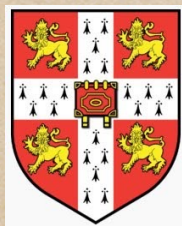


Black-hole collisions in higher-dimensional general relativity

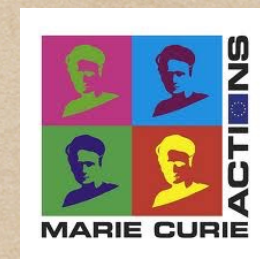
Ulrich Sperhake



DAMTP, University of Cambridge

STAG workshop: Holography, gauge theory & black holes
Southampton UK, 5 Apr 2017

with W Cook, E Berti, V Cardoso, F Pretorius + others



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Overview

- Introduction: Why black holes in higher dimensions?
- Numerical Relativity in >4 dimensions
- High-energy collisions in $3+1$ dimensions
 - Head-on collisions
 - Grazing collisions
- Black-hole collisions in >4 dimensions
 - Collisions from rest
 - Boosted collisions
- Conclusions and outlook

1. Introduction and motivation

Motivation: The hierarchy problem

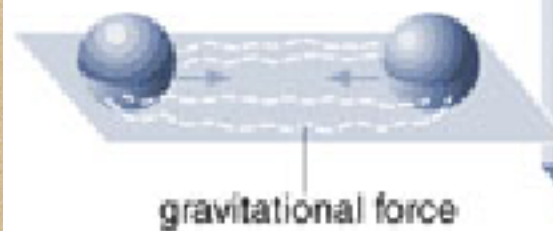
- Gravity $\approx 10^{-39} \times$ other forces
- Higgs field $\mu_{\text{obs}} \approx 125 \text{ GeV} = \sqrt{\mu^2 - \Lambda^2}$
where $\Lambda \approx 10^{16} \text{ GeV} =$ grand unification energy
- Requires enormous fine-tuning
- Fine tuning exists: $\frac{987654321}{123456789} = 8.00000000729$
- Or E_{Planck} much lower? Gravity strong at small r ?
- Gravity not measured below $\sim 0.1 \text{ mm}$. Diluted due to
 - Large extra dimensions Arkani-Hamed, Dimopoulos, Dvali '98
 - Extra dimensions with warp factor Randall & Sundrum '99

Motivation: TeV Gravity

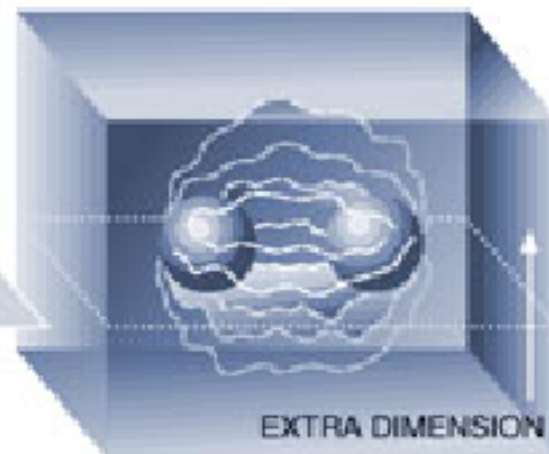
Black Holes on Demand

Scientists are exploring the possibility of producing miniature black holes on demand by smashing particles together. Their plans hinge on the theory that the universe contains more than the three dimensions of everyday life. Here's the idea:

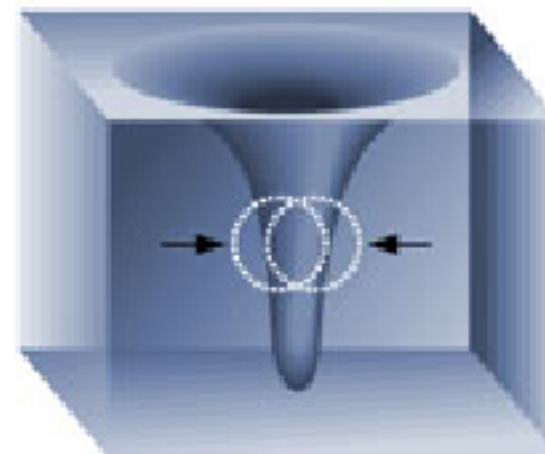
Particles collide in three dimensional space, shown below as a flat plane.



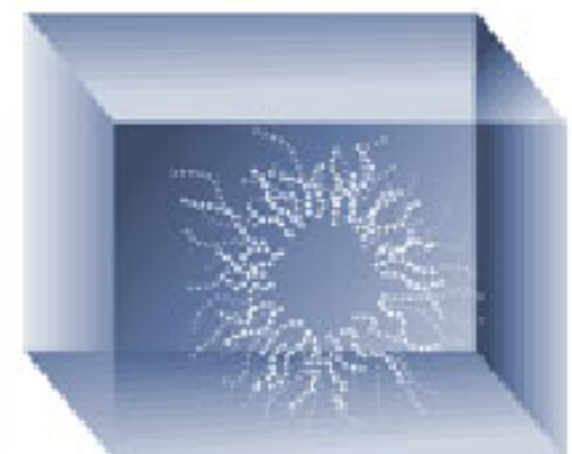
As the particles approach in a particle accelerator, their gravitational attraction increases steadily.



When the particles are extremely close, they may enter space with more dimensions, shown above as a cube.



The extra dimensions would allow gravity to increase more rapidly so a black hole can form.



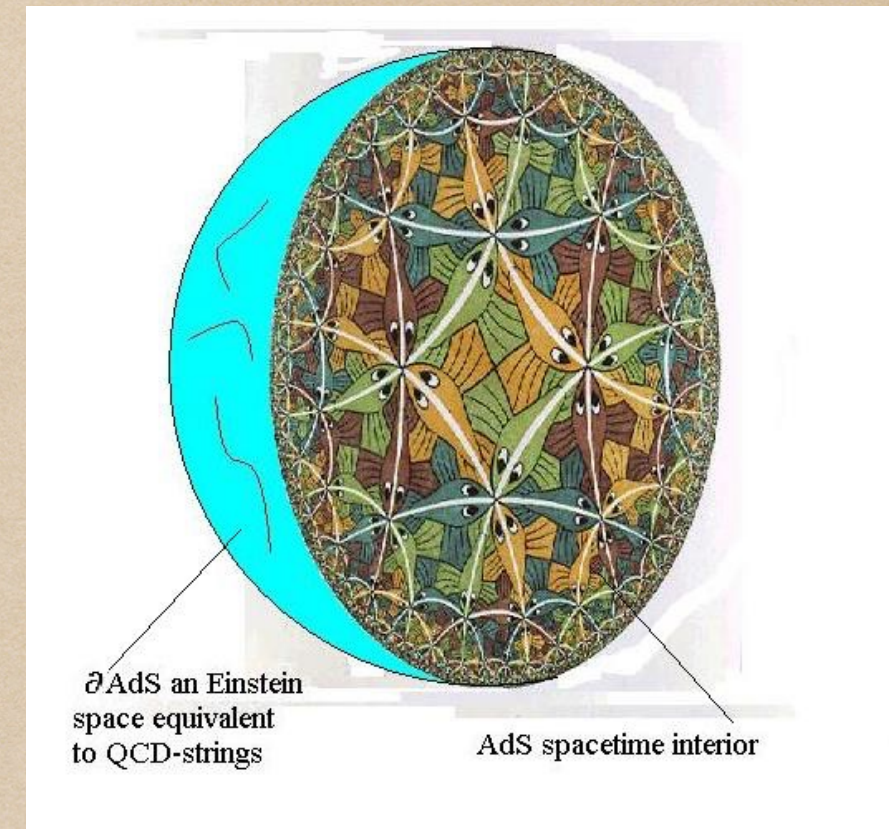
Such a black hole would immediately evaporate, sending out a unique pattern of radiation.

- Particle collisions may form BHs

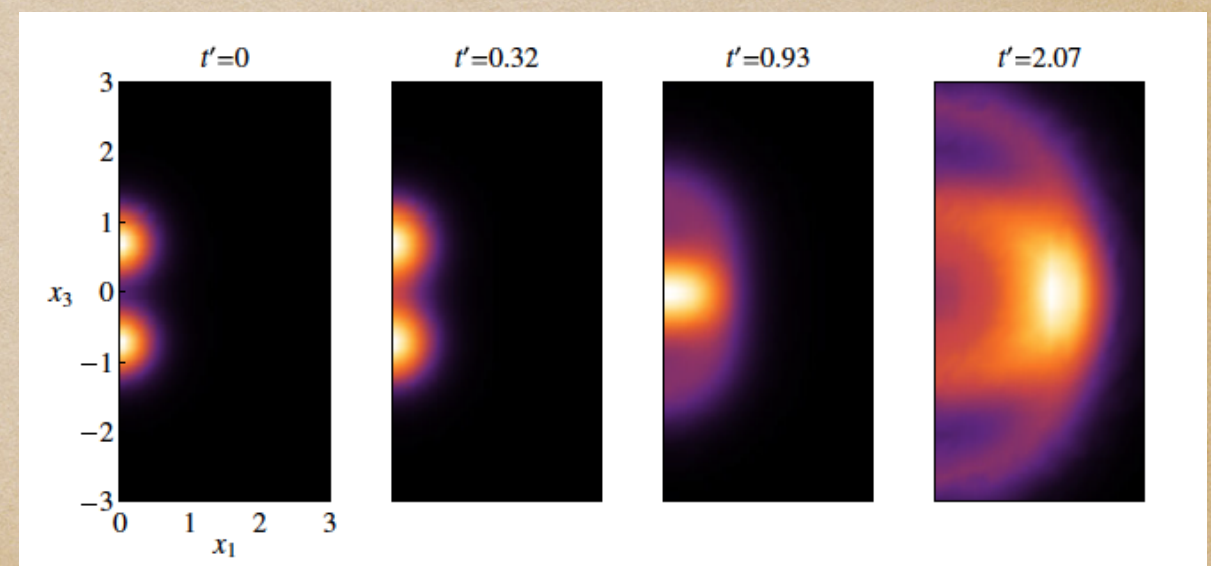
E.g. Dimopoulos & Landsberg '01, Giddings & Thomas '01

Motivation: Holography (AdS/CFT)

- Holography
 - BH entropy $\propto A_{\text{Hor}}$
 - Local field theory:
entropy $\propto V$
 - Gravity in D dims
 $\Leftrightarrow$ local FT in $D - 1$ dims



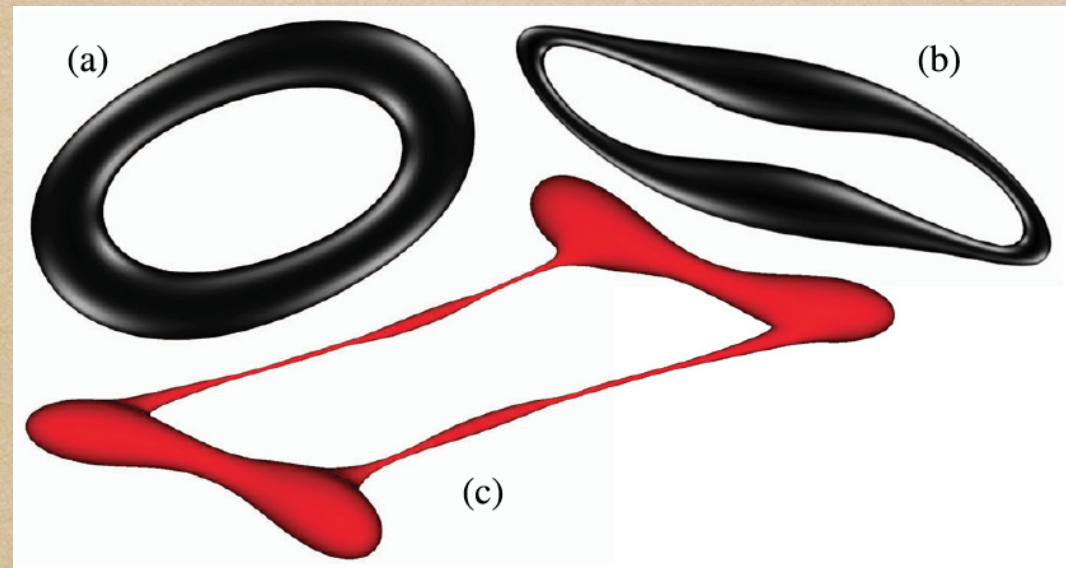
- Model heavy ion collisions through BH collisions in 5D
Bantilan & Romatschke '14



Motivation: Fundamental BH properties

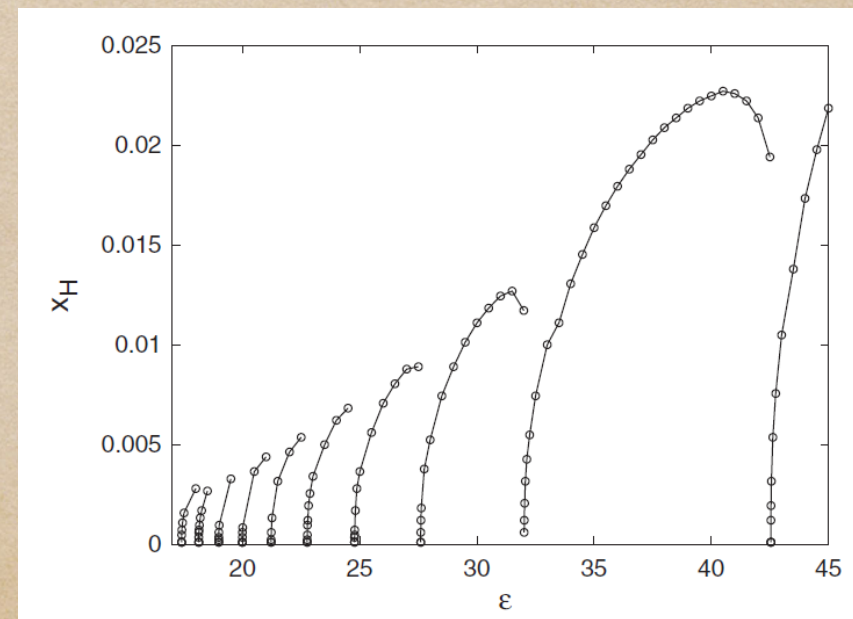
- Cosmic censorship

Lehner & Pretorius '10, Figueras, Kunesch, Tunyasuvunakool et al. '16, '17



- Stability of AdS

Bizon & Rostworowski '11



2. Numerical relativity in D dimensions

The spacetime split of general relativity

- Einstein equations:

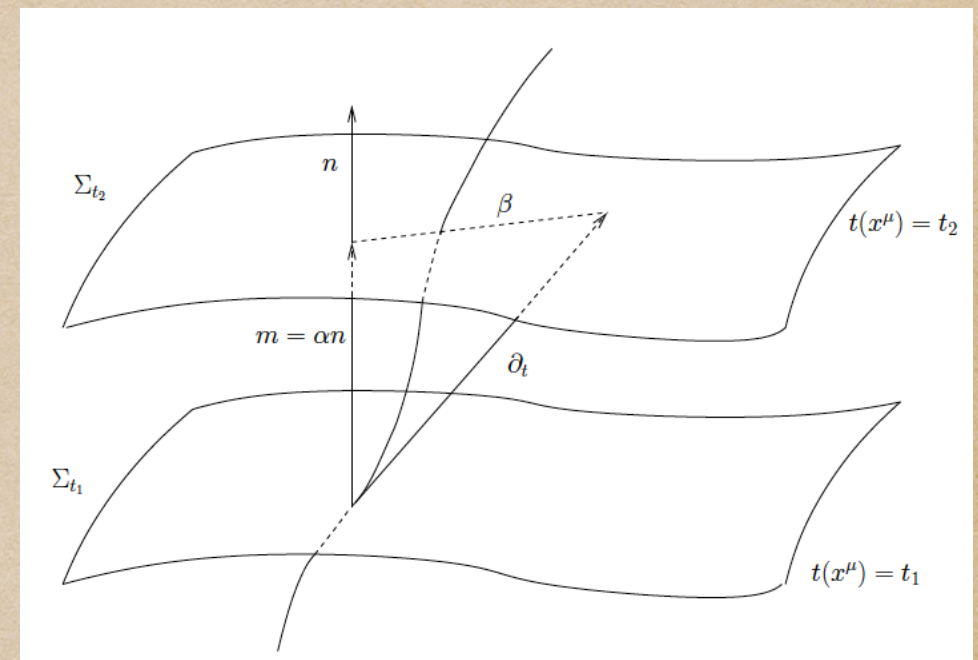
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \Leftrightarrow R_{\mu\nu} = 8\pi \left(T_{\mu\nu} - \frac{1}{D-2}Tg_{\mu\nu} \right) + \frac{2}{D-2}\Lambda g_{\mu\nu}$$

- $(D-1) + 1$ decomposition

$$g_{\alpha\beta} = \left(\begin{array}{c|c} -\alpha^2 + \beta_m\beta^m & \beta_j \\ \hline \beta_i & \gamma_{ij} \end{array} \right)$$

- $\alpha = \text{Lapse}$, $\beta^i = \text{shift}$

$\gamma_{ij} = \text{spatial metric}$



- Extrinsic curvature $K_{\alpha\beta} := -\perp \nabla_{\beta} n_{\alpha}$, $\perp^{\mu}_{\alpha} = \delta^{\mu}_{\alpha} - n^{\mu}n_{\alpha}$

- Energy momentum tensor

$$\rho = T_{\mu\nu}n^{\mu}n^{\nu} , \quad j_{\alpha} = -\perp^{\mu}_{\alpha}T_{\mu\nu}n^{\nu} , \quad S_{\alpha\beta} = \perp T_{\alpha\beta}$$

Arnowitt, Deser & Misner '62, York '77

The ADM version of the Einstein eqs.

- Introduction of the extrinsic curvature:

$$\mathcal{L}_m \gamma_{ij} = -2\alpha K_{ij}$$

- $\perp^\mu_\alpha \perp^\nu_\beta$ projection

$$\mathcal{L}_m K_{ij} = -D_i D_j \alpha + \alpha (\mathcal{R}_{ij} + K K_{ij} - 2K_{im} K^m_j) + 8\pi \alpha \left(\frac{S - \rho}{D - 2} \gamma_{ij} - S_{ij} \right) - \frac{2}{D - 2} \Lambda \gamma_{ij}$$

“Evolution equations”

- $n^\mu n^\nu$ projection

$$\mathcal{R} + K^2 - K^{mn} K_{mn} = 2\Lambda + 16\pi\rho$$

“Hamiltonian constraint”

- $\perp^\mu_\alpha n^\nu$ projection

$$D_i K - D_m K^m_i = -8\pi j_i$$

“Momentum constraints”

The BSSN system

- Goal: Modify ADM eqs. to get a strongly hyperbolic system

Shibata & Nakamura PRD 52 (1995), Baumgarte & Shapiro PRD gr-qc/9810065

- Use (i) conformal decomposition, (ii) trace split, (iii) aux. variables

$$\gamma := \det \gamma_{ij}, \quad \chi = \gamma^{-1/(D-1)}, \quad K = \gamma^{mn} K_{mn},$$

$$\tilde{\gamma}_{ij} = \chi \gamma_{ij} \quad \Leftrightarrow \quad \tilde{\gamma}^{ij} = \frac{1}{\chi} \gamma^{ij}$$

$$\tilde{A}_{ij} = \chi \left(K_{ij} - \frac{1}{D-1} \gamma_{ij} K \right) \quad \Leftrightarrow \quad K_{ij} = \frac{1}{\chi} \left(\tilde{A}_{ij} + \frac{1}{D-1} \tilde{\gamma}_{ij} K \right)$$

$$\tilde{\Gamma}^i = \tilde{\gamma}^{mn} \tilde{\Gamma}_{mn}^i$$

- Auxiliary constraints

$$\tilde{\gamma} = 1, \quad \tilde{\gamma}^{mn} \tilde{A}_{mn} = 0, \quad \mathcal{G}^i \equiv \tilde{\Gamma}^i - \tilde{\gamma}^{mn} \tilde{\Gamma}_{mn}^i = 0.$$

The BSSN equations

$$\mathcal{H} := \mathcal{R} + \frac{D-2}{D-1}K^2 - \tilde{A}^{mn}\tilde{A}_{mn} - 16\pi\rho - 2\Lambda = 0,$$

$$\mathcal{M}_i := \tilde{\gamma}^{mn}\tilde{D}_m\tilde{A}_{ni} - \frac{D-2}{D-1}\partial_i K - \frac{D-1}{2}\tilde{A}^m{}_i\frac{\partial_m\chi}{\chi} - 8\pi j_i = 0,$$

$$\partial_t\chi = \beta^m\partial_m\chi + \frac{2}{D-1}\chi(\alpha K - \partial_m\beta^m),$$

$$\partial_t\tilde{\gamma}_{ij} = \beta^m\partial_m\tilde{\gamma}_{ij} + 2\tilde{\gamma}_{m(i}\partial_{j)}\beta^m - \frac{2}{D-1}\tilde{\gamma}_{ij}\partial_m\beta^m - 2\alpha\tilde{A}_{ij},$$

$$\partial_t K = \beta^m\partial_m K - \chi\tilde{\gamma}^{mn}D_m D_n\alpha + \alpha\tilde{A}^{mn}\tilde{A}_{mn} + \frac{1}{D-1}\alpha K^2 + \frac{8\pi}{D-2}\alpha[S + (D-3)\rho] - \frac{2}{D-2}\alpha\Lambda,$$

$$\partial_t\tilde{A}_{ij} = \beta^m\partial_m\tilde{A}_{ij} + 2\tilde{A}_{m(i}\partial_{j)}\beta^m - \frac{2}{D-1}\tilde{A}_{ij}\partial_m\beta^m + \alpha K\tilde{A}_{ij} - 2\alpha\tilde{A}_{im}\tilde{A}^m{}_j$$

$$+ \chi(\alpha\mathcal{R}_{ij} - D_i D_j\alpha - 8\pi\alpha S_{ij})^{\text{TF}},$$

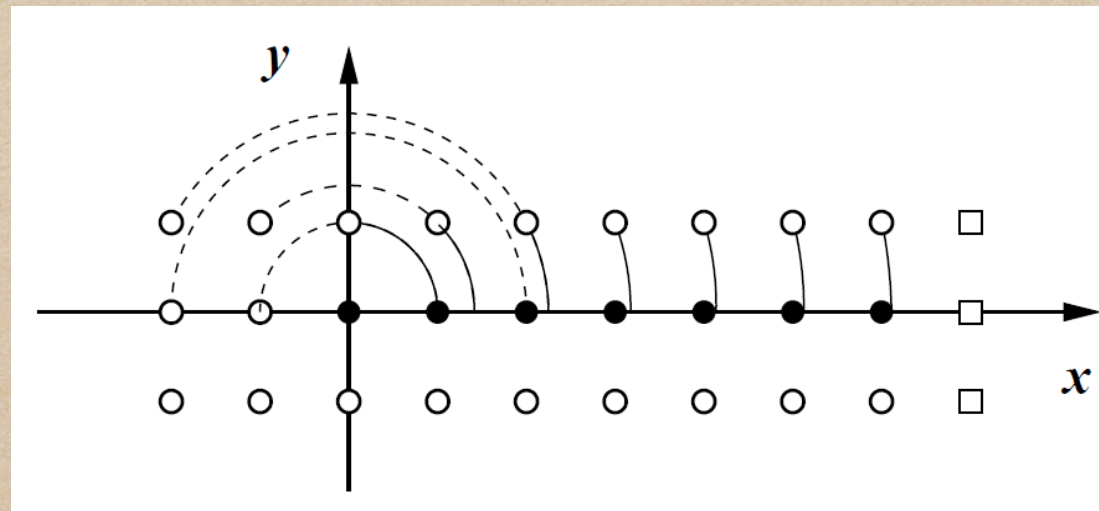
$$\partial_t\tilde{\Gamma}^i = \beta^m\partial_m\tilde{\Gamma}^i + \frac{2}{D-1}\tilde{\Gamma}^i\partial_m\beta^m - \tilde{\Gamma}^m\partial_m\beta^i + \tilde{\gamma}^{mn}\partial_m\partial_n\beta^i + \frac{D-3}{D-1}\tilde{\gamma}^{im}\partial_m\partial_n\beta^n$$

$$- \tilde{A}^{im}\left[(D-1)\alpha\frac{\partial_m\chi}{\chi} + 2\partial_m\alpha\right] + 2\alpha\tilde{\Gamma}^i{}_{mn}\tilde{A}^{mn} - 2\frac{D-2}{D-1}\alpha\tilde{\gamma}^{im}\partial_m K - 16\pi\frac{\alpha}{\chi}j^i - \sigma\mathcal{G}^i\partial_m\beta^m.$$

- Note: there exist slight variations of the exact equations

The original Cartoon method

- Developed for axisymmetry around z in 3+1 GR
Alcubierre et al IJMPD gr-qc/9908012
- Coordinates $(z, x, y) \leftrightarrow (z, \rho, \phi)$ where $x = \rho \cos \phi$, $y = r \sin \phi$
- Killing vector: $\partial_\phi = x\partial_y - y\partial_x$
- Extend 2D grid by ghostzones for derivatives; rotate, interpolate



- Problem: For large D Cartoon ghostzones require lots of memory

Dimensional reduction: Modified Cartoon

- Solution: Use symmetry to relate "off-domain" to "on-domain"

1) Coordinates: $X^i = (x^{\hat{i}}, z, w^a) \leftrightarrow \bar{X}^i = (x^{\hat{i}}, \rho, \phi, w^5, \dots, w^{D-1})$

2) Tensor components: $\bar{T}_{\hat{i}\phi} = \frac{\partial X^\alpha}{\partial \bar{X}^{\hat{i}}} \frac{\partial X^\beta}{\partial \phi} T_{\alpha\beta} = -w T_{\hat{i}z} + z T_{\hat{i}w}, \quad \boxed{w := w^4}$

3) By symmetry $\bar{T}_{\hat{i}\phi} = 0 \Rightarrow T_{\hat{i}w} = \frac{w}{z} T_{\hat{i}z}$

4) Computational domain is $w = 0 \Rightarrow \boxed{T_{\hat{i}w} = 0}$

- Play same game for other tensor components, scalars, vectors and deriv's using also that Lie deriv's along $\xi = z\partial_w - w\partial_z$ vanish
 $\Rightarrow$ express all w^a components and deriv's in terms of components and deriv's in the computational domain and one new func.

- E.g.: $\boxed{\partial_w T_{iw} = \frac{T_{iz} - \delta_{iz} T_{ww}}{z}}$; works for metric, ADM, BSSN variables

Pretorius gr-qc/0407110; Yoshino & Shibata PTPS 189/269; Cook et al 1603.00362

Initial data

- Conformal metric $\gamma_{ij} = \psi^4 \bar{\gamma}_{ij}$

Lichnerowicz '44, York '71

- Conformal traceless split of the extrinsic curvature

- Assume $K = 0$, $\bar{\gamma}_{ij} = f_{ij}$, $\lim_{r \rightarrow \infty} \psi = 0$

In words: Traceless E.Curv., conformal flatness, asymptotic flatness

- Analytic solution of momentum constraints Bowen & York '80

- Puncture data: $\psi = \psi_{\text{BL}} + u$, numerically solve Ham. constr.

Brandt & Bügmann '97, Ansorg et al. '04, "TwoPunctures"

- Generalized to higher dimensions

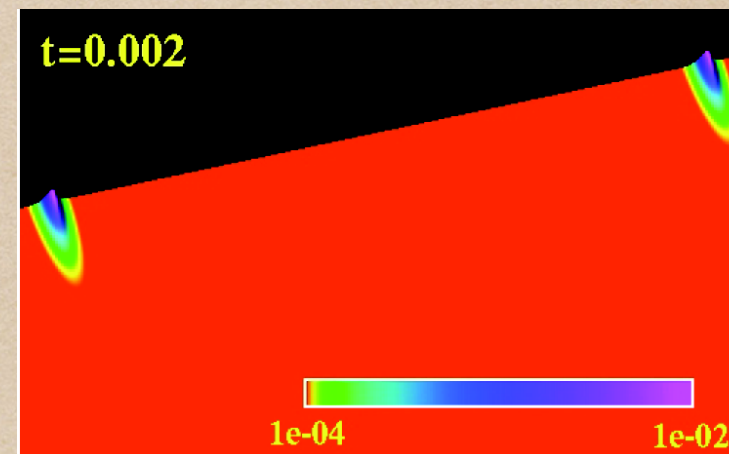
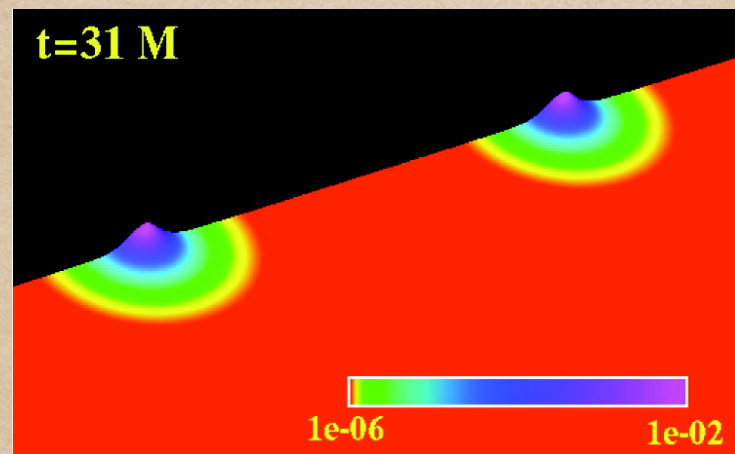
Zilhao et al. '11

3. Results in $D=4$

Does matter matter?

- Hoop conjecture $\Rightarrow$ kinetic energy triggers BH formation
- Einstein + minimally coupled, massive complex scalar field

“Boson stars” Pretorius & Choptuik '10

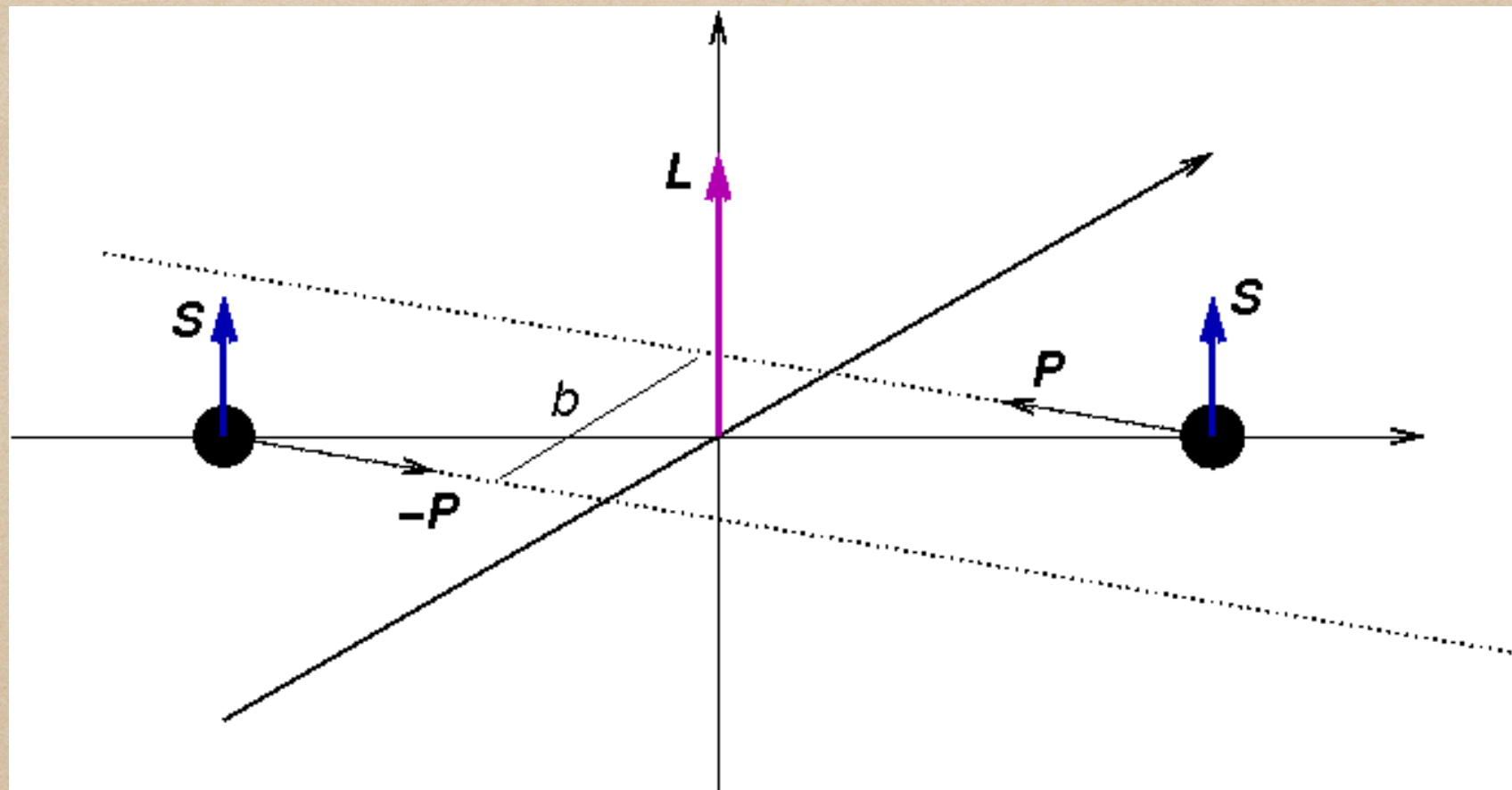


- BH formation threshold $\gamma_{\text{thr}} = 2.9 \pm 10 \% \sim 1/3 \gamma_{\text{hoop}}$
- Model point particle collision by BH collisions
- Similar results for collisions of perfect fluid balls

East & Pretorius '13, Rezzolla & Tanaki '13

Collisions of BHs in D=4

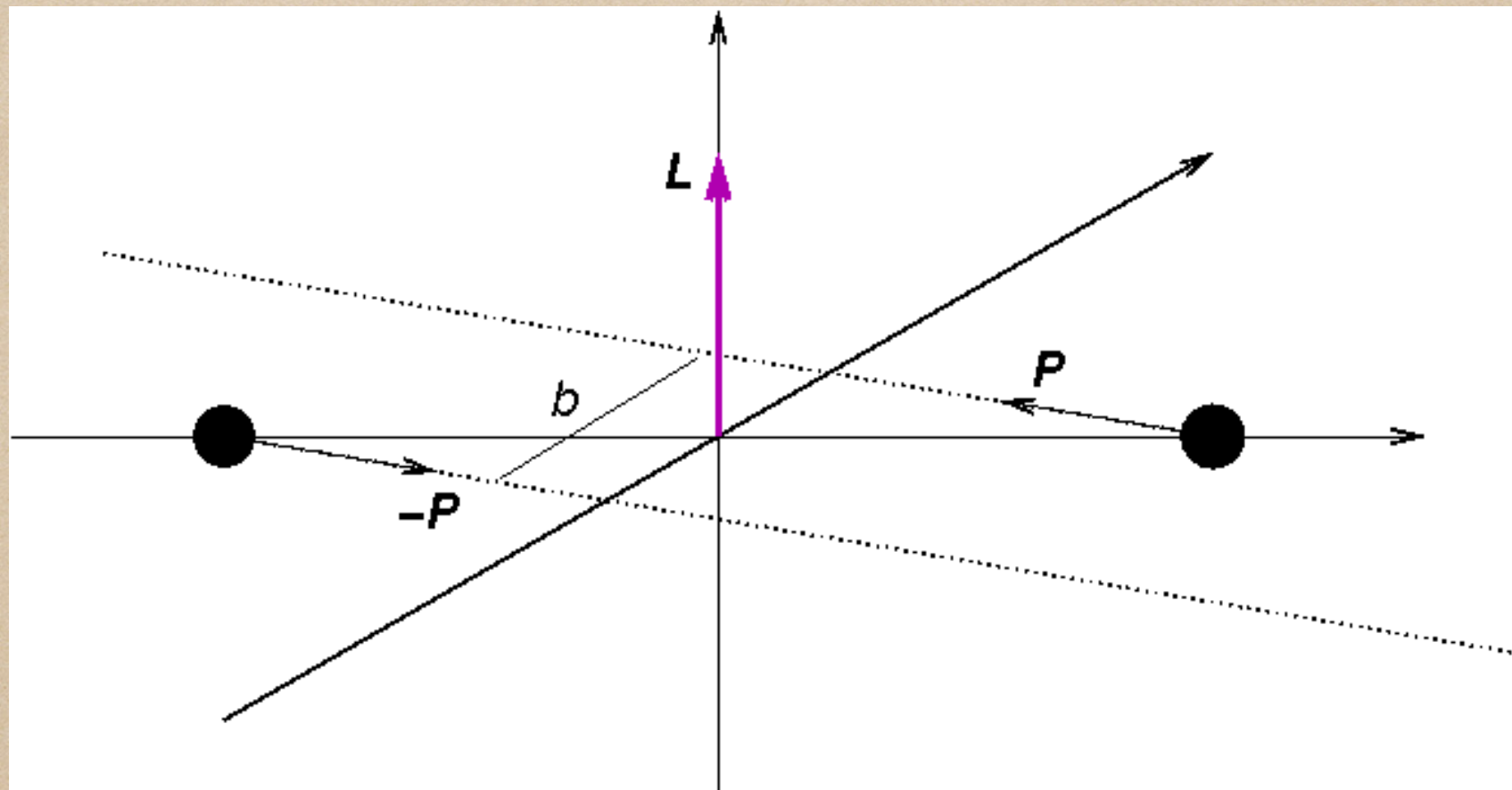
- Orbital hang-up: Campanelli et al. PRD (2006)
- Equal-mass BHs, Boost $\gamma = 1/\sqrt{1-v^2}$
Impact parameter $b = L/P$



- How are scattering threshold and radiated GW energy affected?

Collisions of BHs in D=4

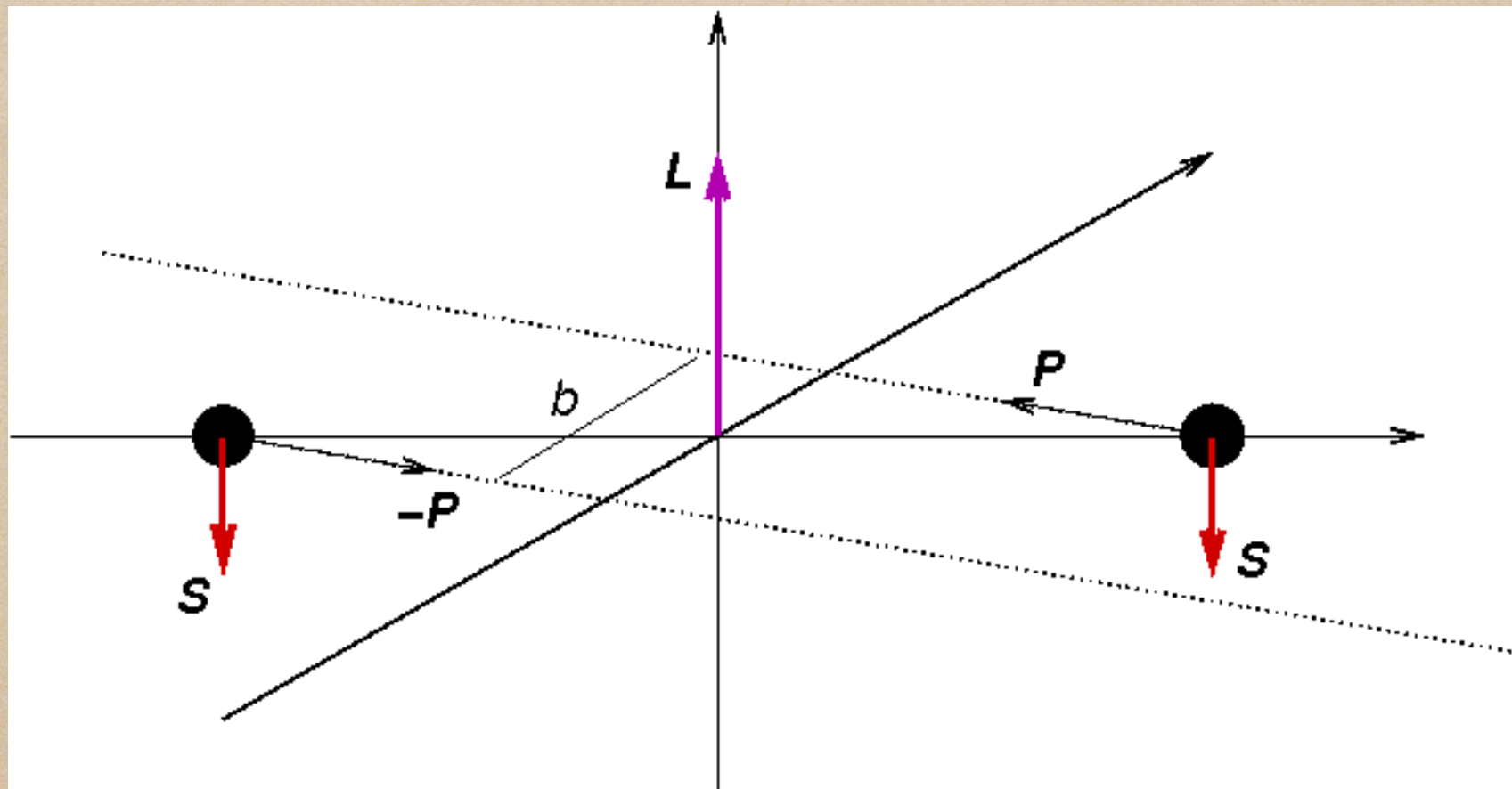
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Collisions of BHs in D=4

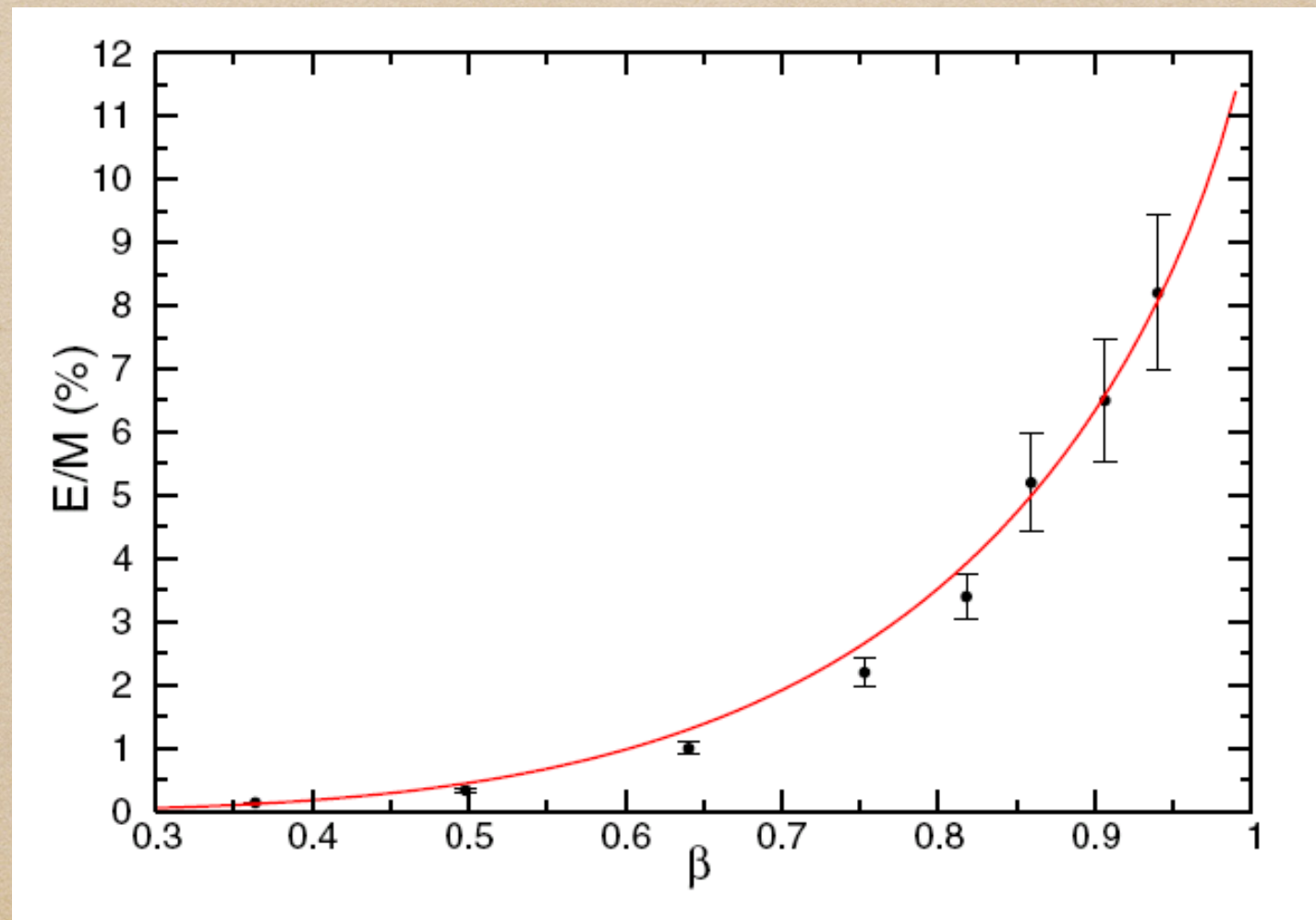
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- Equal-mass BHs, Boost $\gamma = 1/\sqrt{1-v^2}$
Impact parameter $b = L/P$



- How are scattering threshold and radiated GW energy affected?

Boosted BH head-on collisions in D=4

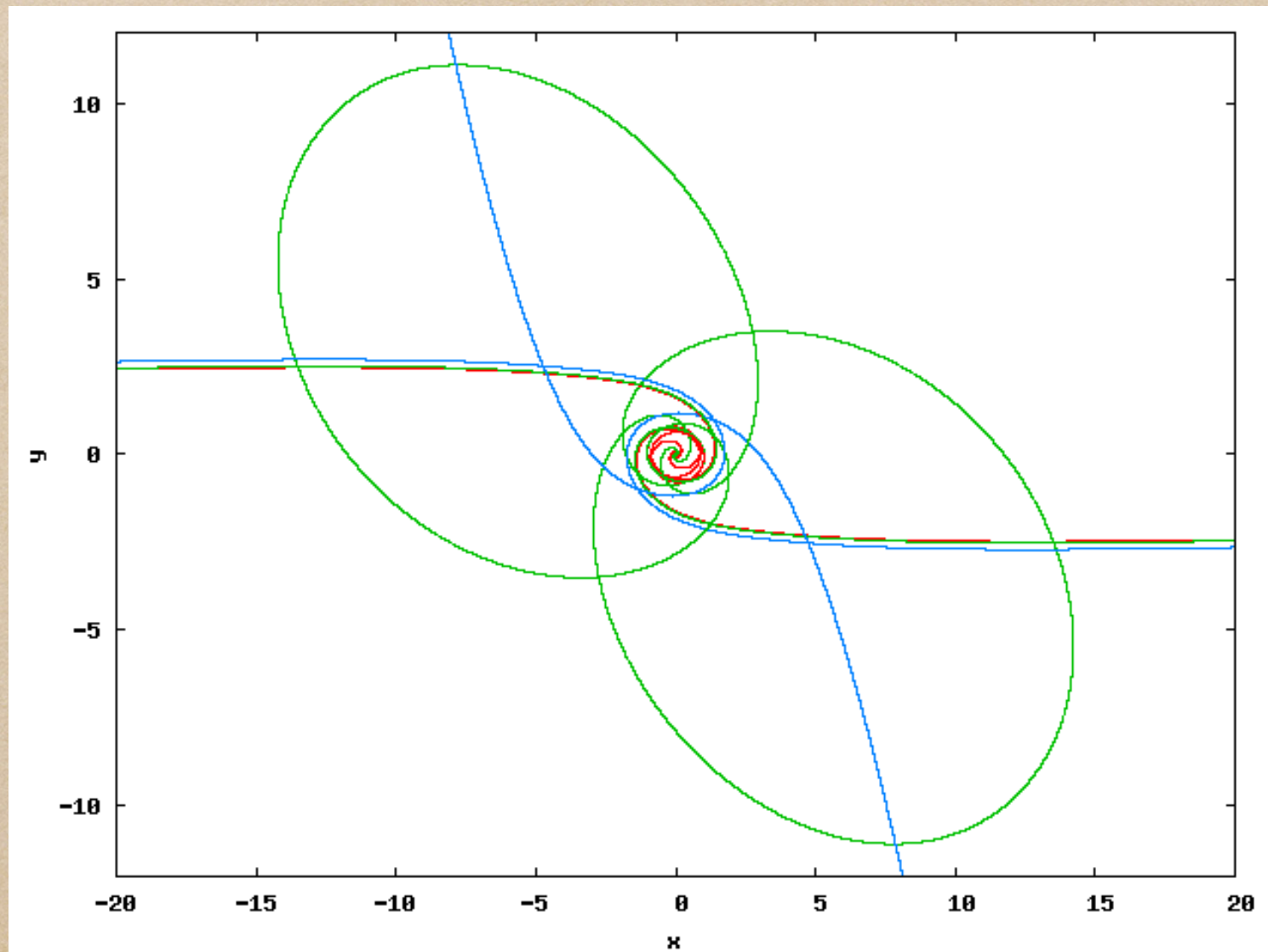
- BSSN, Cactus, Carpet, Moving Puncture, TwoPunctures, AHFinderDirect
- Equal-mass BHs, no spin $\lim_{\beta \rightarrow 1} E_{\text{rad}} = 14 \pm 3 \%$
- Agrees well with perturbative studies Berti et al PRD 1003.0812



Sperhake et al PRL 0806.1738; Healy et al 1506.06153

D=4 grazing collisions: $b = 0$, $\vec{S} = 0$, $\gamma = 1.52$

- Radiated energy up to at least $\approx 35\% M$
- Immediate **vs.** Delayed **vs.** No merger



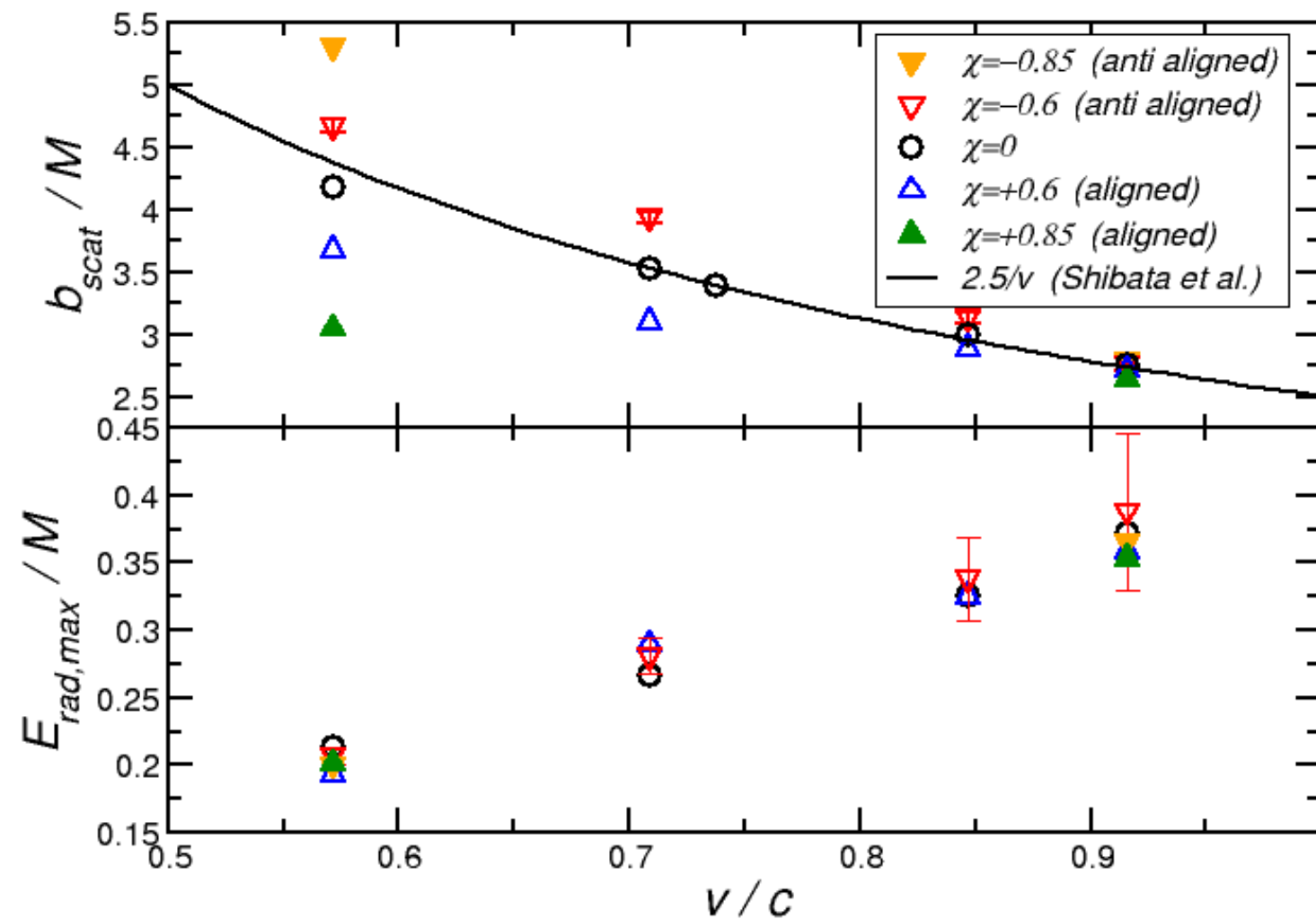
Sperhake et al PRL 0907.1252

Scattering threshold

- $b < b_{\text{scat}} \Rightarrow$ Merger
 $b > b_{\text{scat}} \Rightarrow$ Scattering
- Numerical study: $b_{\text{scat}} = \frac{2.5 \pm 0.05}{v} M$
Shibata et al PRD 0810.4735
- Limit from Penrose construction: $b_{\text{scat}} = 1.685 M$
Yoshino & Rychkov PRD hep-th/0503171
- Impact of structure of the colliding BHs?
→ Collide spinning BHs

Grazing collisions in D=4

- Spins: aligned, zero, anti aligned Sperhake et al PRL 1211.6114
- $b_{\text{scat}}, E_{\text{rad}}$: spin effects washed out as $v \rightarrow c$



3. Results in $D > 4$

GW extraction in $D > 4$

- Generalization of Regge-Wheeler-Zerilli-Moncrief to higher D

Kodama-Ishibashi formalism

Kodama & Ishibashi hep-th/0305147, hep-th/0308128

Applications in NR:

Witek et al 1006.3081, 1011.0742, 1406.2703

- Landau-Lifshitz pseudo tensor:

Yoshino & Shibata PRD '09

- Generalization of the Newman-Penrose scalars:

Peeling properties of Weyl tensor: Godazgar & Reall '12

Numerical implementation: Cook & Sperhake '17

GW extraction in $D > 4$

Weyl-tensor based extraction in higher D ; Cook & Sperhake '17

- Construct null frame: Gram-Schmidt starting with

$$l = -\frac{\partial}{\partial r}, \quad k = \frac{\partial}{\partial u} - \frac{1}{2} \frac{\partial}{\partial r}, \quad m_{(\alpha)} = \frac{\partial}{\partial \phi^\alpha}$$

- Analog of Newman-Penrose scalar Ψ_4 :

$$\Omega'_{(\alpha)(\beta)} = C_{ABCD} k^A m_{(\alpha)}^B k^C m_{(\beta)}^D$$

- Mass loss

$$\dot{M}(u) = - \lim_{r \rightarrow \infty} \frac{r^{D-2}}{8\pi} \int_{S^{D-2}} \left(\int_{-\infty}^u \Omega'_{(\alpha)(\beta)}(\tilde{u}, r, \phi^\gamma) d\tilde{u} \right)^2 d\omega$$

- Use modified cartoon to accommodate extra dimensions

Collisions starting from rest

- Initial data: Generalized Brill-Lindquist data

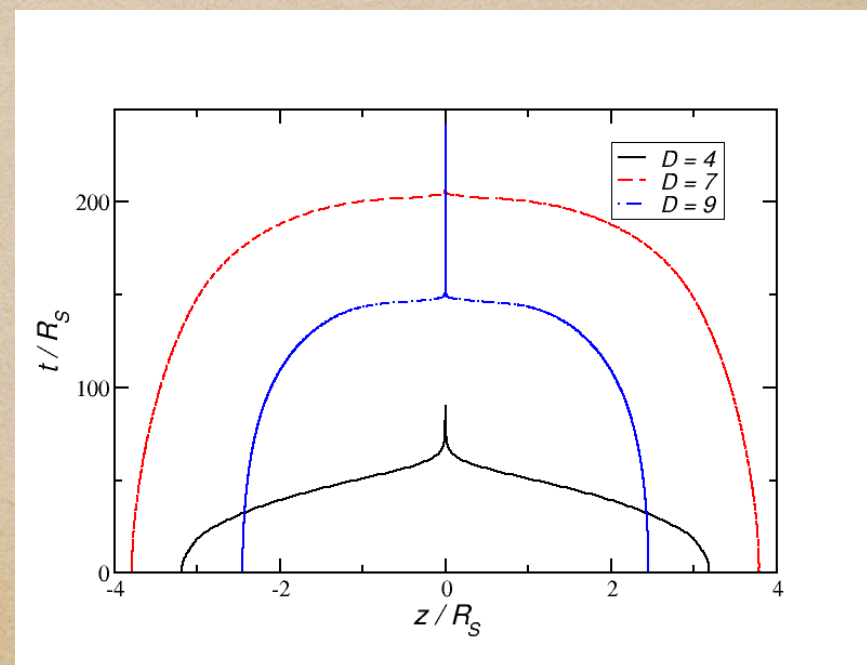
$$K_{IJ} = 0, \quad \gamma_{IJ} = \psi^{4/(D-3)} \delta_{IJ}$$

$$\psi = 1 + \frac{\mu_{\mathcal{A}}}{4 \left[\sum_K (X^K - X_{\mathcal{A}}^K)^2 \right]^{(D-3)/2}} + \frac{\mu_{\mathcal{B}}}{4 \left[\sum_K (X^K - X_{\mathcal{B}}^K)^2 \right]^{(D-3)/2}}$$

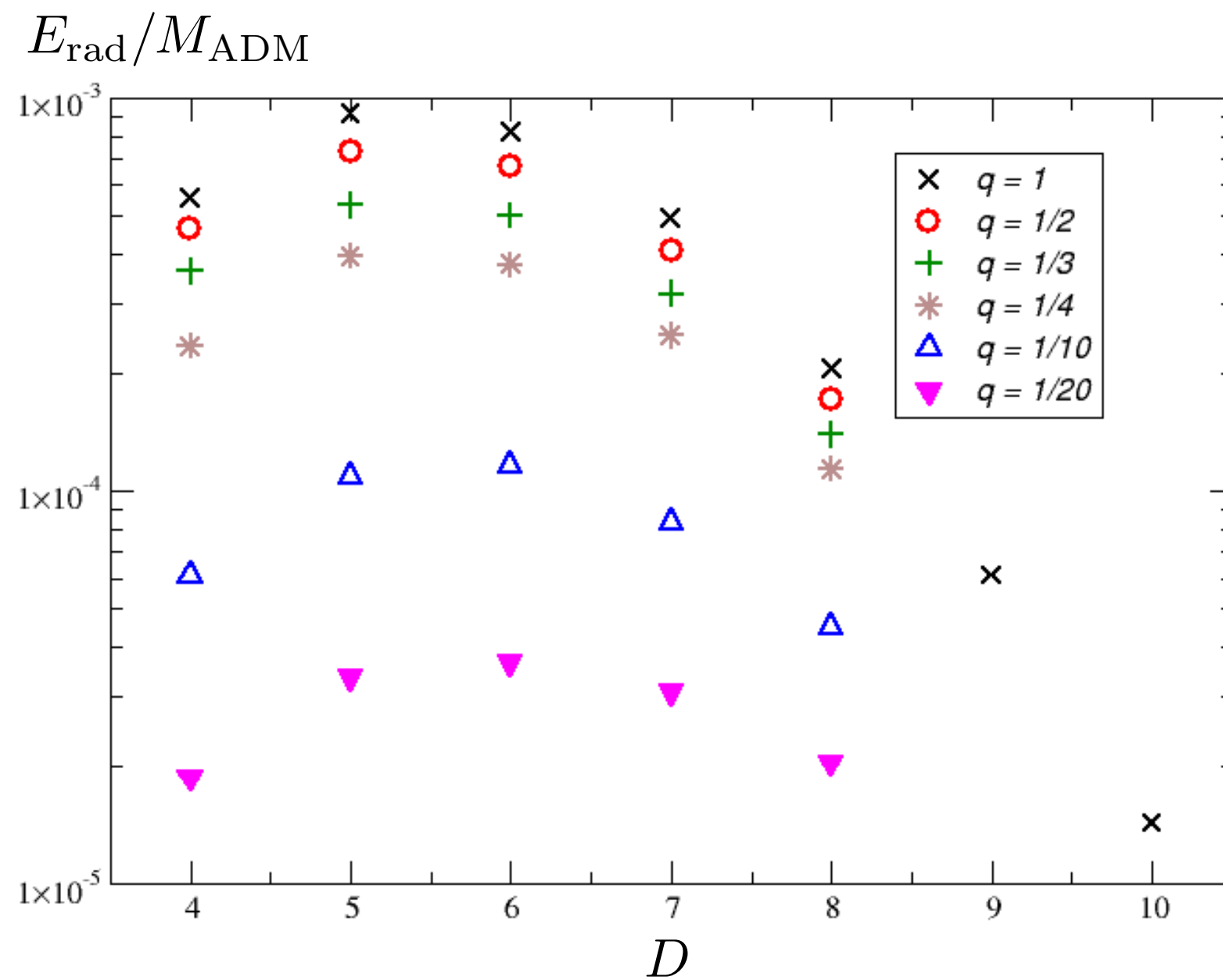
BH mass M , horizon radius R_h :

$$\mu = \frac{16\pi M}{(D-2)\mathcal{A}_{D-2}}, \quad \mu = R_h^{D-3}, \quad \mathcal{A}_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma\left(\frac{D-1}{2}\right)},$$

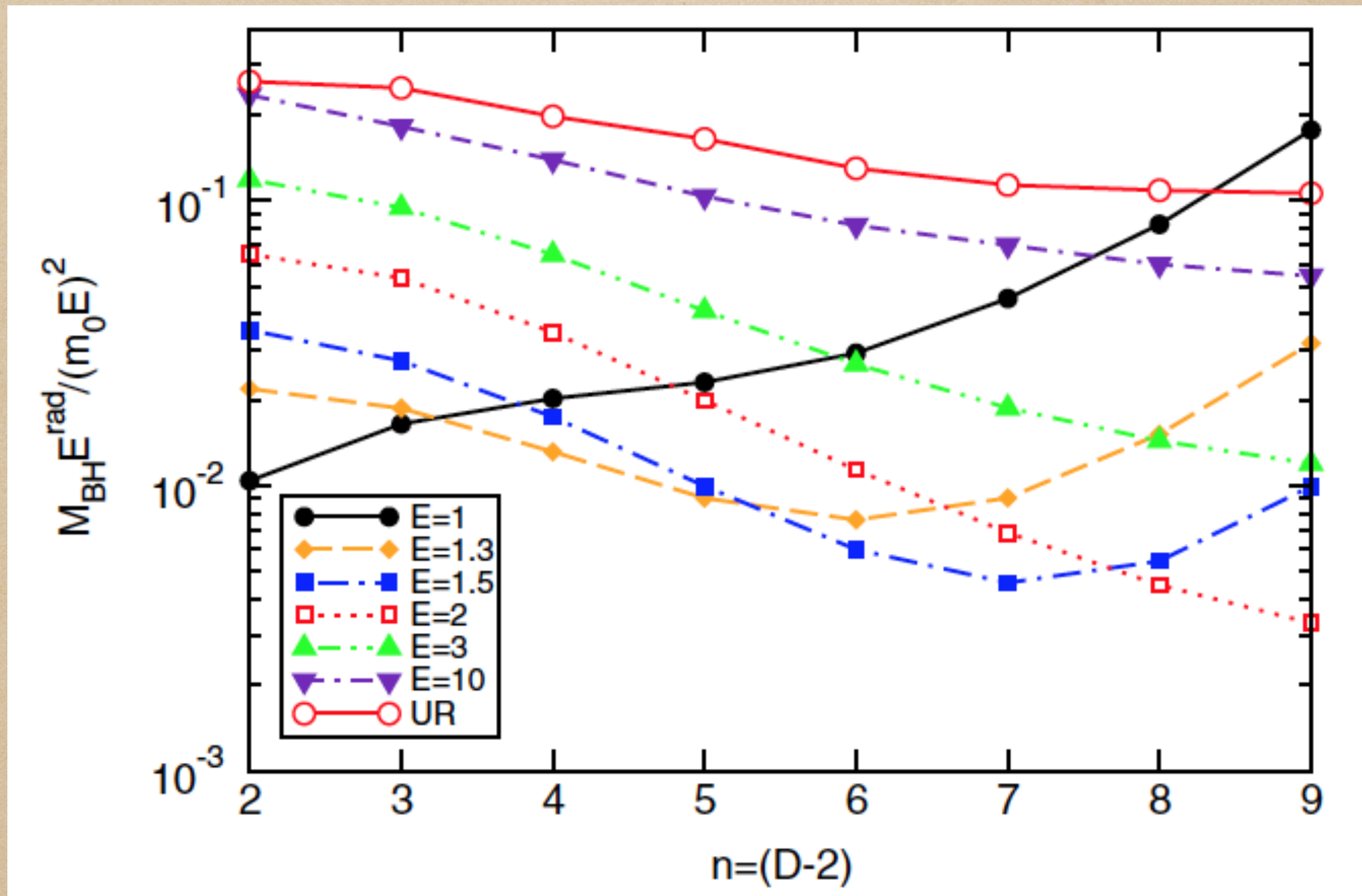
- Note: gravity falls off $\propto 1/r^{D-2}$



Radiated energy

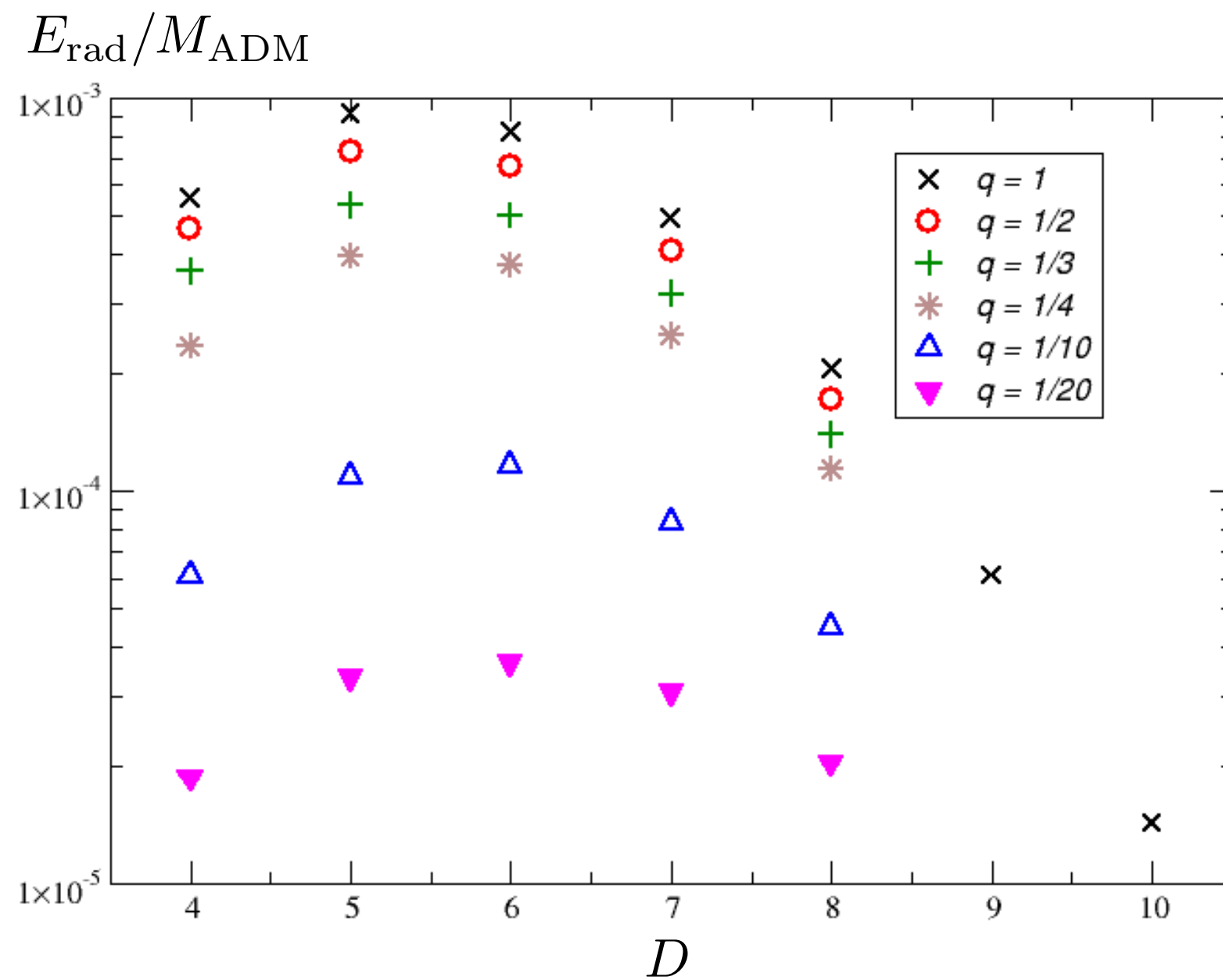


Compare: Point particle calculations



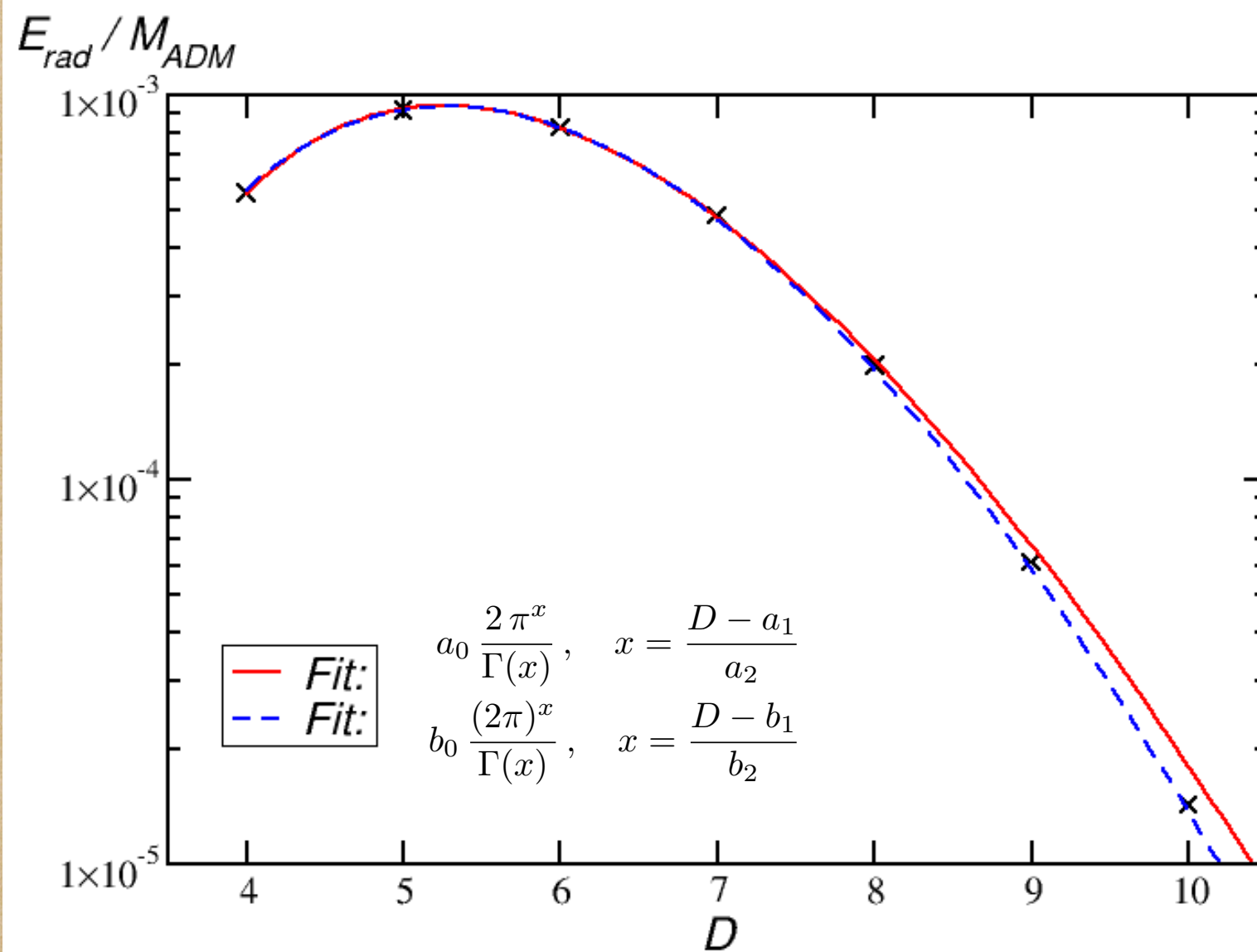
Berti, Cardoso & Kipapa 1003.0812

Radiated energy



Radiated energy: Fit for $q = 1$

Recall: $\mathcal{A}_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma(\frac{D-1}{2})}$ (Thanks to Chris Moore)



$$a_0 = 2.8236 \times 10^{-5}$$

$$a_1 = -2.47721$$

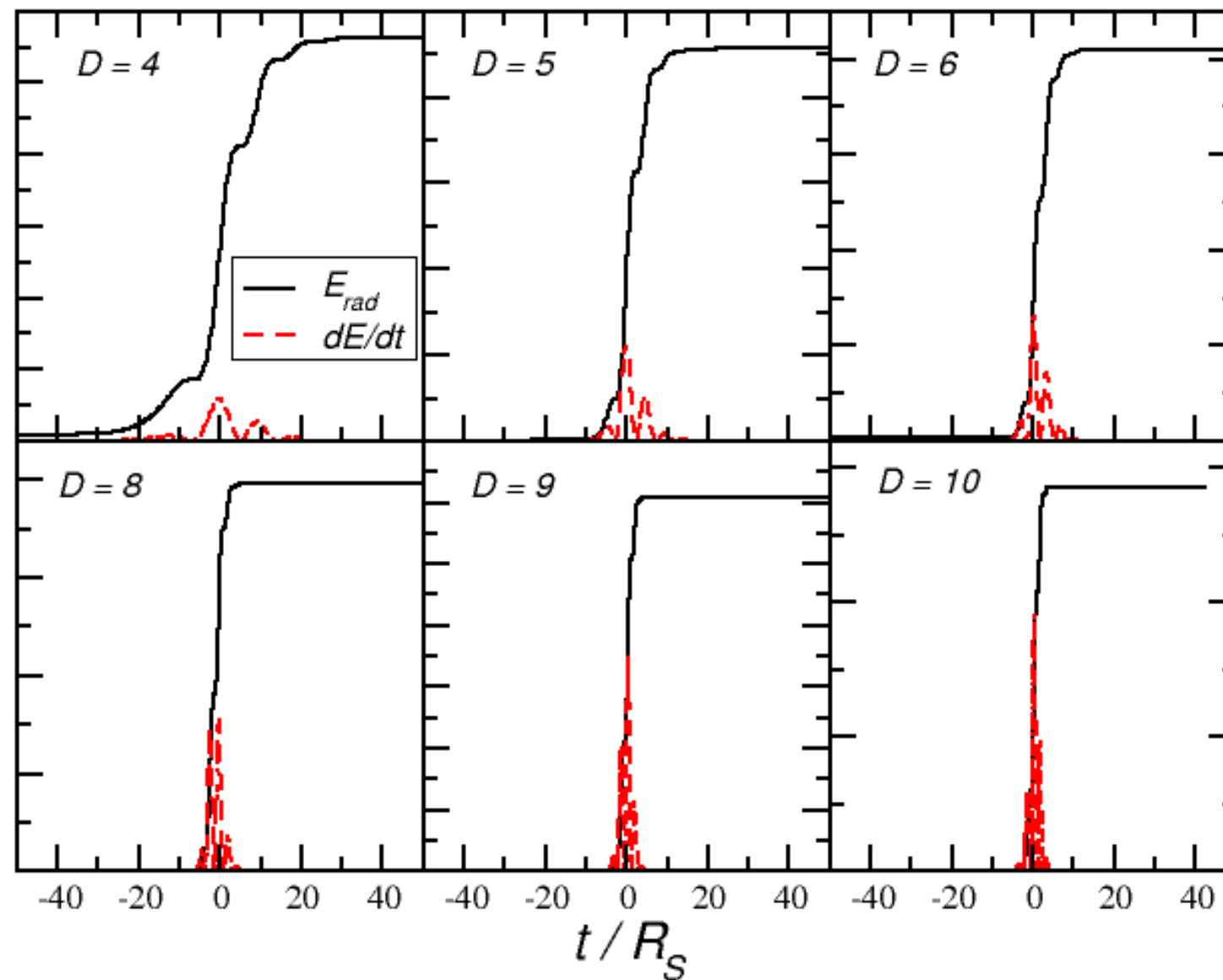
$$a_2 = 0.767106$$

$$b_0 = 1.72877 \times 10^{-6}$$

$$b_1 = -1.57712$$

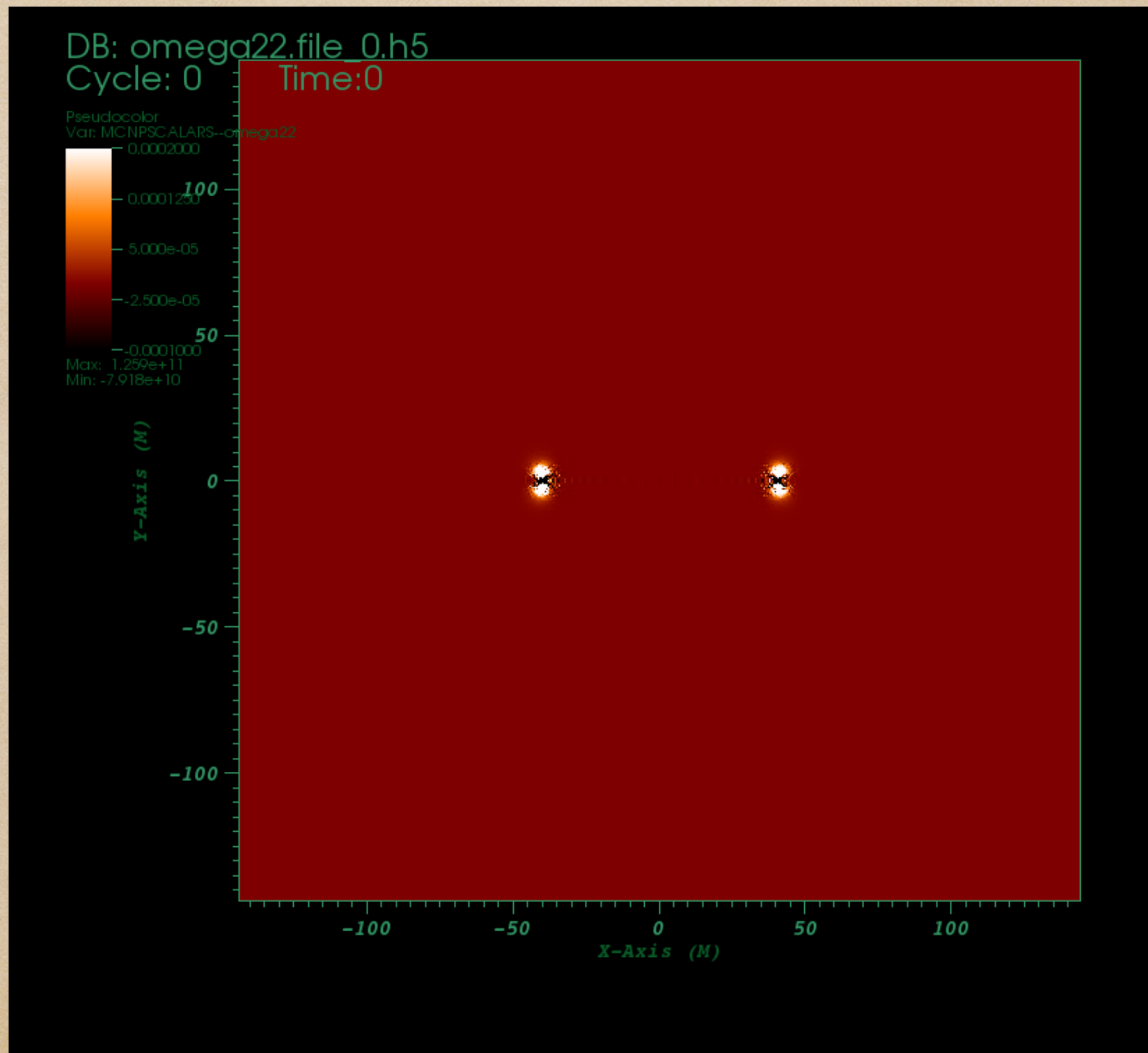
$$b_2 = 0.549696$$

Radiated energy: $E_{\text{rad}}(t)$ for $q = 1$

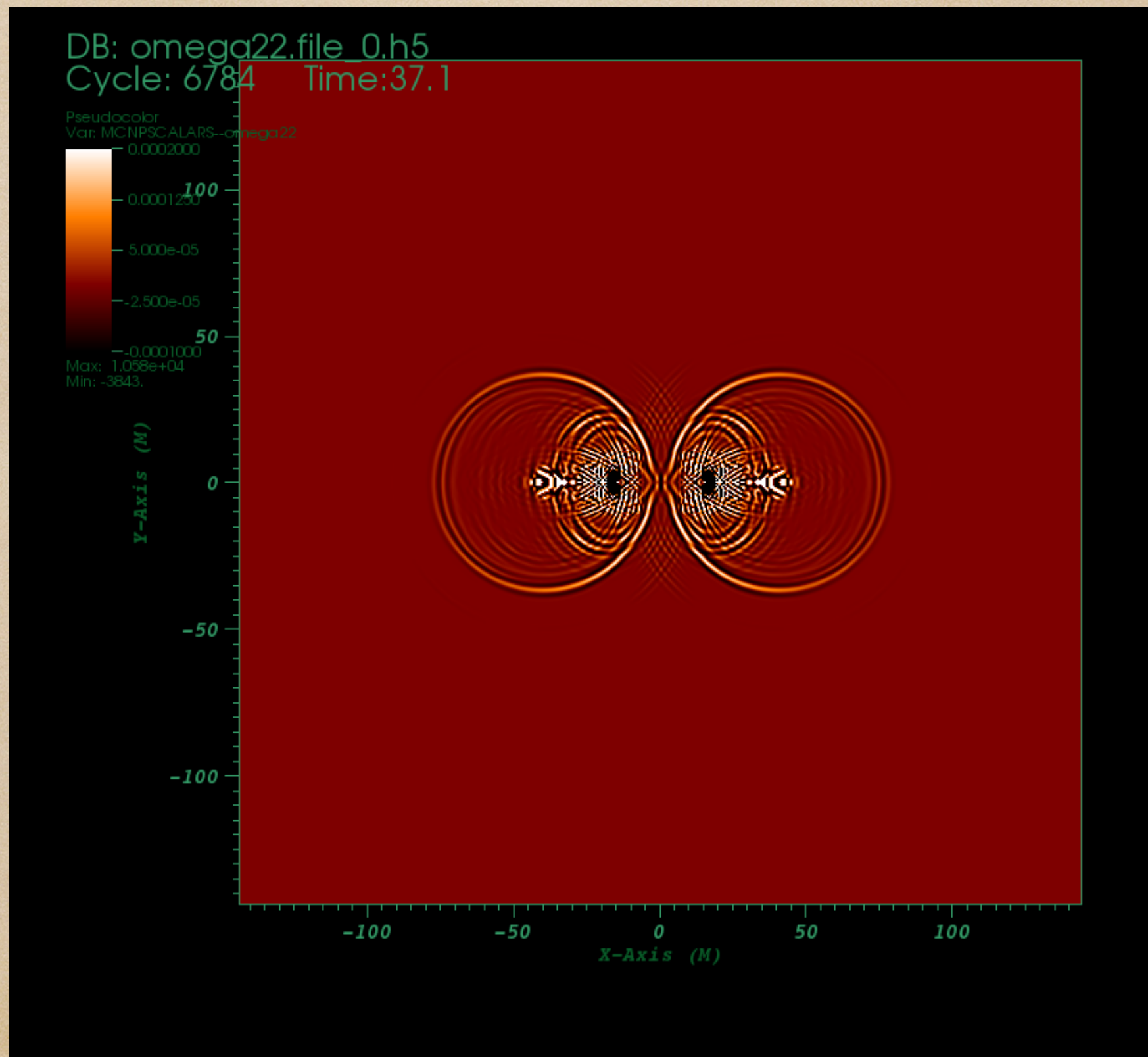


Heaviside and Delta function in the limit $D \rightarrow \infty$?

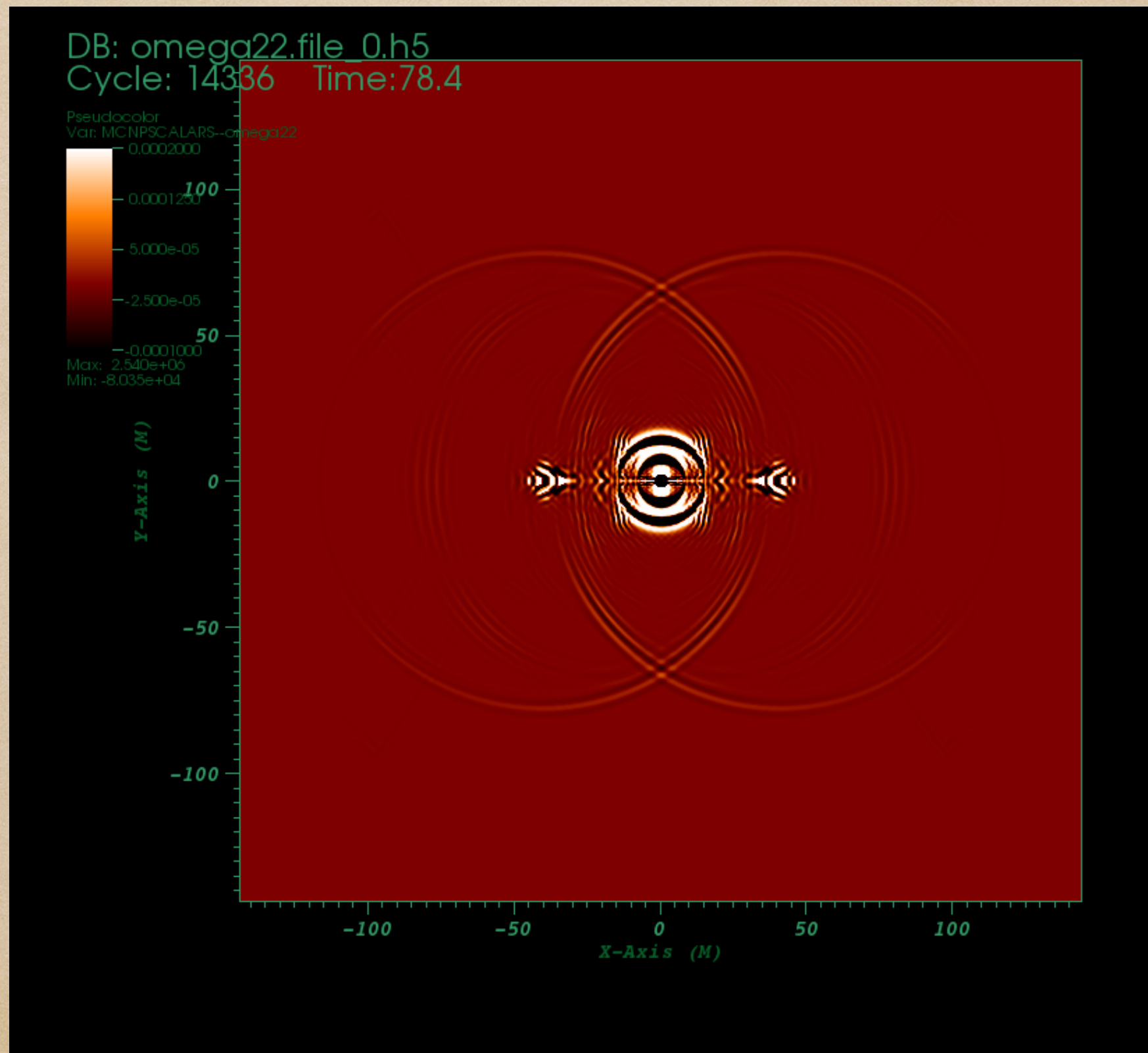
Boosted head-on collision $D = 6$, $v \approx 0.54c$



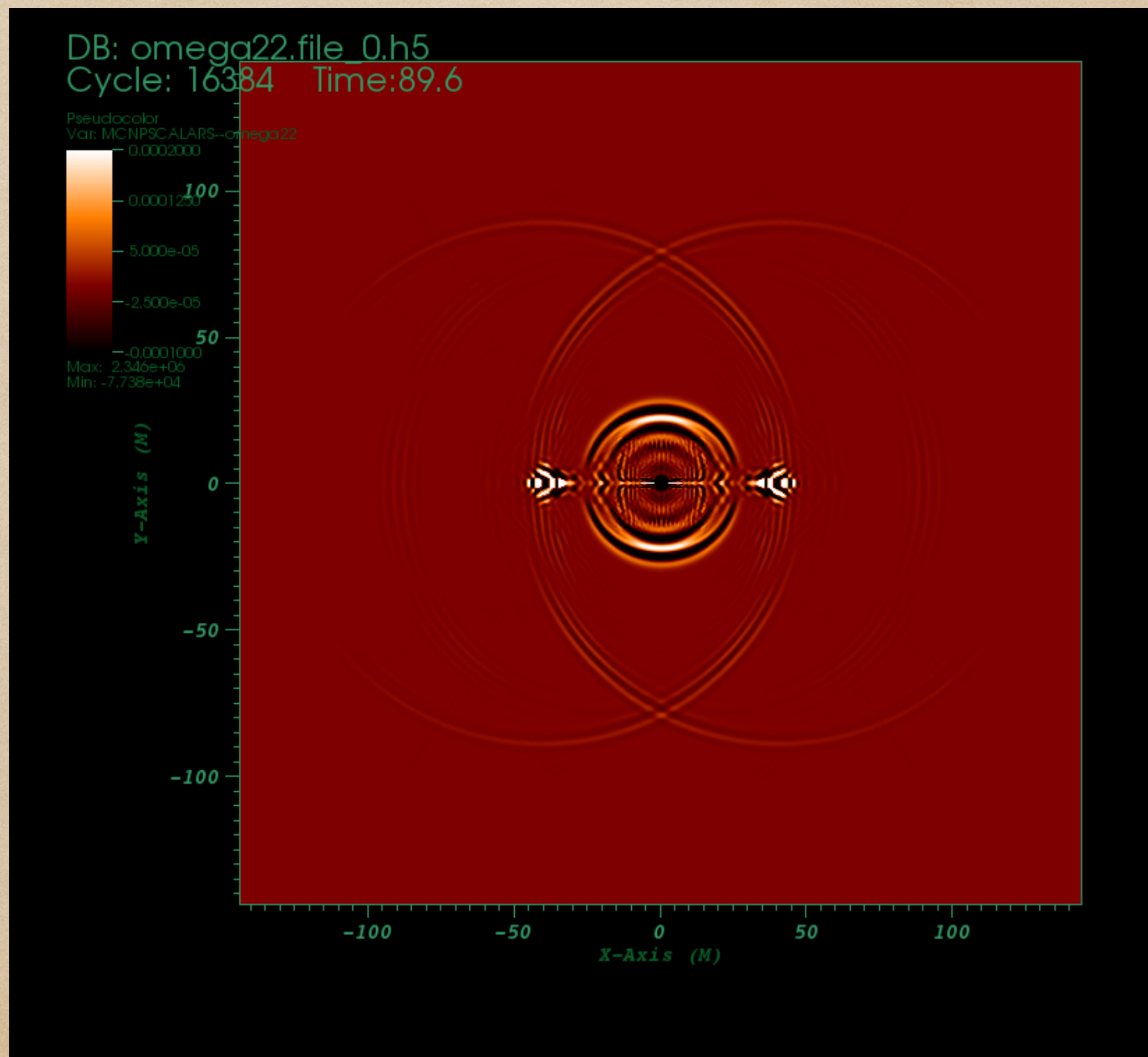
Boosted head-on collision $D = 6$, $v \approx 0.54c$



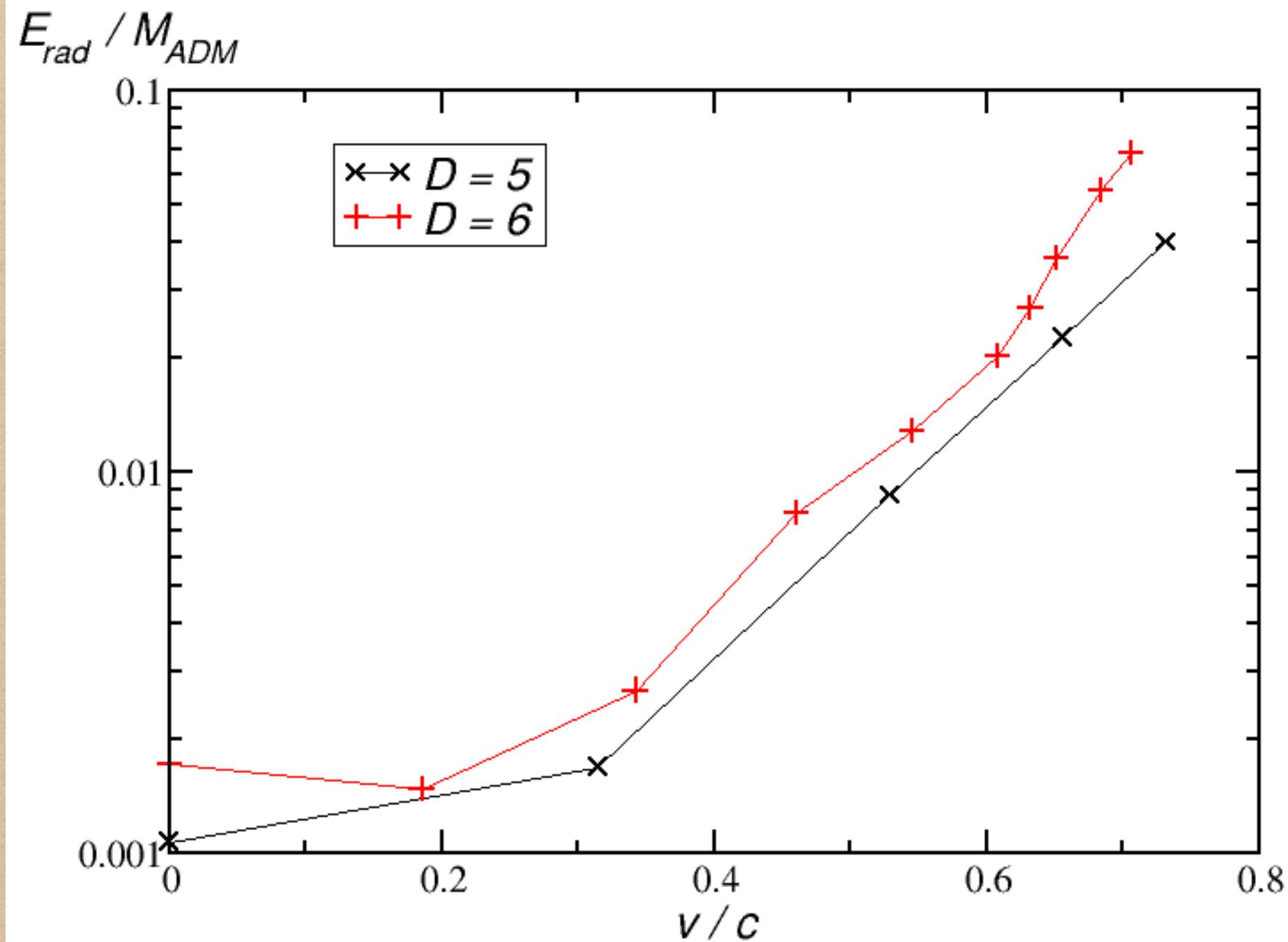
Boosted head-on collision $D = 6$, $v \approx 0.54c$



Boosted head-on collision $D = 6$, $v \approx 0.54c$



Radiated GW energy in head-on collisions



Conclusions

- $D = 4$ well understood
 - Maximal radiation $E_{\text{max}} \approx 0.5 M$
 - Scattering threshold $b_{\text{scat}} \approx \frac{2.5}{v} M$
 - Structure hardly matters
 - Zoom-whirl behavior
- $D > 4$: work in progress
 - Wave extraction works
 - Collisions from rest: Particle limit reliable only at high q
 - Boosted head-on collisions in $D = 5, 6$:
 E_{rad} same ballpark as $D = 4$