Searching for smoking gun effects of modified gravity in supernova core collapse

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Conclusions

- Introduction and motivation
- Supernova core collapse
- Formalism
- Code tests
- Results
- Conclusions and outlook

1. Introduction and motivation

Curvature

- A map of Cambridge

Curvature

- A map of Cambridge
- Grafton Centre -

  Trinity College -

  St.Catherine's College
- Pythagoras works well!

Curvature

- A map of the Earth: Pythagoras doesn't really work…

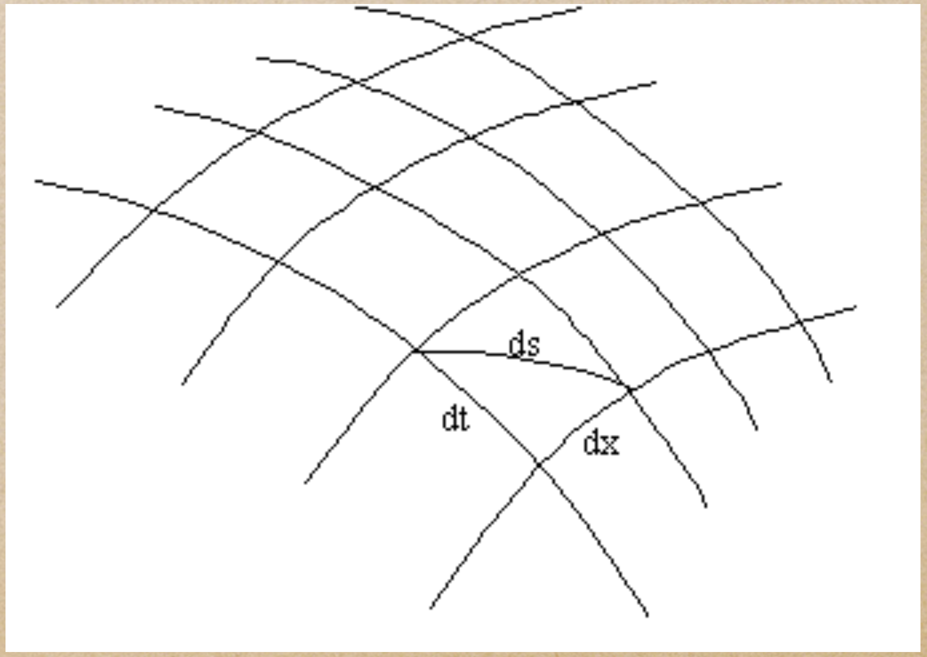
Differential geometry: Non-flat manifolds

- Manifold: A set of points labeled by coordinates $x^\alpha\,, \quad \alpha = 0 \ldots n-1$
- Think of house numbers in a street!
- Measure for real distance: Metric
- E.g. 2D Euclidean: $ds^2 = dx^2 + dy^2 = dr^2 + r^2 d\phi^2$
- In general: $n \times n$ matrix valued function
- Time directions count negative:

Special relativity: $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta$

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$dx^\alpha = \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$



General relativity

- Spacetime as a curved manifold
- Key quantity: spacetime metric $g_{\alpha\beta}$
- Curvature, geodesics etc. all follow
- Einstein equations

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R + \Lambda g_{\alpha\beta} = \frac{8\pi G}{c^4}T_{\alpha\beta}$$

10 non-linear PDEs for $g_{\alpha\beta}$

$T_{\alpha\beta} =$ Matter fields

- Conceptually simple, hard in practice
- E.g. Schwarzschild

$$g_{\mu\nu} = \begin{pmatrix} \left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & -\left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}$$

$$ds^2 = c^2dt^2\left(1-\frac{2GM}{rc^2}\right) - \frac{dr^2}{1-2GM/rc^2} - r^2d\theta^2 - r^2\sin^2\theta d\phi^2$$

General relativity is non-linear!

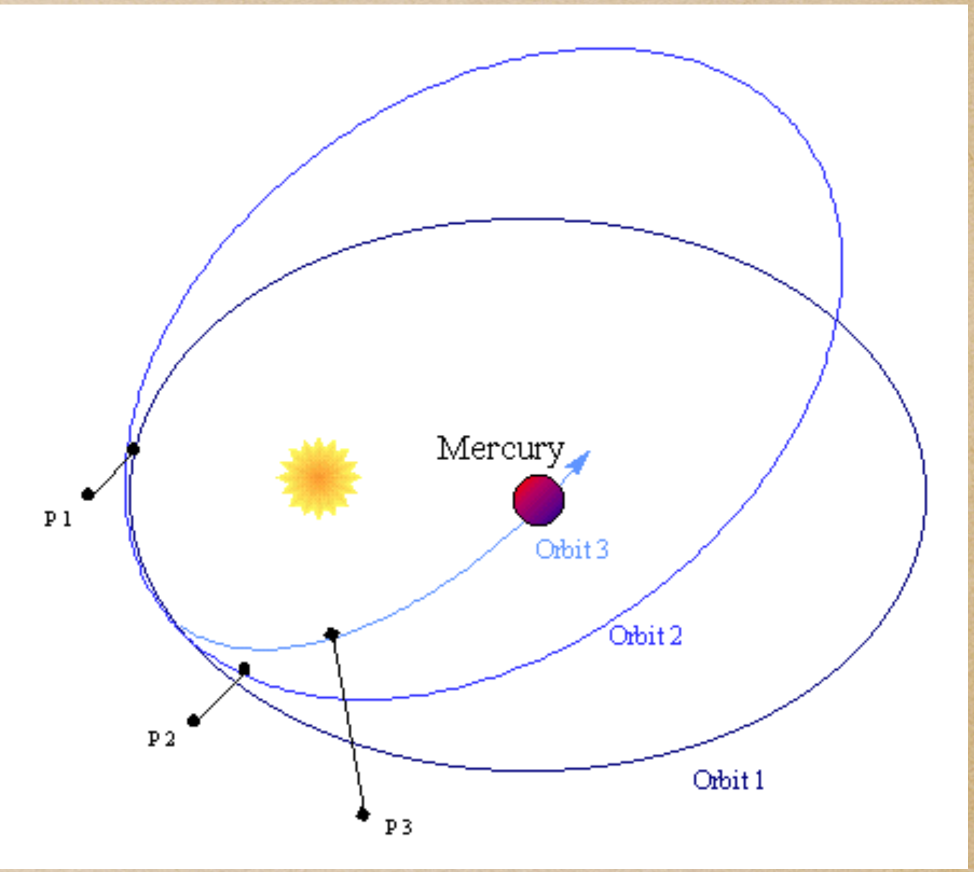
- John Wheeler: Spacetime tells matter how to move; matter tells spacetime how to curve
- What is non-linearity? Think of the stock market

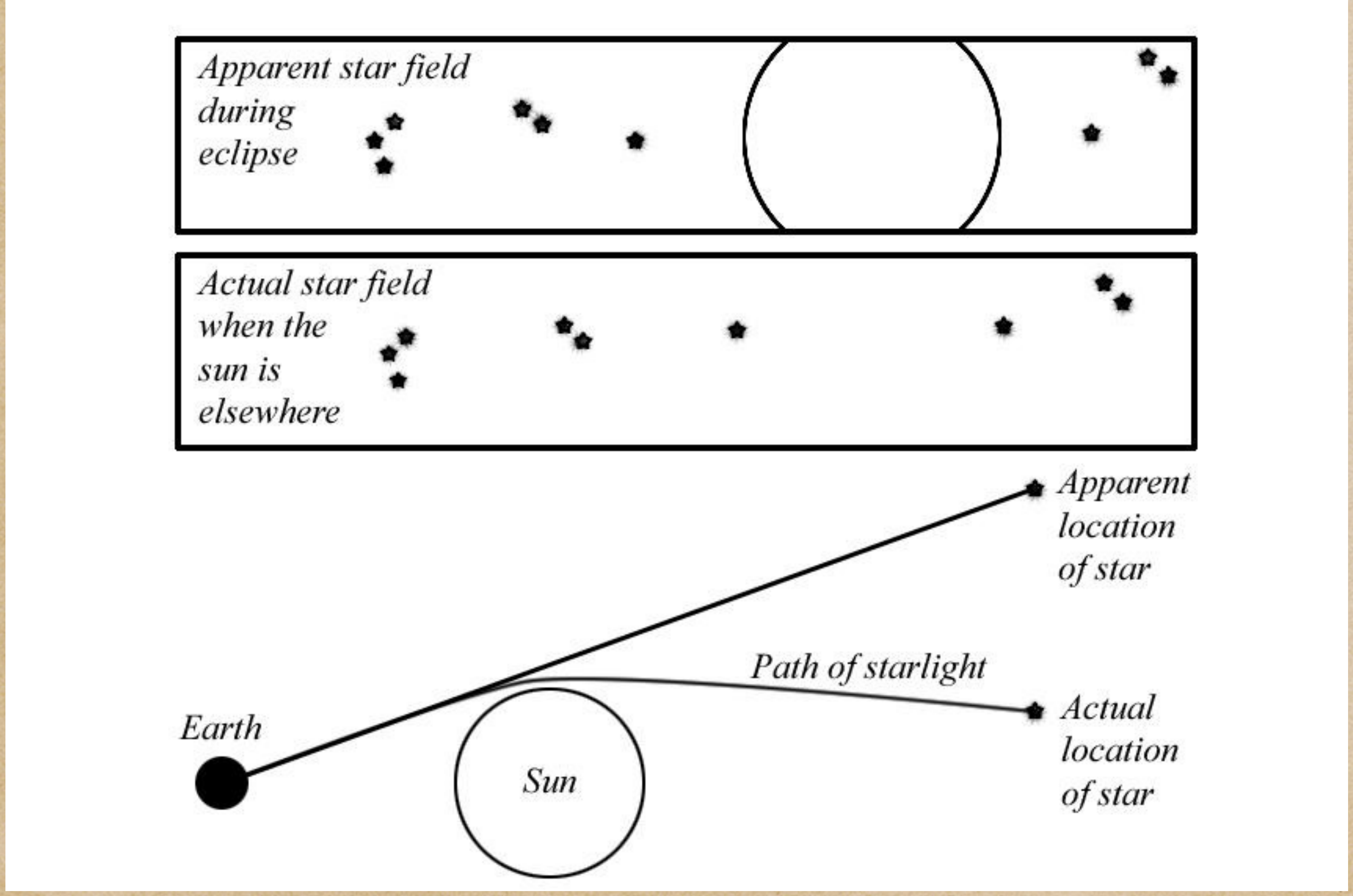
⇒ linear

⇒ NON-LINEAR!

Classical tests of general relativity

- Light bending
- Gravitational redshift
- Mercury's perihelion precession
- Shapiro time delay



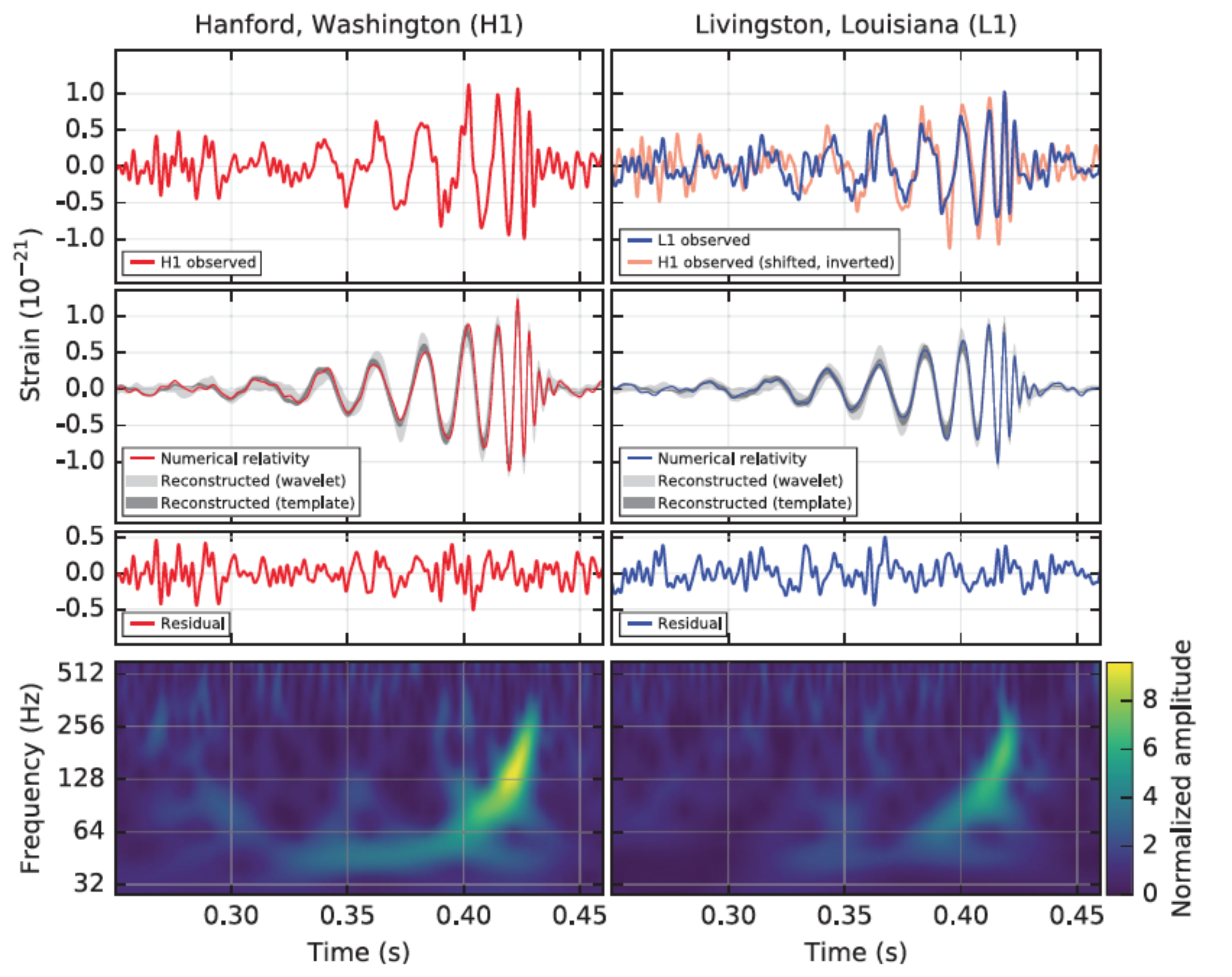


LIGO's gravitational wave detection

- Sep 14, 2015 at 09:50:45 UTC: SNR ~ 24

  Abbott et al. PRL 2016, Abbott et al. 1606.01210

- BBH inspiral, merger and ringdown: $m_1 = 35^{+5}_{-3}\, m_\odot\,, \quad m_2 = 30^{+3}_{-4}\, M_\odot$



GW150914 compatible with GR

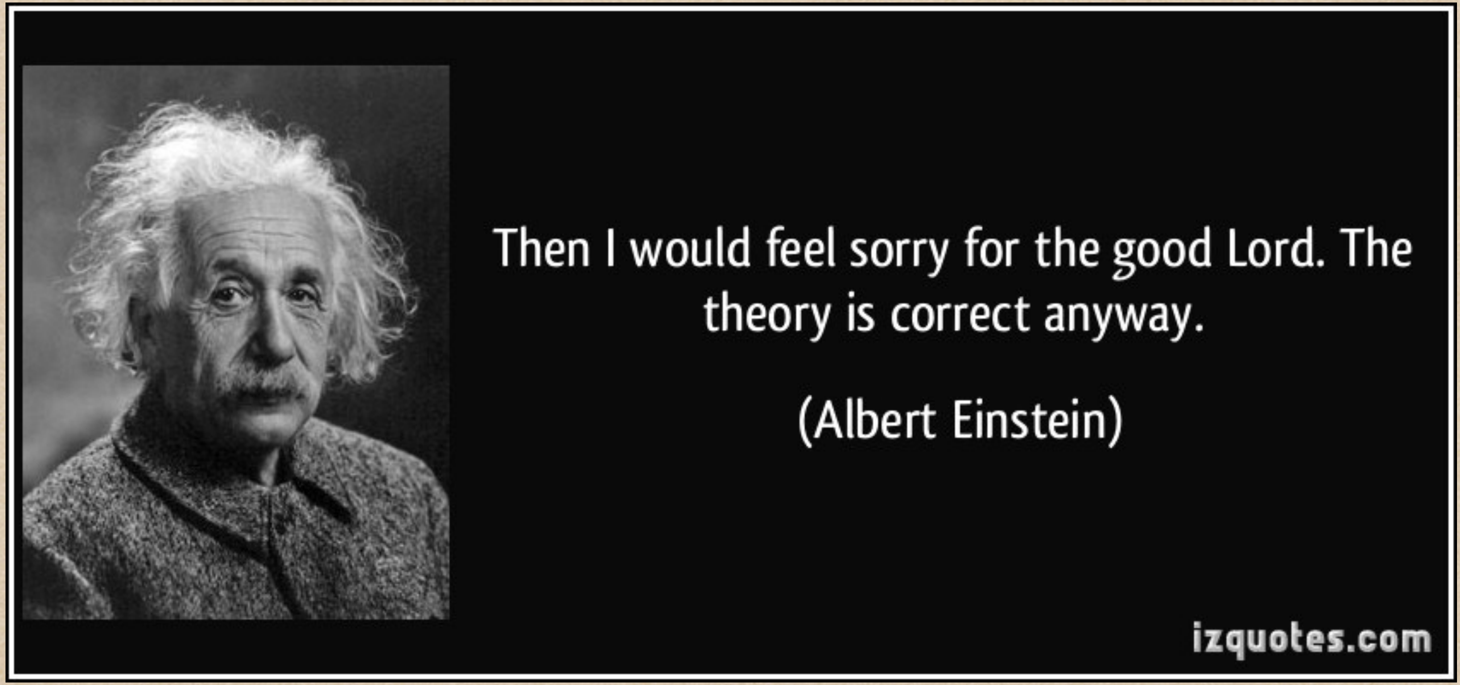
> Here we perform several studies of GW150914, aimed at detecting deviations from the predictions of GR. Within the limits set by LIGO's sensitivity and by the nature of GW150914, we find no statistically significant evidence against the hypothesis that, indeed, GW150914 was emitted by a binary system composed of two black holes (i.e., by the Schwarzschild [17] or Kerr [18] GR solutions), that the binary evolved dynamically toward merger, and that it formed a merged rotating black hole consistent with the GR solution.

Taken from Abbott et al. PRL 2016 1602.03841 "Testing GR"

- GR waveform templates work stunningly well!
- So why bother looking for something other than GR?

Do we need a theory beyond GR?

- When asked what he would do if Eddington's mission failed…

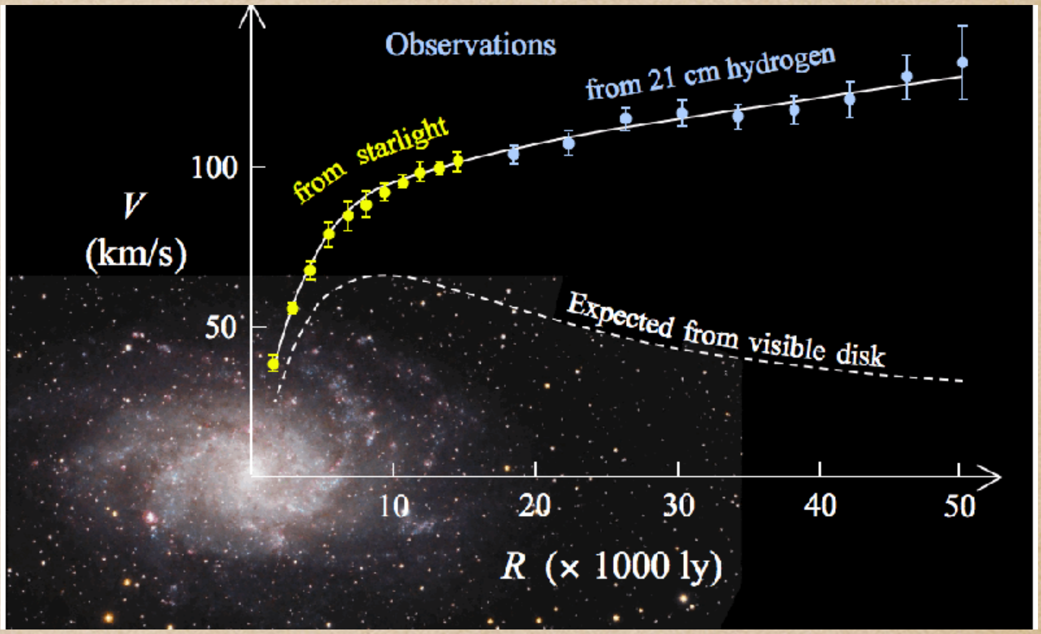


- But we have reasons to search for "beyond GR"
  - Renormalization: Requires, e.g., higher curvature terms.

    $\to$ GR is low-energy limit of more fundamental theory
  - Dark energy: Why is Λ so small and why $\rho_{\rm dark} \sim \rho_{\rm mat}$
  - Dark matter: "Neptune" or "Vulcan" ?

Selected open questions in gravity

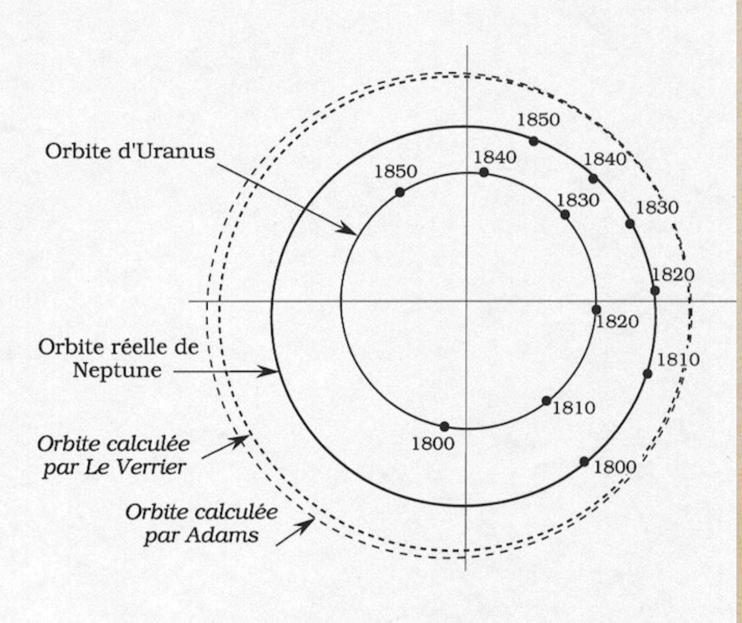
- Rotation curves in galaxies



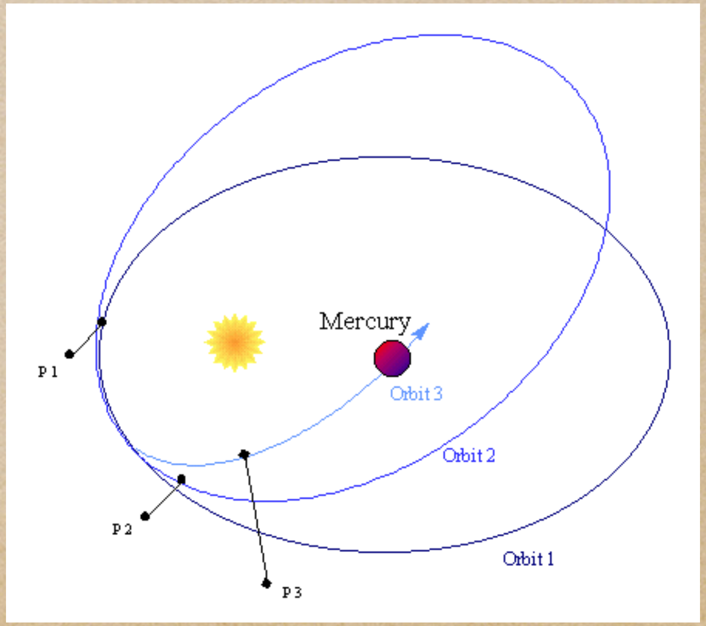
- Cosmological constant problem $\Lambda_{\rm QM} \gg \Lambda_{\rm cos}$

- We've been there before…

Le Verrier 1846 $\rightarrow$ Neptune



Le Verrier 1859 $\rightarrow$ Vulcan



A hard task

- (Weak) Gravity well tested on length scales $1\ \mu\mathrm{m} \lesssim \ell \lesssim 10^{11}\ \mathrm{m}$

  A modified theory needs to be consistent with these tests!

- How to interpret constraints?

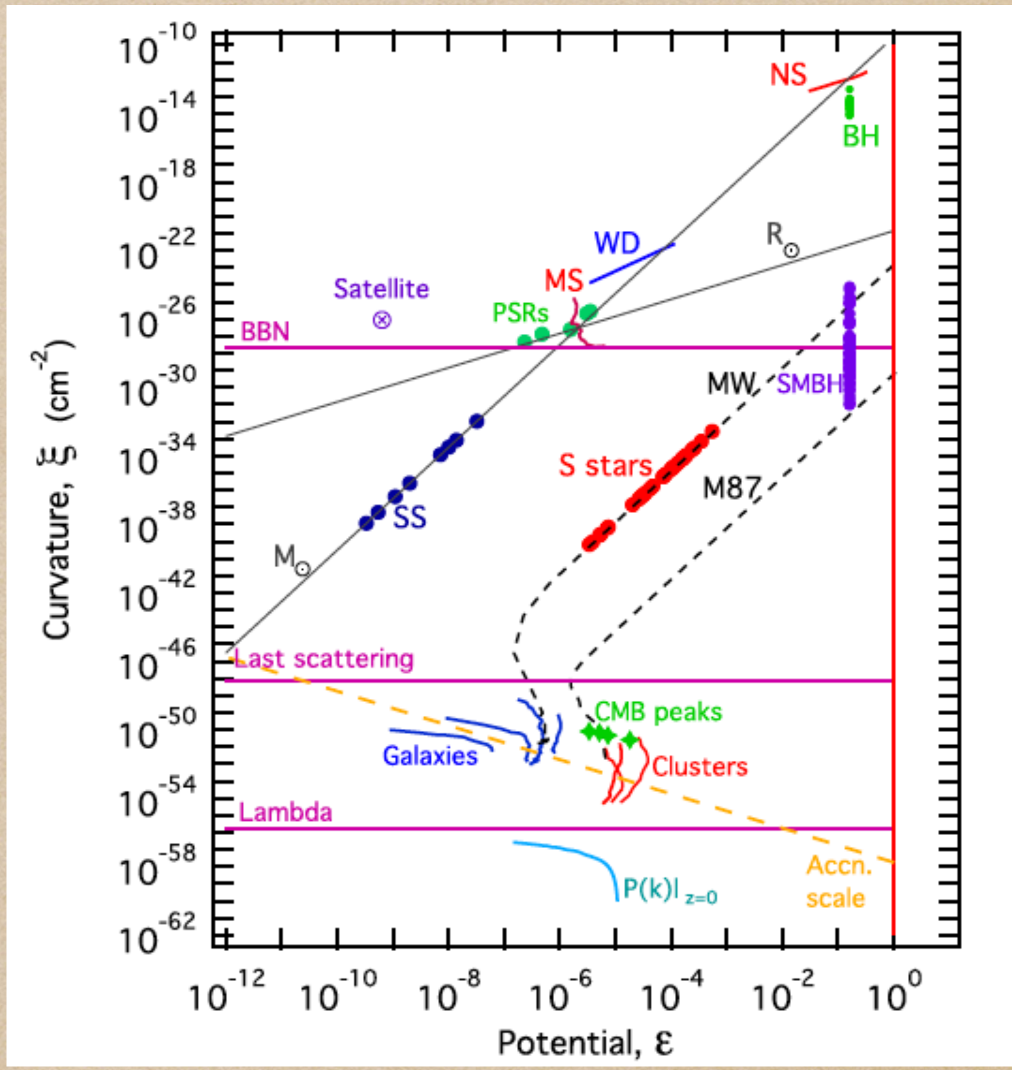
  Strong vs. weak field tests

- There's a zoo of theories!

  Berti et al CQG 1501.07274

- Full modeling requires a well-posed formulation

  e.g. Delsate et al PRD 1407.6727



Baker et al. ApJ 1412.3455

Predictions for GW tests: 2 approaches

- Parameterized post Newtonian / Einsteinium (PPN, PPE)
  - Introduce "phenomenological" extra terms to the theory and quantify these through extra parameters
  - No specific theory in mind
  - Very general

  e.g. Will ApJ 1971, Yunes & Pretorius PRD 0909.3328

- Smoking gun effects
  - Choose specific theory
  - Model physical systems; look for deviations from GR
  - Very concrete

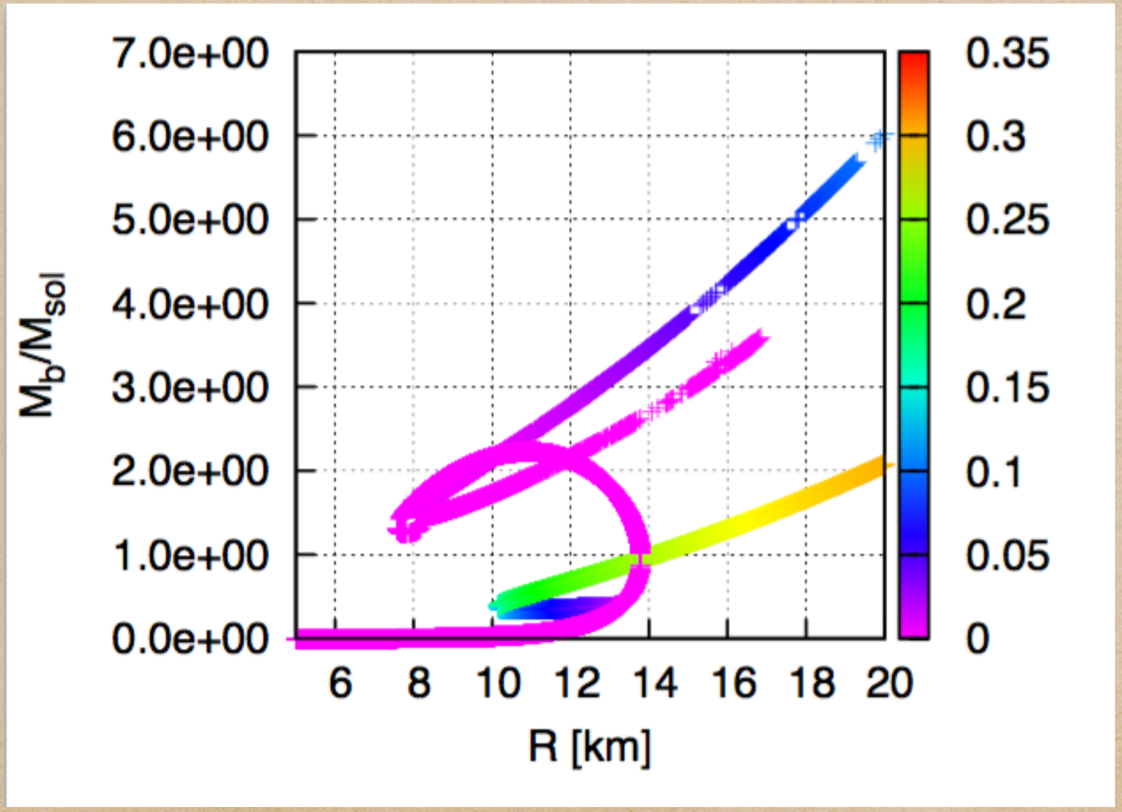
  Topic of this talk

What a modified theory must satisfy

- Consistency with the weak-field tests; Newtonian limit
- Mathematical well-posedness
- Deviations from GR in strong-field regime

Only case: Spontaneous secularization in Scalar tensor theory

Damour & Esposito Farese PRL 1993



Thanks to

Roxana Rosca

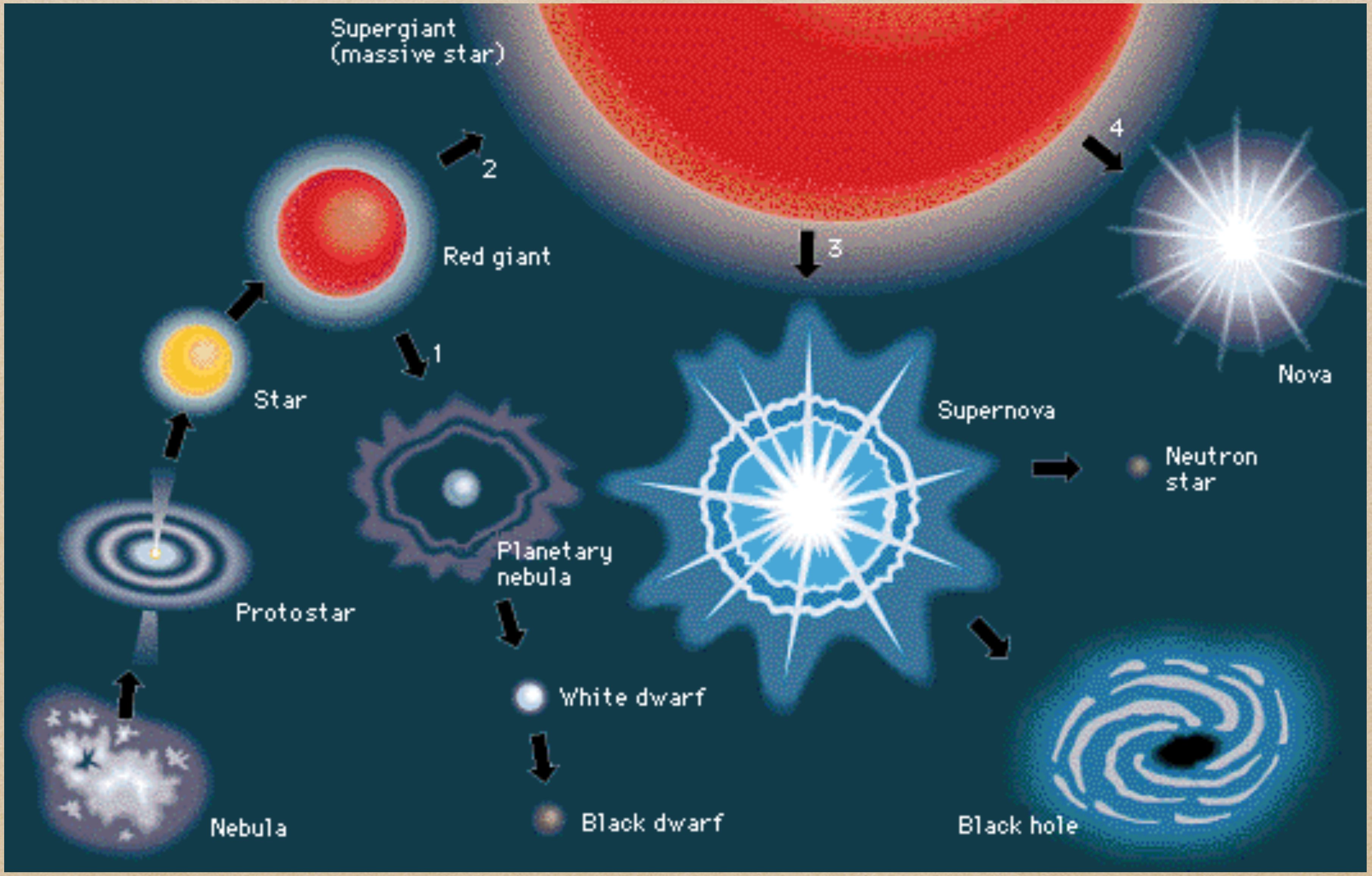
No-hair theorems

- Stationary BHs are the same in ST theory as in GR
  - For Brans-Dicke: Hawking ’72, Thorne & Dykla ’71, Chase ‘70
  - For Bergmann-Wagoner, $f(R)$: Sotirou & Faraoni 1109.6324
  - Supported by numerics of grav.collapse: e.g. Scheel et al ‘95
- How about BH binaries is ST theory?
  - In Brans-Dicke to leading PN order: Will & Zaglauer ‘89
  - No dipolar radiation in EMR limit: Yunes et al 1112.3351
- Generalized no-hair theorems rely on 4 assumptions
  - No matter
  - Vanishing scalar potential
  - Action truncated at second derivatives
  - Metric is asymptotically flat, scalar field assympt. constant

2. Supernova core collapse

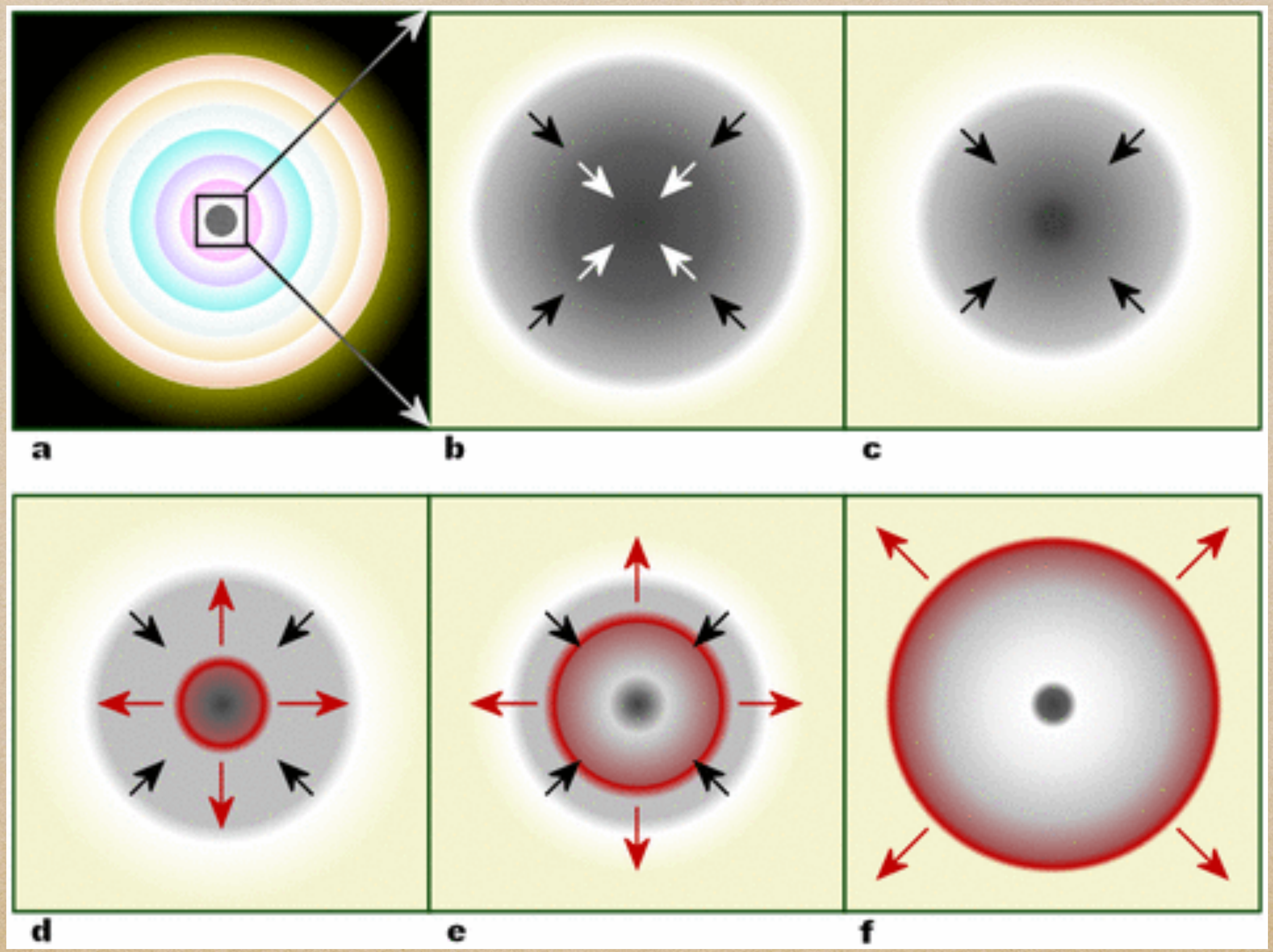
The end point of stellar evolution

- Nuclear fusion above iron requires energy
- Stars with $M_{\rm ZAMS} \gtrsim 8\ M_\odot$ explode as SN $\rightarrow$ BHs, NSs



Core-collapse scenario to 0th order

- Nickel-iron core reaches Chandrasekhar mass $\rightarrow$ Collapse
- EOS stiffens at $\rho \gtrsim \rho_{\rm nuc}$ $\rightarrow$ Bounce
- Outgoing shock, reinvigorated by $\nu_{\rm e}$ $\rightarrow$ Outer layers blast away



Core-collapse scenario to 0th order

- Massive stars: $M_{\rm ZAMS} = 8 \ldots 100\ M_\odot$
- Core compressed from $\sim 1\,500$ km to ~ 15 km

  $\sim 10^{10}$ g/cm^3 to $\gtrsim 10^{15}$ g/cm^3
- Released gravitational energy: $\mathcal{O}(10^{53})$ erg

  $\sim 99\ \%$ in neutrinos, $\sim 10^{51}$ erg in outgoing shock, explosion
- Explosion mechanism: still uncertainties...
- Failed explosions lead to BH formation

  "Collapsar": possible engine for long-soft GRBs

3. Formalism

Theoretical framework

Jordan frame: Physical metric $\tilde{g}_{\alpha\beta}$

- Action $$S = \int dx^4 \frac{\sqrt{-\tilde{g}}}{16\pi G} \left[F(\phi)\tilde{R} - 8\pi G Z(\phi) \tilde{g}^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - U(\phi) \right]$$
- GWs: 3 degrees of freedom
- Matter couples to $\tilde{g}_{\alpha\beta}$

Einstein frame: Conformal metric $g_{\alpha\beta} = F(\phi)\tilde{g}_{\alpha\beta}$

- Scalar field: $\varphi(\phi) = \int d\phi \left[\frac{3}{2} \frac{F'(\phi)^2}{F(\phi)^2} + \frac{8\pi G Z(\phi)}{F(\phi)} \right]^{1/2}$
- Action $$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} \left[R - g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi - W(\varphi) \right]$$
- Price: Matter couples to $g_{\alpha\beta}/F$

From now on: vanishing potential $U(\phi) = W(\varphi) = 0\,.$

A choice of frames

Pro Einstein

- Minimally coupled scalar field $\Rightarrow$ Numerics straightforward
- F, Z not explicitly present in evolutions

  $\Rightarrow$ Evolve entire class of theories at once

Pro Jordan

- Strongly hyperbolic formulations also available

  Salgado gr-qc/0509001; Salgado et al 0801.2372

- Matter couples to the evolved metric $\tilde{g}_{\alpha\beta}$

Here: Mixed approach

Field equations

- Jordan frame

$$G_{\mu\nu} = \frac{8\pi}{F}(T^F_{\mu\nu} T^\phi_{\mu\nu} + T_{\mu\nu})$$

$$T^F_{\mu\nu} = \frac{1}{8\pi}(\nabla_\mu \nabla_\nu F - g_{\mu\nu}\nabla^\rho \nabla_\rho F)$$

$$T^\phi_{\mu\nu} = \partial_\mu \phi \, \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial^\rho \phi \, \partial_\rho \phi$$

$$\nabla^\rho \nabla_\rho \phi = -\frac{1}{16\pi} R \, F_{,\phi}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

- Scalar field: Use φ with $\frac{\partial \varphi}{\partial \phi} = \sqrt{\frac{3}{4}\frac{F_{,\phi}{}^2}{F^2} + \frac{4\pi}{F}}$, $\boxed{F = e^{-2\alpha_0 \varphi - \beta_0 \varphi^2}}$

- Line element for spherical symmetry

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + X^2 dr^2 + \frac{r^2}{F}(d\theta^2 + \sin^2\theta \, d\tilde{\phi}^2)$$

Evolution variables

- Metric: $\Phi = \ln(\sqrt{F}\alpha)\,, \quad m = \frac{r}{2}\left(1 - \frac{1}{FX^2}\right)$
- Matter: $T_{\alpha\beta} = \rho h u_\alpha u_\beta + P g_{\alpha\beta}$

$$J^\alpha = \rho u^\alpha \;, \quad \nabla_\mu J^\mu = 0$$

$$u^\mu = \frac{1}{\sqrt{1-v^2}}\left[\frac{1}{\alpha},\; \frac{v}{X},\; 0,\; 0\right]$$

- Primitive versus conserved variables $(\rho,\; h,\; v) \quad \leftrightarrow \quad (D,\; S^r,\; \tau)$

$$D = \frac{\rho X}{F\sqrt{F}\sqrt{1-v^2}}$$

$$S^r = \frac{\rho h v}{F^2(1-v^2)}$$

$$\tau = \frac{\rho h}{F^2(1-v^2)} - \frac{P}{F^2} - D$$

- Scalar field: $\eta = \frac{\partial_r \varphi}{X}\,, \quad \psi = \frac{\partial_t \varphi}{\alpha}$

Evolution equations in spherical symmetry

- Metric: $\partial_r \Phi = \ldots$

  $$\partial_r m = \ldots$$

- Scalar field: $\partial_t \varphi = \alpha \psi$

  $$\partial_t \psi = \ldots$$

  $$\partial_t \eta = \ldots$$

- HRSC for matter: $\partial_t D + \frac{1}{r^2}\partial_r \left(r^2 \frac{\alpha}{X} f_D \right) = s_D \,, \qquad f_D = Dv \,,$

  $$\partial_t S^r + \frac{1}{r^2}\partial_r \left(r^2 \frac{\alpha}{X} f_{S^r} \right) = s_{S^r} \,, \quad f_{S^r} = S^r v + \frac{P}{F^2} \,,$$

  $$\partial_t \tau + \frac{1}{r^2}\partial_r \left(r^2 \frac{\alpha}{X} f_\tau \right) = s_\tau \,, \qquad f_\tau = S^r - Dv \,.$$

- Flux conservative form: NO derivatives in $s_D,\ s_{S^r},\ s_\tau$

- Use extension of GR1D code O'Connor & Ott CQG 0912.2393

Equation of state

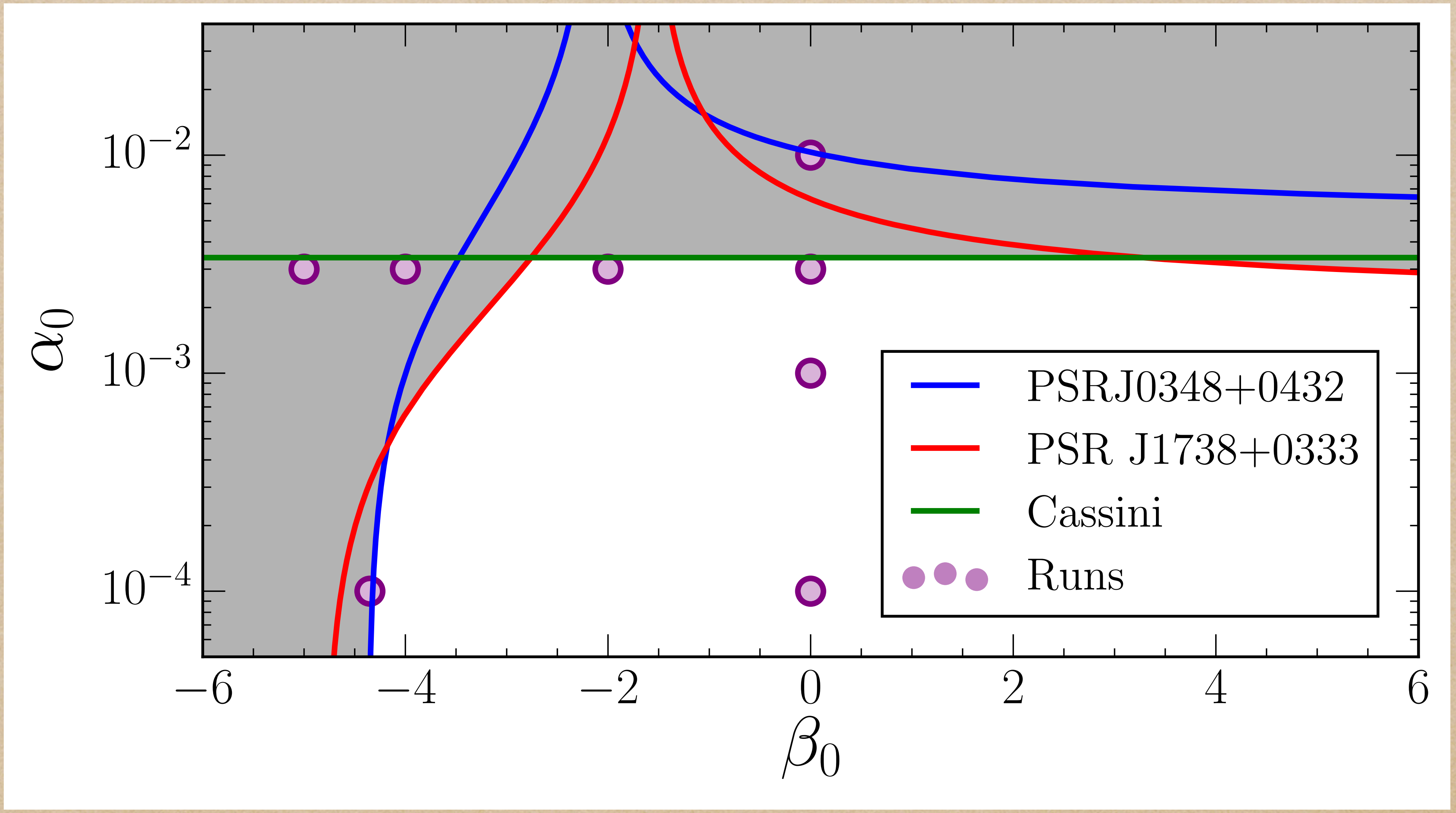
- Pressure: "cold" + "thermal" contribution: $P = P_{\rm c} + P_{\rm th}$
- Hybrid EOS for cold part: $P_c = \begin{cases} K_1 \rho^{\Gamma_1} & \text{if} \quad \rho \le \rho_{\rm nuc} \\ K_2 \rho^{\Gamma_2} & \text{if} \quad \rho > \rho_{\rm nuc} \end{cases}$
- Internal energy from 1st law: $\epsilon_c = \begin{cases} \frac{K_1}{\Gamma_1 - 1} \rho^{\Gamma_1 - 1} & \text{if} \quad \rho \le \rho_{\rm nuc} \\ \frac{K_2}{\Gamma_2 - 1} \rho^{\Gamma_2 - 1} + E_3 & \text{if} \quad \rho > \rho_{\rm nuc} \end{cases}$
- Thermal pressure: $P_{\rm th} = (\Gamma_{\rm th} - 1)\rho(\epsilon - \epsilon_{\rm th})$
- Parameters: $\Gamma_1 = 1.3\,, \quad \Gamma_2 = 2.5\,, \quad \Gamma_{\rm th} = 1.35$

  $K_1 = 4.9345 \times 10^{14}\ [\mathrm{cgs}]\,, \quad \rho_{\rm nuc} = 2 \times 10^{14}\ \mathrm{g\,cm^{-3}}$

  $K_2\,, \quad E_3$ from continuity at $\rho = \rho_{\rm nuc}$

The coupling function

- Coupling function $F = e^{-2\alpha_0\varphi - \beta_0\varphi^2}$
- Free parameters $\alpha_0,\ \beta_0$: Capture all modifications at 1PN



4. Code tests

Static models: Spontaneous scalarization

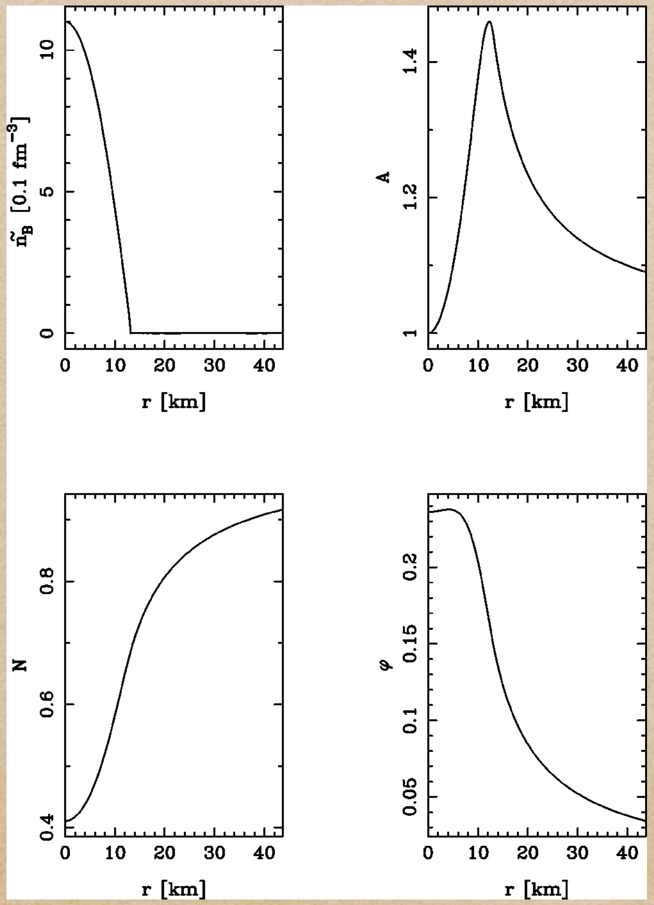
- Spontaneous scalarization: Non-linear effect for $\beta_0 \lesssim -4.35$

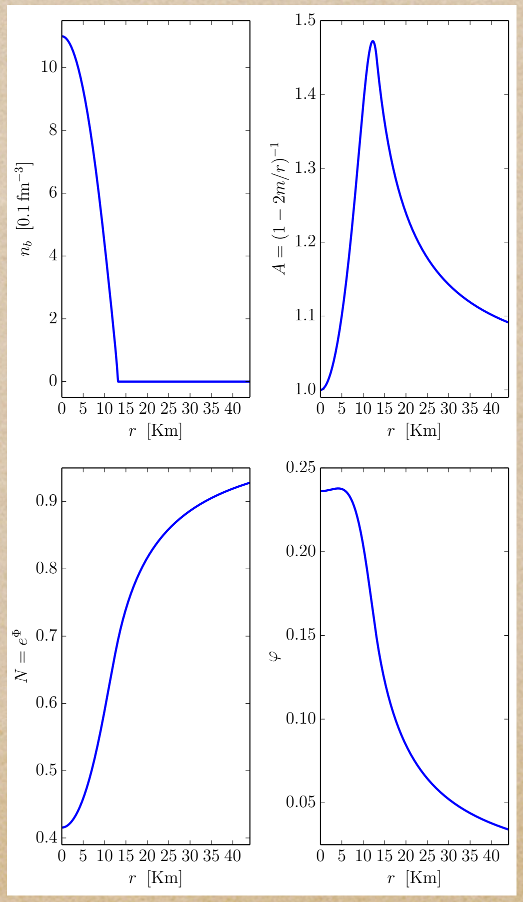
  Damour & Esposito-Farese PRL '93

- $M = 2.4\ M_\odot\,, \quad R = 13.1\ \mathrm{km}$ Model with $\alpha_0 = 0\,, \quad \beta = -6$

  Novak PRD gr-qc/9707041

- Baryon density, metric functions, scalar field



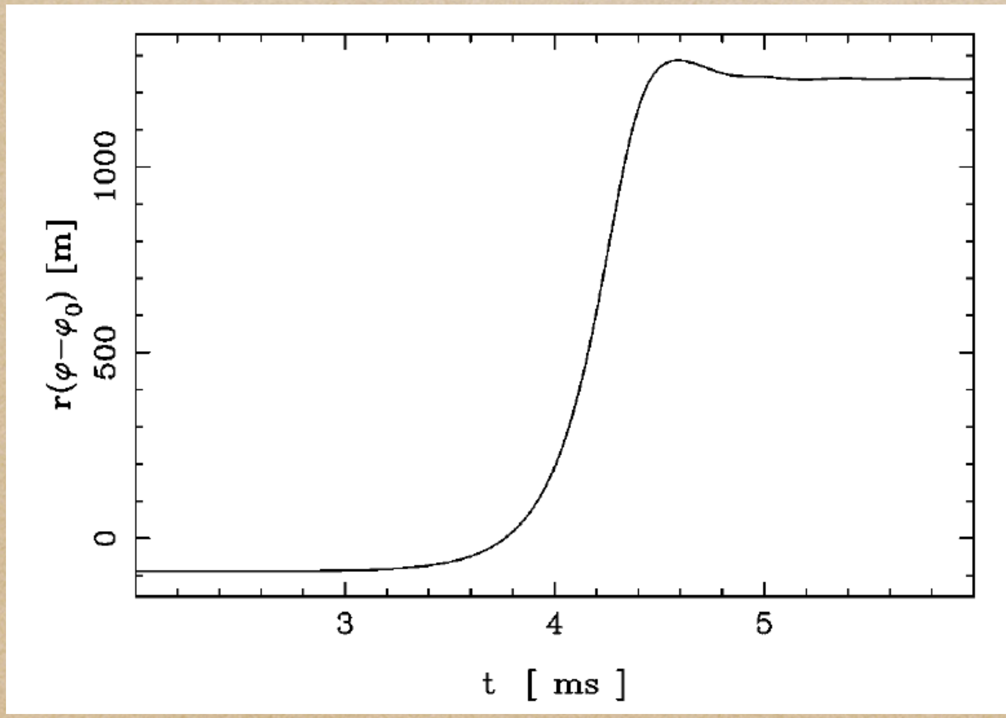


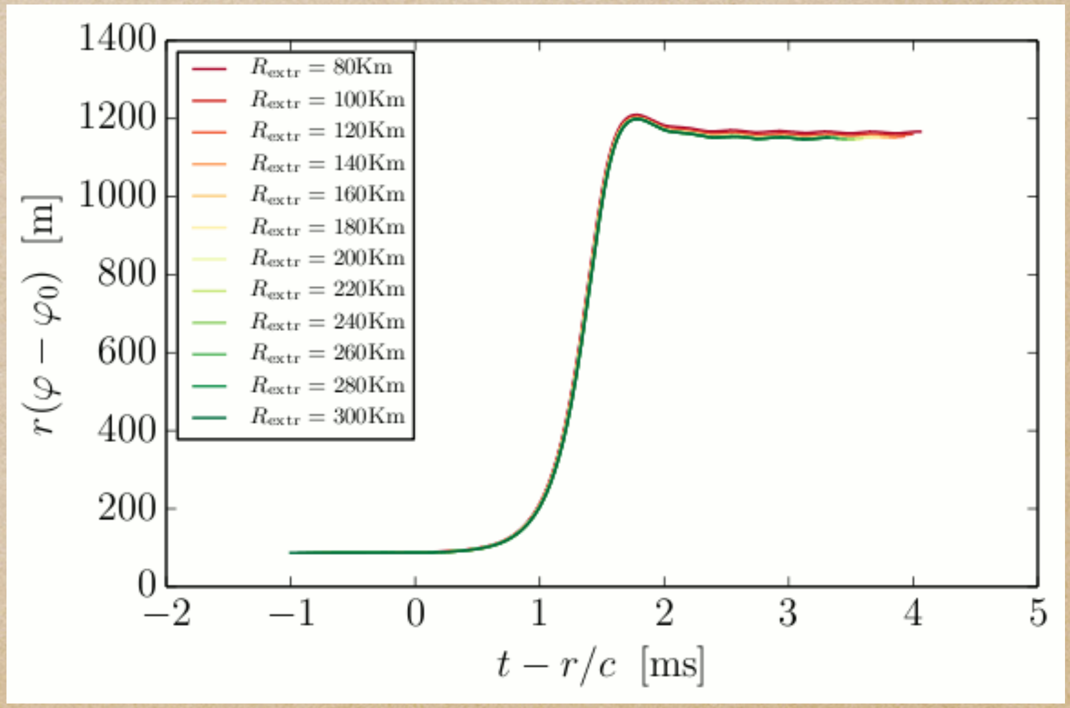
Transition from GR to secularized star

- Here: $\alpha_0 = 0.01\,, \quad \beta_0 = -6$
- Unstable GR-like model: $M = 1.378\ M_\odot\,, \quad R = 13.2\ \text{km}$
- … migrates to scalarized model: $M = 1.373\ M_\odot\,, \quad R = 13.0\ \text{km}$

Novak PRD gr-qc/9806022

- Baryon density, metric functions, scalar field



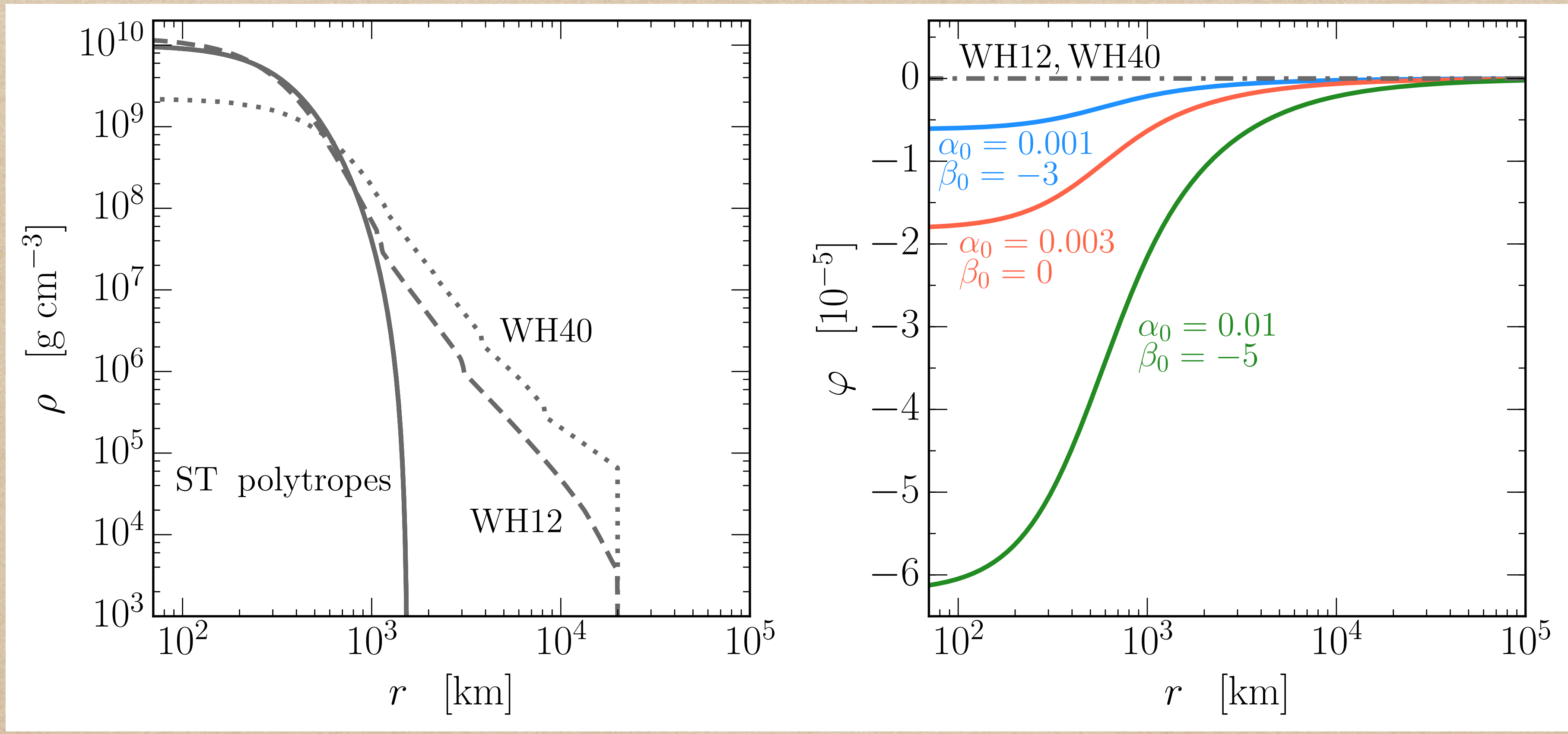


5. Results

Static models: Spontaneous scalarization

- 3 types of initial profiles: Woosley & Heger Phys.Rep. astro-ph/0702176

  (i) Polytrope, (ii) "realistic" $12\ M_\odot$, (iii) "realistic" $40\ M_\odot$



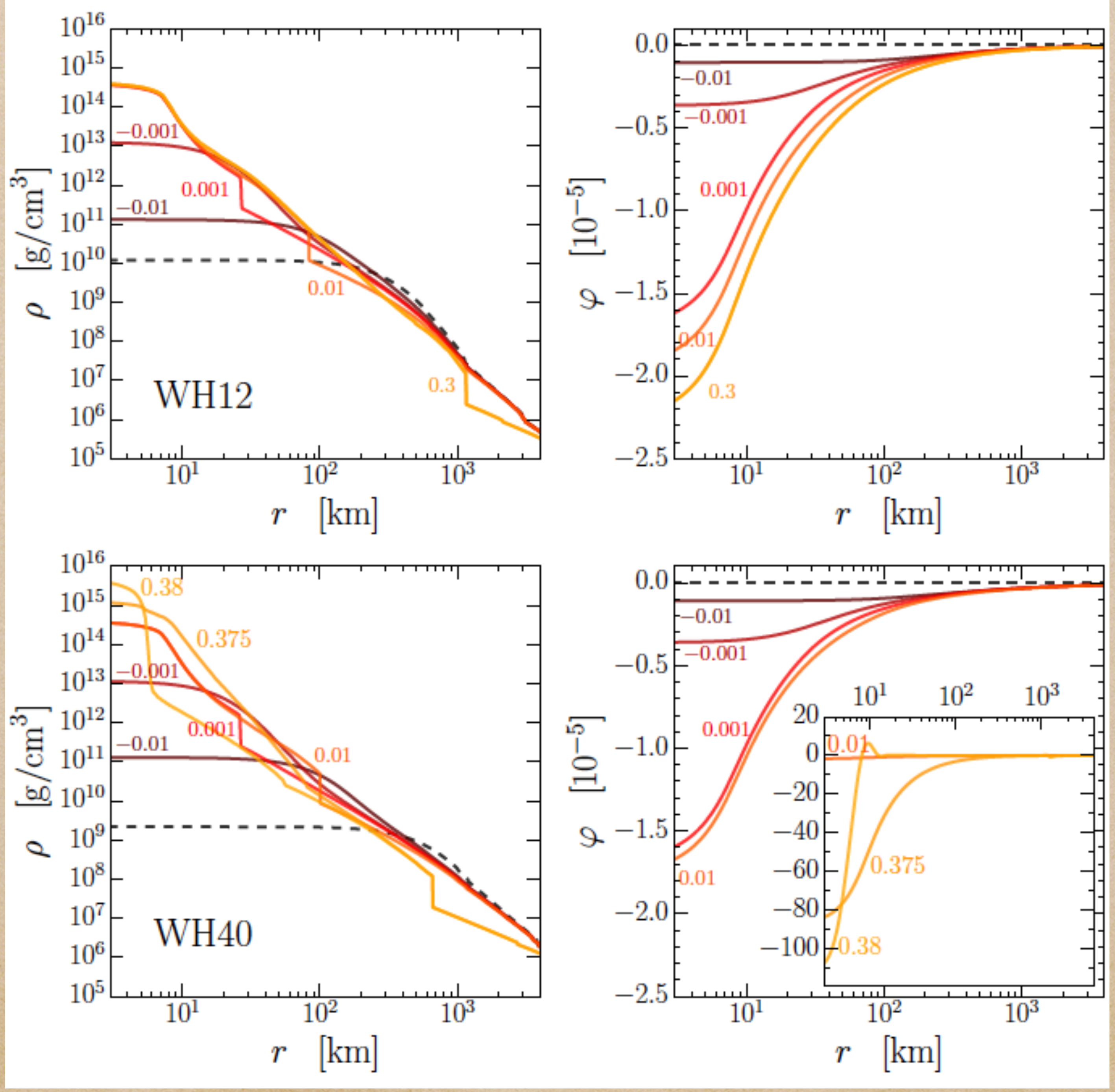
- Polytropes: Mostly for tests

  $12\ M_\odot$: Neutron star formation

  $40\ M_\odot$: Black-hole formation

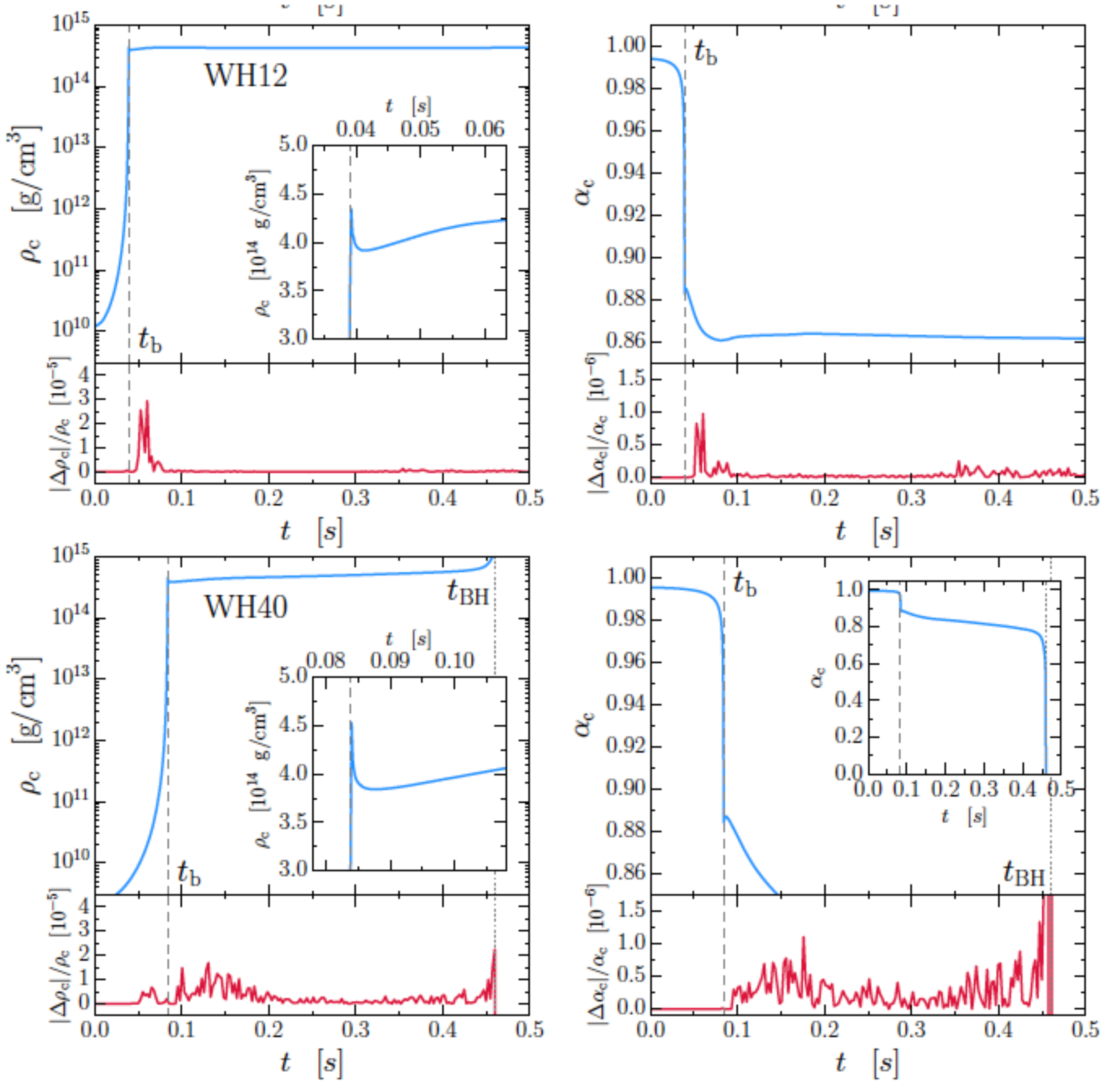
Time evolution

- ST theory with: $\alpha_0 = 10^{-4}\,, \quad \beta = -4.35$
- End product: WH12 $\rightarrow$ Neutron star, WH40 $\rightarrow$ Black hole



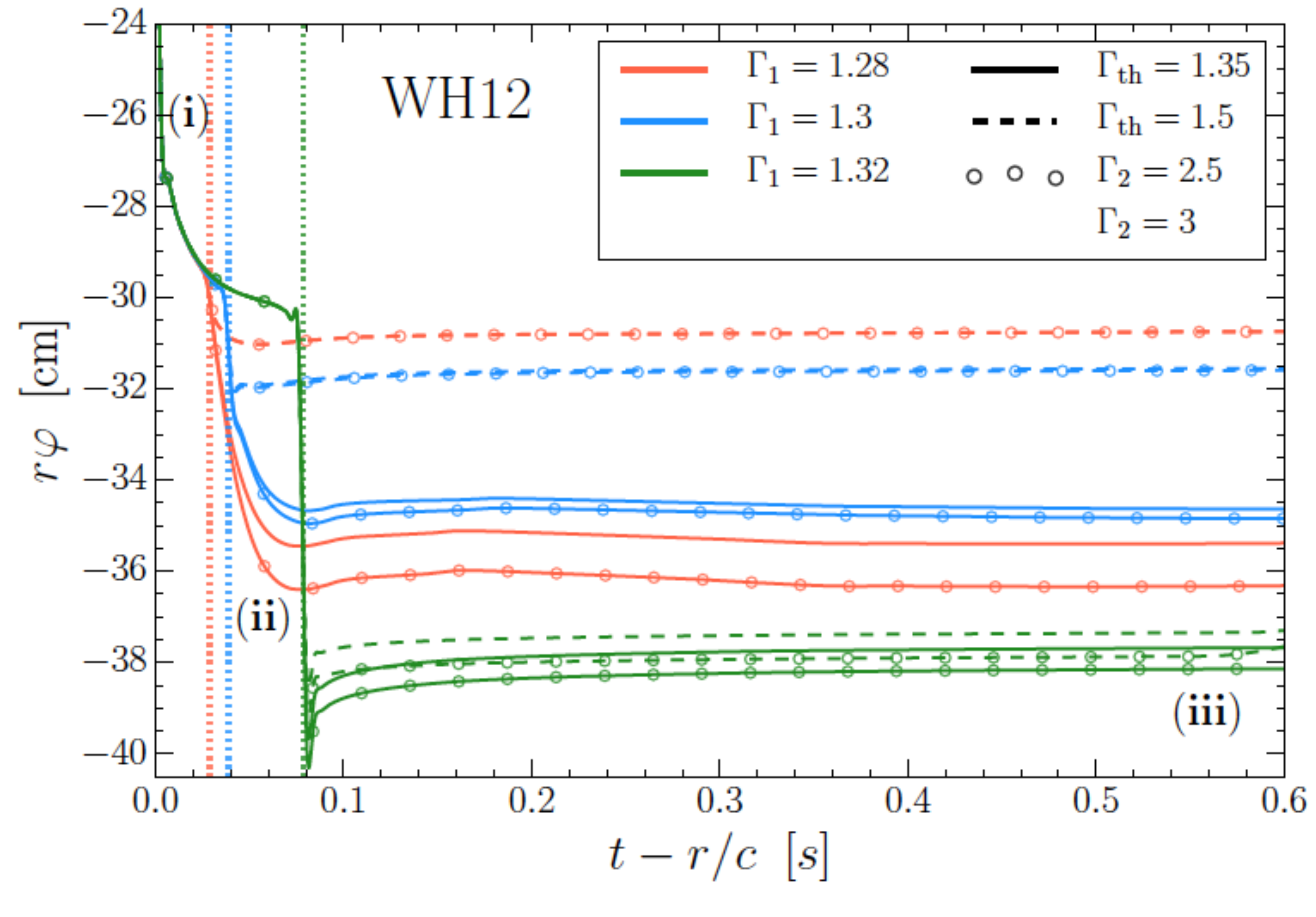
Time evolution: Central values

- Stellar dynamics barely affected by scalar field
- Accretion onto WH40 model leads to BH



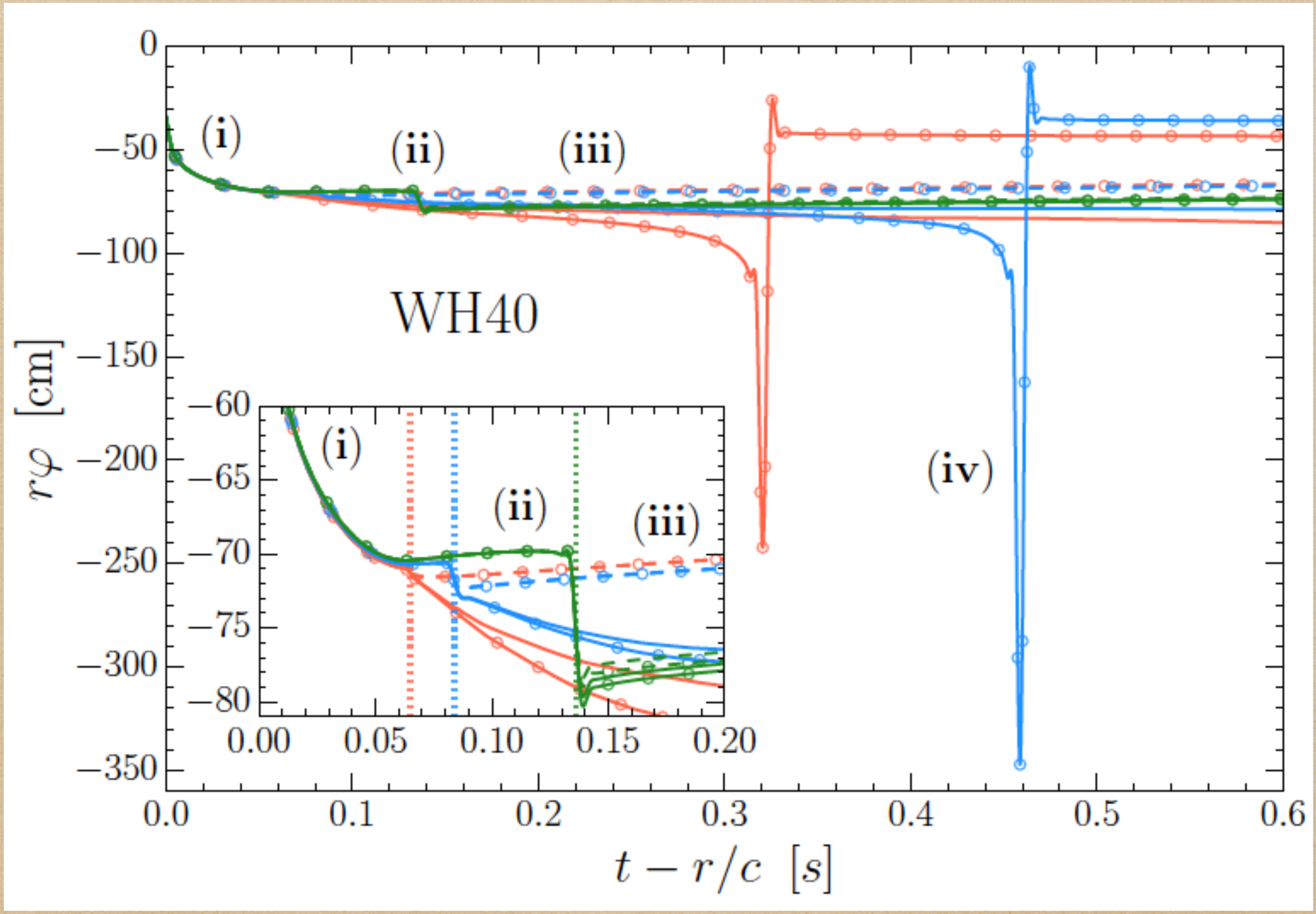
Scalar radiation: WH12

- Vary EOS



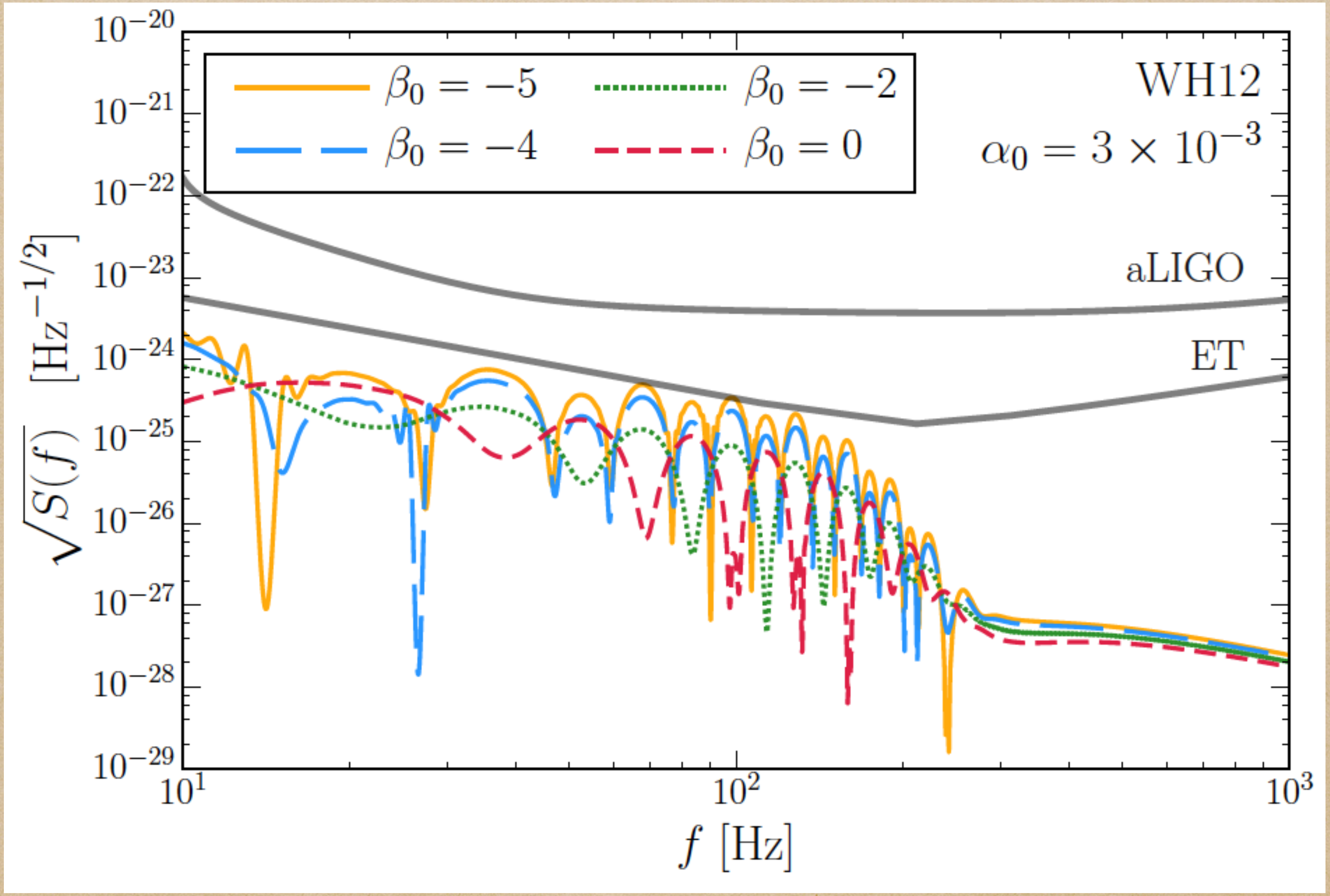
Scalar radiation: WH40

- Vary EOS



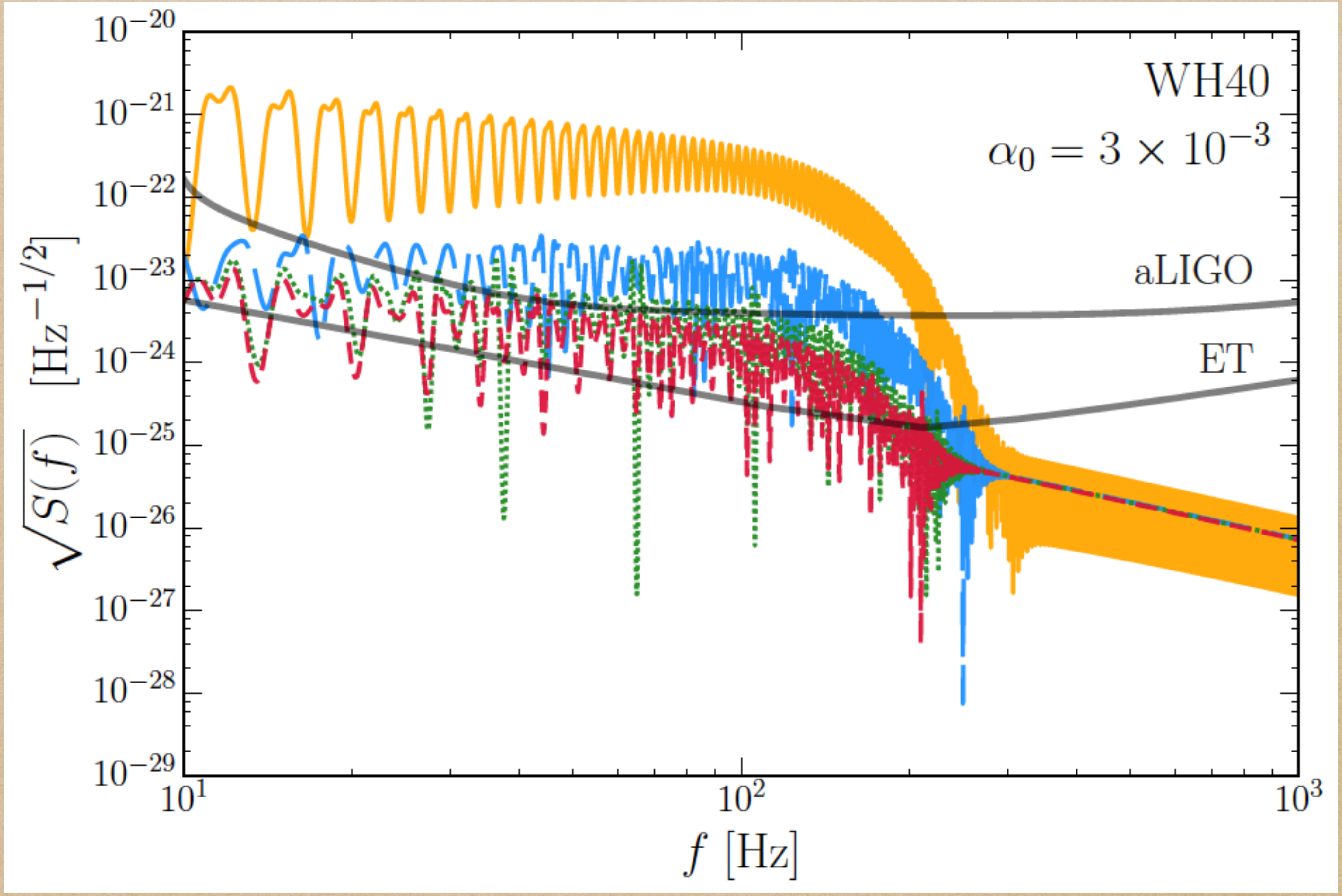
Detectability with GW observatories WH12

- At distance $D = 10$ kpc



Detectability with GW observatories: WH40

- At distance $D = 10$ kpc



6. Conclusions and outlook

Conclusions

- Core collapse
  - Collapse dynamics as in GR, but scalar radiation generated
  - Collapse to NSs: Compactness too low for spontaneous scal.
  - Most promising source: BH formation (high compactness!)
  - Optimistic cases detectable in galactic events
- Still a lot of uncharted territory! Future work:
  - Include neutrino cooling; more sophisticated EOS
  - Massive scalar tensor theory
  - Other modified theories of gravity
  - Go beyond spherical symmetry