Anomalies in the recoil from non-spinning eccentric black-hole binaries

U Sperhake

with M Radia, E Berti, R Croft

DAMTP, University of Cambridge

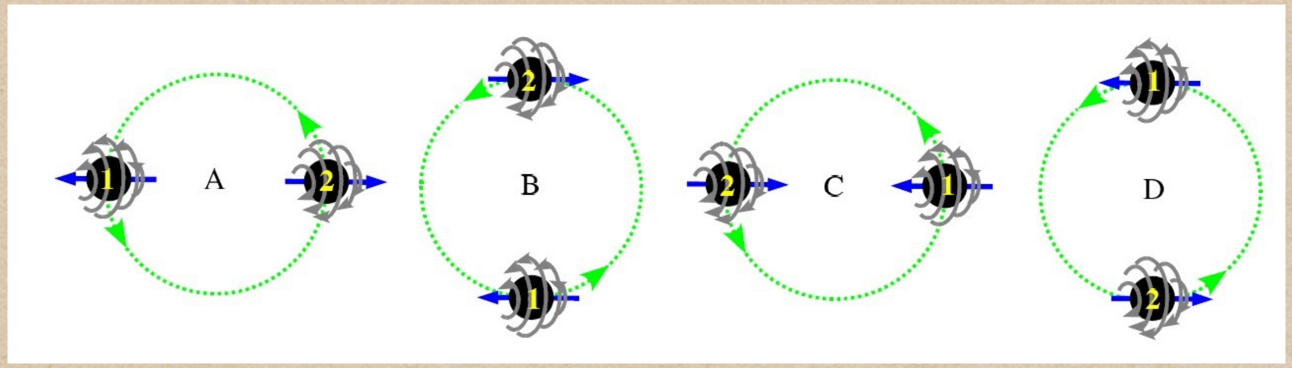
LSC Waveform Call

17 Dec 2017

Original Motivation: Black-hole kicks

- Anisotropic GW emission $\Rightarrow$ recoil of remnant BH
  - Asymmetry through spin; super kick

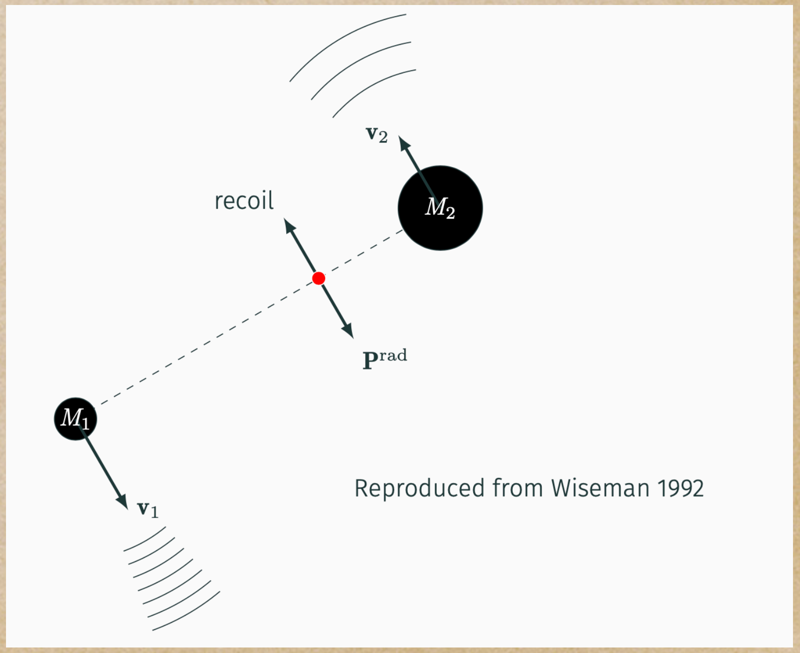
    González et al gr-qc/0702052, Campanelli et al gr-qc/0702133



Pretorius 0710.1338

  - Asymmetry through unequal masses

    González et al gr-qc/0610154



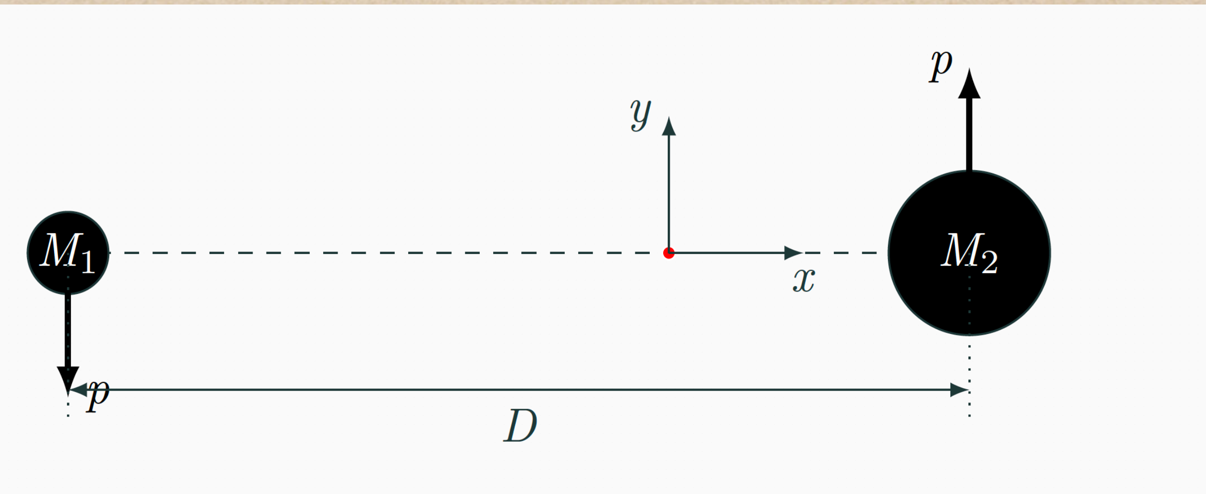
- Kick important for SMBH formation, BH populations, galaxy structure,...
- Eccentricity enhances super kicks

  US et al 1910.01598

  Check the same for unequal-mass kicks

Setup

- Non-spinning BH binaries with masses $M_1 \le M_2\,, \quad q := \frac{M_1}{M_2}$



- Vary $D,\ p$ at const. binding energy $E_b = M_{\mathrm{ADM}} - M_1 - M_2 = \mathrm{const}$
- Four sequences: `sq2:3`, `sq1:2`, `lq1:2`, `sq1:3`

  With 3 mass ratios $q = 2/3, \quad q = 1/2, \quad q = 1/3$
- s = "short" (~ 3 orbits in qc limit), l = "long" (~ 6 Orbits)

  Long sequence `lq1:2` to check for artefacts from short inspiral.
- No rigorous eccentricity estimate in GR

  Use 3PN (harmonic gauge) e_t Memmesheimer at al gr-qc/0407049

⇒

Numerical time evolutions

- GRChombo Clough et al 1503.034036 : sq2:3, sq1:3
  - CCZ4 formulation
  - Fully adaptive MR through Chombo
- Lean
  - BSSN formulation US gr-qc/0606079 : sq1:2, lq1:2
  - Box-in-a-box MR through Carpet
  - Based on Cactus, AHs from AHFinderDirect
- Initial data from Ansorg et al gr-qc/0404056 (TwoPunctures)
- Codes compared for qc $q = 1/2$
  - Both codes convergent $\sim 4\mathrm{th}$ order
  - Total error $(dx,\ R_{\mathrm{ex}})$: $\lesssim 4\,\%$
  - Agreement $\approx 1\,\%$

Diagnostics

- Radiated energy E_{rad}, momenta. $\boldsymbol{P}_{\mathrm{rad}}$, $\boldsymbol{J}_{\mathrm{rad}}$ from Ψ_4 ; e.g.

$$\boldsymbol{P}_{\mathrm{rad}}(t) = \lim_{r\to\infty} \frac{r^2}{16\pi} \int_{t_0}^{t} \mathrm{d}t' \oint_{S_r^2} \mathrm{d}\Omega\, \hat{\boldsymbol{e}}_r \left| \int_{-\infty}^{t'} \mathrm{d}t''\, \Psi_4 \right|^2$$

- Total recoil from $\boldsymbol{v} = -\boldsymbol{P}_{\mathrm{rad}}/M_{\mathrm{fin}}$

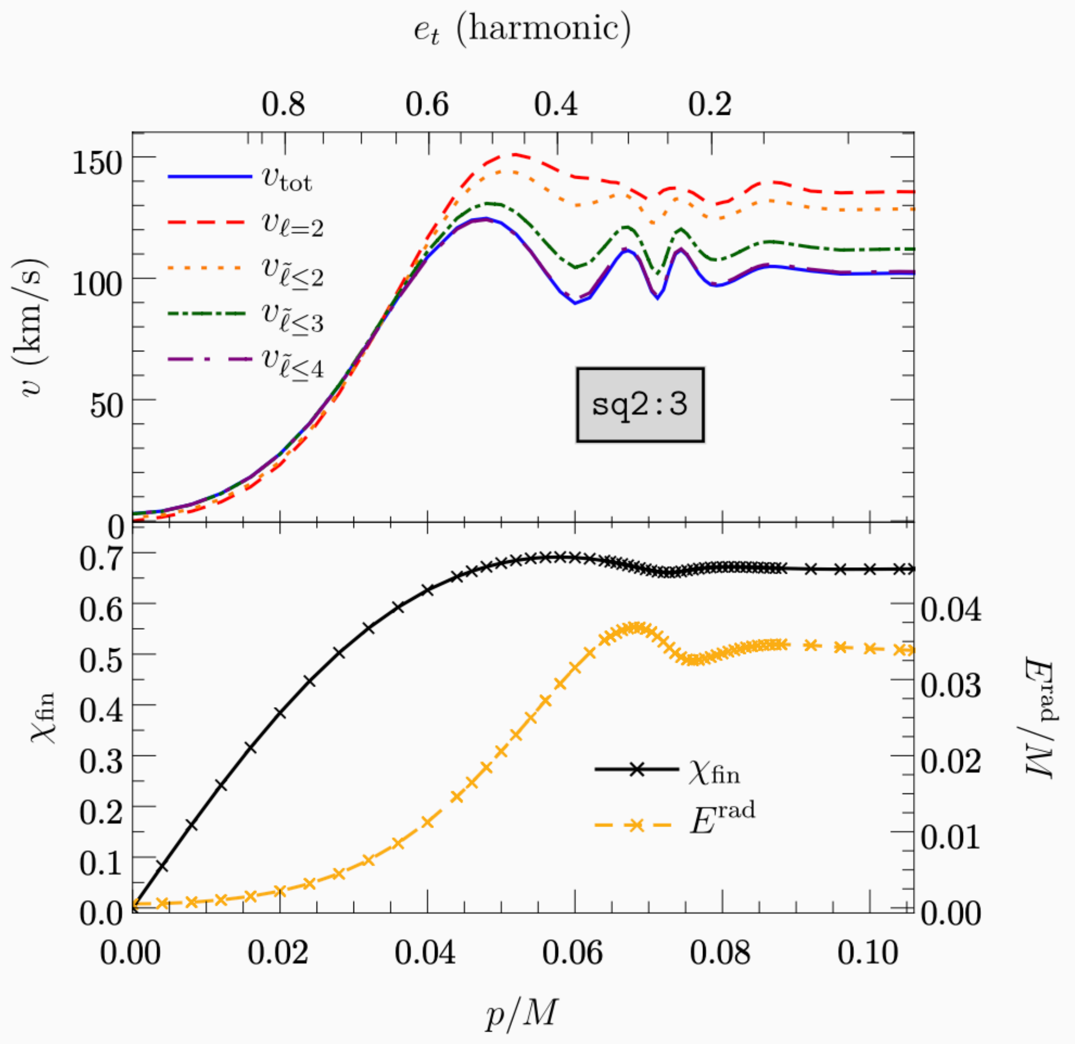
- Also computed partial kick from Ψ_4 multipoles

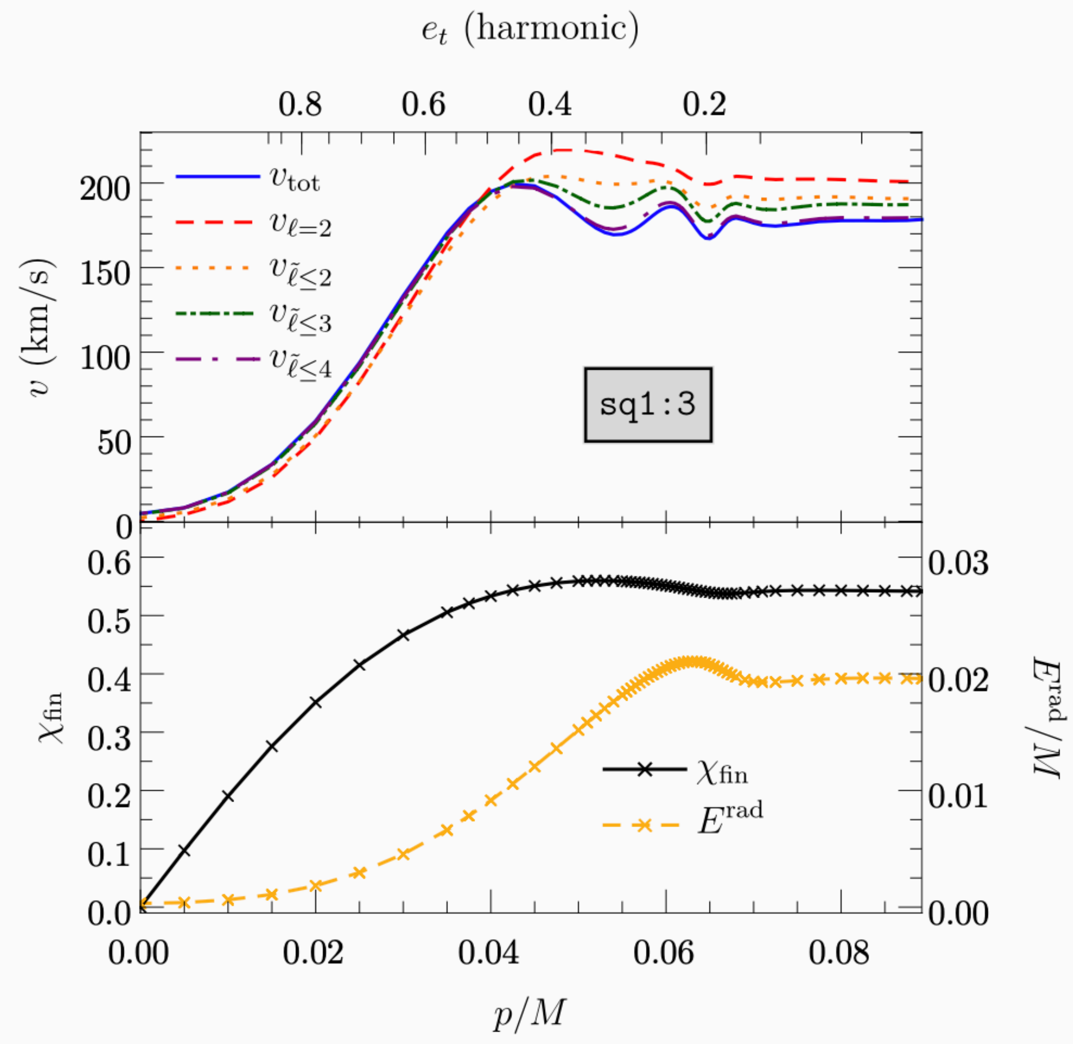
$$P_x^{\mathrm{rad}}(t) + \mathrm{i}P_y^{\mathrm{rad}}(t) = \lim_{r\to\infty} \frac{r^2}{8\pi} \sum_{\tilde{\ell}=2}^{\infty} \sum_{\tilde{m}=-\tilde{\ell}}^{\tilde{\ell}} \int_{t_0}^{t} \mathrm{d}t' \left\{ \left(\int_{-\infty}^{t'} \mathrm{d}t'' \psi_{\tilde{\ell},\tilde{m}} \right) \right.$$
$$\left. \times \left(\int_{-\infty}^{t'} \left[a_{\tilde{\ell},\tilde{m}} \bar{\psi}_{\tilde{\ell},\tilde{m}+1} + b_{\tilde{\ell},-\tilde{m}} \bar{\psi}_{\tilde{\ell}-1,\tilde{m}+1} - b_{\tilde{\ell}+1,\tilde{m}+1} \bar{\psi}_{\tilde{\ell}+1\tilde{m}+1} \right] \mathrm{d}t'' \right) \right\}$$

Ruiz et al 0707.4654

- By symmetry $P_z^{\mathrm{rad}} = 0$

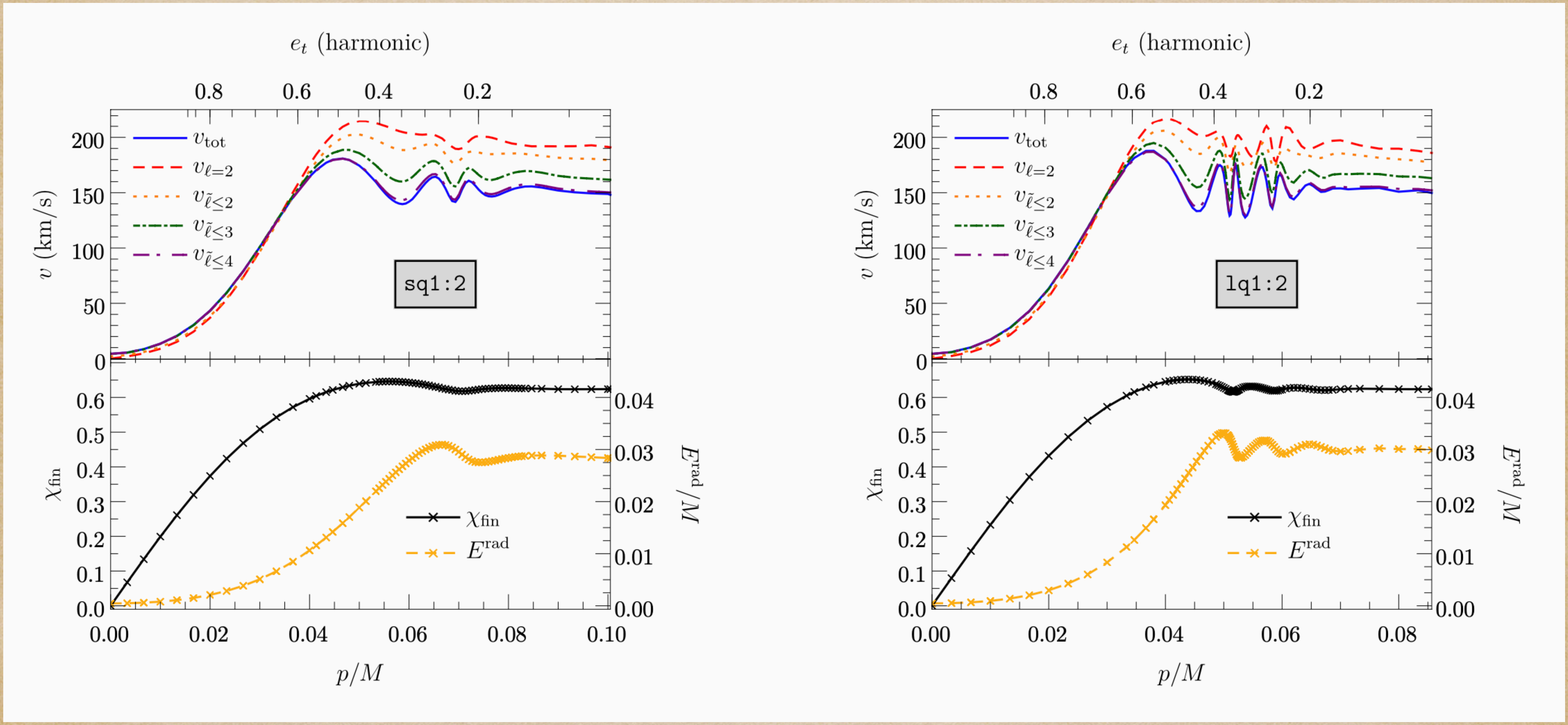
Results: sq2:3, sq1:3





- Oscillatory dependence on eccentricity!

Results: sq1:2, lq1:2



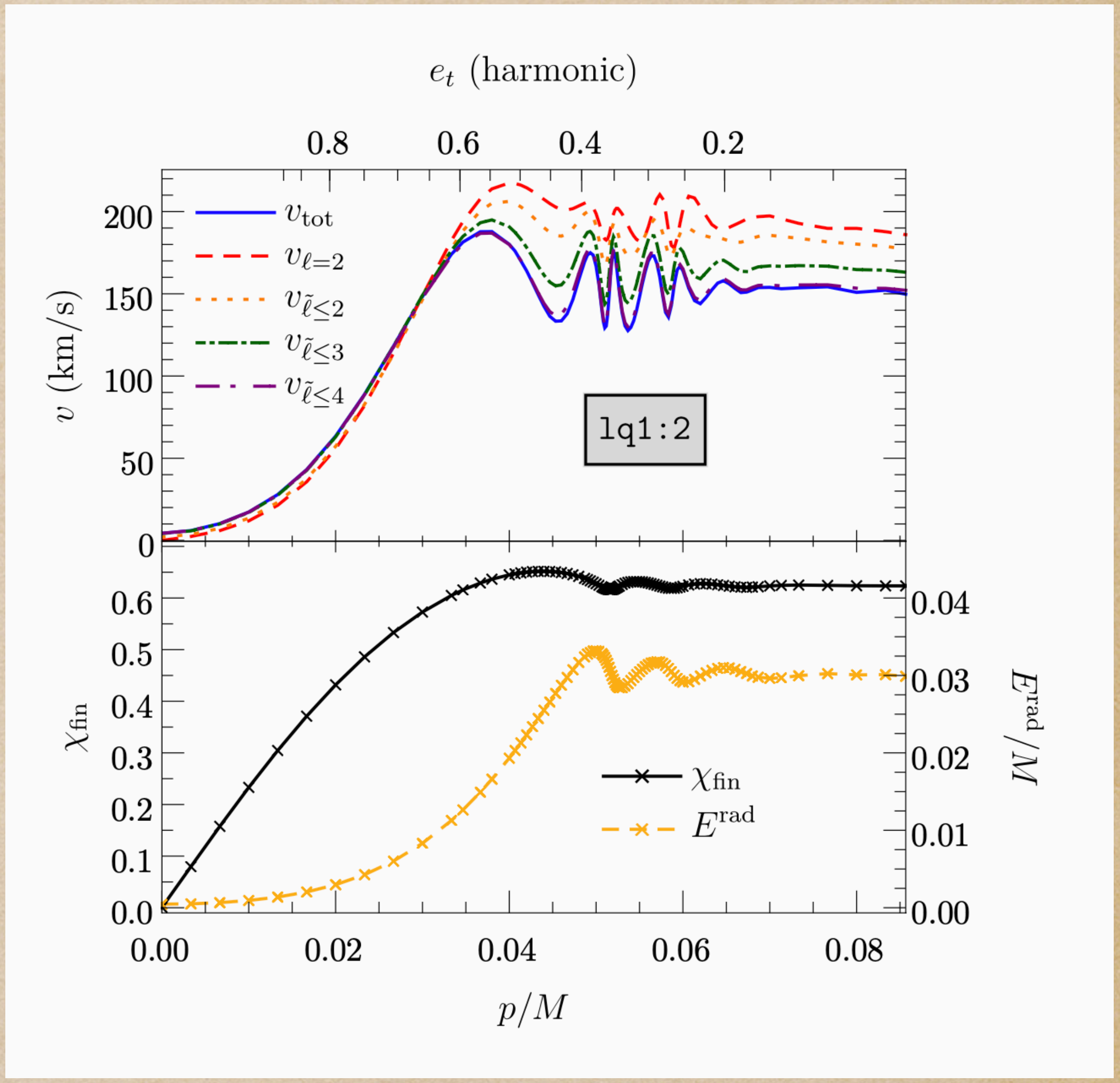
- Oscillatory dependence on eccentricity!

Summary of observations

- Max kick at $e_t \sim 0.5$

  Exceeds qc kick by

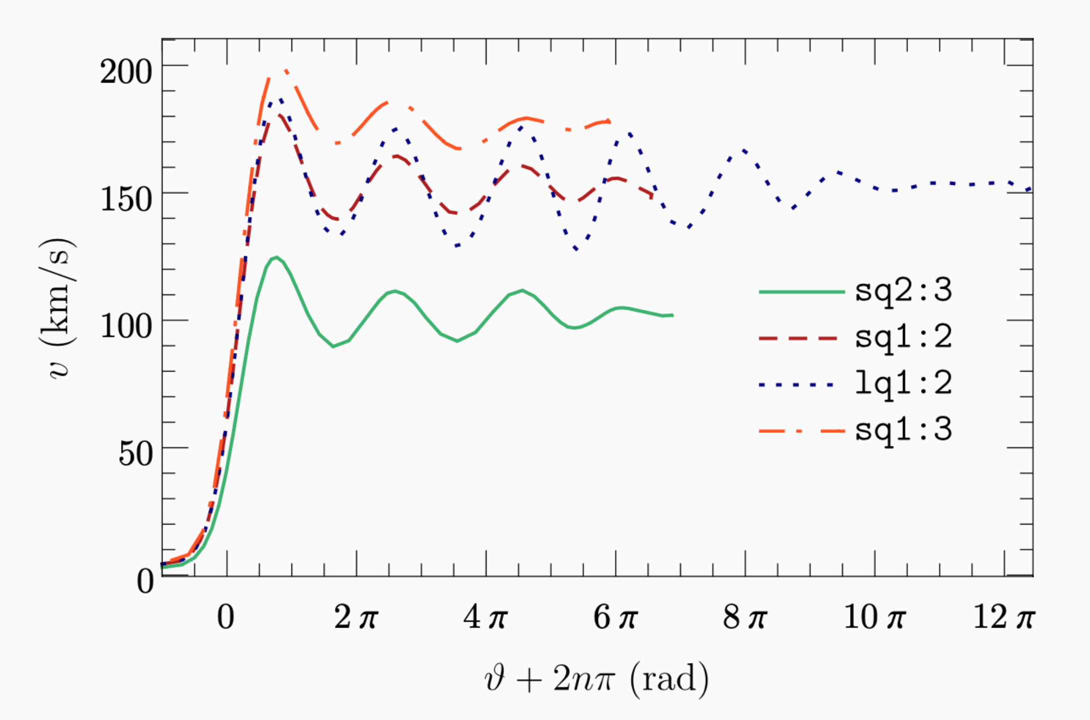
sq2:3	sq1:2	lq1:2	sq1:3
22 %	22 %	25 %	12 %

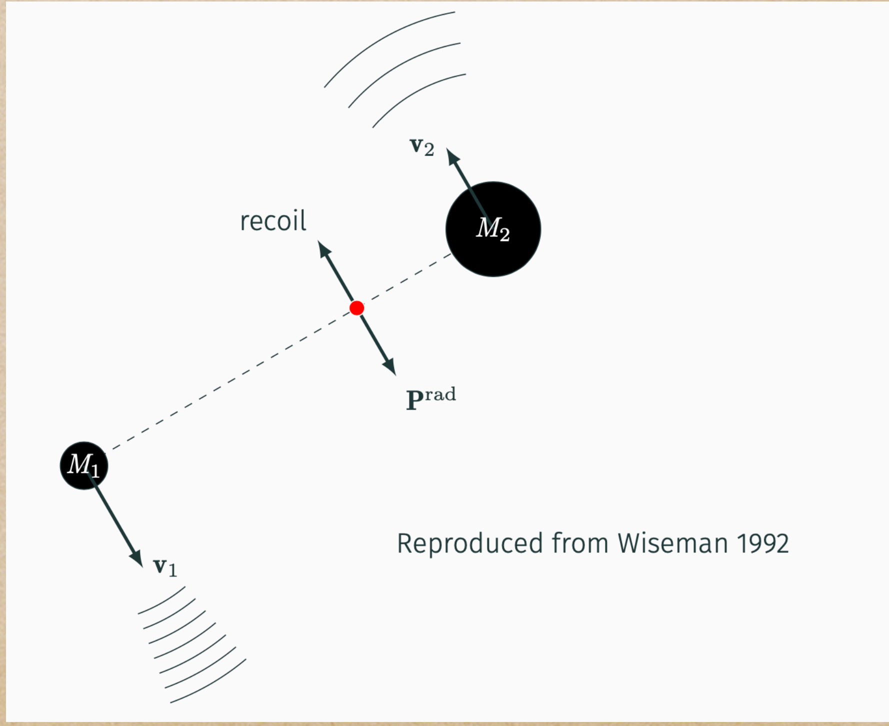


- Oscillatory variation increases in frequency, magnitude for longer inspiral; high sensitivity for long inspirals?
- Fewer/less pronounced Oscillations in E_{rad}, χ_{fin} ; Extrema not aligned.
- Oscillations in all partial recoils $v_{\tilde{\ell}\leq\tilde{\ell}_0}$
- Higher-order terms $v_{\tilde{\ell}>2}$ systematically reduce kick

Interpretation

- Only "special" direction: apoapsis
- Goal: Measure BH infall direction relative to apoapsis
- Problem: neither rigorously defined. So approximate apoapsis $\approx x$ Axis; infall $\sim$ kick direction (beaming!)
- Ideally expect 2π periodicity of $v_{\rm kick}(\vartheta)$
- Must not forget apsidal precession (Mercury!), so only $\lesssim 2\pi$

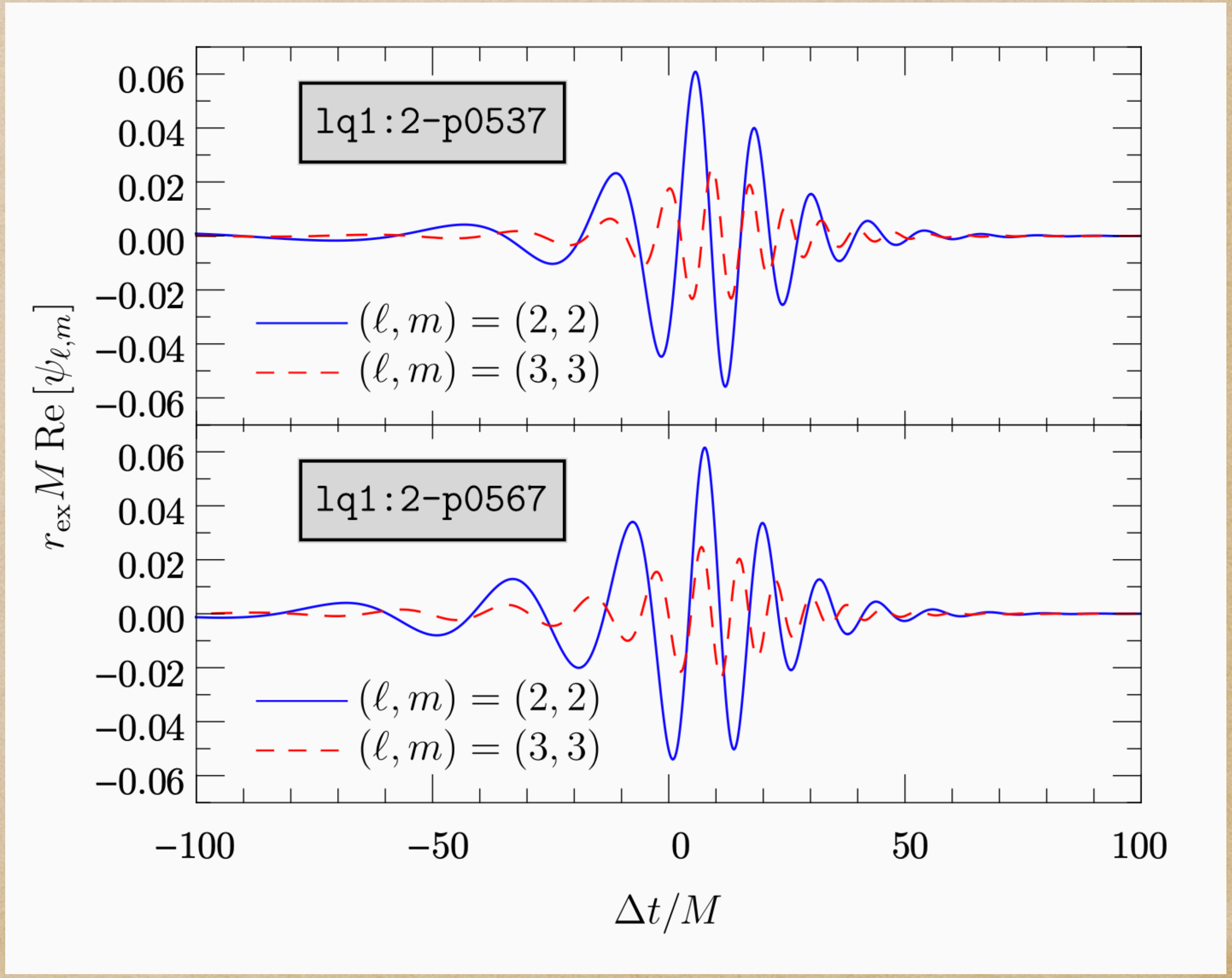




First glimpse at waveform features

- Kick arises from overlap of multipoles!
- So expect features in $\psi_{\ell' m'}$ **relative** to $\psi_{\ell m}$
- Example: ψ_{33} versus ψ_{22} for:
- Better alignment for large kick
- Similar pattern for other $v_{\min} - v_{\max}$ configurations and other multipoles

lq1:2-p0537	lq1:2-p0567
$v = 128\,\mathrm{km/s}$	$v = 173\,\mathrm{km/s}$



Summary

- Measured $v_{\rm kick}$ as function of eccentricity for non-spinnning BBHs in a total of 274 BBH simulations.
- Max kick at $e_t \sim 0.5$; $\approx 12-25\,\%$ larger than qc result.
- Additional oscillations in $v_{\rm kick}(e_t)$
- Oscillations stronger and more rapid in long inspirals. Suggests that kick and GW sensitively depend on, e_t i.e. Infinitesimal ${\rm d}e_t$ may cause finite change in $v_{\rm kick}$, $\psi_{\ell m}$
- Kick variation due to angle of infall vs. apsidal direction
- Manifestation in GW signals: phasing $\psi_{\ell>2,m}$ vs. ψ_{22}