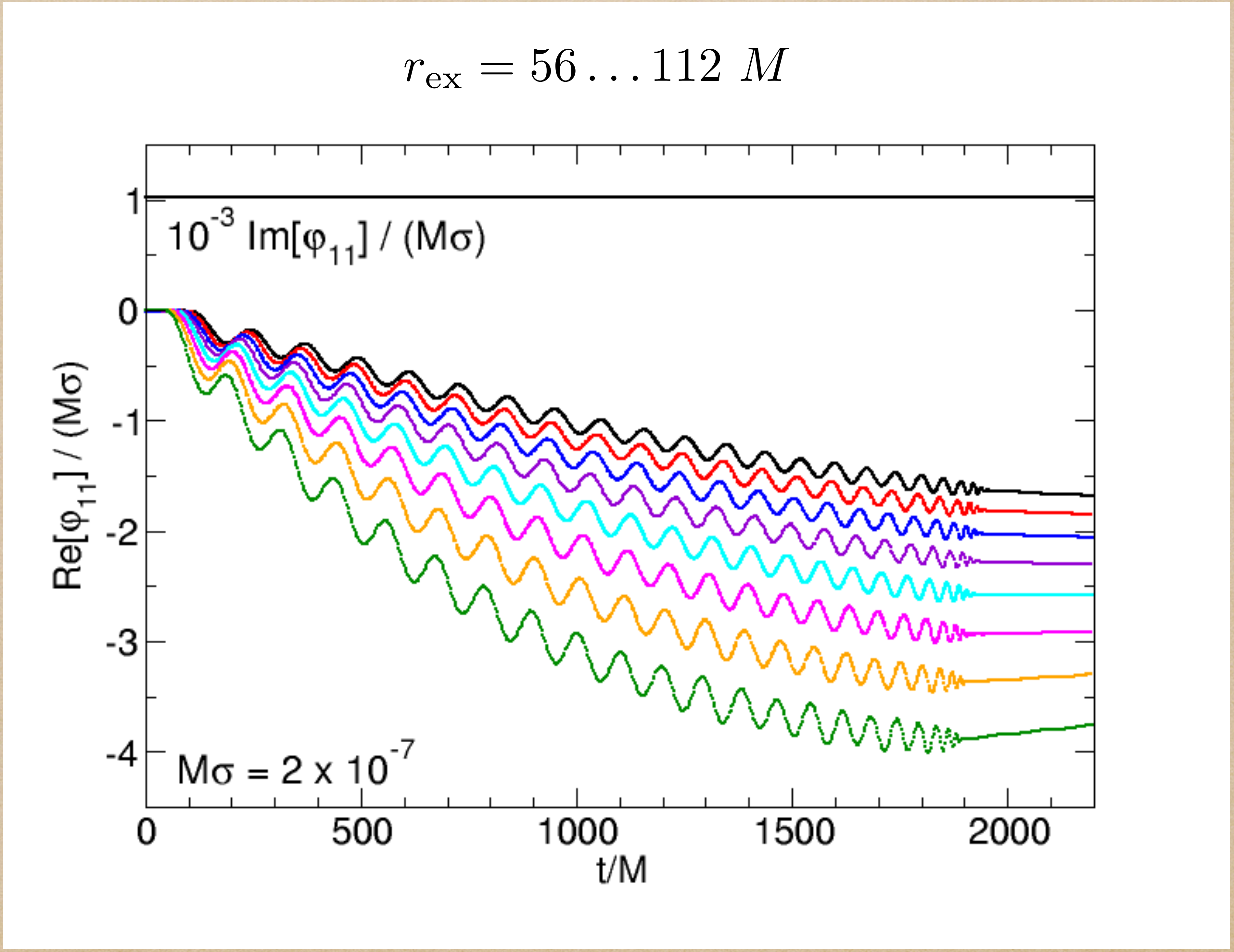


BH binary GW signal: $M\sigma = 2 \times 10^{-7}$

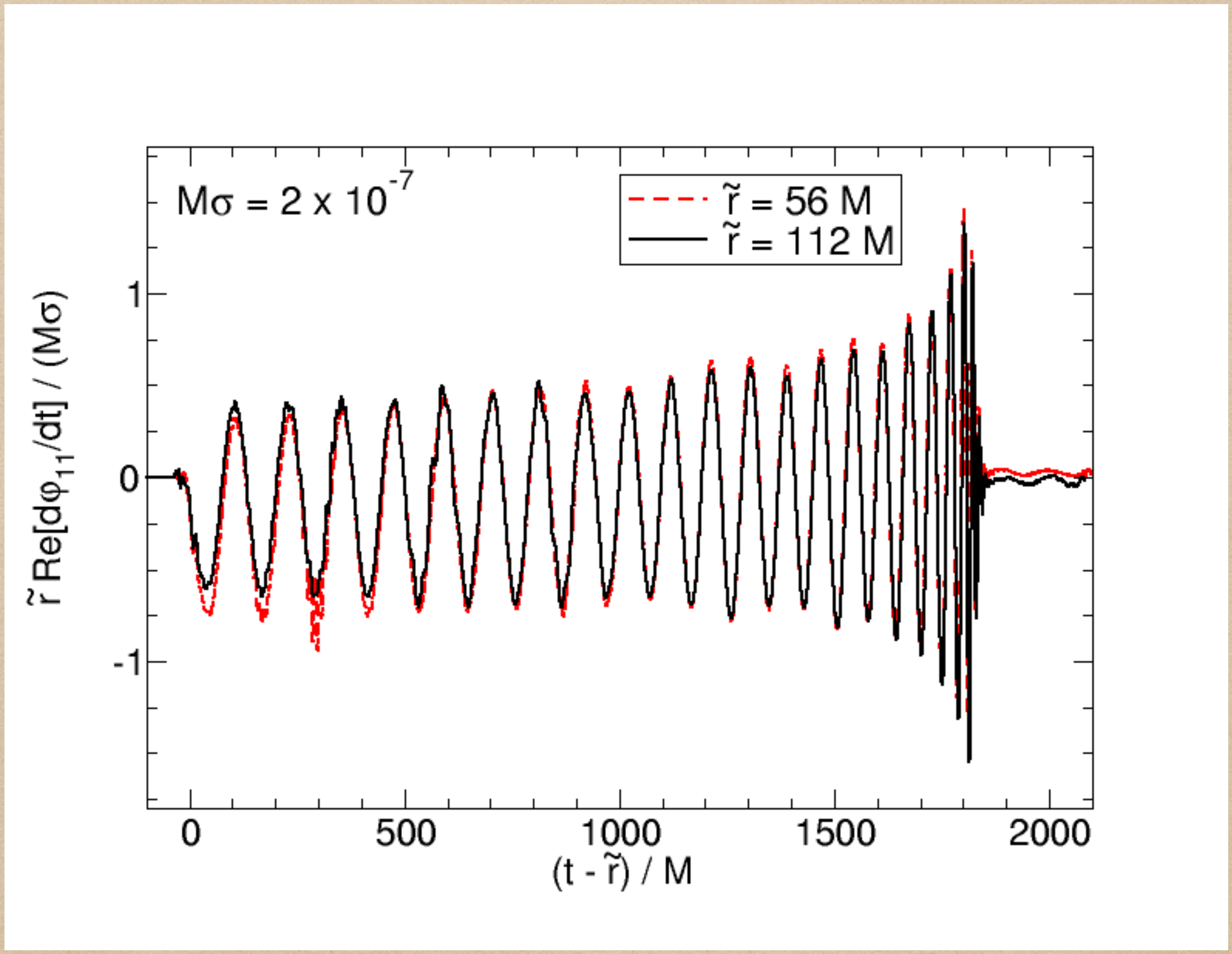
$q = 1/3\,,\quad \boldsymbol{S} = 0\,,\quad yz$ -plane; Multipoles of Ψ_4



BH binary scalar dipol: $M\sigma = 2 \times 10^{-7}$



BH binary scalar dipol: $M\sigma = 2 \times 10^{-7}$



Dipole oscillates with $2\Omega_{\mathrm{orb}}$ as expected

Features of the radiation

- Ringdown of a $a/M = 0.543$ black hole
  - GWs: $M\omega_{11,\ \mathrm{lin}} = 0.476 - 0.0849i\,, \quad M\omega_{11,\,\mathrm{num}} = 0.48 - 0.081i$
  - Scal: $M\omega_{11,\ \mathrm{lin}} = 0.351 - 0.0936i\,, \quad M\omega_{11,\,\mathrm{num}} = 0.36 - 0.070i$
- Drift in φ_{11}
  - Effective-field theory predicts some drift
  - Contribution from BH kick expected but should be very small
  - Frame dragging: order of magnitude ok, but r dependence not
  - Injection of scalar field energy through BCs

  Probably combination of all (+ more?) effects

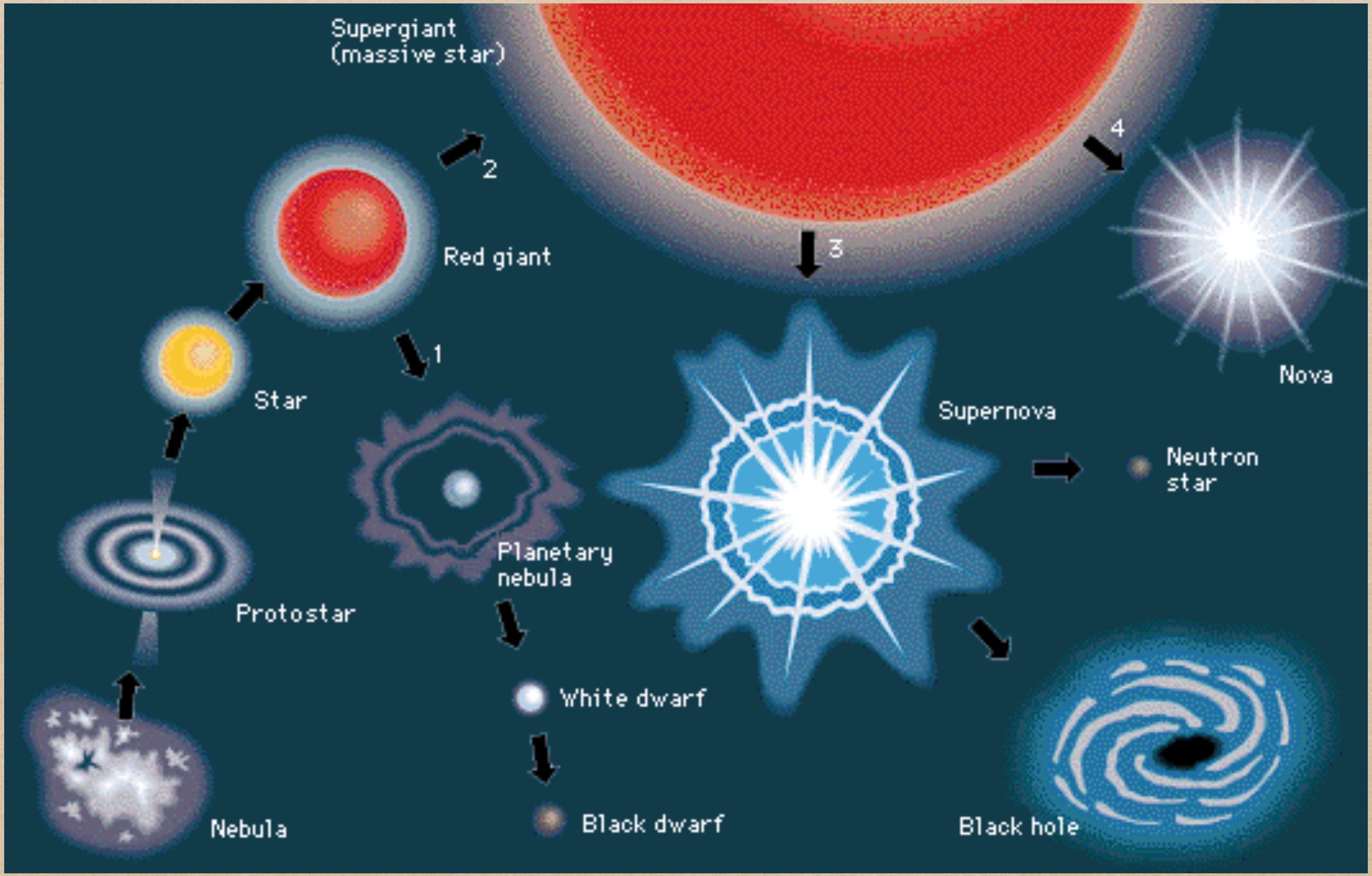
Part B: Supernova core collapse

Gerosa, Sperhake & Ott   CQG 1602.06952

B1. Introduction

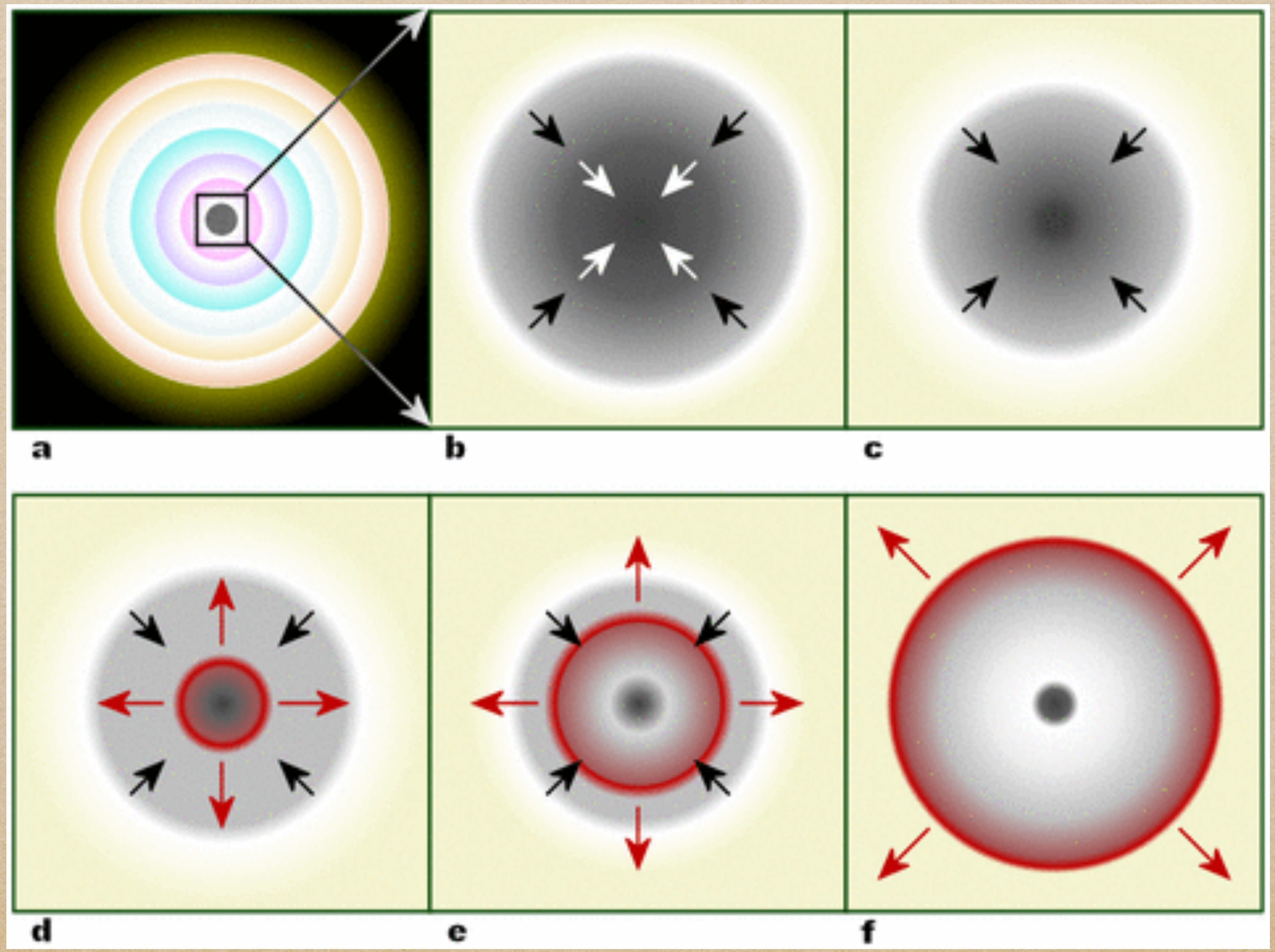
The end point of stellar evolution

- Nuclear fusion above iron requires energy
- Stars with $M_{\mathrm{ZAMS}} \gtrsim 8\ M_{\odot}$ explode as SN $\rightarrow$ BHs, NSs



Core-collapse scenario to 0th order

- Nickel-iron core reaches Chandrasekhar mass $\rightarrow$ Collapse
- EOS stiffens at $\rho \gtrsim \rho_{\rm nuc}$ $\rightarrow$ Bounce
- Outgoing shock, reinvigorated by $\nu_{\rm e}$ $\rightarrow$ Outer layers blast away



Core-collapse scenario to 0th order

- Massive stars: $M_{\rm ZAMS} = 8 \ldots 100\ M_\odot$
- Core compressed from $\sim 1\,500\ \rm km$ to $\sim 15\ \rm km$

  $\sim 10^{10}\ \rm g/cm^3$ to $\gtrsim 10^{15}\ \rm g/cm^3$
- Released gravitational energy: $\mathcal{O}(10^{53})\ \rm erg$

  $\sim 99\ \%$ in neutrinos, $\sim 10^{51}\ \rm erg$ in outgoing shock, explosion
- Explosion mechanism: still uncertainties...
- Failed explosions lead to BH formation

  "Collapsar": possible engine for long-soft GRBs

B2. Formalism

Field equations

- Jordan frame $$G_{\mu\nu} = \frac{8\pi}{F}(T^F_{\mu\nu} T^\phi_{\mu\nu} + T_{\mu\nu})$$

$$T^F_{\mu\nu} = \frac{1}{8\pi}(\nabla_\mu \nabla_\nu F - g_{\mu\nu}\nabla^\rho \nabla_\rho F)$$

$$T^\phi_{\mu\nu} = \partial_\mu \phi\, \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial^\rho \phi\, \partial_\rho \phi$$

$$\nabla^\rho \nabla_\rho \phi = -\frac{1}{16\pi} R\, F_{,\phi}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

- Scalar field: Use φ with $\dfrac{\partial\varphi}{\partial\phi} = \sqrt{\dfrac{3}{4}\dfrac{F_{,\phi}{}^2}{F^2} + \dfrac{4\pi}{F}}$, $\boxed{F = e^{-2\alpha_0\varphi - \beta_0\varphi^2}}$

- Line element for spherical symmetry

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + X^2 dr^2 + \frac{r^2}{F}(d\theta^2 + \sin^2\theta\, d\tilde{\phi}^2)$$

Evolution variables

- Metric: $\Phi = \ln(\sqrt{F}\alpha)\,, \quad m = \frac{r}{2}\left(1 - \frac{1}{FX^2}\right)$
- Matter: $T_{\alpha\beta} = \rho h u_\alpha u_\beta + P g_{\alpha\beta}$

$$J^\alpha = \rho u^\alpha \;, \quad \nabla_\mu J^\mu = 0$$

$$u^\mu = \frac{1}{\sqrt{1 - v^2}}\left[\frac{1}{\alpha},\; \frac{v}{X},\; 0,\; 0\right]$$

- Primitive versus conserved variables $(\rho,\; h,\; v) \quad \leftrightarrow \quad (D,\; S^r,\; \tau)$

$$D = \frac{\rho X}{F\sqrt{F}\sqrt{1 - v^2}}$$

$$S^r = \frac{\rho h v}{F^2(1 - v^2)}$$

$$\tau = \frac{\rho h}{F^2(1 - v^2)} - \frac{P}{F^2} - D$$

- Scalar field: $\eta = \frac{\partial_r \varphi}{X}\,, \quad \psi = \frac{\partial_t \varphi}{\alpha}$

Evolution equations in spherical symmetry

- Metric: $\partial_r \Phi = \ldots$

  $$\partial_r m = \ldots$$

- Scalar field: $\partial_t \varphi = \alpha \psi$

  $$\partial_t \psi = \ldots$$

  $$\partial_t \eta = \ldots$$

- HRSC for matter: $\partial_t D + \frac{1}{r^2}\partial_r \left(r^2 \frac{\alpha}{X} f_D \right) = s_D \,, \qquad f_D = Dv \,,$

  $$\partial_t S^r + \frac{1}{r^2}\partial_r \left(r^2 \frac{\alpha}{X} f_{S^r} \right) = s_{S^r} \,, \quad f_{S^r} = S^r v + \frac{P}{F^2} \,,$$

  $$\partial_t \tau + \frac{1}{r^2}\partial_r \left(r^2 \frac{\alpha}{X} f_\tau \right) = s_\tau \,, \qquad f_\tau = S^r - Dv \,.$$

- Flux conservative form: NO derivatives in $s_D, \; s_{S^r}, \; s_\tau$

- Use extension of GR1D code O'Connor & Ott CQG 0912.2393

Equation of state

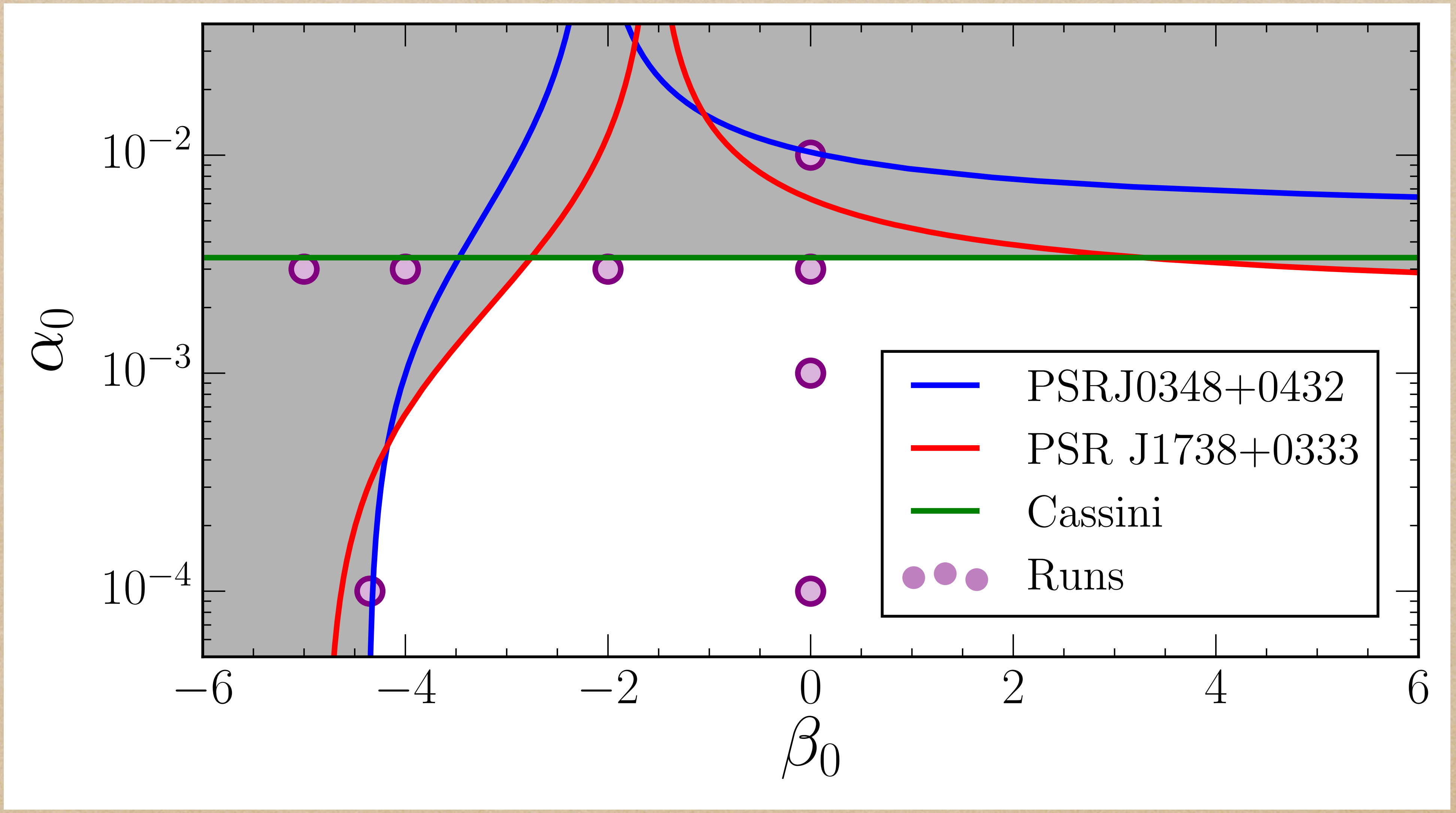
- Pressure: "cold" + "thermal" contribution: $P = P_{\rm c} + P_{\rm th}$
- Hybrid EOS for cold part: $P_c = \begin{cases} K_1 \rho^{\Gamma_1} & \text{if} \quad \rho \leq \rho_{\rm nuc} \\ K_2 \rho^{\Gamma_2} & \text{if} \quad \rho > \rho_{\rm nuc} \end{cases}$
- Internal energy from 1st law: $\epsilon_c = \begin{cases} \frac{K_1}{\Gamma_1 - 1} \rho^{\Gamma_1 - 1} & \text{if} \quad \rho \leq \rho_{\rm nuc} \\ \frac{K_2}{\Gamma_2 - 1} \rho^{\Gamma_2 - 1} + E_3 & \text{if} \quad \rho > \rho_{\rm nuc} \end{cases}$
- Thermal pressure: $P_{\rm th} = (\Gamma_{\rm th} - 1)\rho(\epsilon - \epsilon_{\rm th})$
- Parameters: $\Gamma_1 = 1.3\,, \quad \Gamma_2 = 2.5\,, \quad \Gamma_{\rm th} = 1.35$

  $K_1 = 4.9345 \times 10^{14}\ [\text{cgs}]\,, \quad \rho_{\rm nuc} = 2 \times 10^{14}\ {\rm g\,cm^{-3}}$

  $K_2\,, \quad E_3$ from continuity at $\rho = \rho_{\rm nuc}$

The coupling function

- Coupling function $\boxed{F = e^{-2\alpha_0\varphi - \beta_0\varphi^2}}$
- Free parameters $\alpha_0,\ \beta_0$: Capture all modifications at 1PN



B3. Code tests

Static models: Spontaneous scalarization

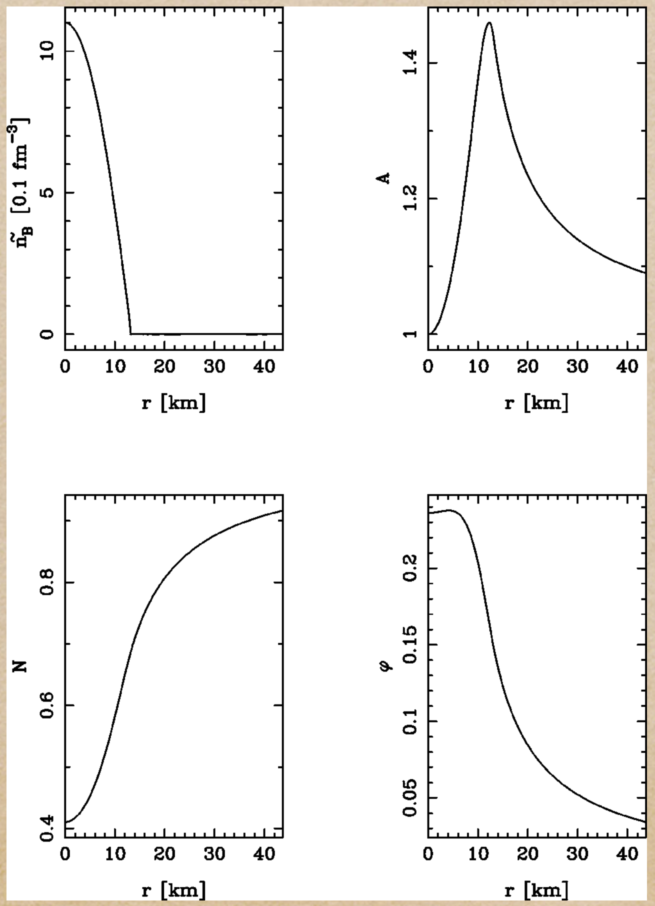
- Spontaneous scalarization: Non-linear effect for $\beta_0 \lesssim -4.35$

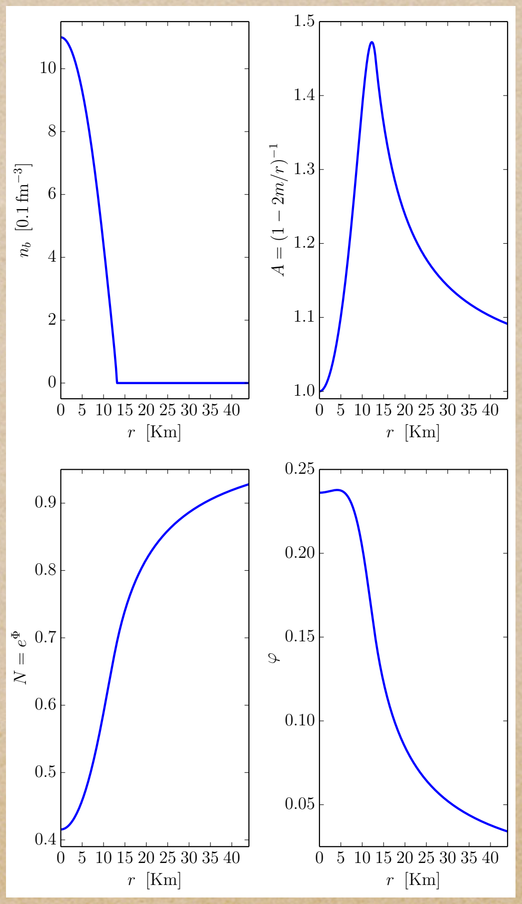
  Damour & Esposito-Farese PRL '93

- $M = 2.4\ M_\odot\,,\quad R = 13.1\ \mathrm{km}$ Model with $\alpha_0 = 0\,,\quad \beta = -6$

  Novak PRD gr-qc/9707041

- Baryon density, metric functions, scalar field



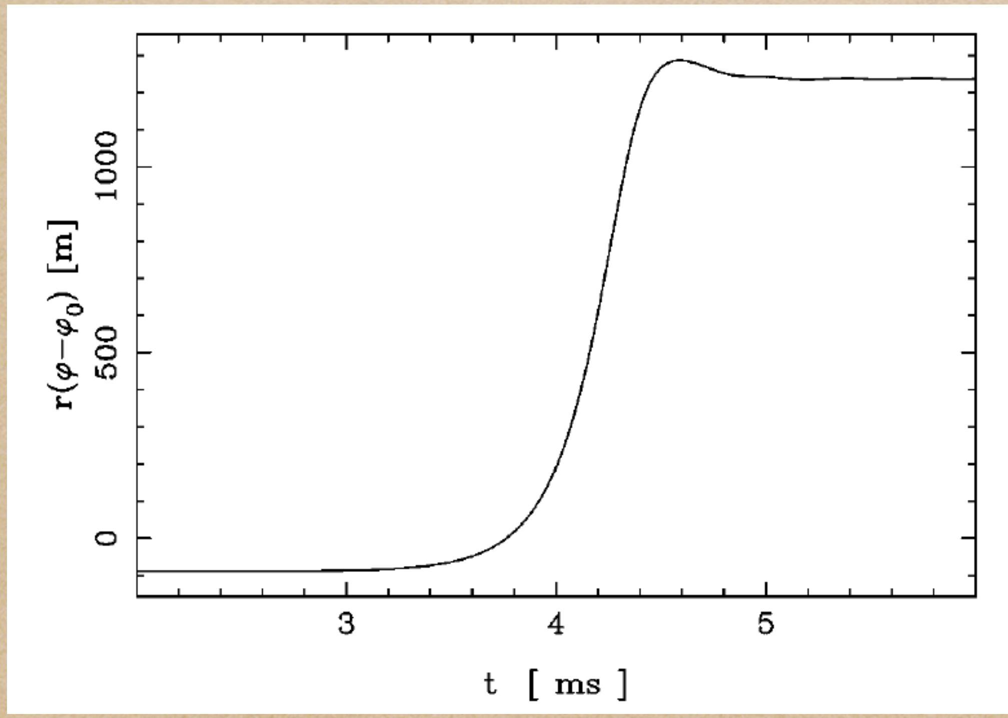


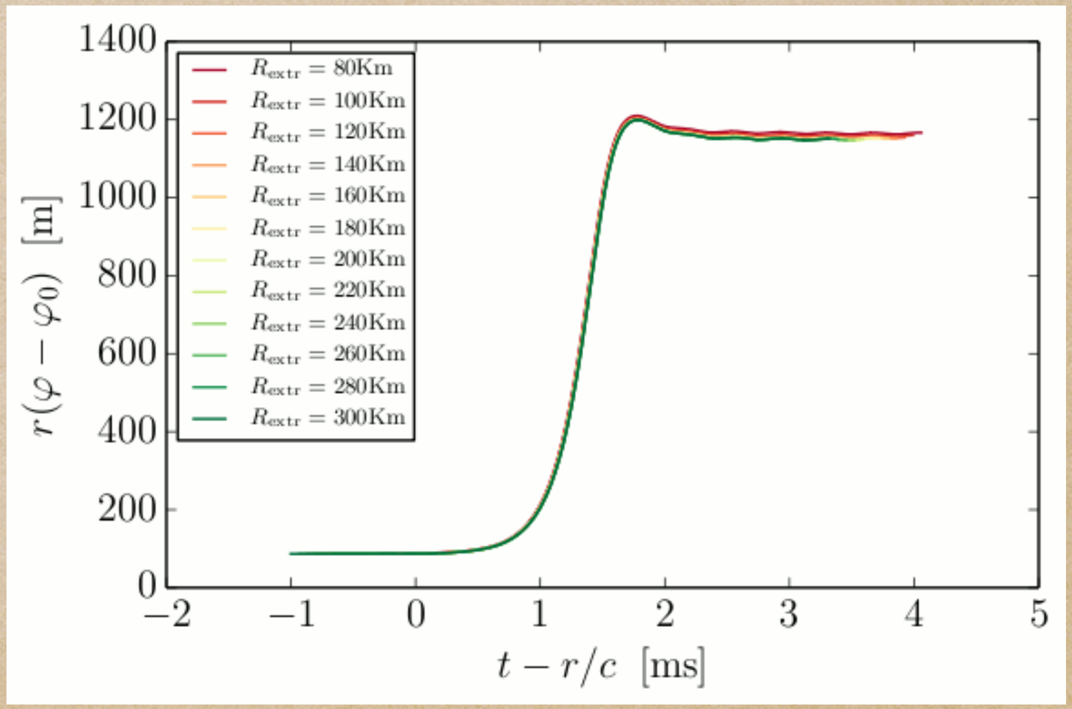
Transition from GR to secularized star

- Here: $\alpha_0 = 0.01\,, \quad \beta_0 = -6$
- Unstable GR-like model: $M = 1.378\ M_\odot\,, \quad R = 13.2\ \mathrm{km}$
- ... migrates to scalarized model: $M = 1.373\ M_\odot\,, \quad R = 13.0\ \mathrm{km}$

  Novak PRD gr-qc/9806022

- Baryon density, metric functions, scalar field



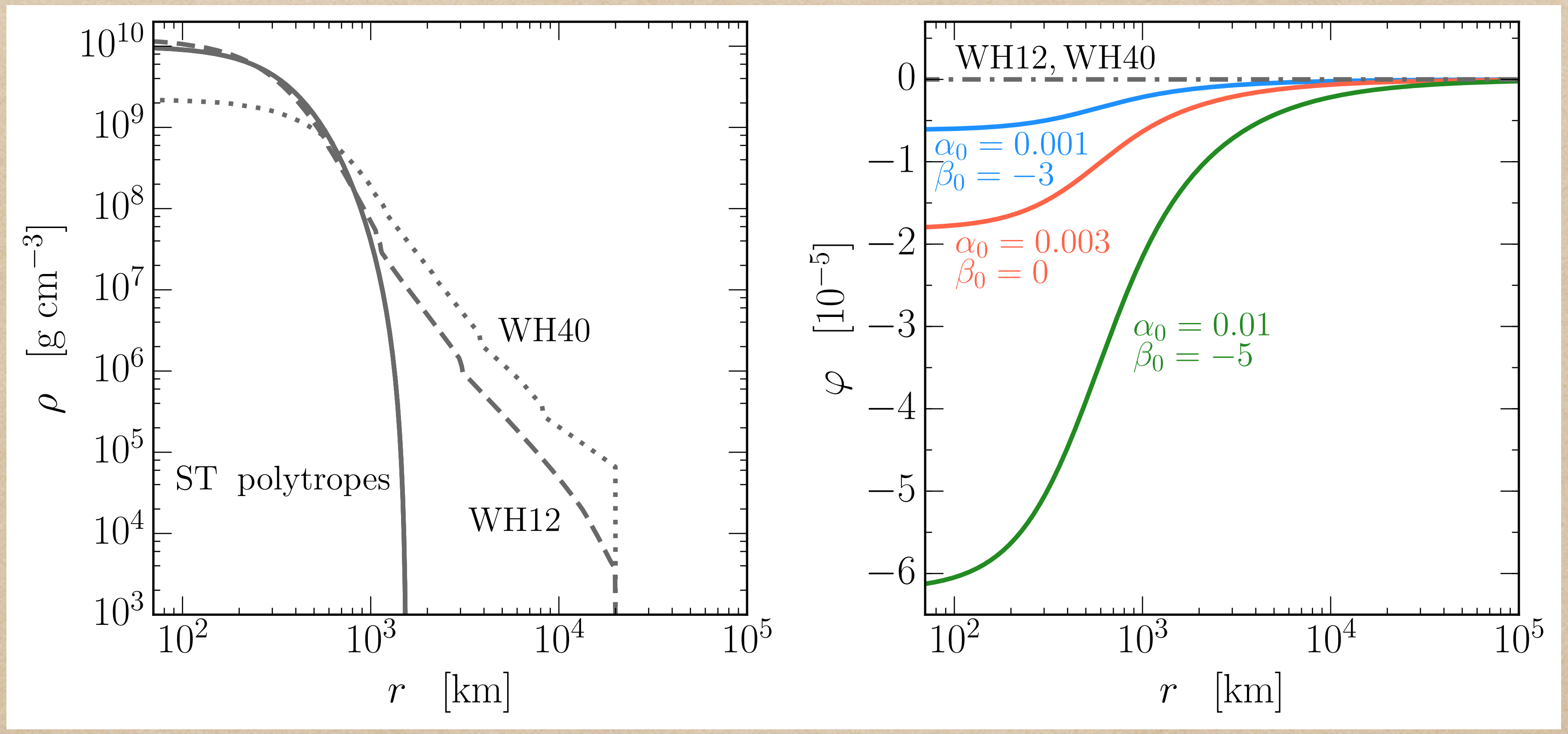


B4. Results

Static models: Spontaneous scalarization

- 3 types of initial profiles: Woosley & Heger Phys.Rep. astro-ph/0702176

  (i) Polytrope, (ii) "realistic" $12\ M_\odot$, (iii) "realistic" $40\ M_\odot$



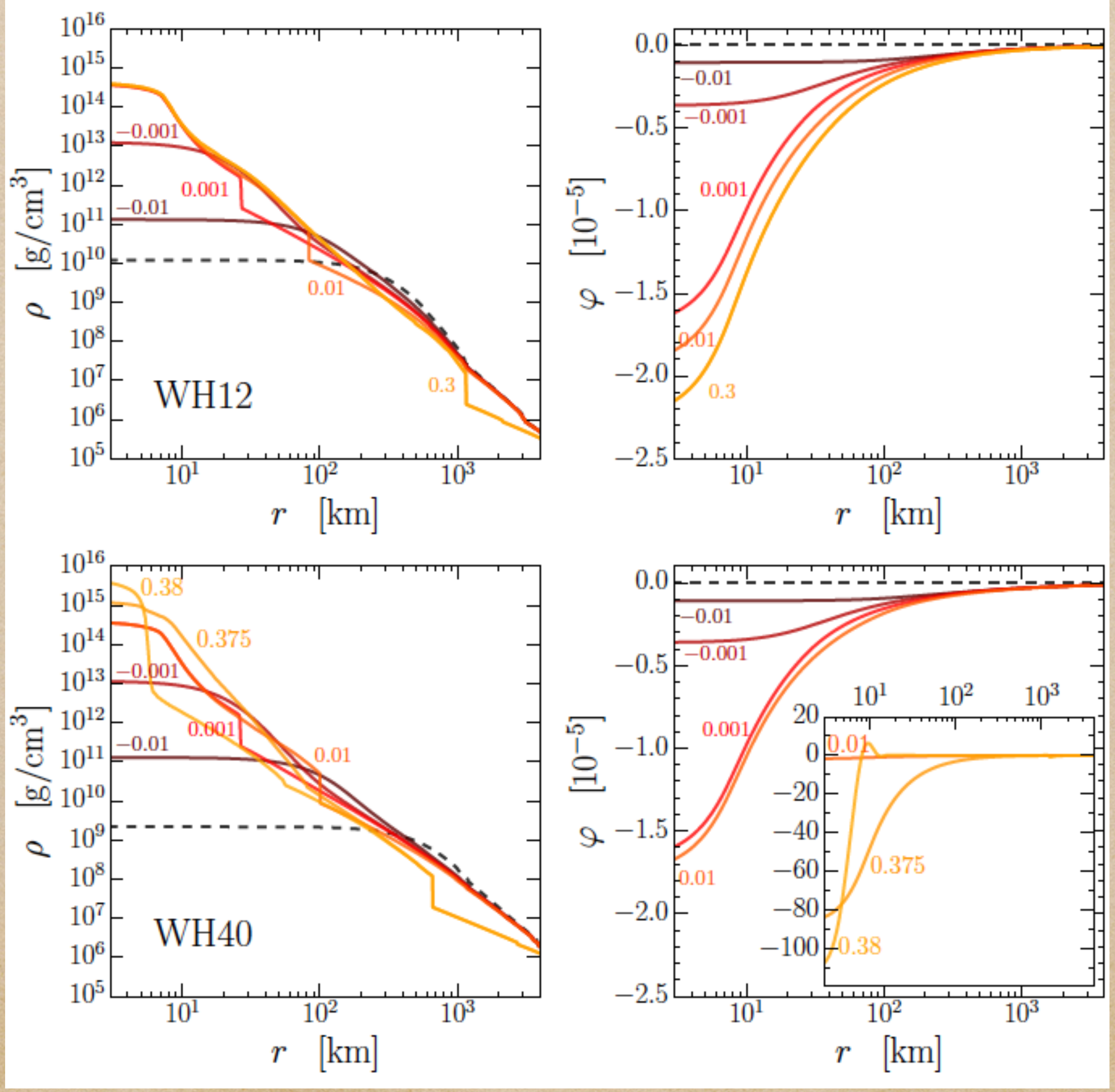
- Polytropes: Mostly for tests

  $12\ M_\odot$: Neutron star formation

  $40\ M_\odot$: Black-hole formation

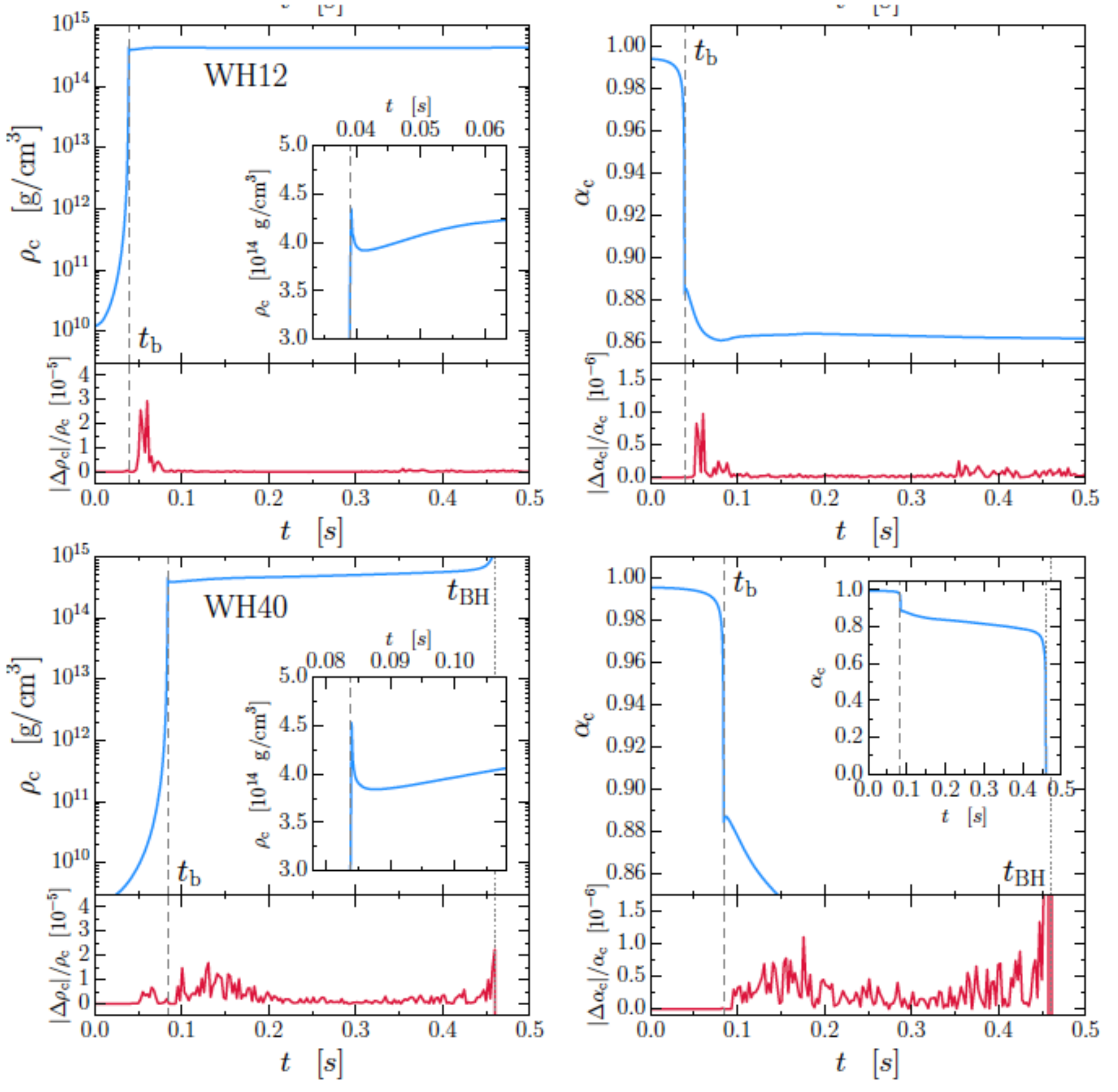
Time evolution

- ST theory with: $\alpha_0 = 10^{-4}\,, \quad \beta = -4.35$
- End product: WH12 $\rightarrow$ Neutron star, WH40 $\rightarrow$ Black hole



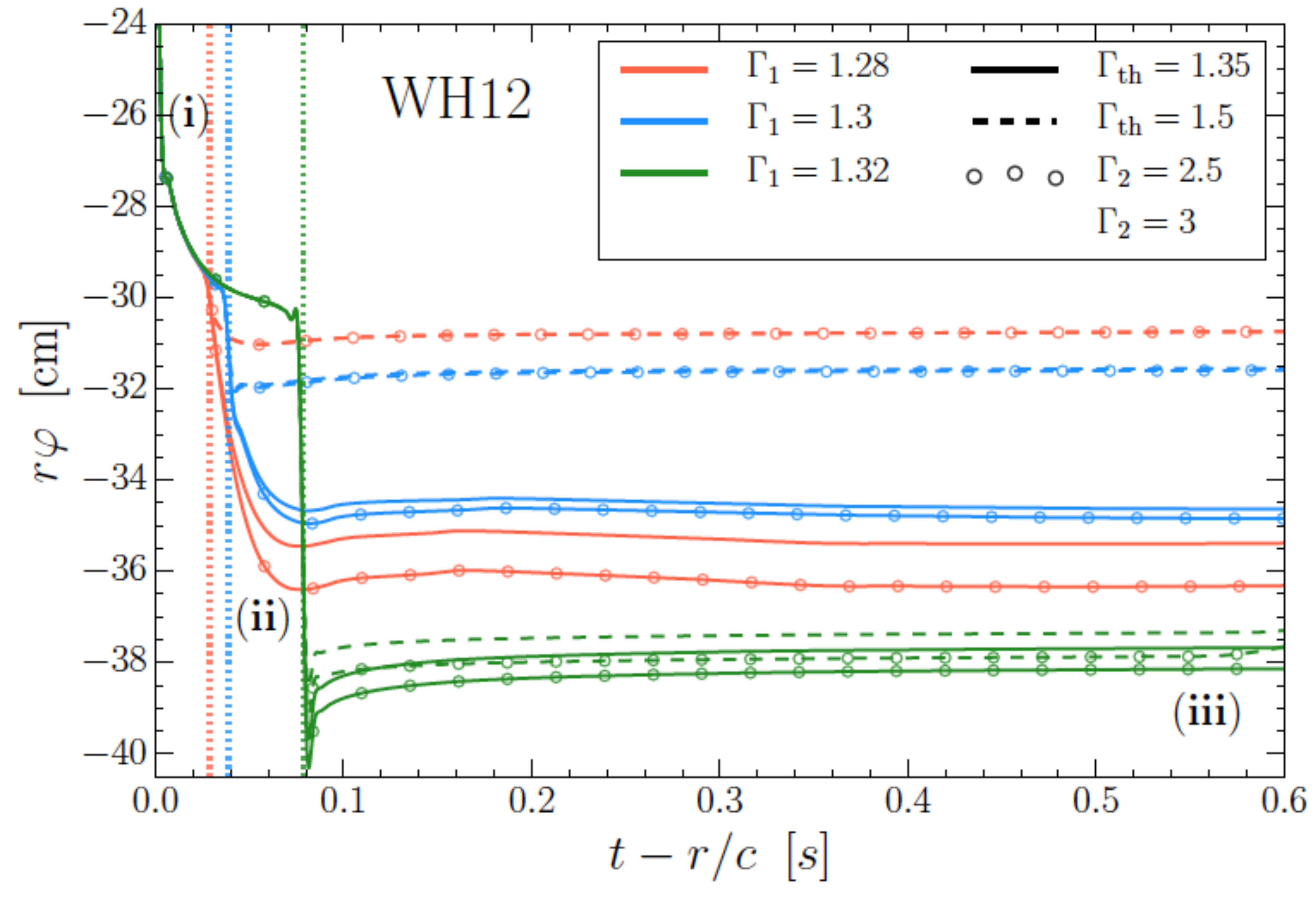
Time evolution: Central values

- Stellar dynamics barely affected by scalar field
- Accretion onto WH40 model leads to BH



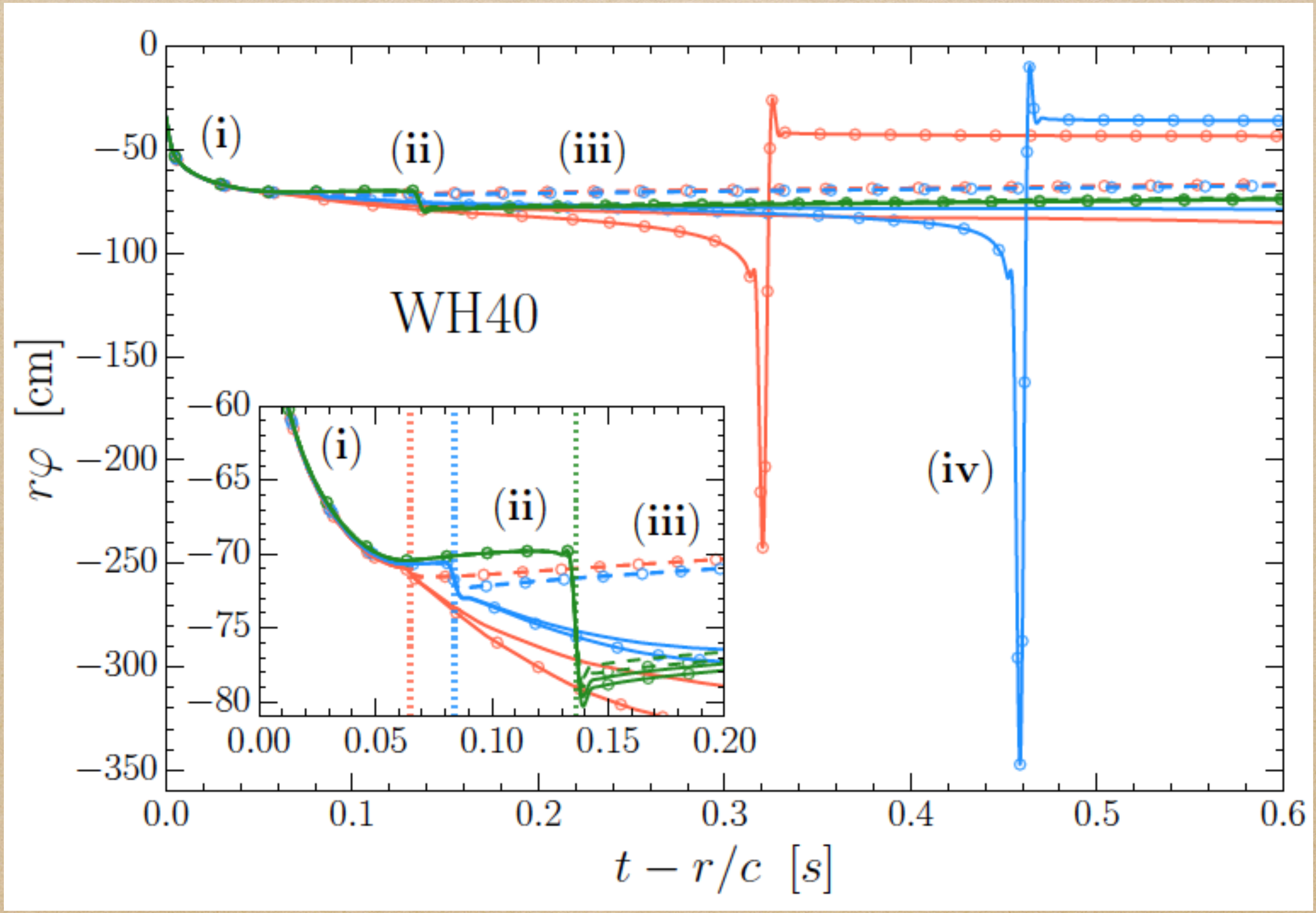
Scalar radiation: WH12

- Vary EOS



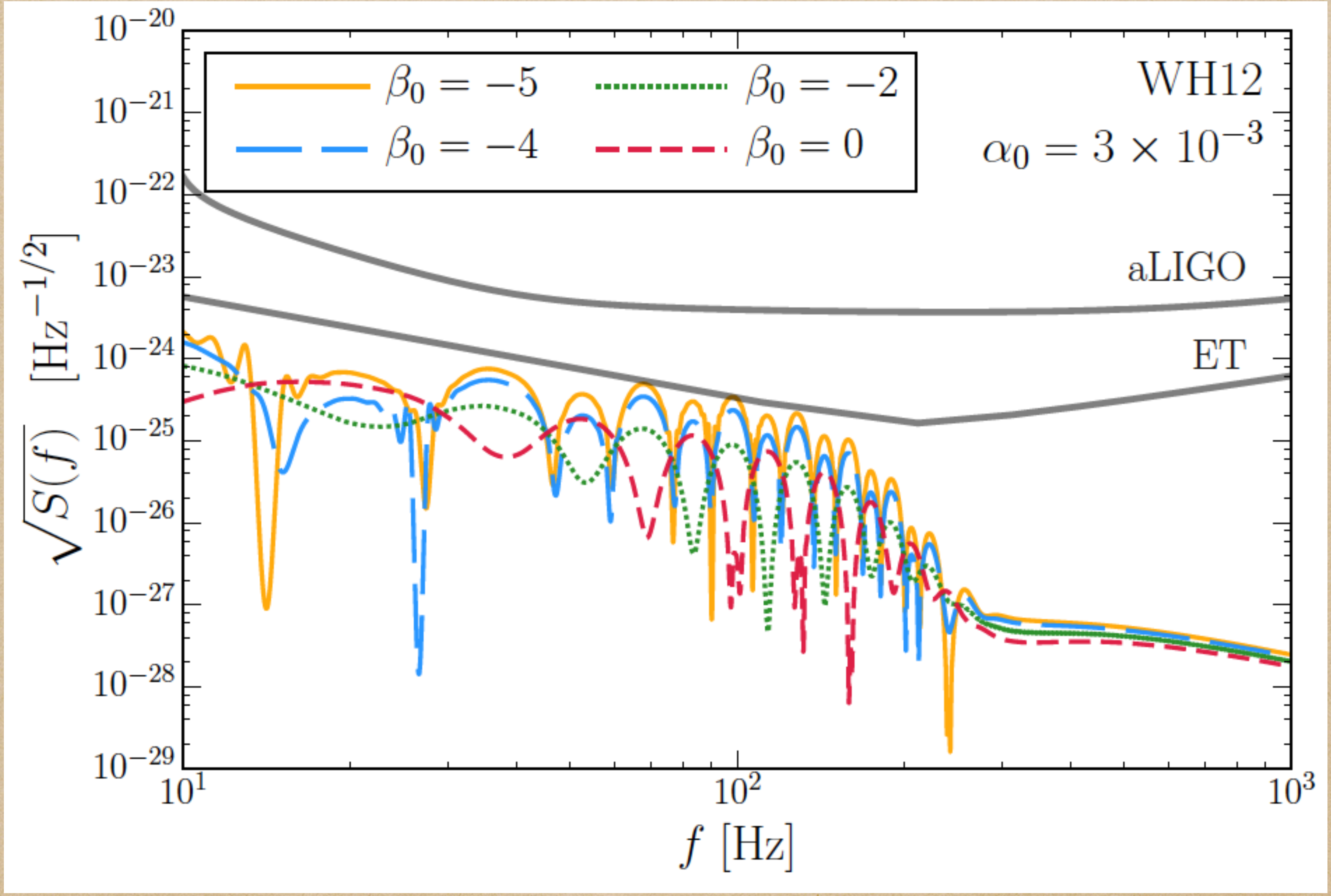
Scalar radiation: WH40

- Vary EOS



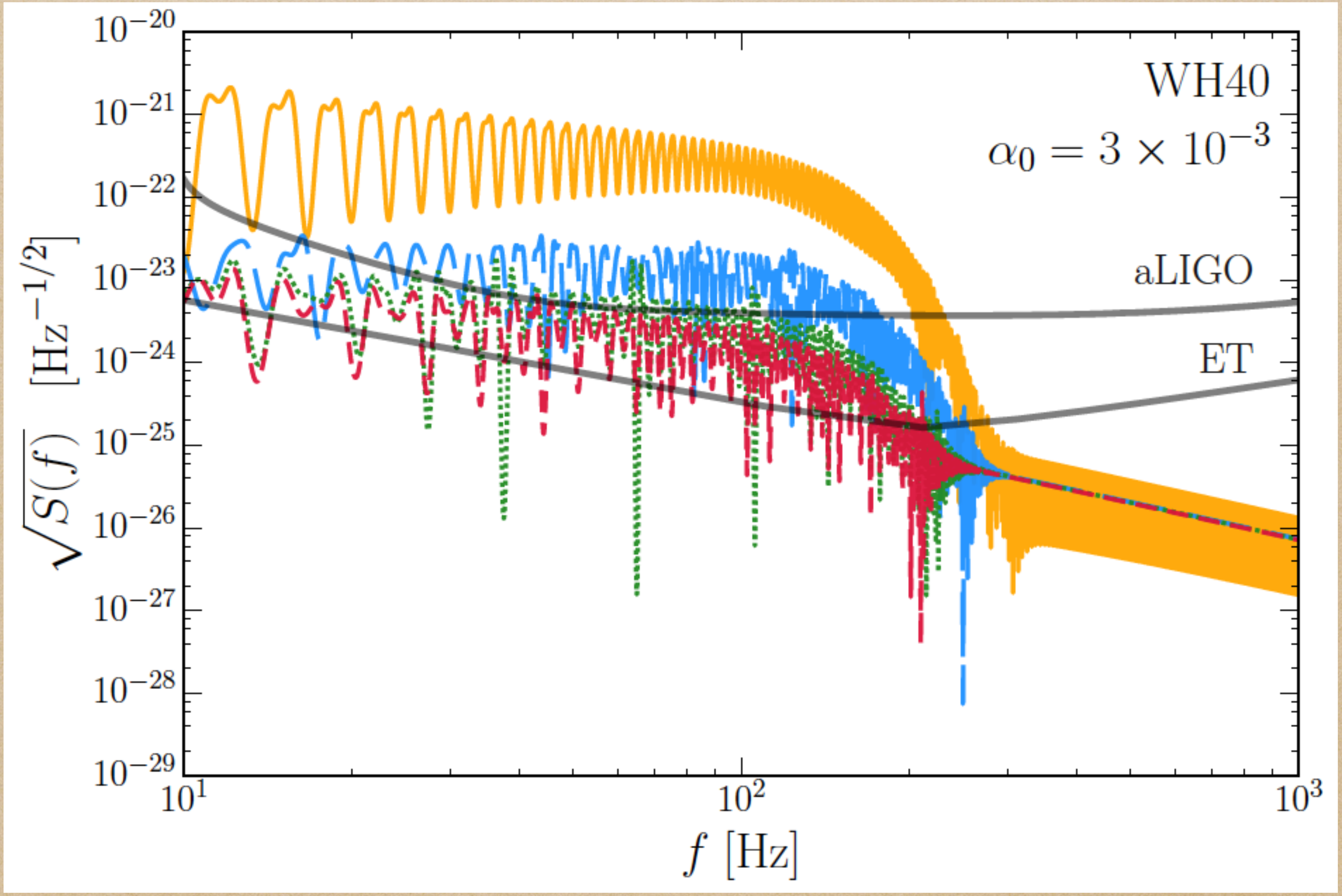
Detectability with GW observatories WH12

- At distance $D = 10$ kpc



Detectability with GW observatories: WH40

- At distance $D = 10$ kpc



B4. Results

Conclusions

- BH binaries
  - Circumvent the no-hair theorems with a scalar gradient
  - Dipole radiation is emitted at twice the orbital frequency
  - Effect very likely too small for LIGO: $M\sigma \ll 10^{-7}$
  - For SMBHs $M\sigma \sim 10^{-7}$ may be possible
- Core collapse
  - Collapse dynamics as in GR, but scalar radiation generated
  - Collapse to NSs: Compactness too low for spontaneous scal.
  - Most promising source: BH formation (high compactness!)
  - Optimistic cases detectable in galactic events
- Still a lot of uncharted territory! Much understanding lacking…