Boson-Star Binaries and Gravitational Waves

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Frontiers in Numerical Relativity (FNR 2022)

University of Jena, 28 July 2022

Who really deserves the credit...

Robin
Croft

Thomas
Helfer

Bo-Xuan
Ge

Miren
Radia

Talk at 18:00

Initial data for unequal-mass boson-star collisions

Tamara
Evstafyeva

Eugene
Lim

Katy
Clough

Interlocutor

$\subset$ GRChombo https://www.grchombo.org

Overview

- Introduction and Motivation
- Gravitational Afterglow of Boson Stars
- (Tumbling) Towards accurate Boson-Star Binary Waveforms
- Pan-pan!
- Conclusions

1. Introduction and motivation

The idea of boson stars

- "Gravitational-electromagnetic entities" or Geons

Wheeler 1955

PHYSICAL REVIEW VOLUME 97, NUMBER 2 JANUARY 15, 1955

Geons*

JOHN ARCHIBALD WHEELER
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received September 8, 1954)

Associated with an electromagnetic disturbance is a mass, the gravitational attraction of which under appropriate circumstances is capable of holding the disturbance together for a time long in comparison with the characteristic periods of the system. Such

of equations of self-consistent geon; mass and radius values. 4. Transformations and interactions of electromagnetic geons; evaluation of refractive index barrier penetration integral for spherical geon; photon-photon collision processes as additional

- Energy = mass gravitates $\rightarrow$ Compact (equilibrium?) objects
- Geons are not equilibrium configurations
- Dark matter candidates: QCD axions, ALPs, dark photons,…
- Complex fields (scalar, vector,…)

  $\rightarrow$ Genuine equilibrium states; $T_{\alpha\beta}$ stationary!
- First shown for scalar fields $\rightarrow$ "Boson stars"

Feinblum & McKinley PR 168 (1968), Kaup PR 172 (1968),

Ruffini & Bonazzola PR 187 (1969)

A boson star zoo

- Mini BSs (no self-interaction) Kaup PR (1968) and others
- "Solitonic" BSs (self-interacting scalar field) $\rightarrow$ more compact

  Colpi+ PRL (1986), Lee PRD (1987), ...
- Proca stars Brito+ Phys.Lett.B (2016)
- ℓ -boson stars (multiple scalar fields) Alcubierre+ CQG (2018)
- Multi-oscillating BSs Choptuik+ PRL (2019)
- Thin-shell BSs (one scalar with false vacuum state)

  Collodel & Doneva 2203.08203
- Higher-spin fields Jain & Amin 2109.04892
- Multi-field BSs Sanchis-Gual+ PRL (2021)
- May condense from local over-densities Widdicombe+ JCAP (2018)

Focus here: Single-scalar, solitonic and mini BSs

Formalism and basic features

- GR + minimally coupled complex scalar field φ

$$S = \int \sqrt{-g} \left\{ \frac{1}{16\pi G} R - \frac{1}{2} [g^{\mu\nu} \nabla_\mu \bar{\varphi} \nabla_\nu \varphi + V(\varphi)] \right\} \mathrm{d}x^4$$

$$T_{\alpha\beta} = \partial_{(\alpha} \bar{\varphi} \, \partial_{\beta)} \varphi - \frac{1}{2} g_{\alpha\beta} [g^{\mu\nu} \partial_\mu \bar{\varphi} \, \partial_\nu \varphi + V(\varphi)]$$

- Potential; analogous to EOS:

$$V_{\mathrm{min}}(\varphi) = m^2 |\varphi|^2 \,, \qquad V_{\mathrm{soli}}(\varphi) = m^2 |\varphi|^2 \left(1 - 2 \frac{|\varphi|^2}{\sigma_0^2} \right)^2 \,, \qquad \text{or} \ldots$$

- Spherically symmetric equilibrium models

Ansatz: $\varphi(t, r) = A(r) e^{\mathrm{i}\omega t}$

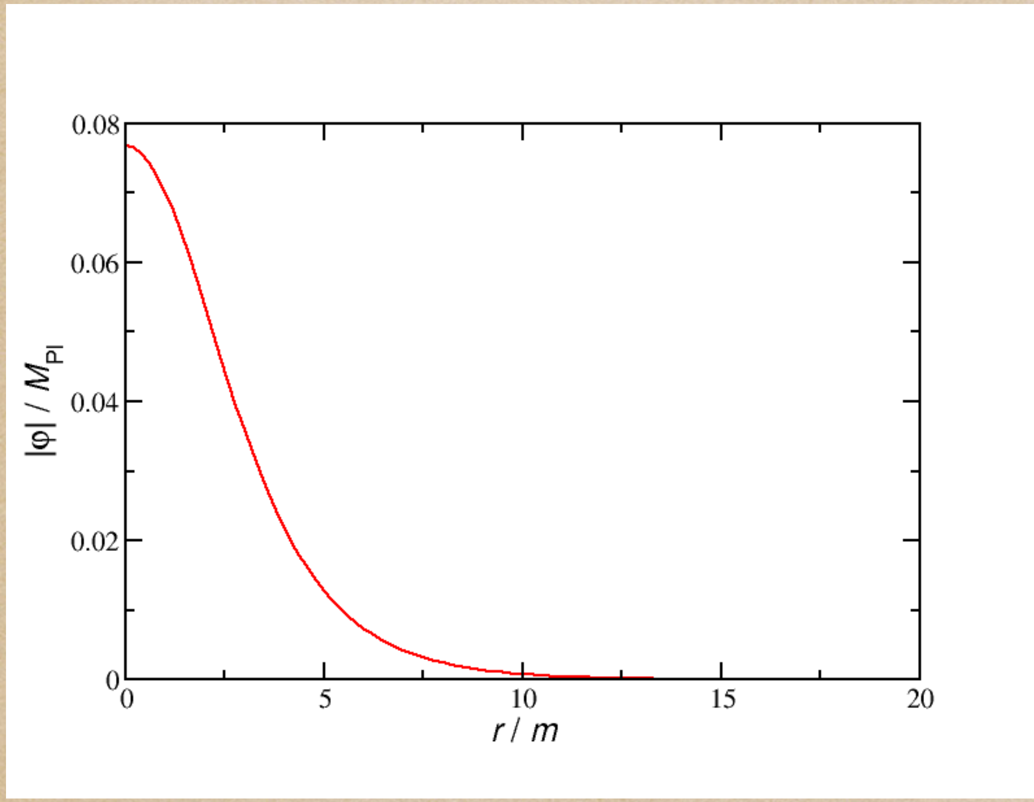
Regular solutions only for countably infinite values

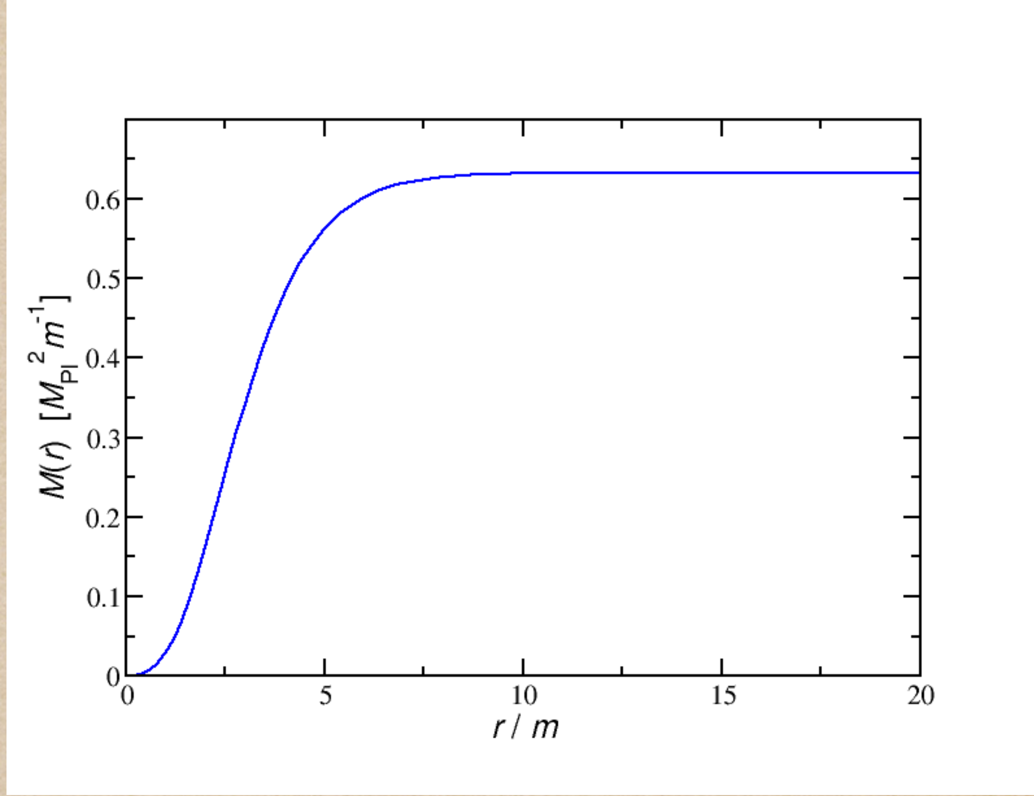
$\omega_0 < \omega_1 < \omega_2 < \ldots$ (ground state, excited states)

Formalism and basic features

- E.g. Maximal-mass mini boson star (Kaup limit)

$$\omega_0 = 0.853\, m\,, \qquad M = 0.633\, M_{\rm Pl}^2/m$$





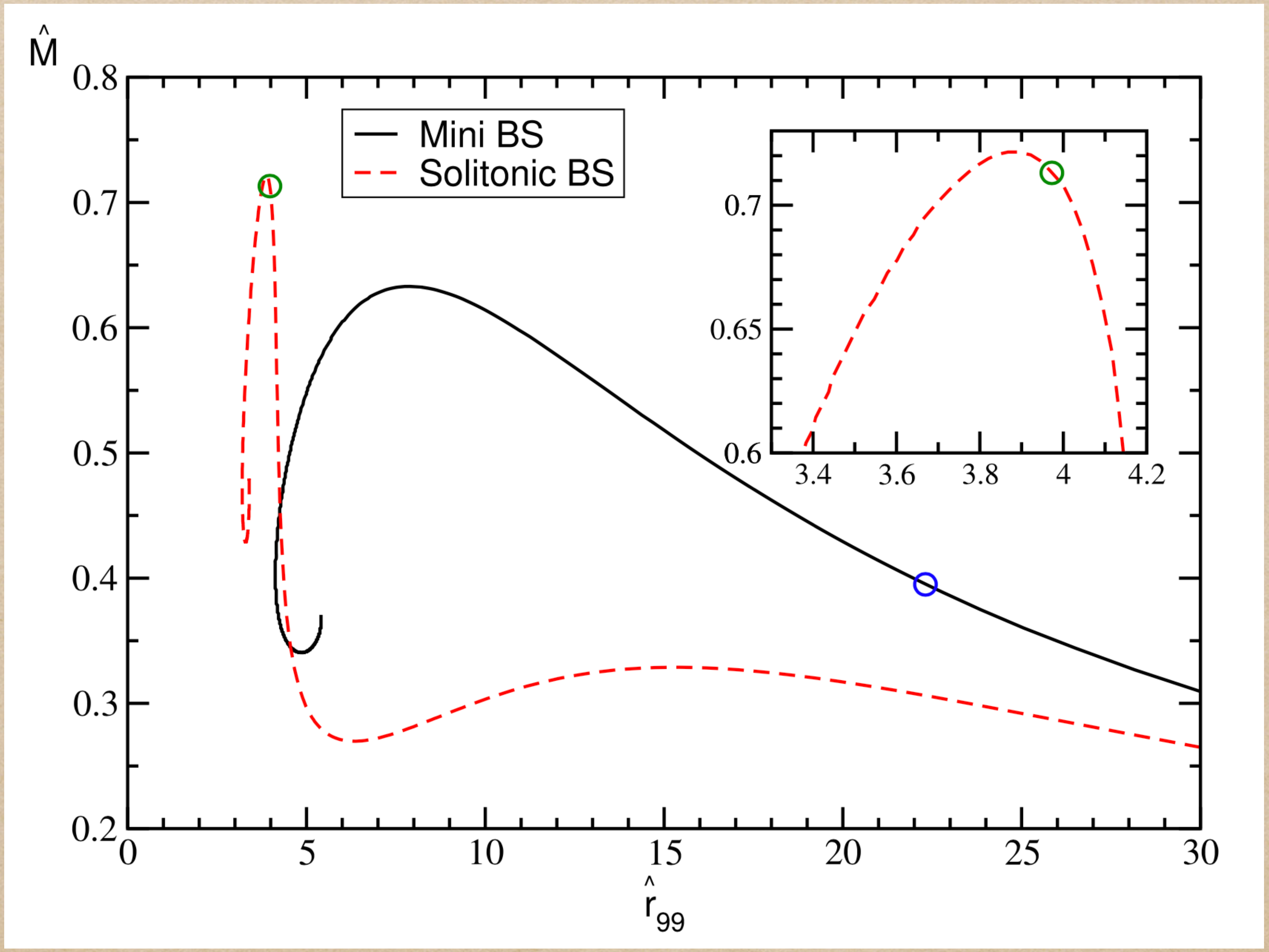
- Excited states unstable:

collapse to BH, dispersion or migration to stable ground-state BS

Balakrishna, Seidel, Suen PRD (1998)

Formalism and basic features

- Mass-Radius curves similar to Tolman-Oppenheimer-Volkoff stars



Formalism and basic features

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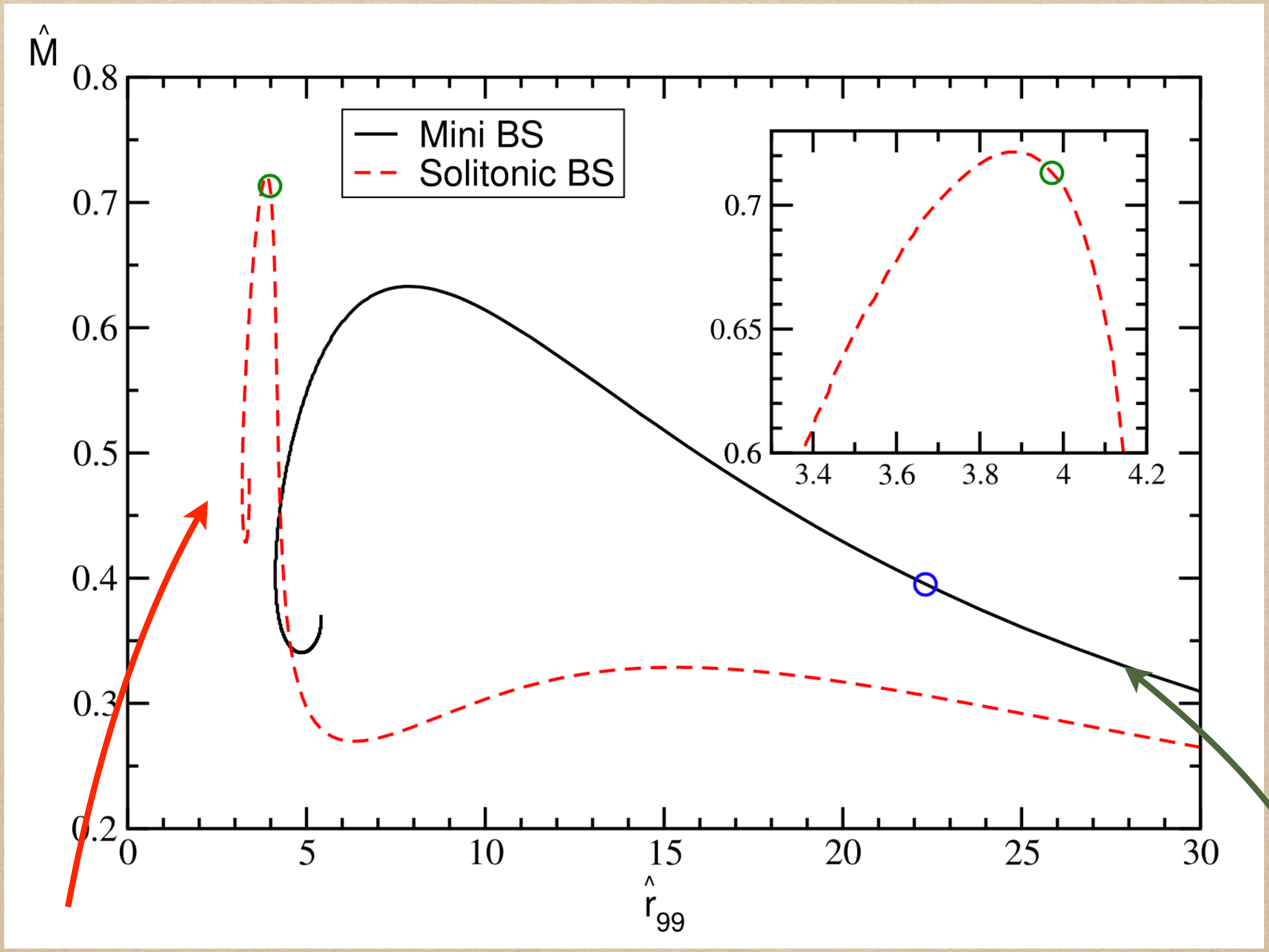


unstable

stable

Spinning Boson Stars

- Scalar BSs cannot spin perturbatively Kobayashi+ PRD (1994)
- Spinning scalar BSs exist with but have quantized spin

  Schunck & Mielke Phys.Lett.A (1998)
- Spinning scalar BSs likely unstable in contrast to

  spinning Proca stars! Sanchis-Gual+ PRL (2019)

  Possibly due to toroidal structure: scalar field vanishes at origin
- What happens in scalar BS inspiral and merger?
  - Kerr BH
  - Non-spinning BS; angular momentum shed
  - Total dispersal
  - Spinning BS with exact angular momentum?

2. Gravitational Afterglow of BSs

The Configuration

Croft, Helfer, Ge, Radia, Evstafyeva, Lim, US & Clough   2207.05690

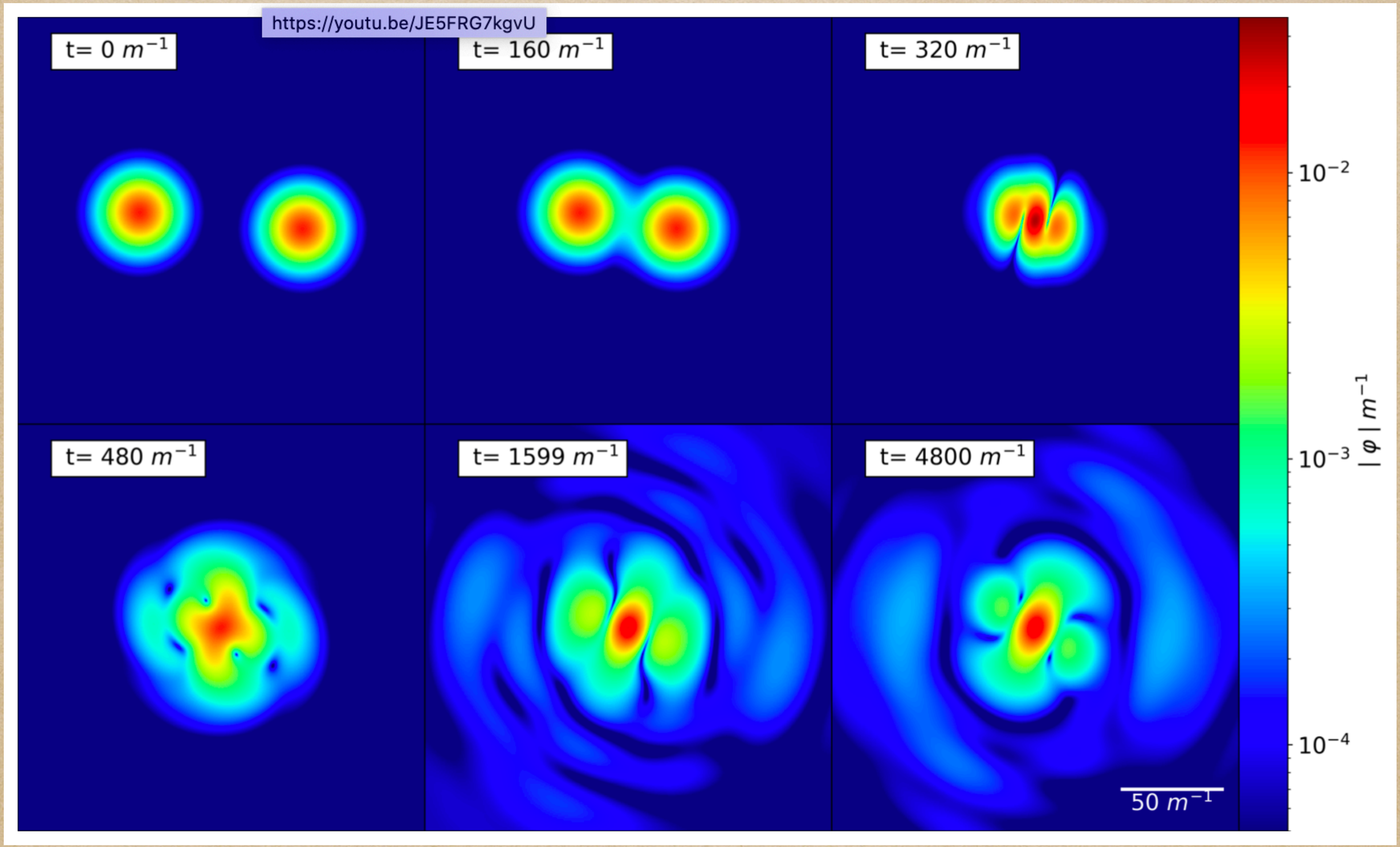
- Equal-mass eccentric (grazing) collision of two mini BSs of compactness ~ 0.025

	Run	N	$d_{\text{init}}\ [m^{-1}]$	$b\ [m^{-1}]$	v_x	$M\ [M_{\text{Pl}}^2\ m^{-1}]$
low	1	256	80	8	0.1	0.395(0)
medium	2	320	80	8	0.1	0.395(0)
high	3	384	80	8	0.1	0.395(0)
ultra-high	4	448	80	8	0.1	0.395(0)

- GRChombo code   Radia+ CQG (2022),   Andrade+ JOSS (2022)
  - Full AMR
  - CCZ4 formulation
  - Moving puncture gauge
- Initial data improved superposition: No Malaise!

Helfer+ PRD (2018), CQG (2021),   *Tamara's talk at 18:00*

Snapshots of the time evolution

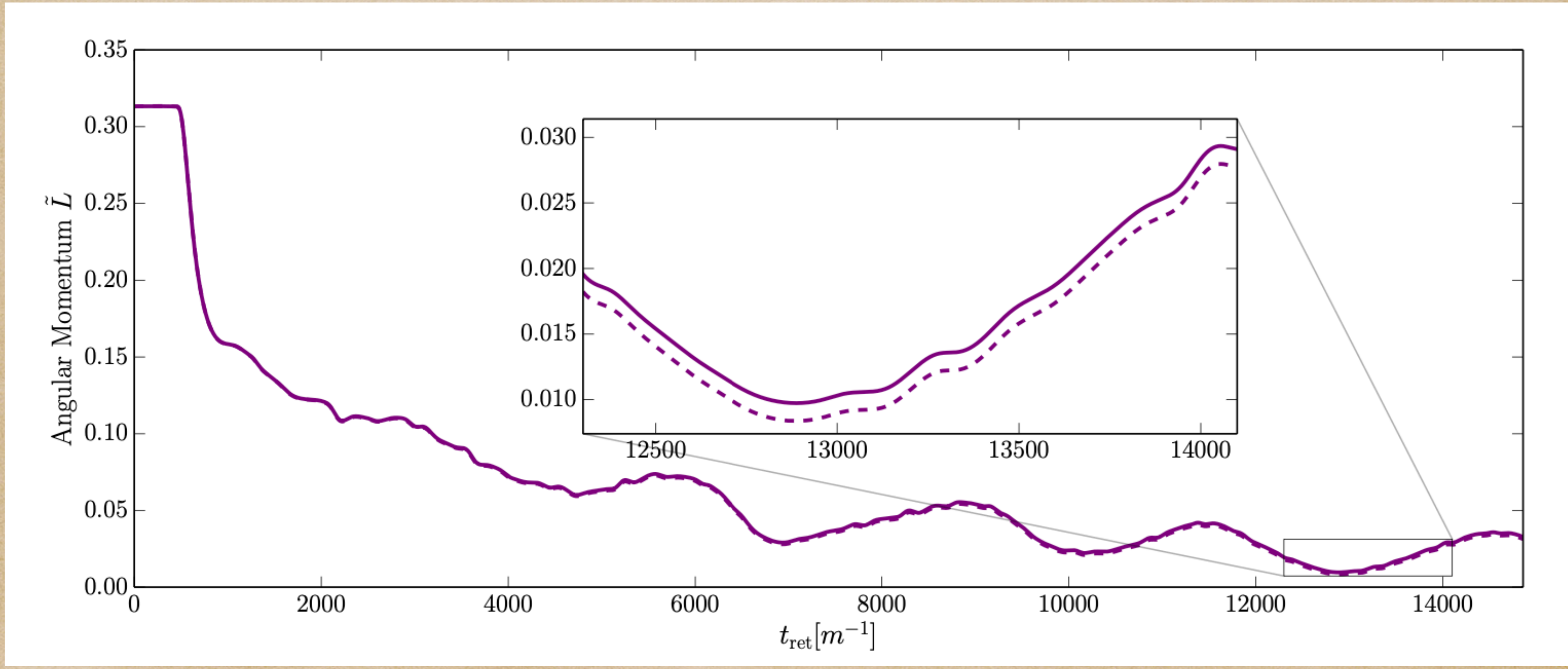


The merger remnant

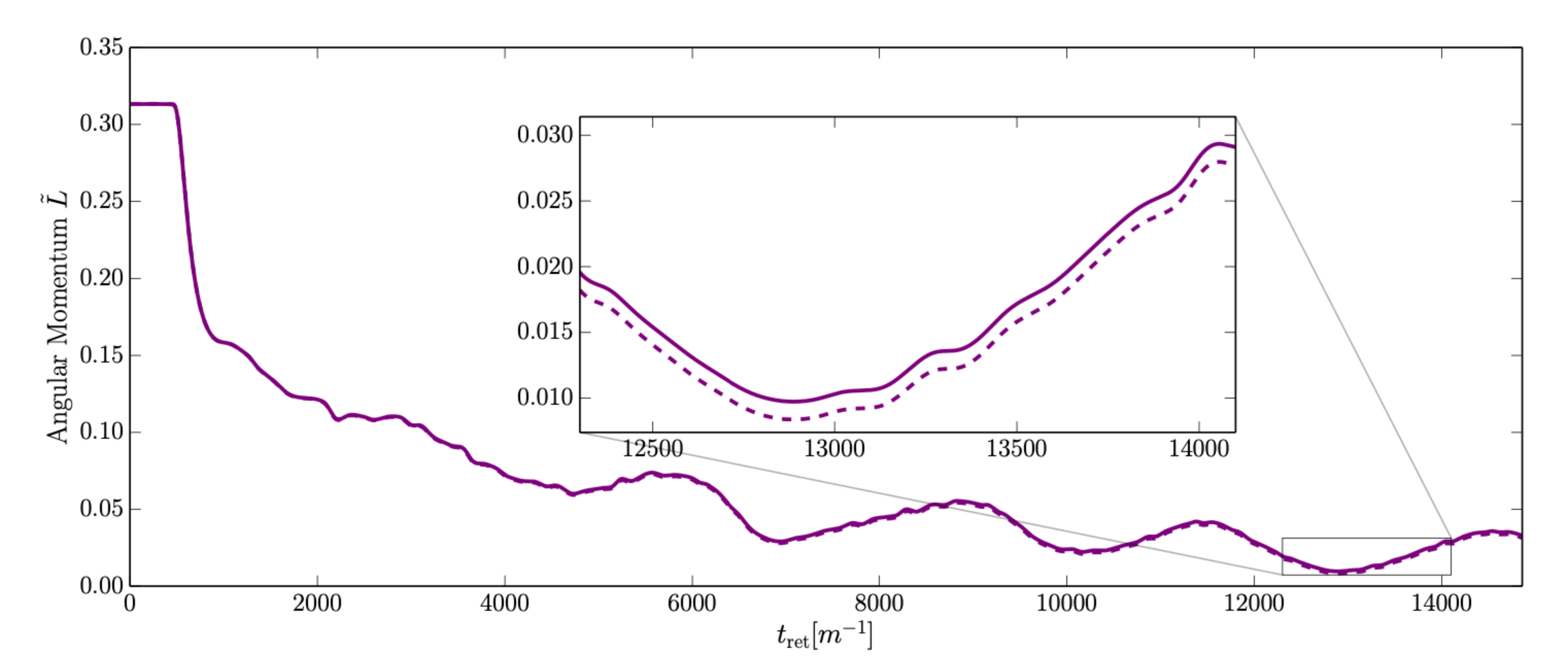
- Boson-star like remnant
- Does it spin?

  Compute Angular-momentum measure including curvature!

  Croft gr-qc/2203.13845; see also Clough CQG (2021)
- L inside sphere of $r \leq 60\ m^{-1}$



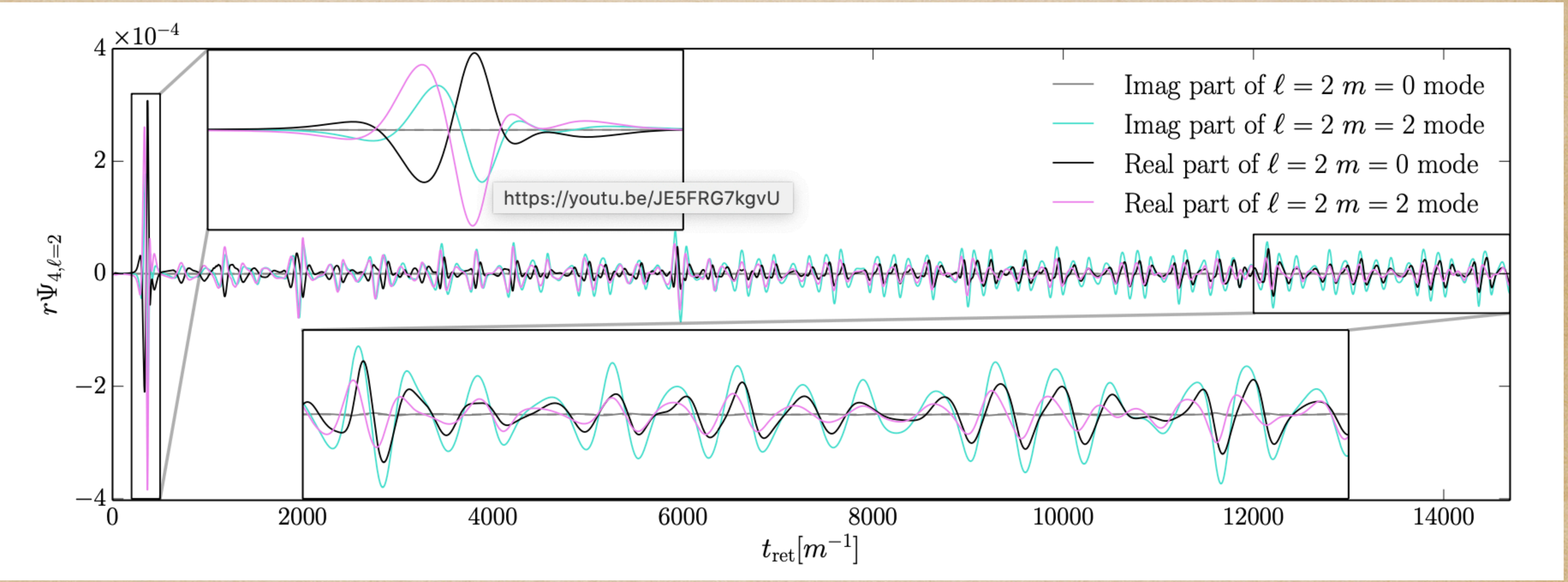
The merger remnant



- Leading order decay: exponential with $t_{\rm half} \approx 4\,000\ m^{-1}$
  - $m = 10^{-14}$ eV $\Rightarrow$ $\approx$ LISA band: $t_{\rm half} \approx 4\,{\rm min}$
  - $m = 10^{-25}\,$eV $\Rightarrow$ $t_{\rm half} = \mathcal{O}({\rm Myr})$
- Oscillatory part due to dynamics of post-merger remnant

The Gravitational-Wave Signal

- $(2,2),\ (2,0)$ multipoles of Ψ_4 at $r_{\rm ex} = 220\ m^{-1}$



The Gravitational-Wave Signal

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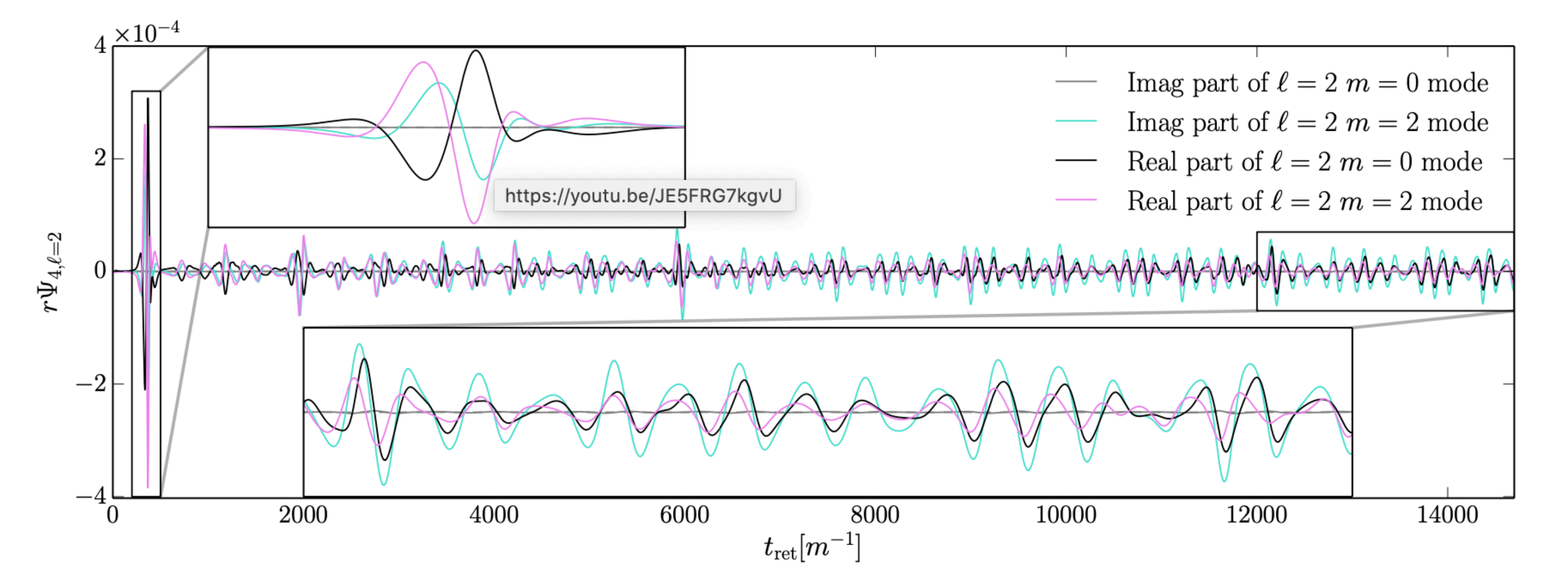
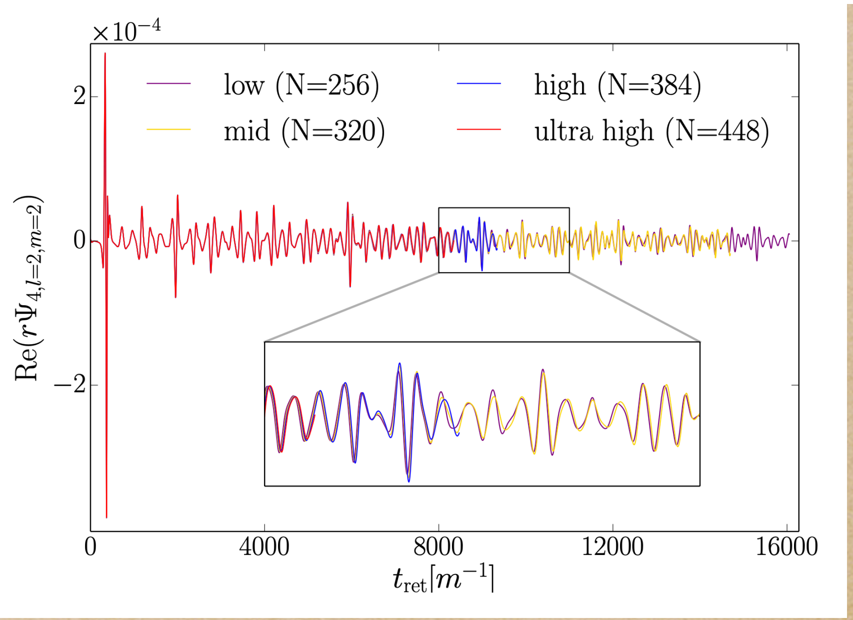


- This is resolved!
- Somewhat similar to GWs from Neutron-Star mergers



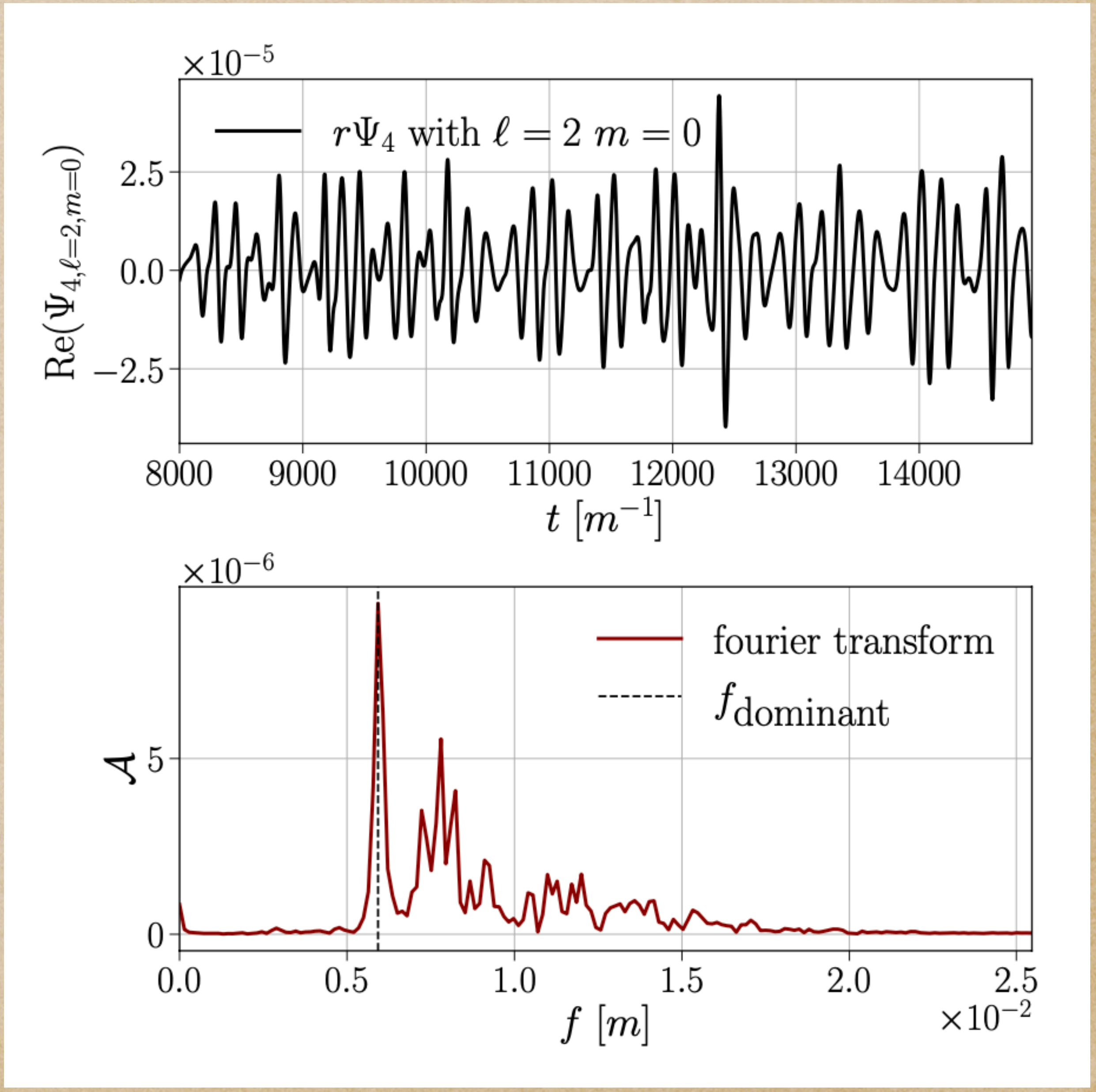
Power Spectrum

- Compute Fourier spectrum of afterglow $(2,0)$ multipole
- $(2,2)$ multipole looks very similar
- Dominant mode $f \approx 0.006\, m$
- Signs of beating
- For reference:

  $10^{-14}\,\mathrm{eV} \approx 2.42\,\mathrm{Hz}$
- Cf. Palenzuela+ PRD (2017) :

  Fundamental frequency

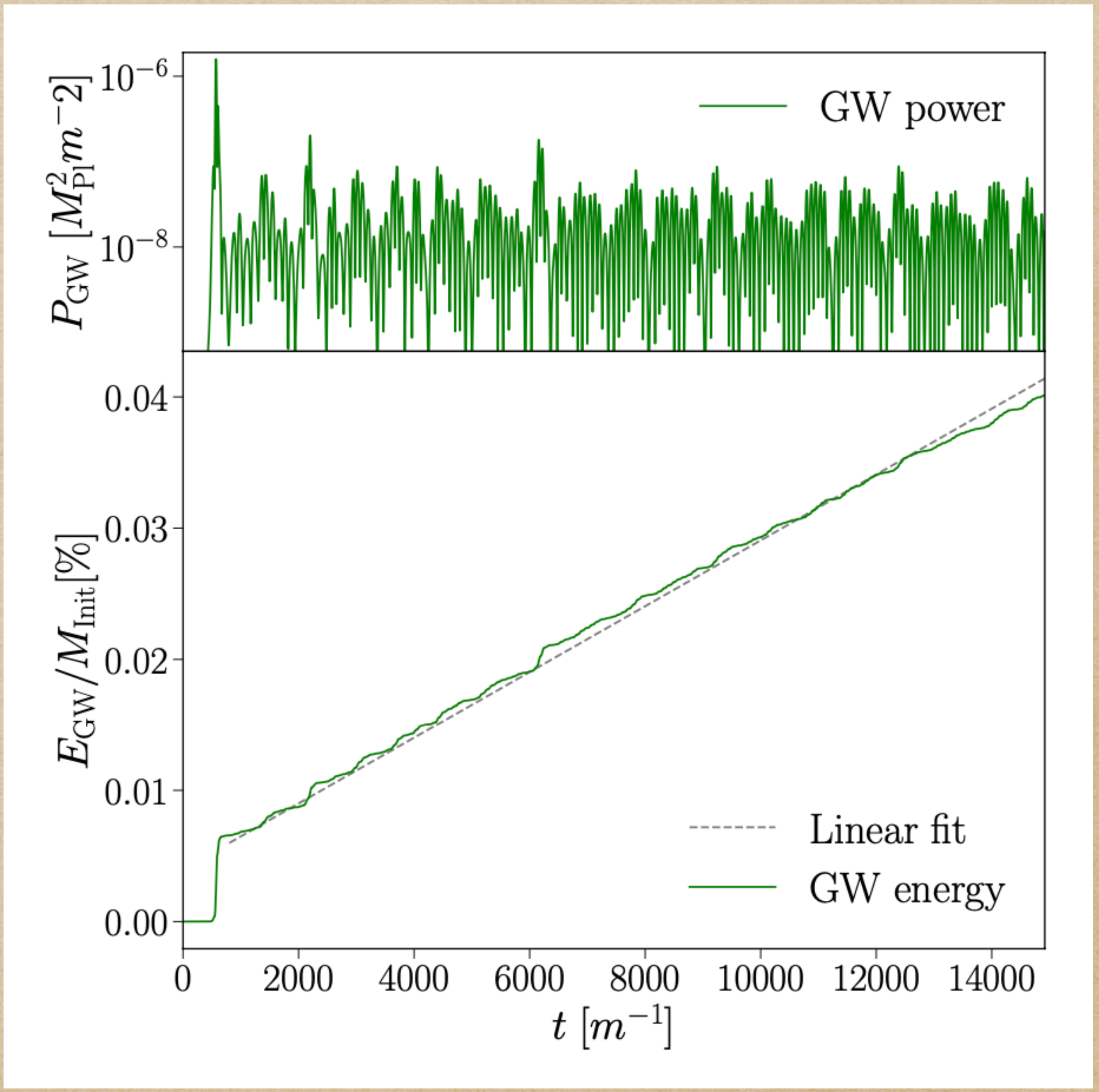
  of remnant



Radiated energy

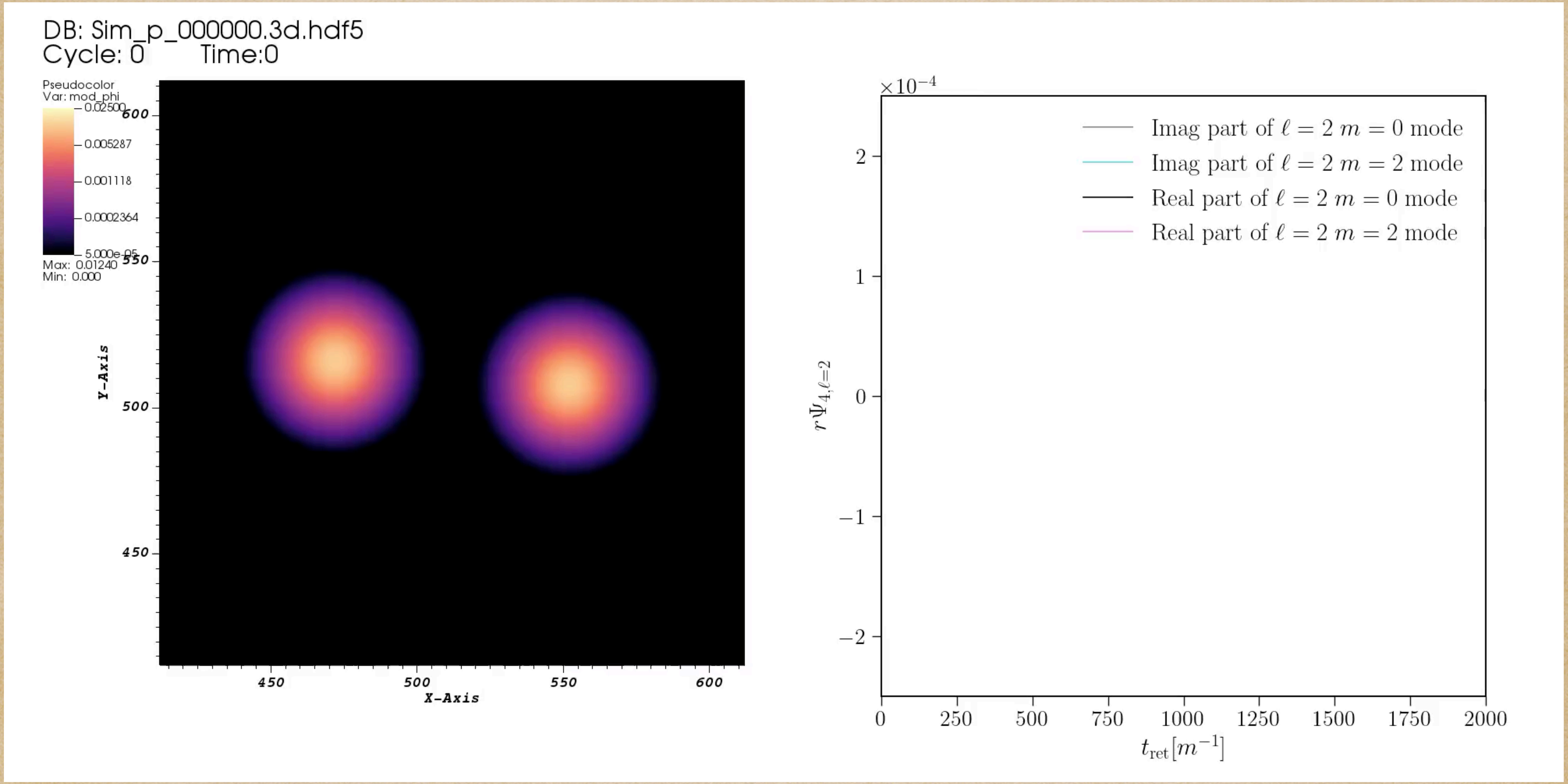
- Radiated GW energy and power
- Power peak at merger
- Barely drops in afterglow
- $E_{\rm tot}/M \approx 4 \times 10^{-4}$

  (squishy BSs)
- But keeps growing!!!



Animation

Credits: Thomas Helfer



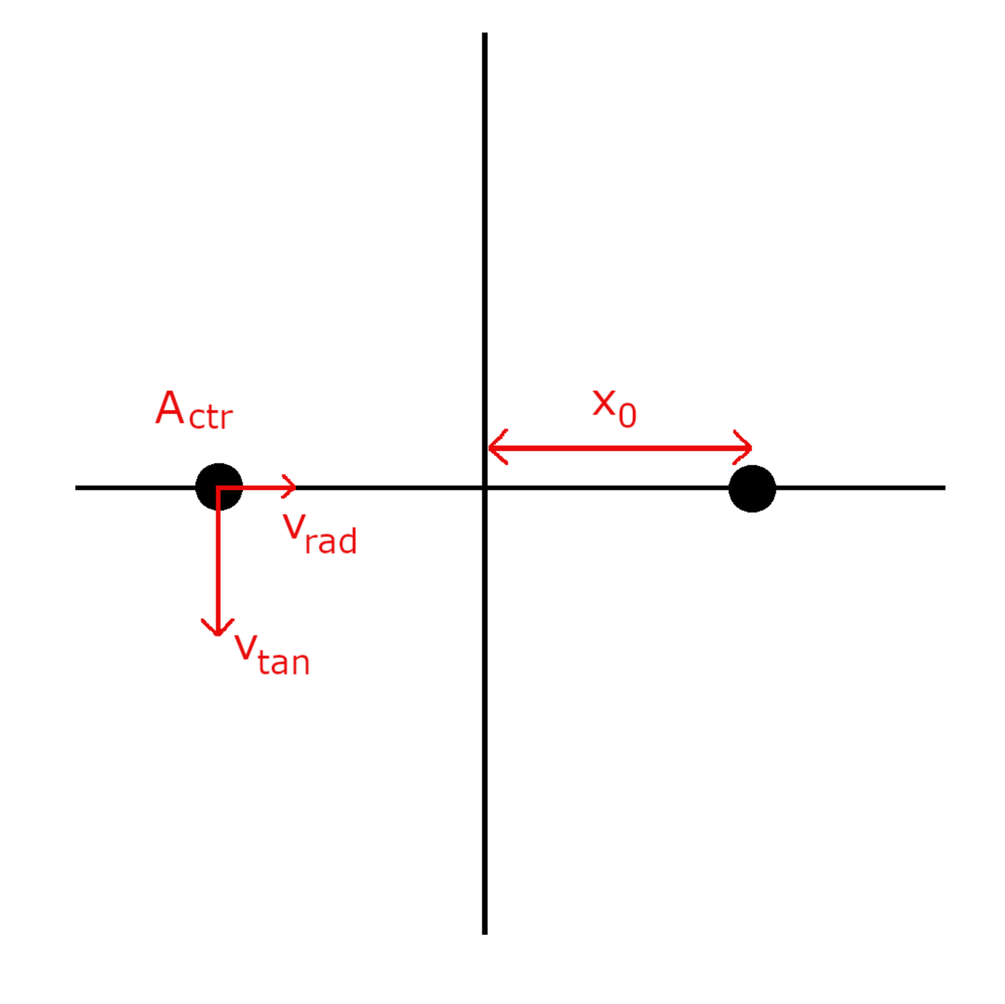
- $(2,2)$ and $(2,0)$ multipoles synchronize as ang.momentum drops.

  Coincidence or causal relation?

3. Binary BS inspiral

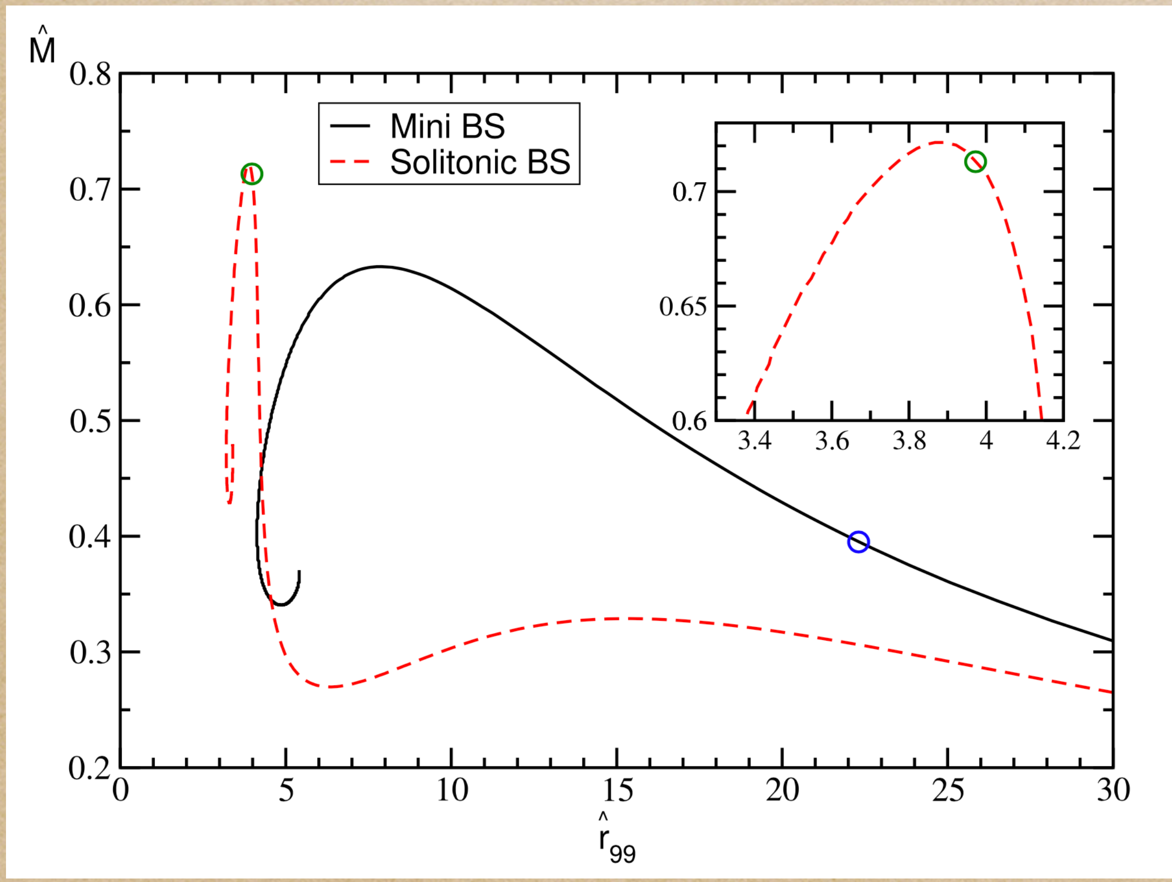
BS binary setup

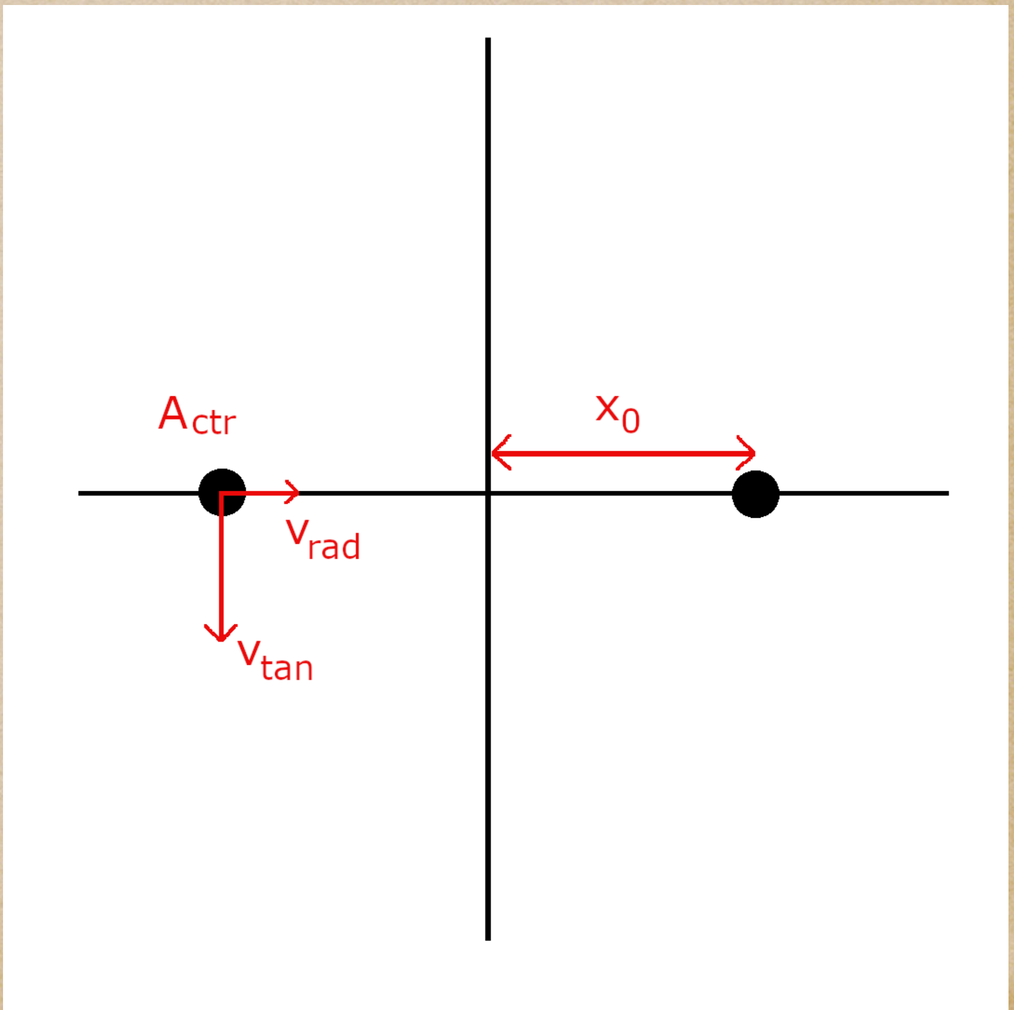
- Potential $V(\varphi) = m^2|\varphi|^2 \left(1 - 2\frac{|\varphi|^2}{\sigma_0^2}\right)^2 , \qquad \sigma_0 = 0.2$
- Equal mass $A_{\rm ctr} := |\varphi_{\rm ctr}| = 0.17\, M_{\rm Pl}$
- Velocity $v_{\rm tan}\,, \quad v_{\rm rad}$
- Distance $2x_0$
- Lean code
  - Cactus / Carpet
  - AHFinderDirect
  - BSSN
  - Moving Puncture Gauge
  - No-Malaise initial data



BS binary setup

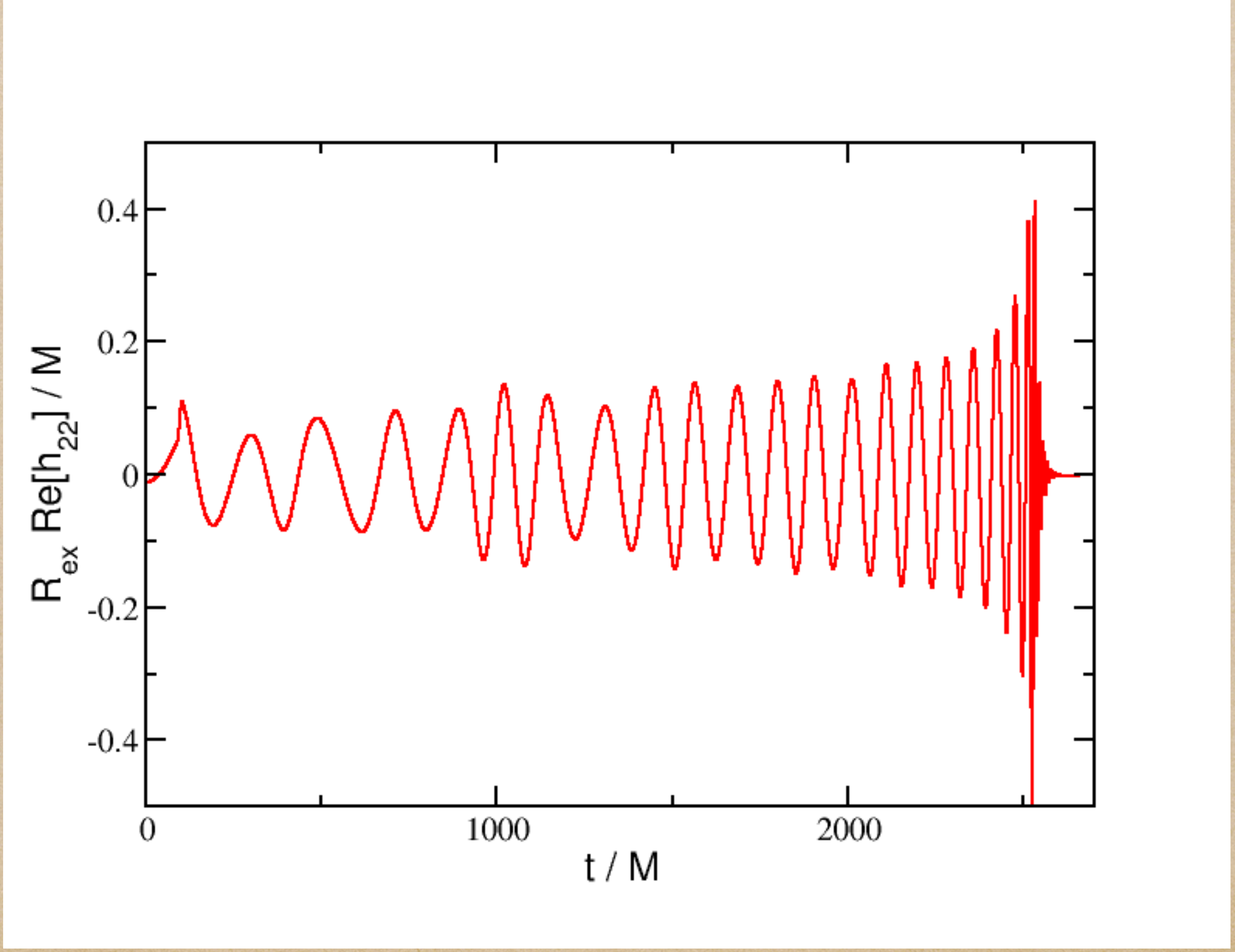
- Potential $V(\varphi) = m^2|\varphi|^2 \left(1 - 2\frac{|\varphi|^2}{\sigma_0^2}\right)^2$, $\sigma_0 = 0.2$
- Equal mass $A_{\rm ctr} := |\varphi_{\rm ctr}| = 0.17\, M_{\rm Pl}$
- Velocity $v_{\rm tan}$, $v_{\rm rad}$
- Distance $2x_0$





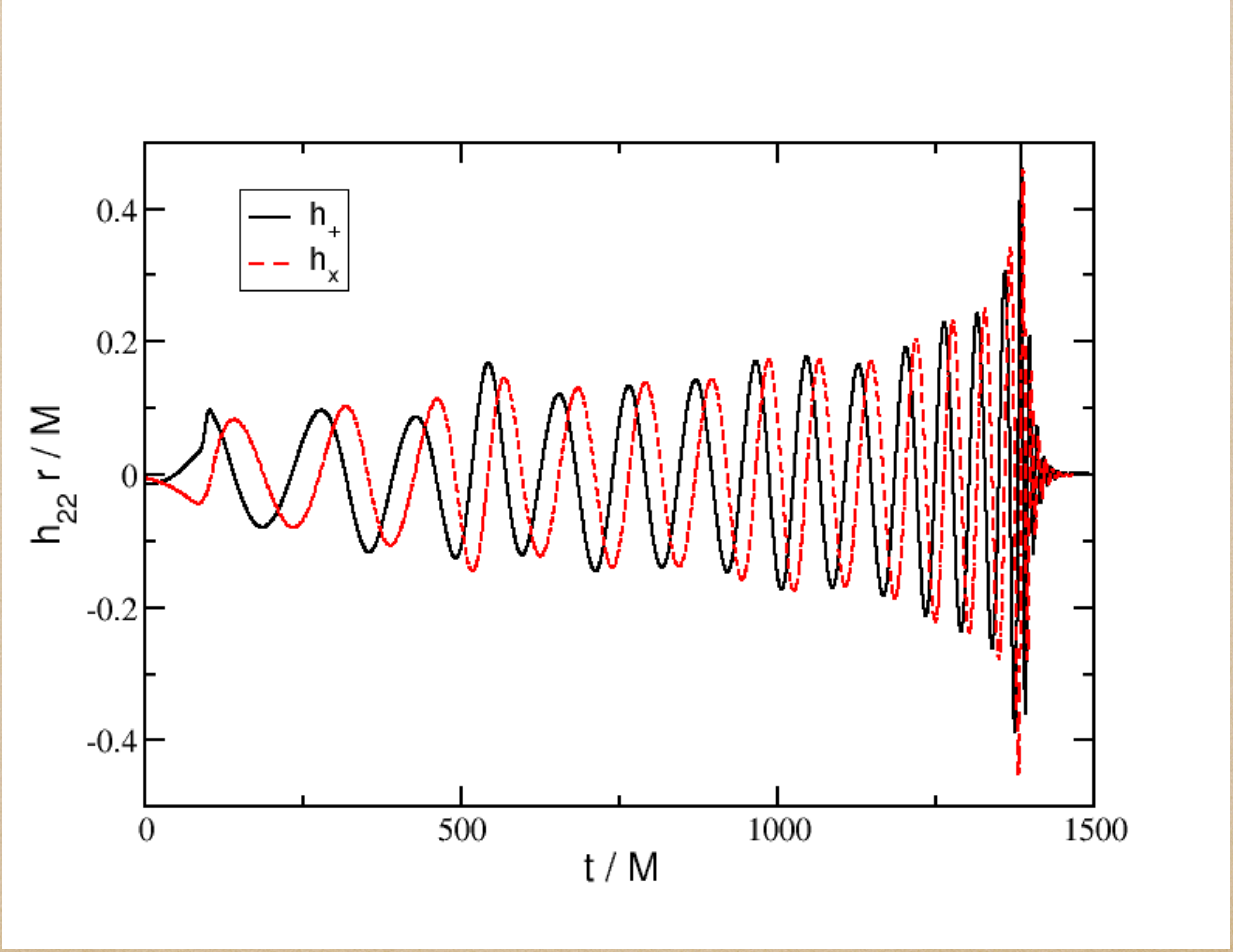
Gravitational-Wave Strain

- $v_{\rm tan} = 0.172\,, \qquad v_{\rm rad} = 0.002\,, \qquad x_0 = 6.1446$



Gravitational-Wave Strain

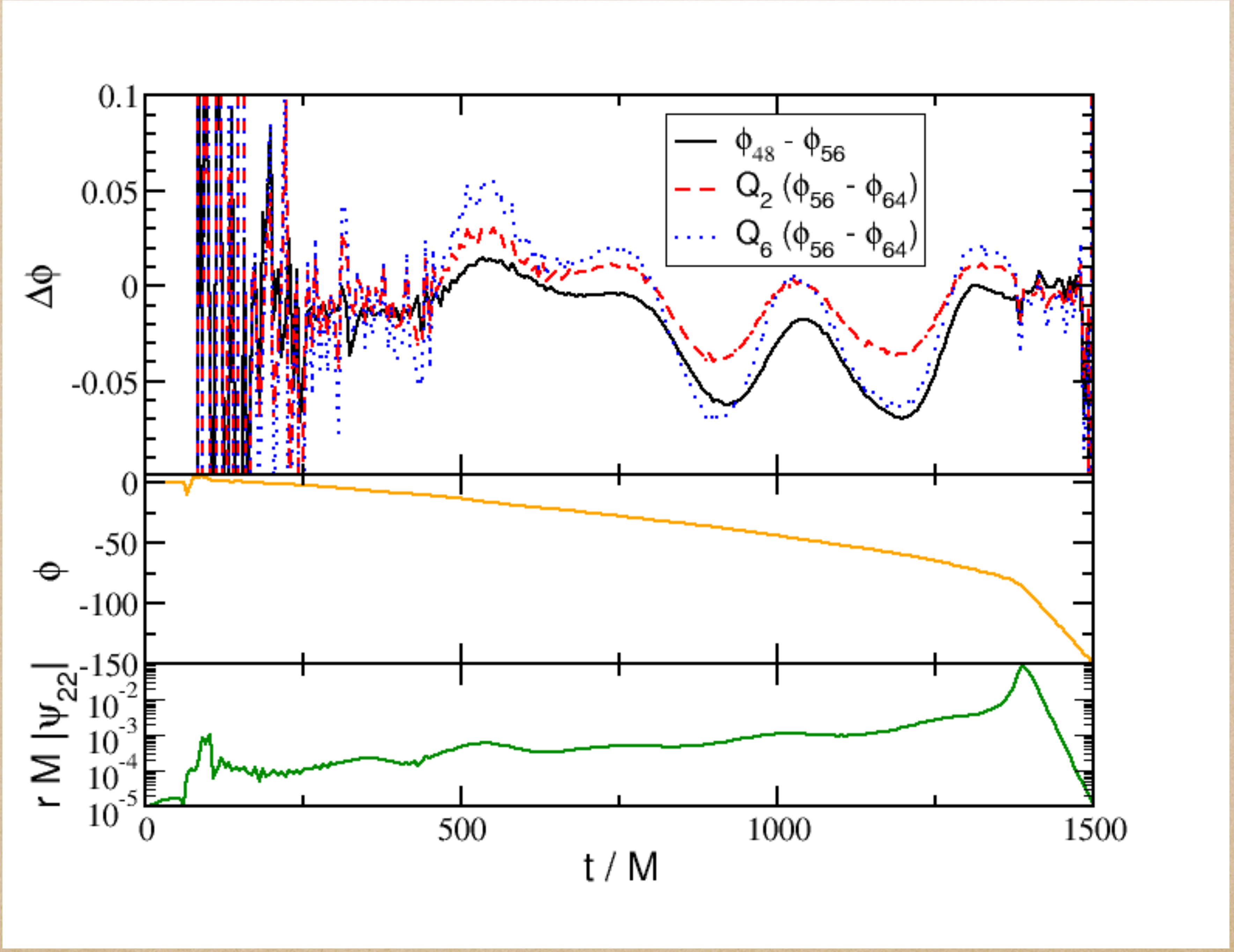
- $v_{\text{tan}} = 0.1684\,, \qquad v_{\text{rad}} = 0\,, \qquad x_0 = 6.1446$



Convergence: Phase Ψ_4

- $v_{\rm tan} = 0.1684\,, \qquad v_{\rm rad} = 0\,, \qquad x_0 = 6.1446$

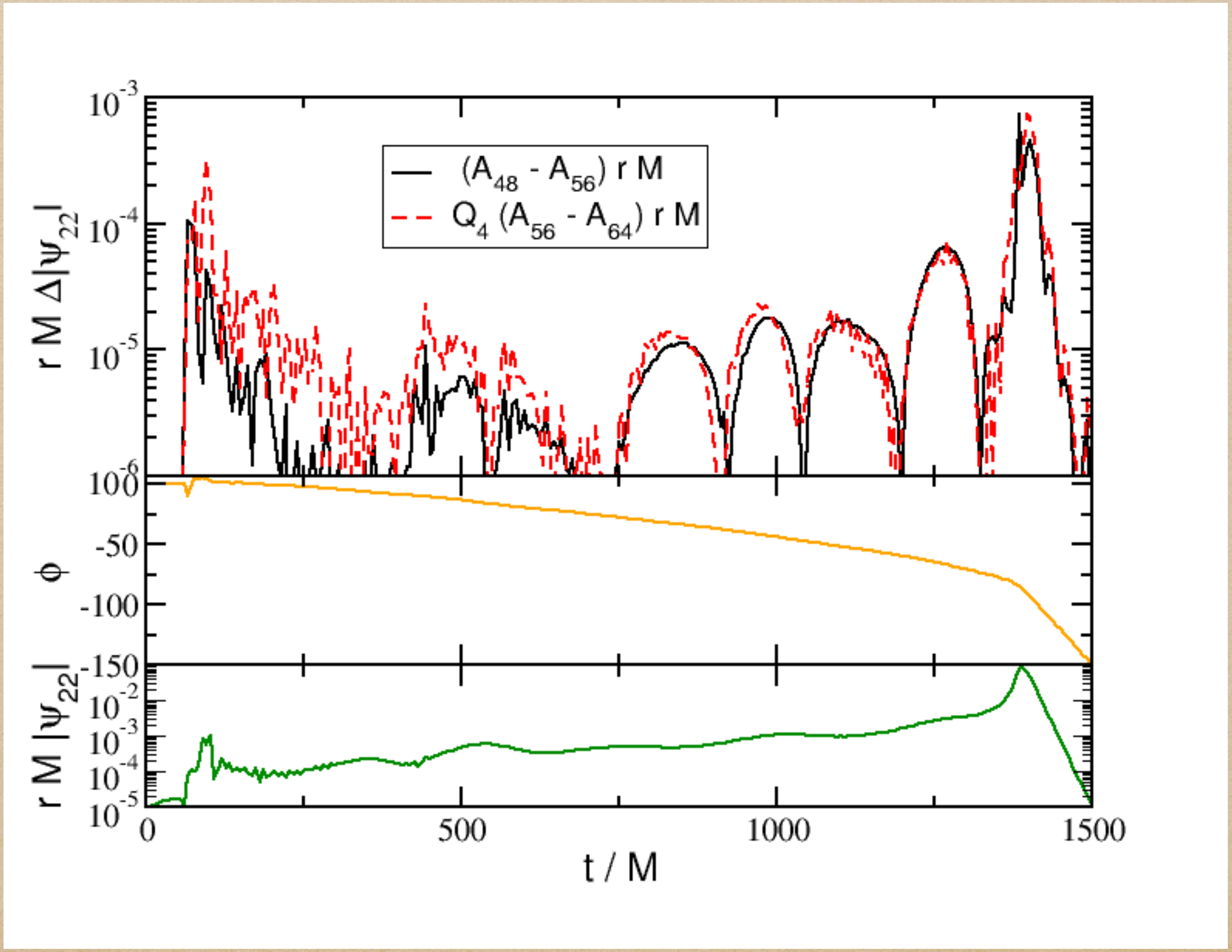
$\mathcal{E}(\phi) \sim 0.1$



Convergence: Amplitude Ψ_4

- $v_{\tan} = 0.1684\,, \qquad v_{\rm rad} = 0\,, \qquad x_0 = 6.1446$

$\mathcal{E}(A) \sim 1\,\%$



4. Pan-pan

Disclaimer

- **Warning:** Only gamble with time you can afford to lose!!!
- E Lim: "Uli's Ahab problem"
- There might be no solution…

What's the problem?

- We model complex scalar fields with $\text{Re}[\varphi]\,, \quad \text{Im}[\varphi]$
- Recall single boson star: $\varphi(t,r) = A(r)e^{\mathrm{i}\omega t}$

  Time evolution: $A(t,r) = A(r)\,, \quad \omega = \text{const}$
- Amplitude + phase easier than Re + Im
- Problem: Phase not defined at $\varphi = 0$

  $\Rightarrow$ Singular evolution equation for φ
- I tried...

Summary of observations

- We model complex scalar fields with $\text{Re}[\varphi]\,, \quad \text{Im}[\varphi]$
- Recall single boson star: $\varphi(t,r) = A(r)e^{\mathrm{i}\omega t}$

  Time evolution: $A(t,r) = A(r)\,, \quad \omega = \text{const}$
- Amplitude + phase easier than Re + Im
- Problem: Phase not defined at $\varphi = 0$

  $\Rightarrow$ Singular evolution equation for φ
- I tried… and failed.
  - Amplitude + phase
  - Log measures
  - Fluid analogy for scalar fields
  - Riemann sphere with patches

5. Summary

Summary

- Motivation: Dark matter, BSs as proxies
- Clean problem
- Long-lived BS afterglow
- Long, accurate inspirals not easy but looks doable
- Find better formalisms?