The stochastic GW background from core collapse SNe in massive ST gravity

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Rosca-Mead et al 2302.04995, 2007.14429, 2005.09728, 1903.09704;

US et al 1708.03651, Gerosa et al 1602.06952

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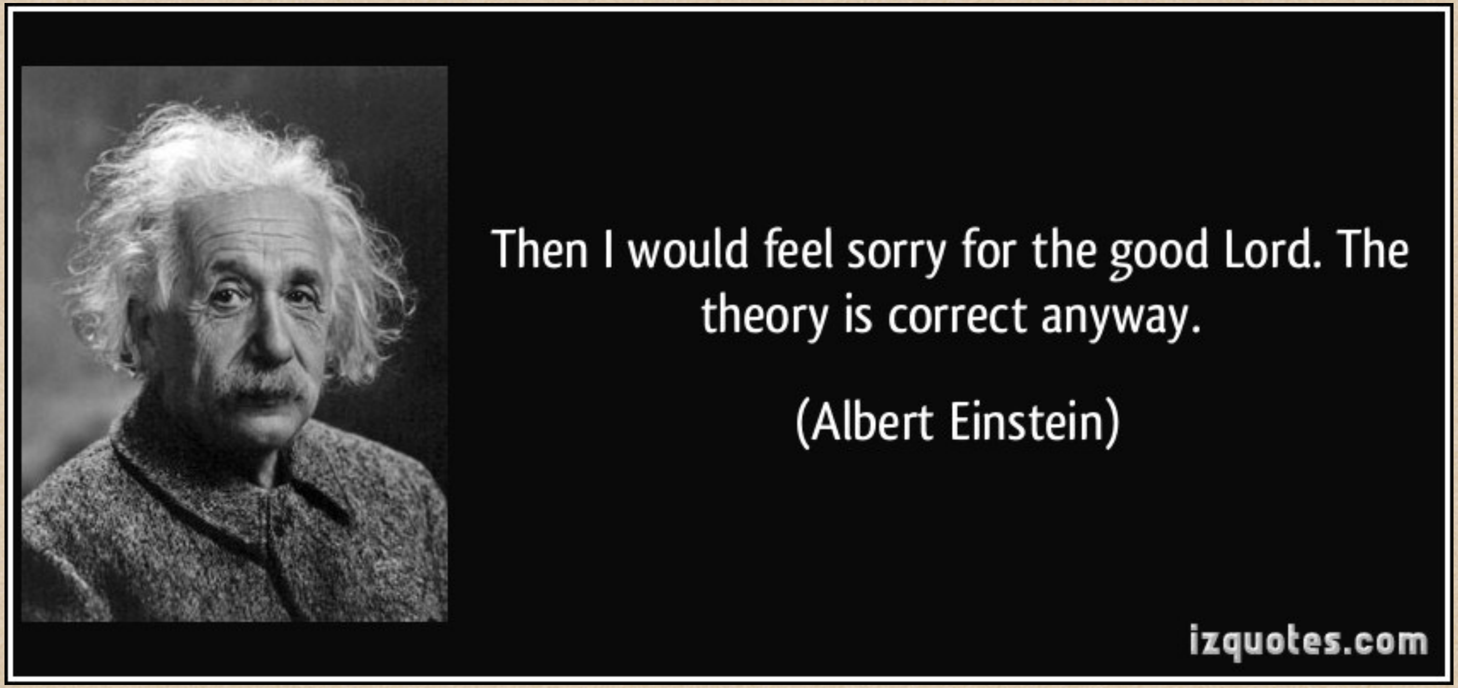
Albert Einstein Institut, Potsdam, *19 Apr 2023*

Do we need a theory beyond GR?

- When asked what he would do if Eddington’s mission failed…



Then I would feel sorry for the good Lord. The theory is correct anyway.

(Albert Einstein)

izquotes.com

- But we have reasons to search for “beyond GR”
  - Renormalization: Requires, e.g., higher curvature terms.

    $\rightarrow$ GR is low-energy limit of more fundamental theory
  - Dark energy: Why is Λ so small and why $\rho_{\rm dark} \sim \rho_{\rm mat}$
  - Dark matter: “Neptun” or “Vulcan” ?

Scalar tensor theory of gravity

- Scalars appear naturally in extra-dimensional theories
- Scalars prominent in cosmology
- ST theory well-posed; fairly well understood mathematically
- No-hair theorems limit potential of black-hole spacetimes
- Matter: Neutron stars, core-collapse
- Best example of smoking gun to date:

  Spontaneous scalarization Damour & Esposito-Farese PRL 1993
- Collapse studies in massless case

  Novak PRD 1998/1999

  Novak & Ibanez ApJ 2000,

  Gerosa+ CQG 2016

$\Rightarrow$

Core-collapse scenario to 0th order

- Massive stars: $M_{\mathrm{ZAMS}} = 8 \ldots 100\ M_\odot$
- Core compressed from $\sim 1\,500$ km to ~ 15 km
  $\sim 10^{10}$ g/cm^3 to $\gtrsim 10^{15}$ g/cm^3
- Released gravitational energy: $\mathcal{O}(10^{53})$ erg
  ~ 99 % in neutrinos, $\sim 10^{51}$ erg in outgoing shock, explosion
- Explosion mechanism: still uncertainties…
- Failed explosions lead to BH formation
- "Collapsar": possible engine for long-soft GRBs
- All of this handled for us by Woosley & Heger Phys.Rept. 2007
  $\rightarrow$ Initial pre-collapse profile

Theoretical framework

Einstein frame: conformal metric $\bar{g}_{\mu\nu} = F(\varphi)\, g_{\mu\nu}$

- Action

$$S = \frac{1}{16\pi} \int dx^4 \sqrt{-\bar{g}}\, [\bar{R} - 2\bar{g}^{\mu\nu} \partial_\mu \varphi\, \partial_\nu \varphi - 4V(\varphi)] + S_m[\psi_m, \bar{g}_{\mu\nu}/F(\varphi)]$$

- Energy momentum tensor: $T_{\alpha\beta} = \rho h u_\alpha u_\beta + P g_{\alpha\beta}$

- Spherical symmetry: $d\bar{s}^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = -F\alpha^2 dt^2 + FX^2 dr^2 + r^2 d\Omega^2$

$$u^\alpha = \frac{1}{\sqrt{1-v^2}} [\alpha^{-1},\ vX^{-1},\ 0,\ 0]$$

- Equations (gravity): $\partial_r \alpha = \dots\,,\qquad \partial_r X = \dots$

  $\partial_t \partial_t \varphi = \dots$

- Equations (matter): $(\rho,\, h,\, v) \leftrightarrow (D,\, S^r,\, \tau) \quad \Rightarrow$ HRSC

  GR1D code O'Connor & Ott CQG 2009

Equation of state

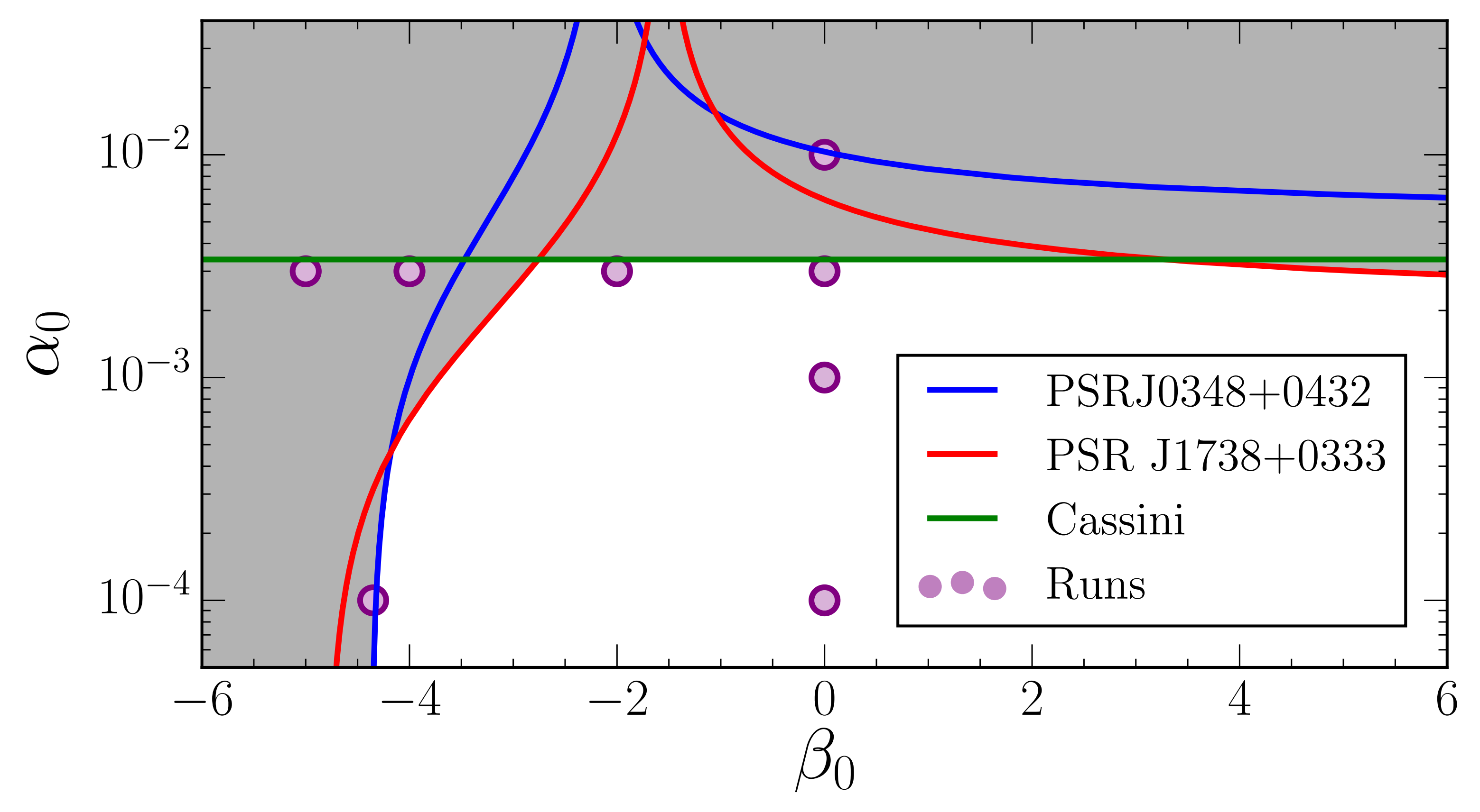
- Pressure: "cold" + "thermal" contribution: $P = P_\mathrm{c} + P_\mathrm{th}$
- Hybrid EOS for cold part: $P_c = \begin{cases} K_1 \rho^{\Gamma_1} & \text{if} \quad \rho \le \rho_\mathrm{nuc} \\ K_2 \rho^{\Gamma_2} & \text{if} \quad \rho > \rho_\mathrm{nuc} \end{cases}$
- Internal energy from 1st law: $\epsilon_c = \begin{cases} \frac{K_1}{\Gamma_1 - 1} \rho^{\Gamma_1 - 1} & \text{if} \quad \rho \le \rho_\mathrm{nuc} \\ \frac{K_2}{\Gamma_2 - 1} \rho^{\Gamma_2 - 1} + E_3 & \text{if} \quad \rho > \rho_\mathrm{nuc} \end{cases}$
- Thermal pressure: $P_\mathrm{th} = (\Gamma_\mathrm{th} - 1)\rho(\epsilon - \epsilon_\mathrm{th})$
- Parameters: $\Gamma_1 = 1.3\,, \quad \Gamma_2 = 2.5\,, \quad \Gamma_\mathrm{th} = 1.35$

  $K_1 = 4.9345 \times 10^{14}\ [\mathrm{cgs}]\,, \quad \rho_\mathrm{nuc} = 2 \times 10^{14}\ \mathrm{g\,cm^{-3}}$

  $K_2\,, \quad E_3$ from continuity at $\rho = \rho_\mathrm{nuc}$

The coupling function and potential

- Coupling function, potential: $\boxed{F(\varphi) = e^{-2\alpha_0\varphi - \beta_0\varphi^2}}$ $\boxed{V(\varphi) = \frac{1}{2}\mu^2\varphi^2}$



- Only for $\mu \lesssim 10^{-19}\,\mathrm{eV}$!! Here: $\mu[\mathrm{eV}] \in [10^{-15}, 10^{-12}]$

Ramazanoglu & Pretorius PRD 2016

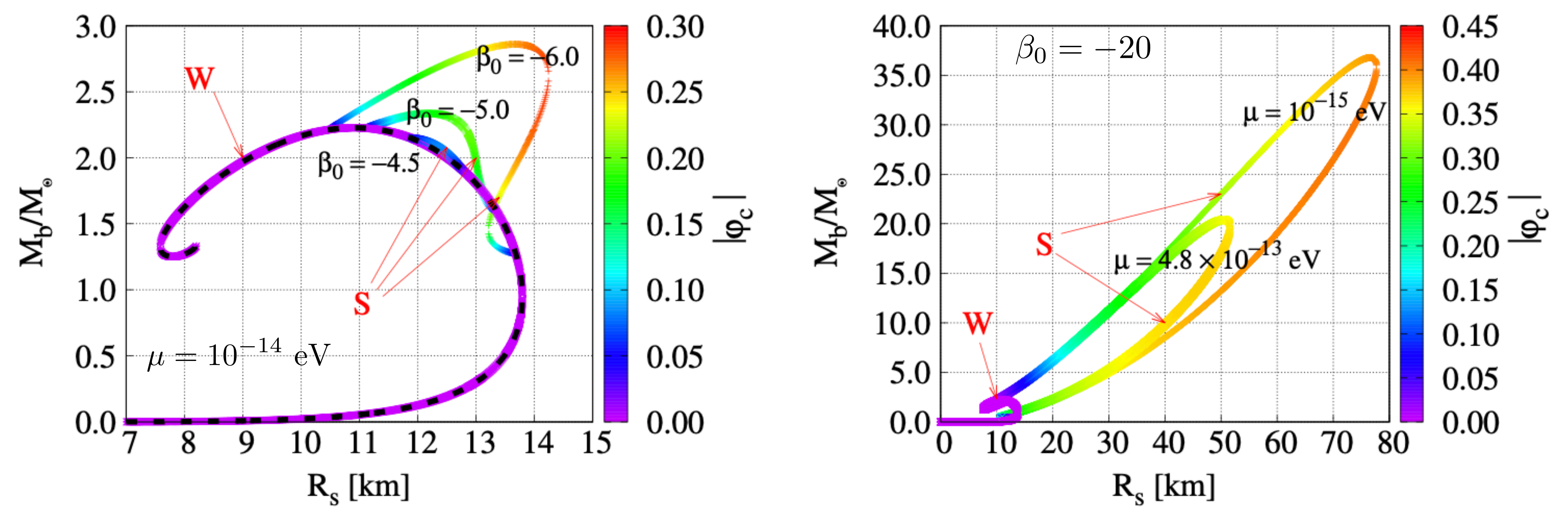
- Free parameters: $\mu,\ \alpha_0,\ \beta_0,\ \Gamma_1,\ \Gamma_2,\ \Gamma_{\rm th}$ + progenitor $M_{\rm ZAMS},\ \zeta$

Spontaneous scalarization

- Phase transition in the solution space as we vary β_0

  Damour & Esposito-Farese PRL 1993

- $\beta_0 \lesssim -4.35 \quad \Rightarrow$ New families of solutions



- Lots of substructure Rosca-Mead+ Symmetry 2020
- Scalarized stars often energetically favored!

Time evolutions cooking book recipe

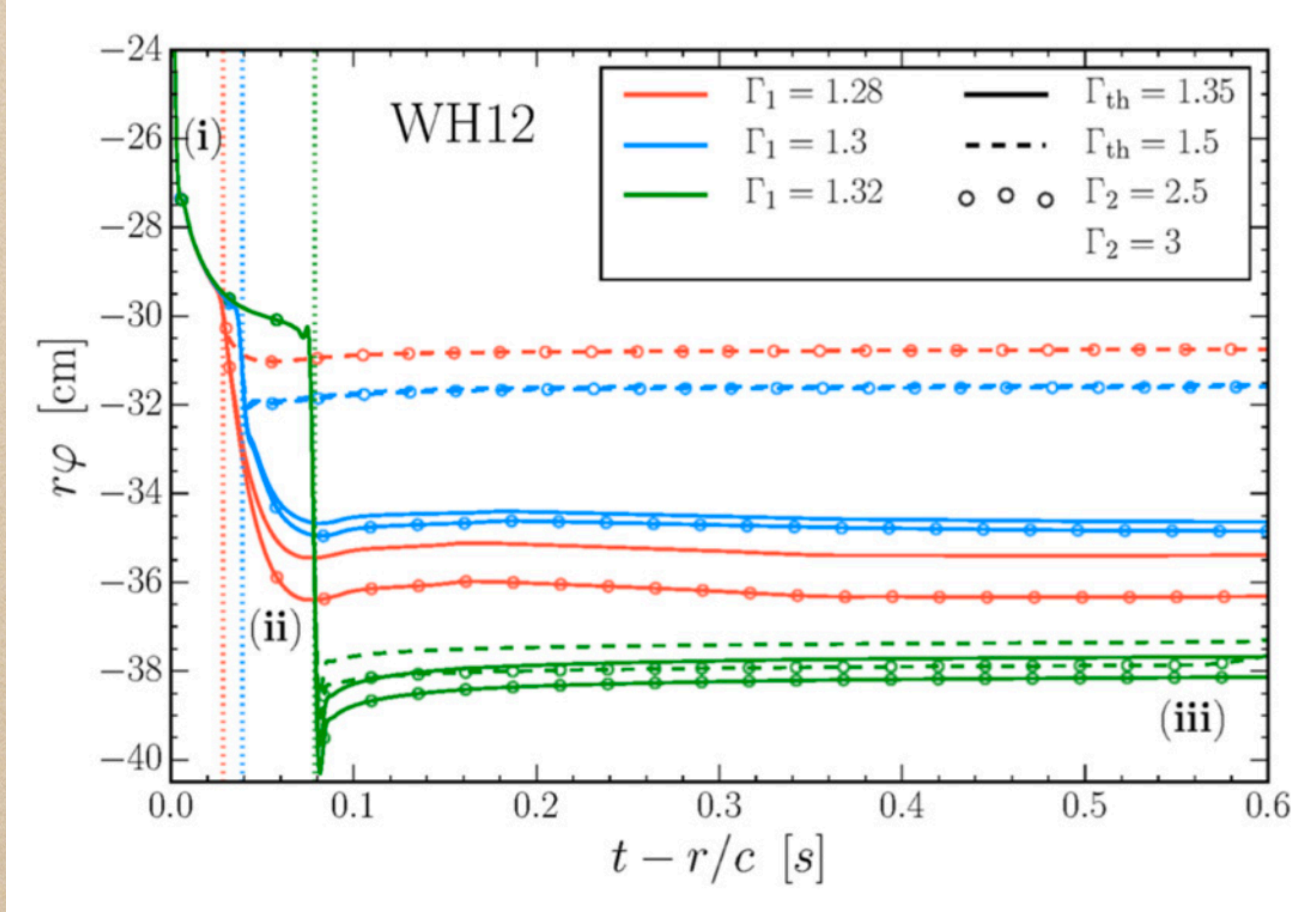
- Choose your Woosley-Heger progenitor $M_{\rm ZAMS},\ \zeta$
- Specify parameters $\mu,\, \alpha_0,\, \beta_0,\, \Gamma_1,\, \Gamma_2,\, \Gamma_{\rm th}$
- Specify the grid
- Run (may need checkpointing, but no Parallelization)
- Extract GW signals at $R_{\rm ex} \sim \mathcal{O}(1)$ light second
- Propagate signal to astrophysical distances;

  easy if $\mu = 0$, not easy if $\mu \neq 0$

Core collapse in massless ST theory

- Here: $\mu = 0 \quad \Rightarrow \quad V(\varphi) = 0$

$$\alpha_0 = 10^{-4}, \quad \beta_0 = -4.35$$

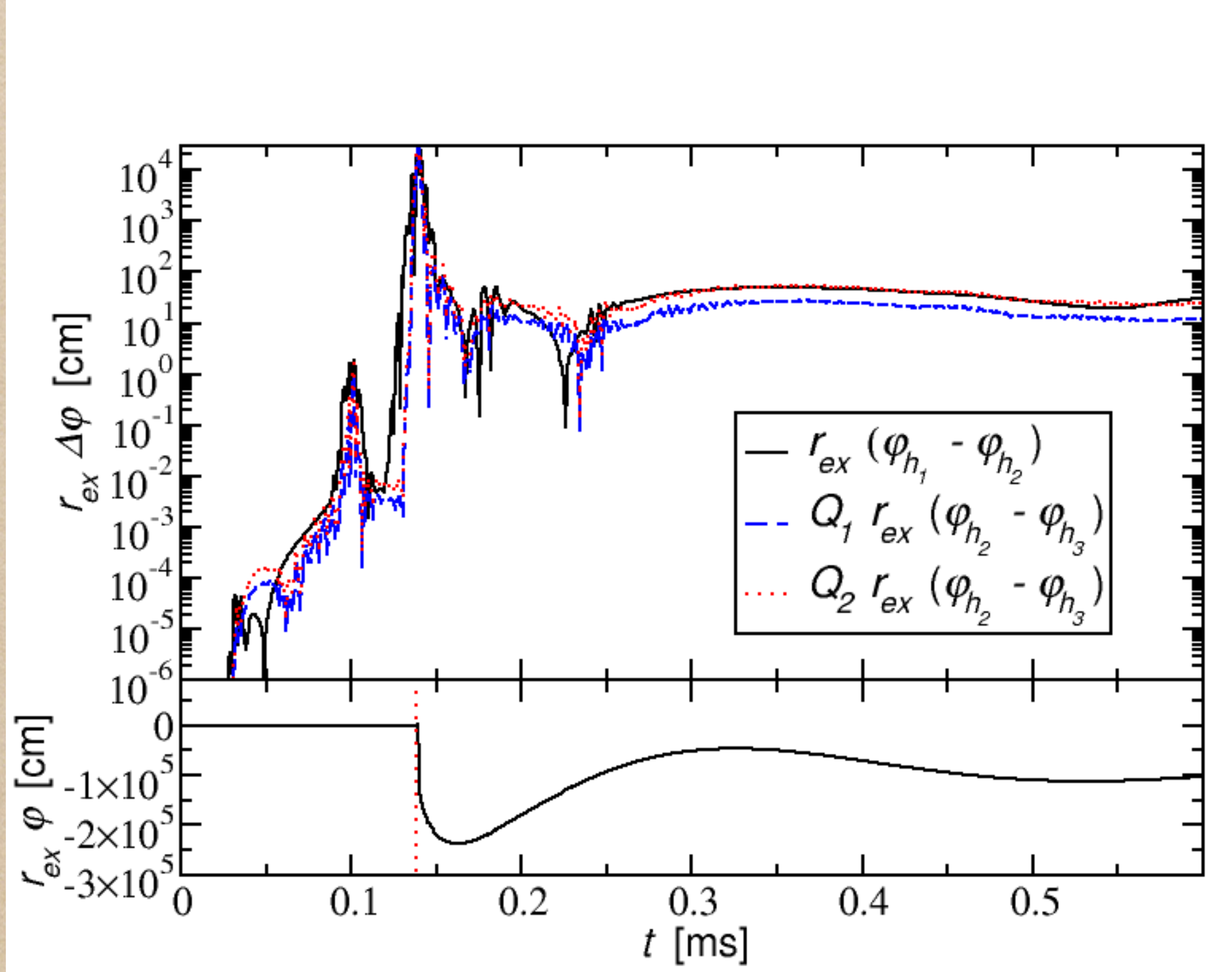


Gerosa, US, Ott CQG 2016

- Weak signals (β_0 constraints!), Heaviside like

Massive ST theory: Convergence test

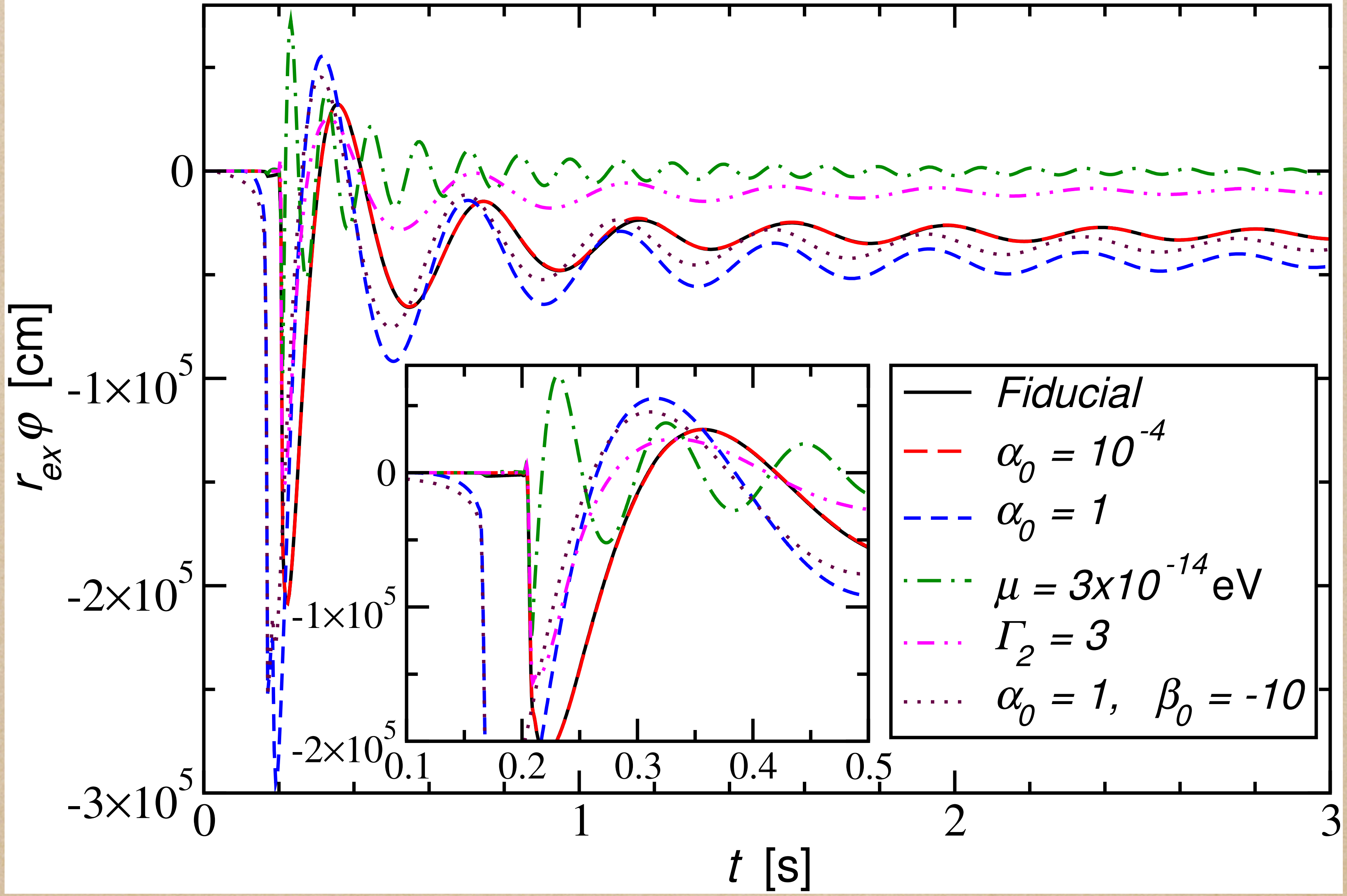
- For $\mu = 10^{-14}\,\mathrm{eV}, \quad \alpha_0 = 10^{-4}, \quad \beta_0 = -20$
  $\Gamma_1 = 1.3, \quad \Gamma_2 = 2.5, \quad \Gamma_{\rm th} = 1.35$
- Using $N_1 = 5000, \quad N_2 = 10000, \quad N_3 = 20000$ points



- Discretization error: $\sim 5\,\%$

Waveforms "close to" the source

- For $\mu = 10^{-14}\,\mathrm{eV}, \quad \alpha_0 = 10^{-2}, \quad \beta_0 = -20$
  $\Gamma_1 = 1.3, \quad \Gamma_2 = 2.5, \quad \Gamma_{\rm th} = 1.35$

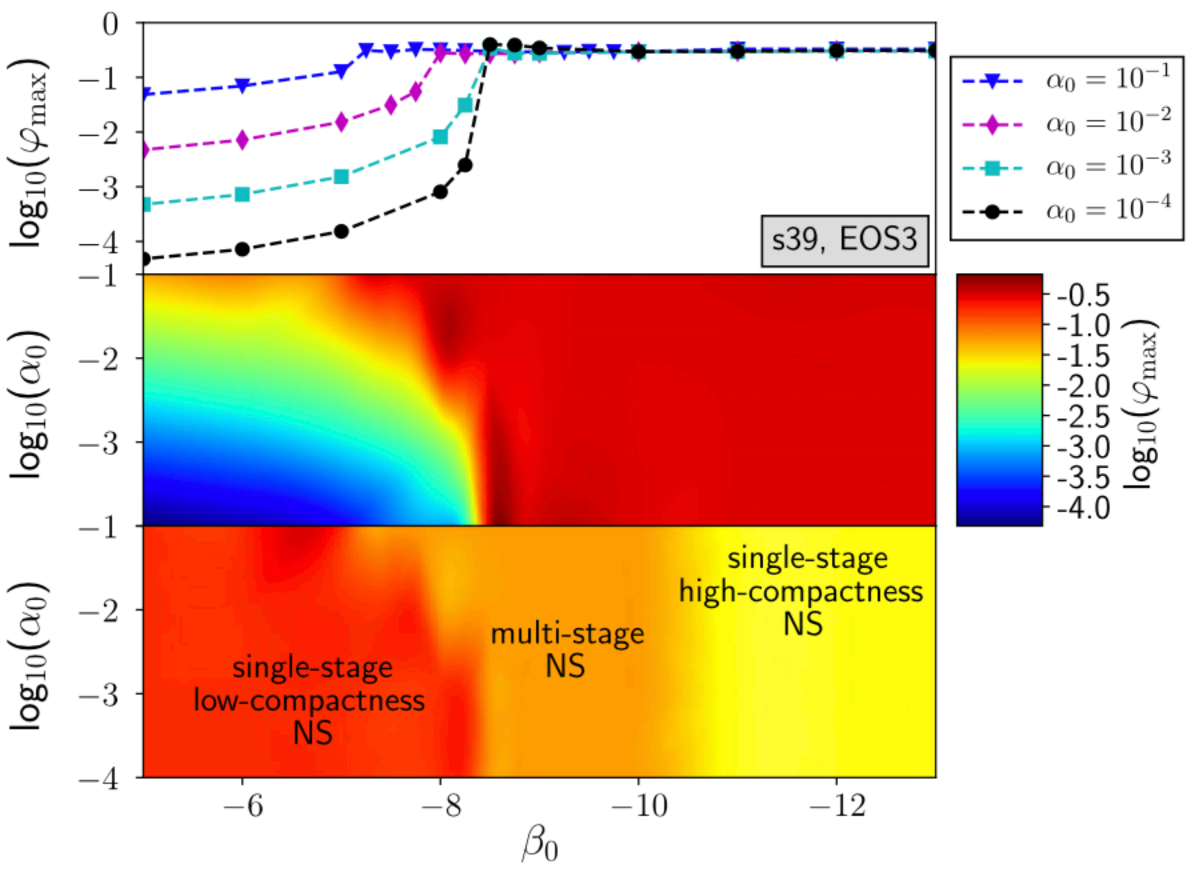


- $r\varphi \gg$ massless case; fairly insensitive to parameters; dispersion!

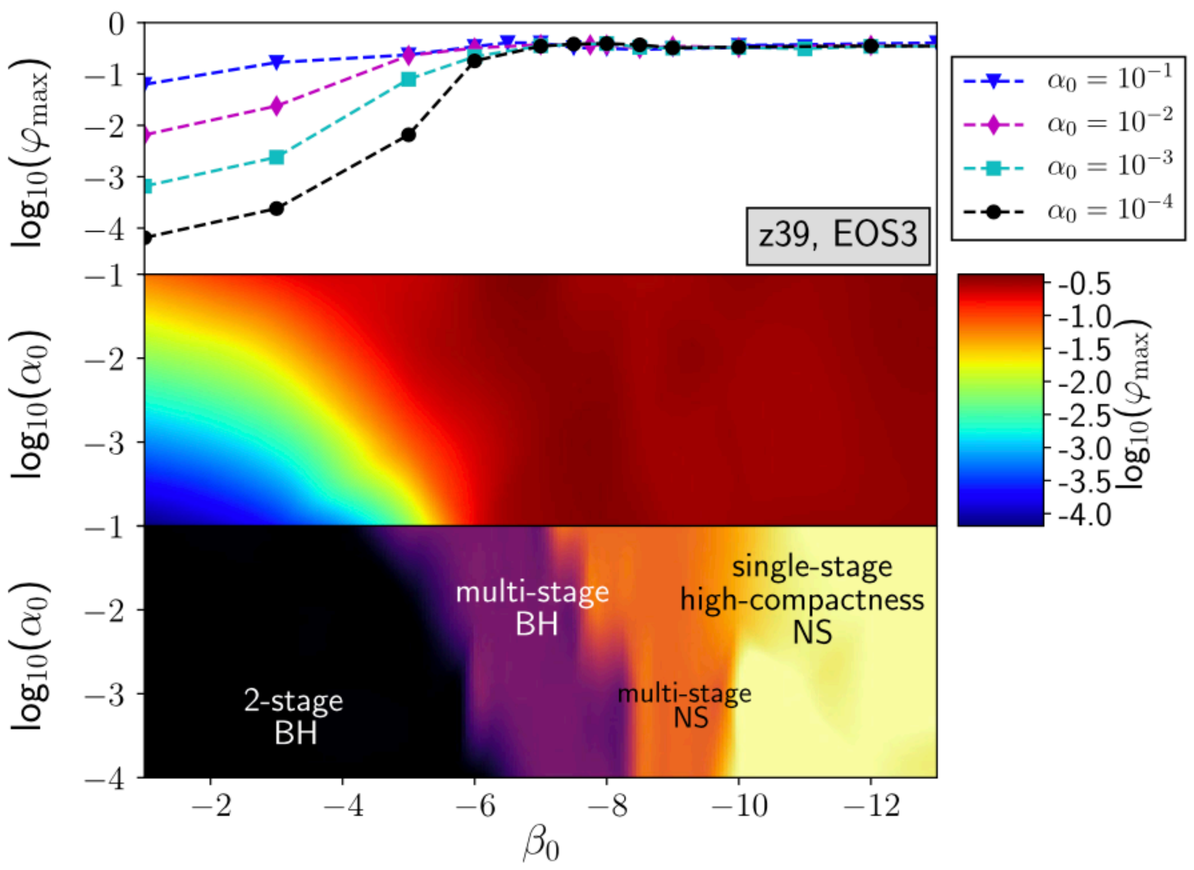
Classification of outcomes

- All collapse scenarios fall into 5 categories
- Check what happens in GR:

Neutron star



Black hole

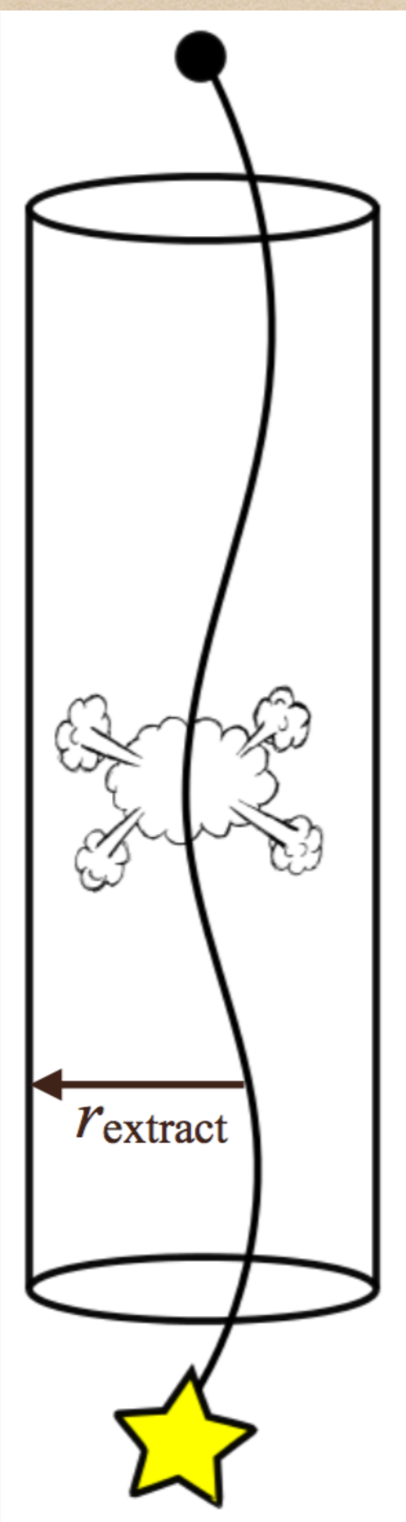


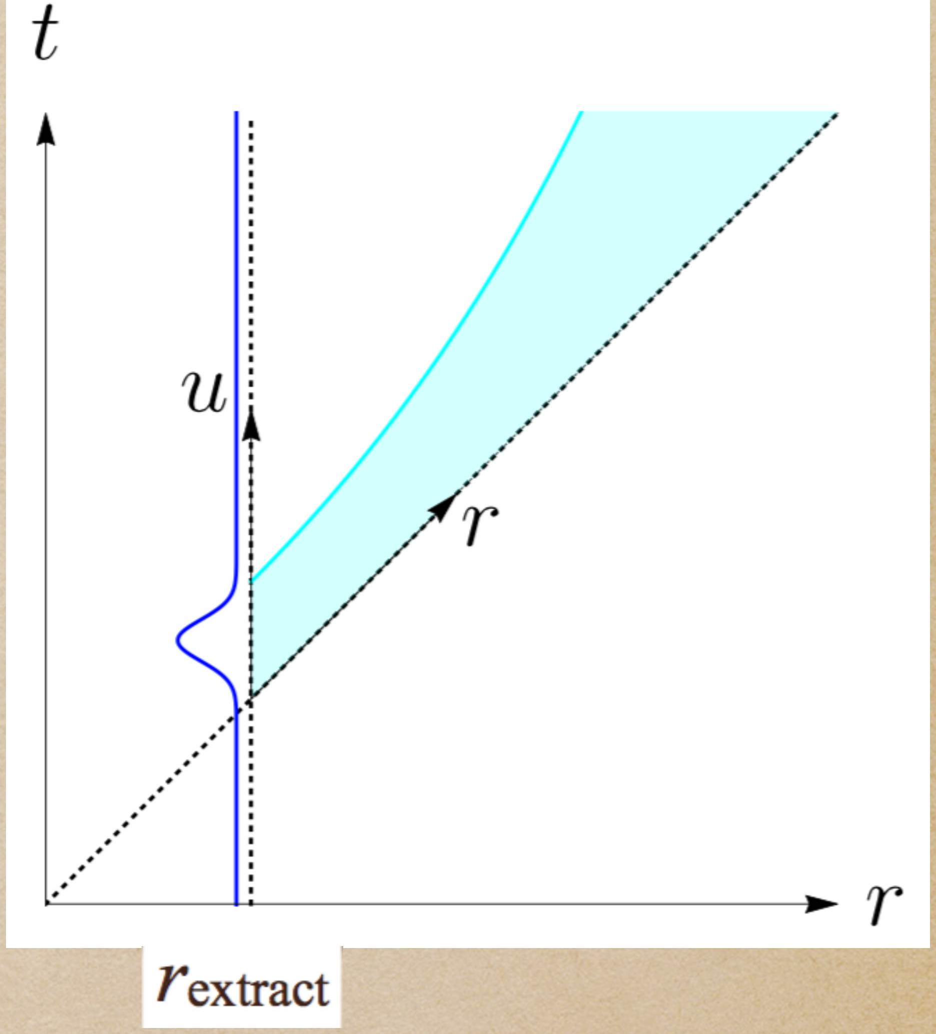
Waveforms "far from" the source

- LIGO will observe the above scalar profiles after they propagate to large distances
- In the massless case this is almost trivial

$$\varphi(t;r) = \frac{1}{r}\varphi(t-r;r_{\text{extract}})$$

- In the massive case things are more complicated: signals propagate with ***dispersion***

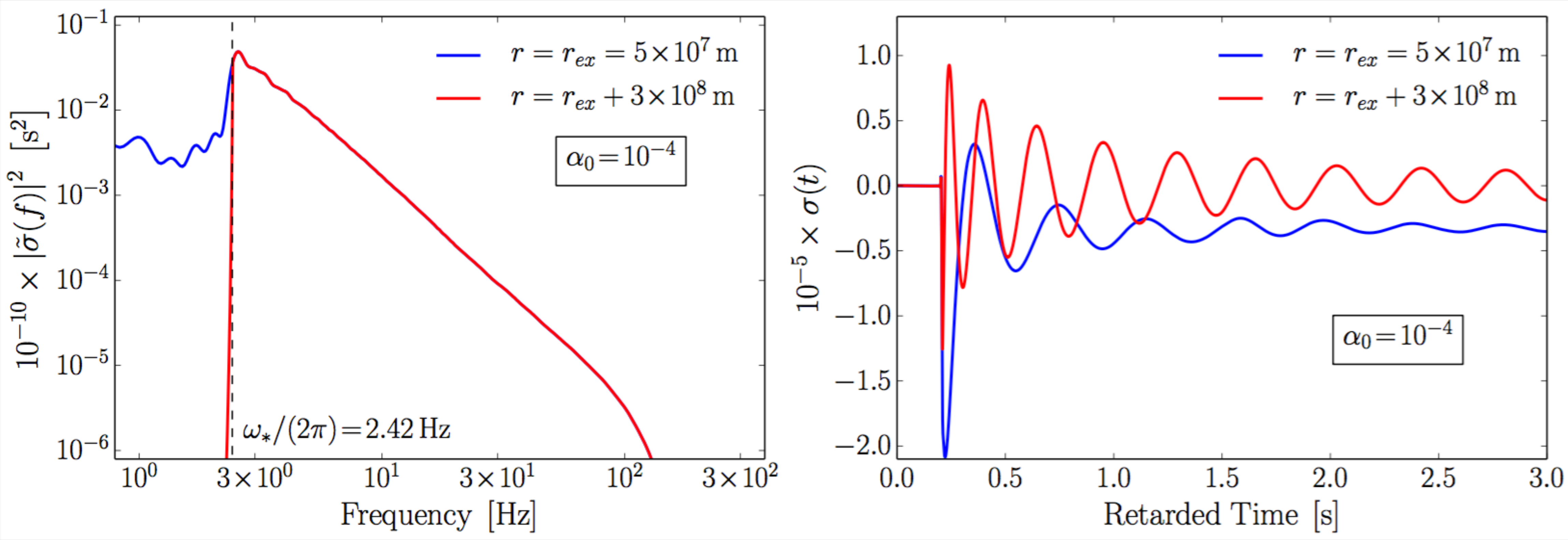




Waveforms "far from" the source

- Far from the source, scalar dynamics are governed by the flat-space Klein-Gordon wave equation $\partial_t^2\varphi - \nabla^2\varphi + \omega_*^2\varphi = 0$
- Easier to work with the radially rescaled field $\sigma \equiv r\varphi$
- As the signal propagates outwards:
  - Low frequencies are suppressed
  - High frequency power spectrum is unaffected
  - Signal spreads out in time
  - High frequencies arrive earlier than low frequencies
  - Signal becomes increasingly oscillatory

The scalar field mass has a natural frequency $\omega_* = c^2\mu/\hbar$



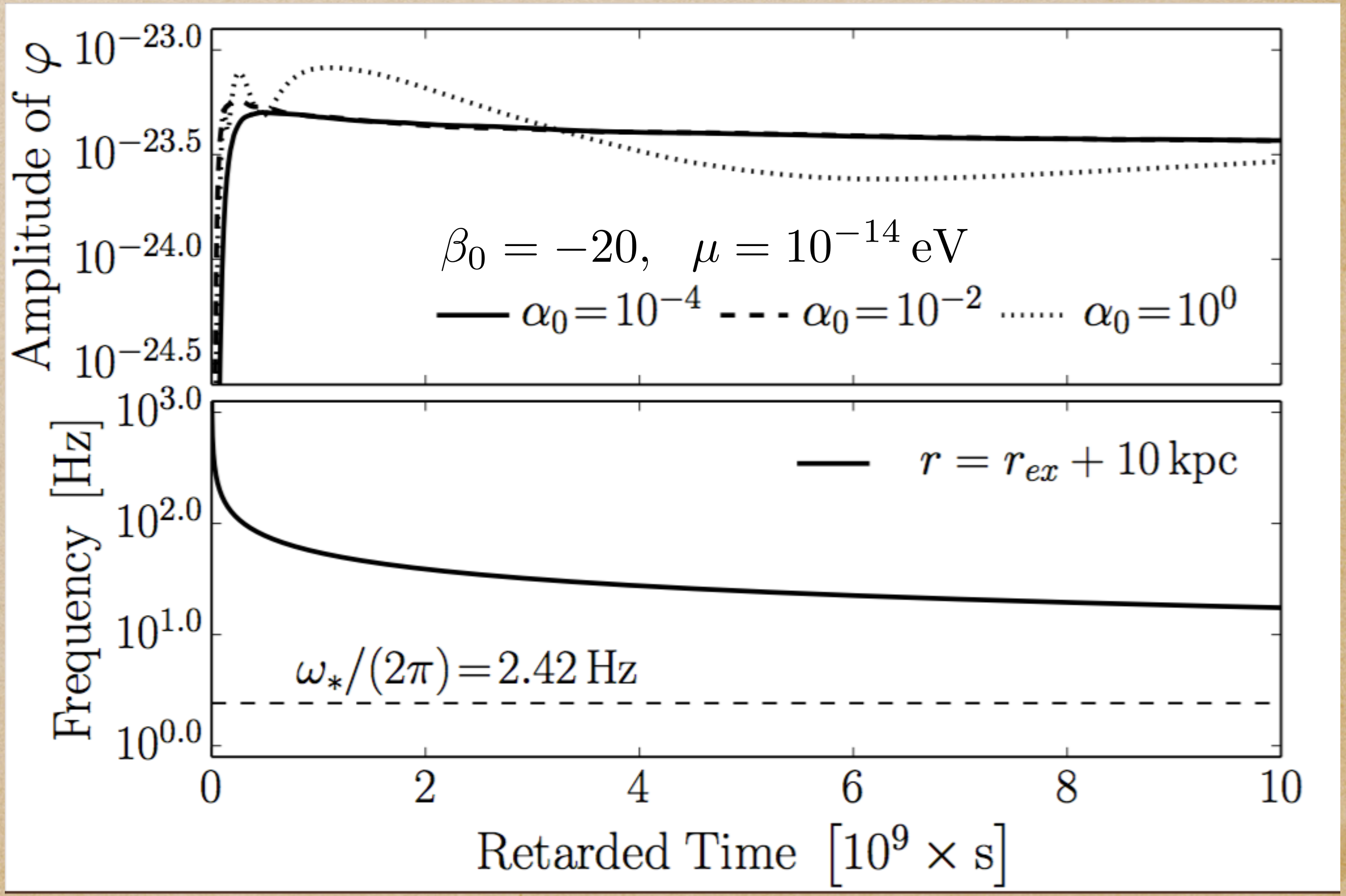
Waveforms "far from" the source

- Signals become more oscillatory as they propagate outwards
- In the large-distance limit the stationary phase approximation applies → analytic expression for the time domain signal
- Signals have a characteristic "inverse chirp" lasting many years
- Strain $h \propto \alpha_0 \varphi$

SPA frequency as function of time (Inverse Chirp)

$$F(t) = \frac{\omega_*}{2\pi} \frac{1}{\sqrt{1-(d/t)^2}}$$

Distance to source $d = 10\,\mathrm{kpc}$



Stochastic background

- Events are stronger in GR and long-lived

  ⇒ signals from the local universe overlap

- Task list:
  - Waveform catalog for parameters

    $\mu,\ \alpha_0,\ \beta_0,\ \Gamma_1,\ \Gamma_2,\ \Gamma_{\rm th},\ M_{\rm ZAMS},\ \zeta$
  - Wave propagation in expanding cosmos
  - SN event rate in local Universe
  - Integrate all events in frequency space

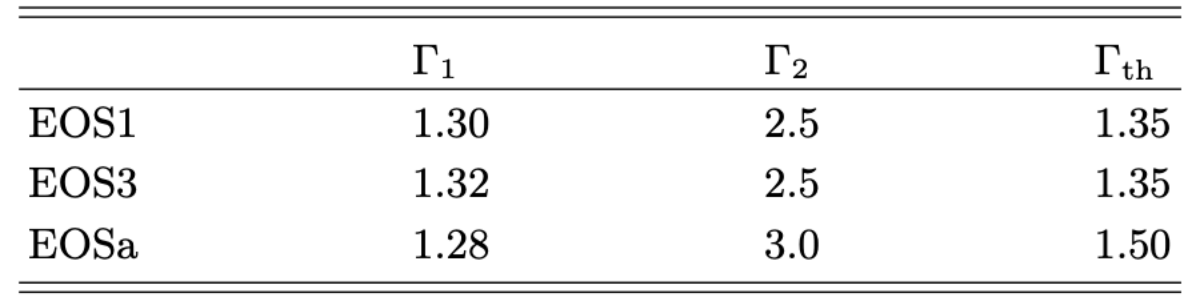
Rosca-Mead, Agathos, Moore, US arXiv:2302.04995

Catalog

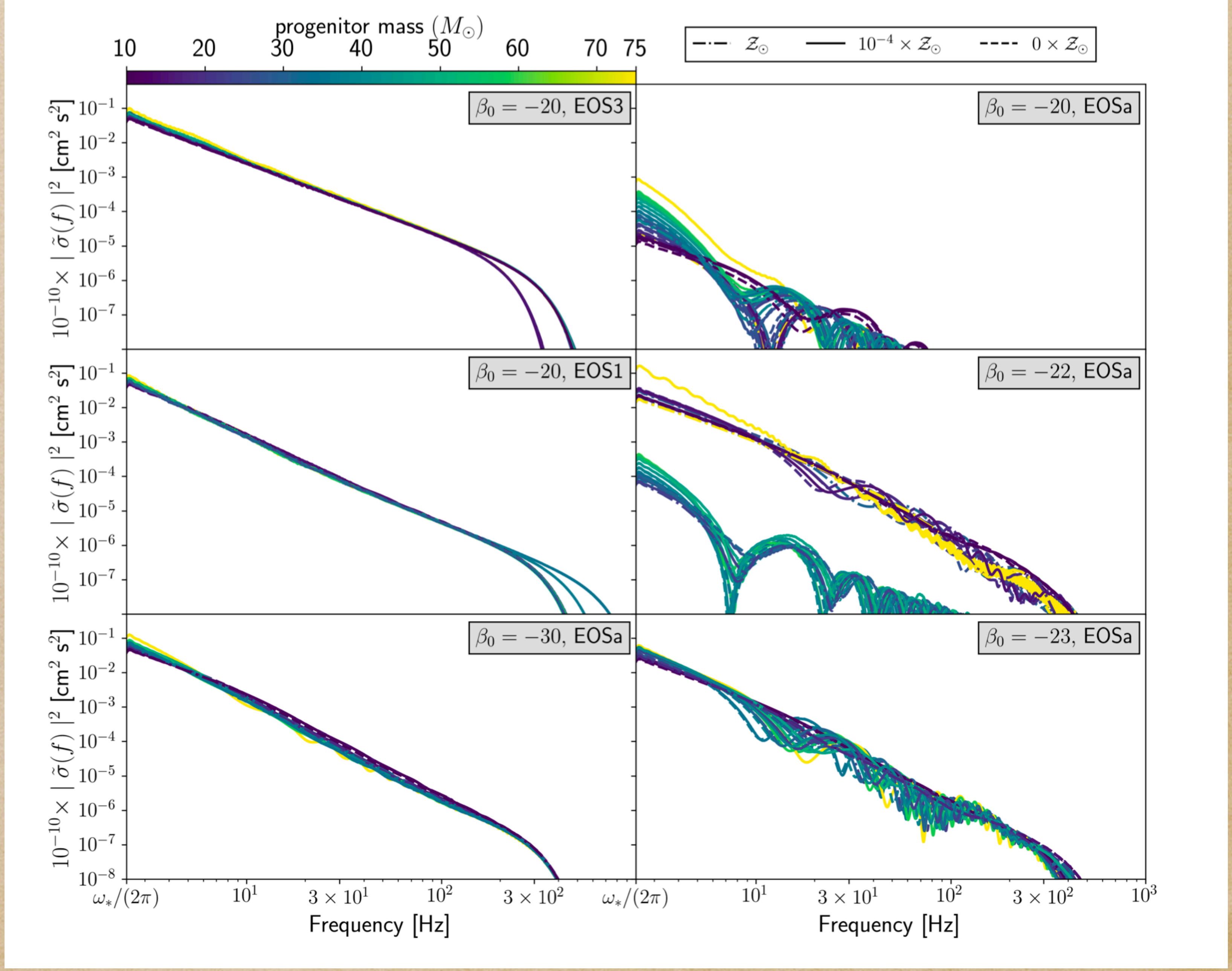
- EOS: soft —> stiff
- ST: $\alpha_0 = 10^{-2}$
- Approximate

  Universality!
- Only exception:

  $\beta_0 \approx \beta_{0,\mathrm{thr}}$
- Focus on **EOS1**



	Γ_1	Γ_2	Γ_{th}
EOS1	1.30	2.5	1.35
EOS3	1.32	2.5	1.35
EOSa	1.28	3.0	1.50



Wave propagation and event rate

- Very similar to flat space; wave equation in $k = 0$ cosmology:

$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2)$; conformal time $\frac{\mathrm{d}\eta}{\mathrm{d}t} = \frac{1}{a}\,, \quad \sigma = ar\varphi$

$$\Rightarrow \boxed{\partial_\eta^2\sigma - \partial_r^2\sigma - a^2H^2(1-q)\sigma + \mu^2a^2\sigma = 0}$$

- Stationary phase approximation:

Frequency $$F(\tau) = \frac{\omega_*}{2\pi}\frac{1}{\sqrt{1 - D_L^2/(\tau + D_L)^2}}\,, \quad \omega_* = (1+z)\mu$$

Amplitude $$A(\tau) = \sqrt{\frac{2\mu}{\pi}}\frac{(1+z)D_L}{[(\tau + D_L)^2 - D_L^2]^{3/4}}\left|\tilde{\sigma}(\Omega, r_e)\right|.$$

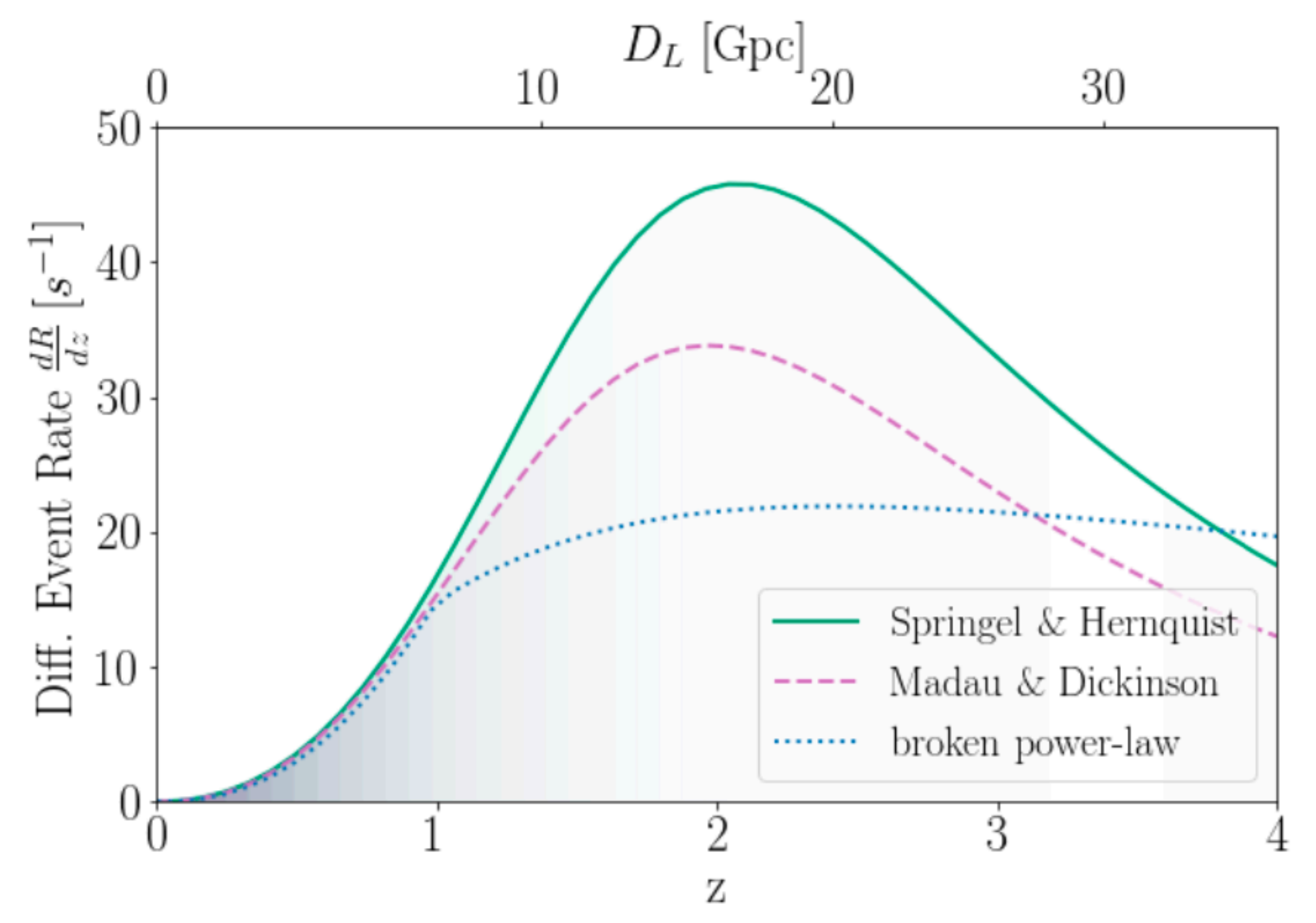
Phase $$\phi(\tau) = -\mu\sqrt{(\tau + D_L)^2 - D_L^2} + \arg\left[\tilde{\sigma}(\Omega, r_e)\right] - \frac{\pi}{4}$$

$$\boxed{\sigma(\tau) = A(\tau)e^{\mathrm{i}\phi(\tau)}}$$

- That’s no blueshift!!!

Astrophysical population statistics

- Need the even rate $\frac{\mathrm{d}R(z)}{\mathrm{d}\theta}$ in the $\theta = (M_{\mathrm{ZAMS}},\ \zeta)$ parameter space
  - Broken power law Buonanno et al astro-ph/0412277
  - Springel, Hernquist astro-ph/0206395, 0209183, 1409.2462
  - Madau, Dickinson astro-ph/1403.0007



Integration of events

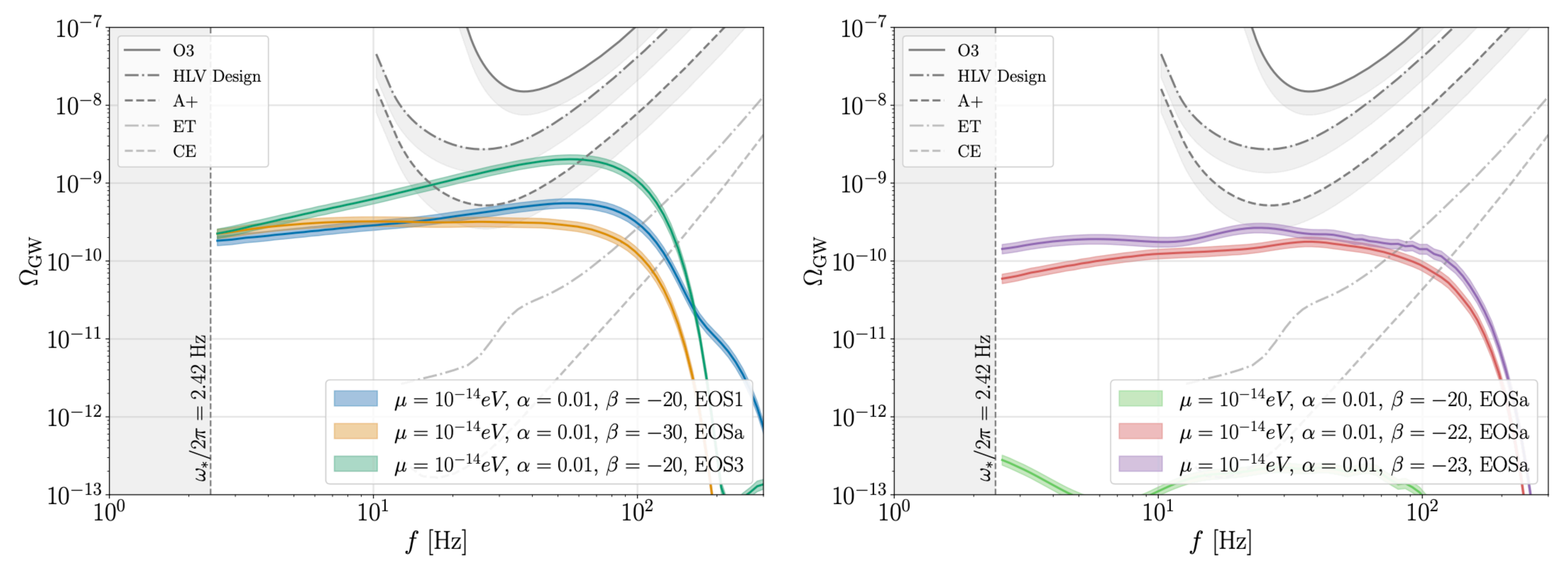
- Energy density frequency space:

$$\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}f_s} = \frac{c^3(2\pi f_s)^2}{16\pi G}\int\left\langle(\tilde{h}_+^{\mathrm{TT}})^2 + (\tilde{h}_\times^{\mathrm{TT}})^2 + (\tilde{h}_S^{\mathrm{TT}})^2\right\rangle \mathrm{d}\Omega$$
$$= \frac{c^3\pi^2 f_s^2}{G}\left\langle\left(\tilde{h}_S^{\mathrm{TT}}(f_s)\right)^2\right\rangle$$

$$H_0 = 67.4\ \mathrm{km/s}$$
$$\Omega_m = 0.315$$
$$\Omega_\Lambda = 0.68$$

- Cosmology $H_0 = 67.4\ \mathrm{km/s}$ Aghanim et al 1807.06209

$\Omega_m = 0.315\,,\quad \Omega_\Lambda = 0.68$



Detection with LIGO-Virgo

GWs from core-collapse in ST gravity may fall into 3 classes:

- Burst signals: For light scalars $(\mu < 10^{-20}\,\mathrm{eV})$ and short distances $(10\,\mathrm{kpc})$, the pulse does not disperse significantly; will look like a $< 1\,\mathrm{s}$ burst
- Continuous wave signal: for heavier scalars, long dispersion turns pulse into a quasi-monochromatic signal
  $\rightarrow$ capture using standard directed CW searches, assuming EM counterpart; e.g. SN1987A, Kepler1604
- Stochastic background:
  - Many quiet sources $+$ very long duration (superposed)
  - Cosmological redshift $+$ mass variation $\rightarrow$ smeared low-f cutoff around $\sim \omega_*$

Conclusions

- We have simulated stellar core collapse in massive ST theory
- Explored combined parameter space of EOS and ST theory. parameters
- Spontaneous scalarization occurs as in massless case, but effect can be more dramatic because the scalar mass “screens” the effect of the scalar, allowing larger values of $\alpha_0,\ \beta_0$ to be compatible with binary pulsar observations
- Signals propagate with dispersion, signals can last for years to centuries at kpc distances
- Signals can show up in LIGO/Virgo burst, CW or stochastic searches