# Numerical and analytic modeling of boson stars

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# 1. Background

# The popularity of boson stars

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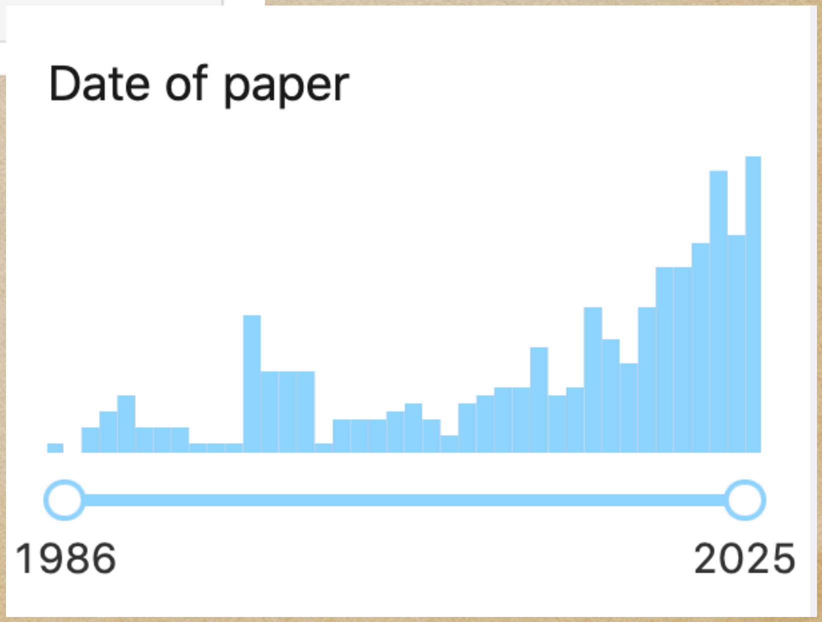
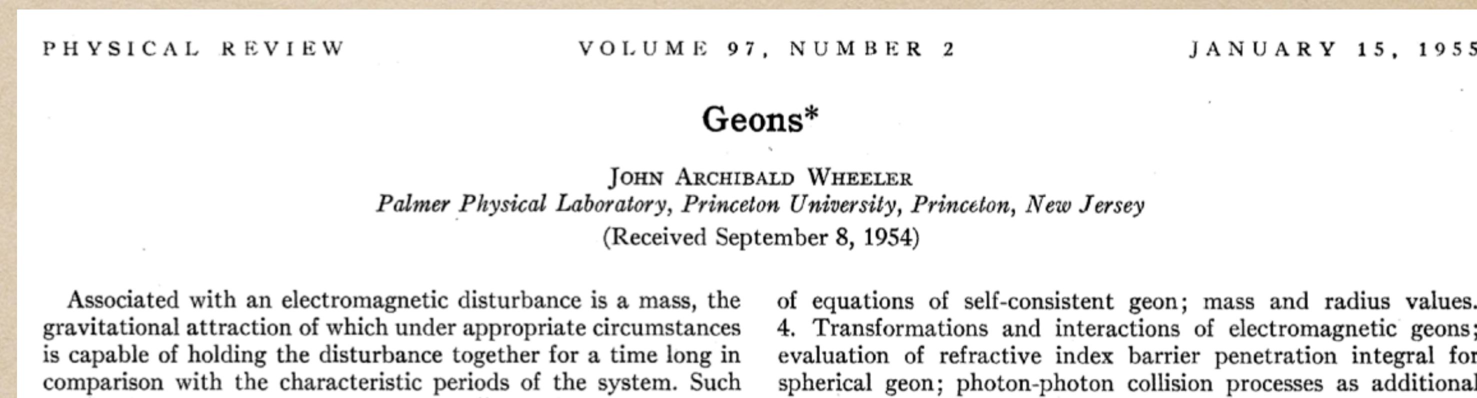
find t boson stars



# The idea of boson stars

- "Gravitational-electromagnetic entities" or Geons
- Wheeler 1955



PHYSICAL REVIEW VOLUME 97, NUMBER 2 JANUARY 15, 1955

**Geons***

JOHN ARCHIBALD WHEELER
*Palmer Physical Laboratory, Princeton University, Princeton, New Jersey*
(Received September 8, 1954)

Associated with an electromagnetic disturbance is a mass, the gravitational attraction of which under appropriate circumstances is capable of holding the disturbance together for a time long in comparison with the characteristic periods of the system. Such

of equations of self-consistent geon; mass and radius values. 4. Transformations and interactions of electromagnetic geons; evaluation of refractive index barrier penetration integral for spherical geon; photon-photon collision processes as additional

- Energy = mass gravitates $\rightarrow$ Compact (equilibrium?) objects
- Geons are not equilibrium configurations
- Dark matter candidates: QCD axions, ALPs, dark photons,...
- Complex fields (scalar, vector,...)

  $\rightarrow$ Genuine equilibrium states; $T_{\alpha\beta}$ stationary!
- First shown for scalar fields $\rightarrow$ "Boson stars"
- Feinblum & McKinley PR 168 (1968), Kaup PR 172 (1968),

  Ruffini & Bonazzola PR 187 (1969)

# A boson star zoo

- Mini BSs (no self-interaction) Kaup PR (1968) and others
- "Solitonic" BSs (self-interacting scalar field) $\rightarrow$ more compact

  Colpi+ PRL (1986), Lee PRD (1987), ...
- Proca stars Brito+ Phys.Lett.B (2016)
- $\ell$ -boson stars (multiple scalar fields) Alcubierre+ CQG (2018)
- Multi-oscillating BSs Choptuik+ PRL (2019)
- Thin-shell BSs Collodel & Doneva 2203.08203
- Axion stars e.g. Helfer et al JCAP 2017
- Higher-spin fields Jain & Amin 2109.04892
- Multi-field BSs Sanchis-Gual+ PRL (2021)
- May condense from local over-densities Widdicombe+ JCAP (2018)

Focus here: Single-scalar BSs

# A boson-star précis

- BS = Classical limit of a quantum field

  Ground state = Boson-Einstein condensate
- Only known scalar field: Higgs, $\mu = 125\,\mathrm{GeV}$
- Units $G = c = \hbar = 1$, scale determined by $\mu$ [inverse length]

  E.g. $\mu = 1.33 \times 10^{-10}\,\mathrm{eV} \;\Rightarrow\; \mu^{-1} = M_\odot = 1.48\,\mathrm{km}$
- Astrophysical stars need ultralight fields → Beyond SM

  axion like, dark matter candidates: $10^{-11} \ldots 10^{-20}\,\mathrm{eV}$
- Properties: Compactness 0 to > NSs, any Mass
- BSs can have light rings → Black-hole mimickers; stable???

Reviews: Liebling & Palenzuela LRR 2023, Visinelli 2109.05481,

Mielke Fund.Theor.Phys. 2016, Schunck & Mielke GRG 1999, ...

# Formalism and basic features

- GR + minimally coupled complex scalar field $\varphi$

$$S = \int \sqrt{-g} \left\{ \frac{1}{16\pi G} R - \frac{1}{2} [g^{\mu\nu} \nabla_\mu \bar{\varphi} \, \nabla_\nu \varphi + V(\varphi)] \right\} \mathrm{d}x^4$$

$$T_{\alpha\beta} = \partial_{(\alpha} \bar{\varphi} \, \partial_{\beta)} \varphi - \frac{1}{2} g_{\alpha\beta} [g^{\mu\nu} \partial_\mu \bar{\varphi} \, \partial_\nu \varphi + V(\varphi)]$$

- Potential; analogous to EOS:

$$V_{\mathrm{mas}}(\varphi) = \mu^2 |\varphi|^2 + \lambda |\varphi|^4, \quad V_{\mathrm{soli}}(\varphi) = \mu^2 |\varphi|^2 \left( 1 - 2 \frac{|\varphi|^2}{\sigma_0^2} \right)^2$$

- Mini Boson Stars: $\lambda \to 0, \quad \sigma_0 \to \infty$

- Ansatz $\varphi(t, r) = A(r) e^{\mathrm{i}\epsilon\omega t + \delta\Phi}, \quad \varphi(t, r, \theta, \phi) = A(r, \theta) e^{\mathrm{i}(\epsilon\omega t + m\phi + \delta\Phi)}$

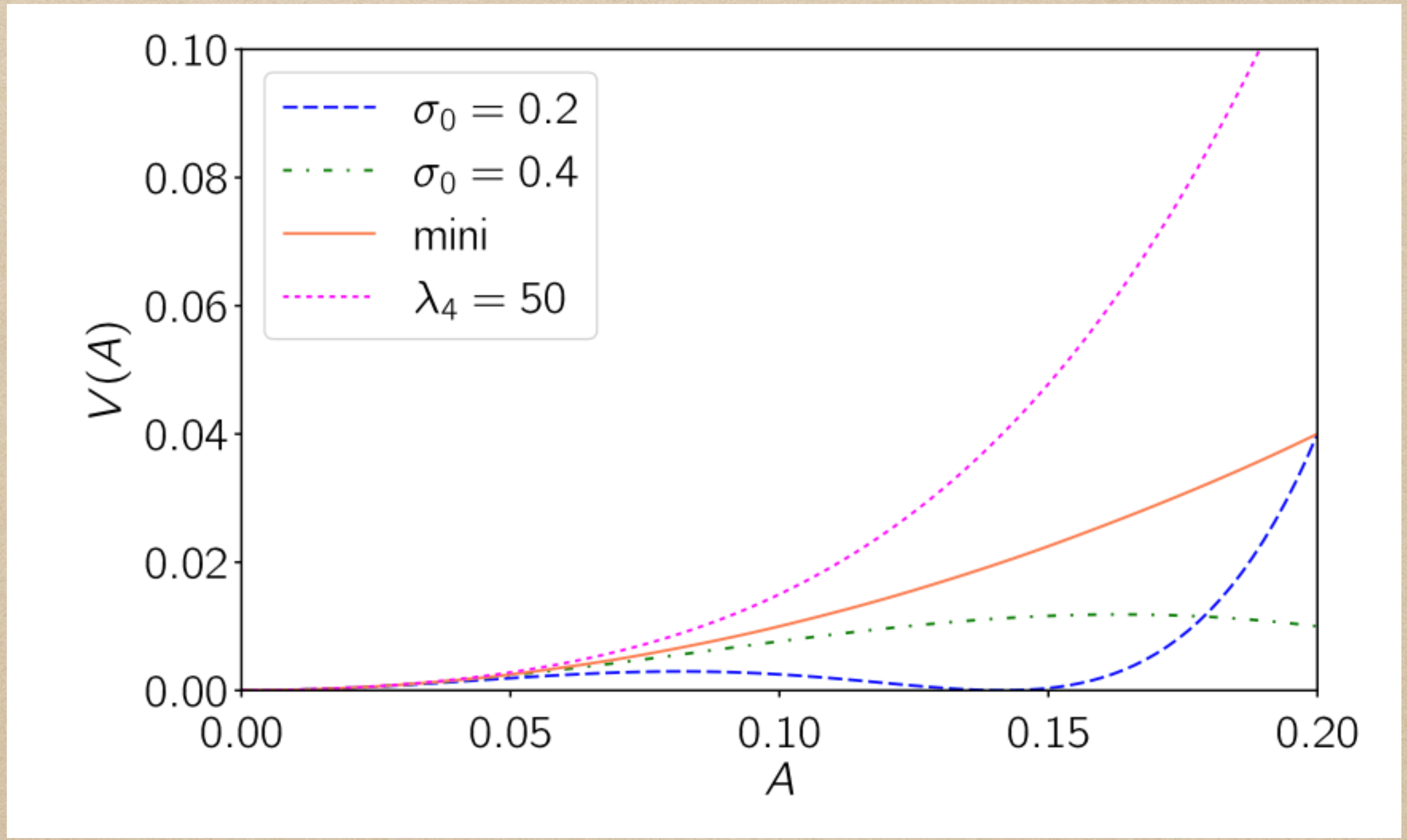
  $\delta\Phi$ = phase offset, $\epsilon$ = BS vs anti-BS

- Regular solutions only for countably infinite values

  $\omega_0 < \omega_1 < \omega_2 < \ldots$ (ground state, excited states)

# Formalism and basic features

- Scalar potentials




Evstafyeva et al PRD 2023

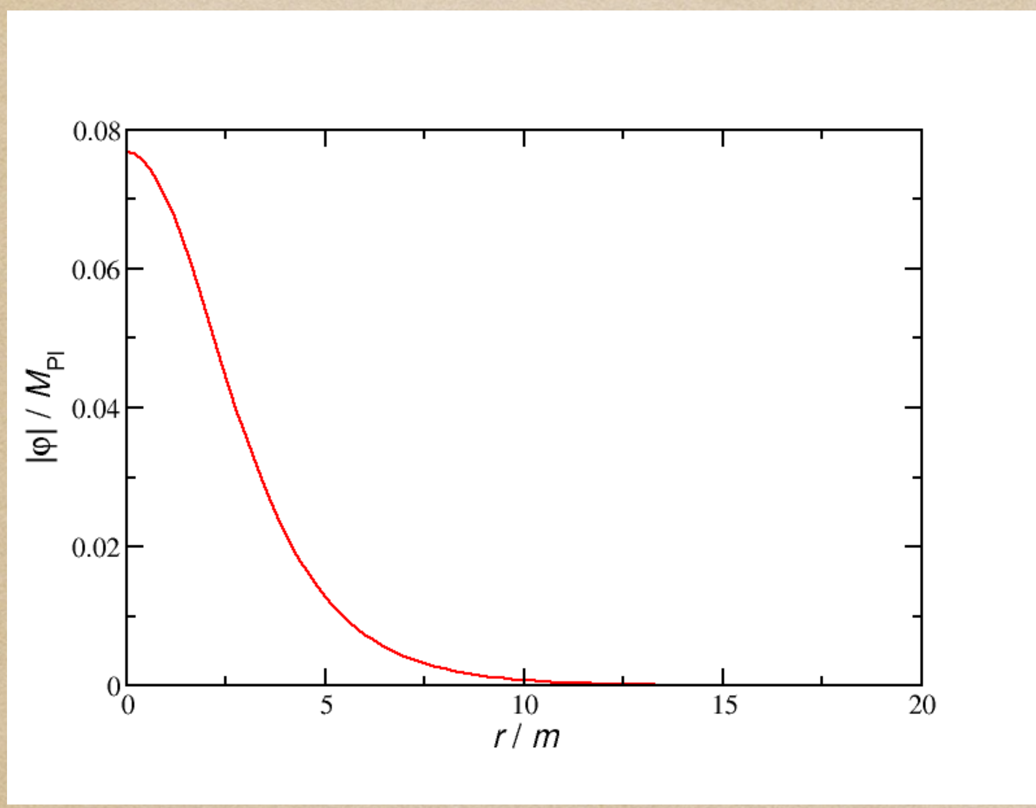
- Massive Potential: Stiff EOS

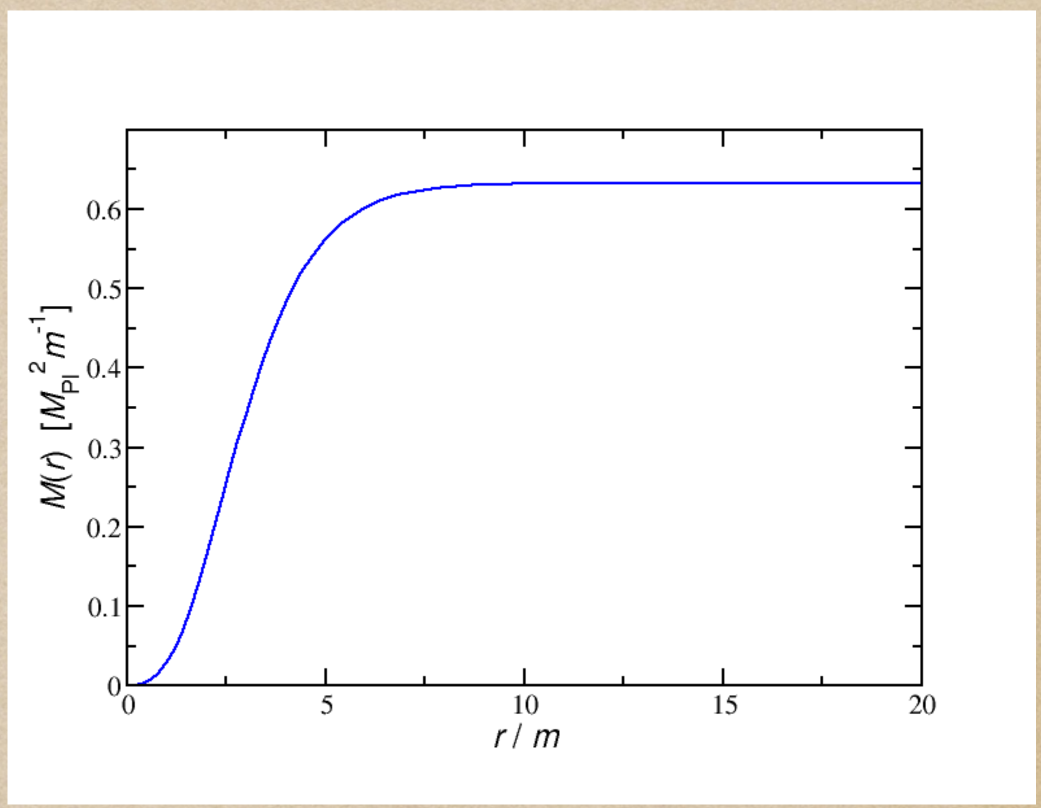
  Solitonic Potential: Soft EOS

# Formalism and basic features

- E.g. Maximal-mass, spherically symm. mini boson star (Kaup limit)

$$\omega_0 = 0.853\, m\,, \qquad M = 0.633\, M_{\rm Pl}^2/m$$



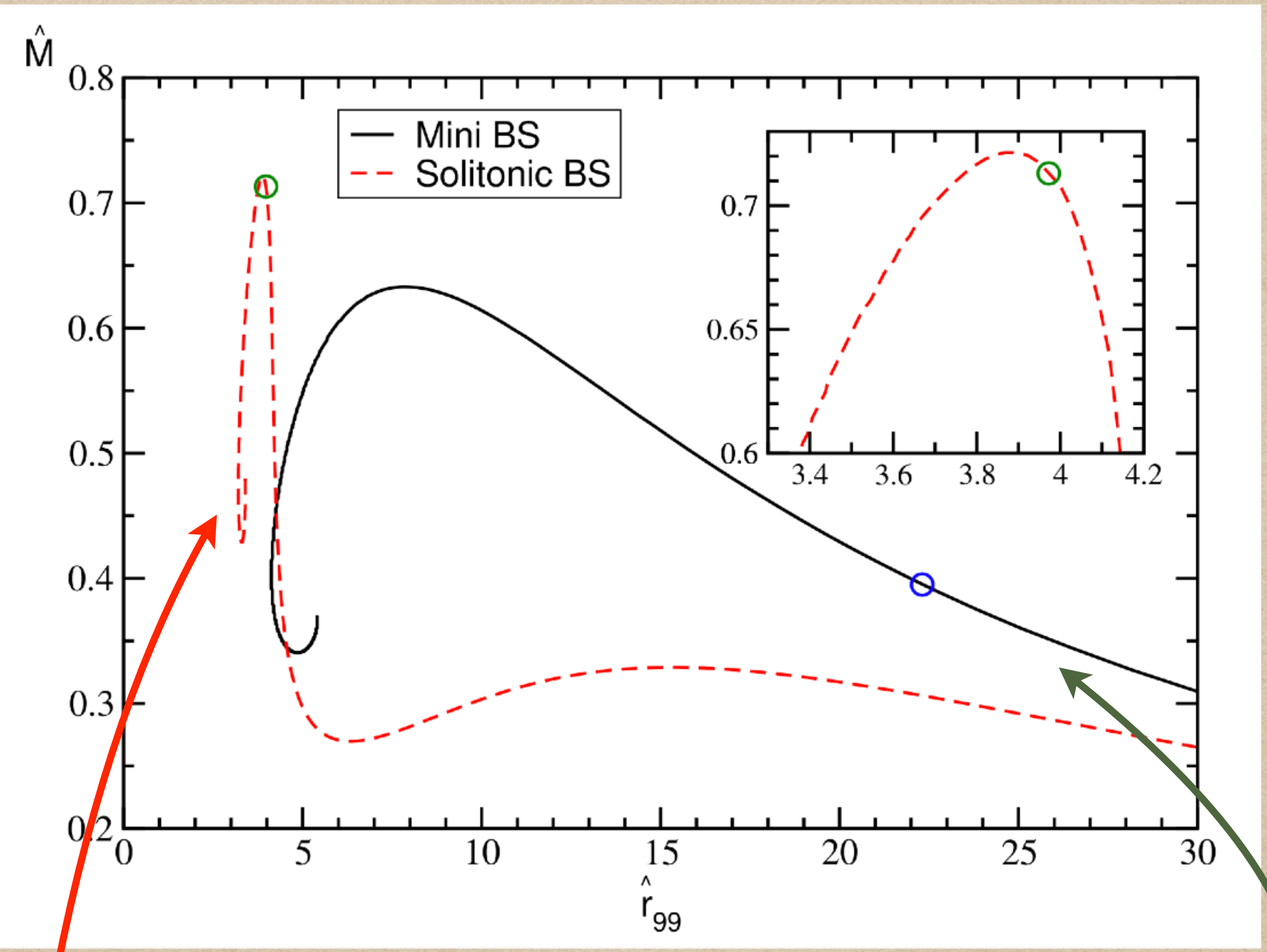

- Excited states unstable

collapse to BH, dispersion or migration to stable ground-state BS

Balakrishna, Seidel, Suen PRD (1998)

# Formalism and basic features

- Mass-Radius curves similar to Tolman-Oppenheimer-Volkoff stars




unstable

stable

- Radial stability similar to TOV stars Marks 2508.11757

# Spinning Boson Stars

- Scalar BSs cannot spin perturbatively    Kobayashi+ PRD (1994)
- Spinning scalar BSs exist with but have quantized spin

  Schunck & Mielke    Phys.Lett.A (1998)
- Spinning scalar BSs may be unstable in contrast to

  spinning Proca stars!    Sanchis-Gual+    PRL (2019)

  Possibly due to toroidal structure: scalar field vanishes at origin
- What happens in scalar BS inspiral and merger?
  - Kerr BH
  - Non-spinning BS; angular momentum shed
  - Total dispersal
  - Spinning BS with exact angular momentum?

# 2. A primer of boson-star dynamics

# Theory and Numerical Modelling

- Massive complex scalar field + GR

$$S = \int \frac{\sqrt{-2}}{2} \left\{ \frac{R}{8\pi G} - [g^{\mu\nu} \nabla_\mu \bar{\varphi} \nabla_\nu \varphi + V(\varphi)] \right\} \mathrm{d}^4 x$$

$\Rightarrow$ Einstein-Klein-Gordon equations

- Space-time formulations: GHG, CCZ4, BSSNOK
- Techniques like for BH binaries. We use:

GRChombo Radia et al CQG 2021

Lean US PRD 2006

- Typical grid setups:

$\mathrm{d}x = \frac{M}{48} \ldots \frac{M}{32}$, domain size $\sim 1024\,M$, 8 refinement levels

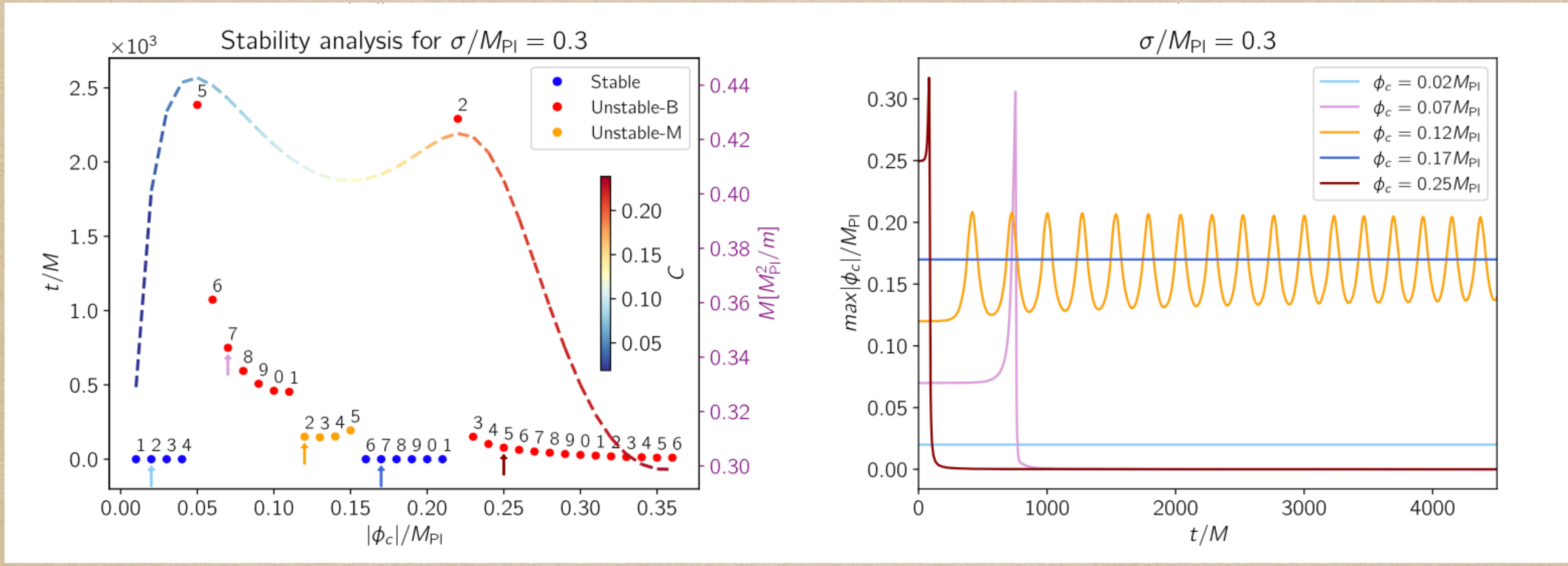
- Improve binary initial data: Constant Volume Element (CVE)

Helfer et al PRD 2019, CQG 2021, Evstafyeva et al CQG 2023,

Atteneder et al PRD 2024

# Stability of nonspinning BSs

- Select potential; here solitonic
- Maxima in $M(|\varphi_c|)$ curve (typically) signals swap in stability
- Radial stability similar to TOV stars Marks 2508.11757

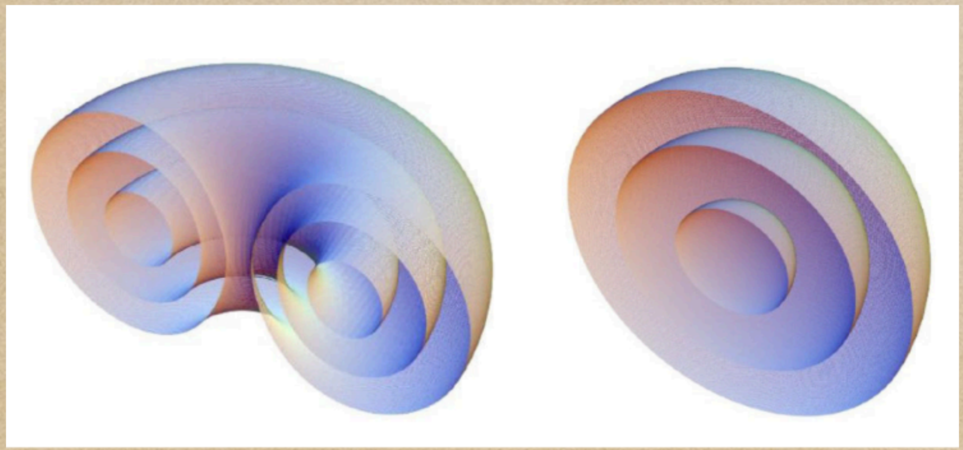



Ge et al. 2410.23839

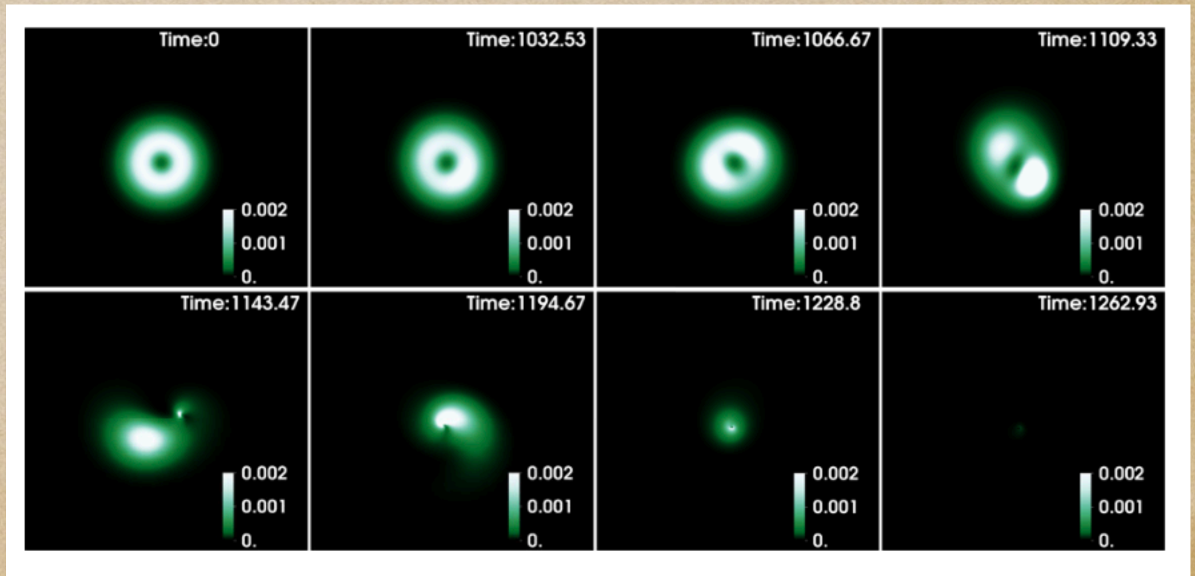
- Evolution of BSs:
  - BH collapse
  - Migration to stable BS
  - Evaporation to infinity (partial or whole)

# Formation and Stability of spinning BSs

- Evolve scalar or vector cloud; no self-interaction!
  - Cloud collapses due to gravitational cooling
  - Scalar case: Instability shedding all angular momentum
  - Vector case: No instability, spinning Proca star forming
- Evolve spinning equilibrium stars
  - Scalar case: Instability → Formation of spinning BH
  - Vector case: Stable evolutions even for explicit perturbations
- May be due toroidal/spherical energy profile

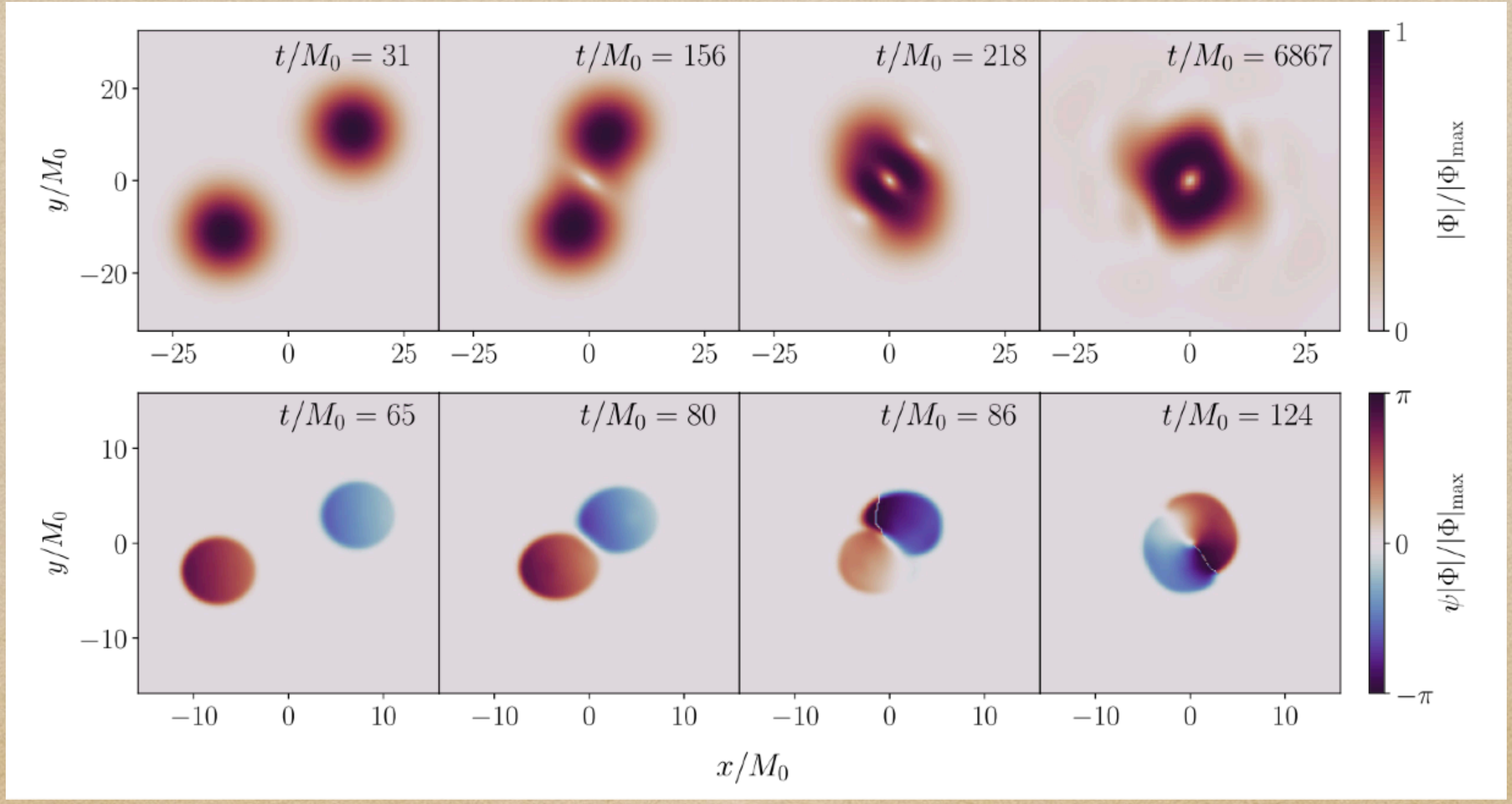


Sanchis-Gual et al PRL 2019



# Spinning scalar BSs with self-interaction

- Nonrotating ($\delta\Phi = 0...\pi$, $M/r \approx 0.037$) scalar BS binary on eccentric orbits Siemonsen & East PRD 2023
- Massive and solitonic potentials
- Potential and angular momentum strongly affects GW signal
- Threshold dephasing $0 < \delta\Phi_{\rm cr} < \pi$ required for forming spinning BS
- Dephasing, scalar interaction, vortex condition $(J/M^2)$ important



# Black-hole mimickers

- Ultracompact objects with light ring but no horizon
- Observational evidence for BHs
  - Accretion
  - GWs + QNM ringdown

Talks Seppe Staelens
Gereth Marks!

Related closely to light rings rather than event horizon

- Theorem: Smooth compact objects with light ring have at least two light rings, one them stable. Cunha, Berti & Herdeiro PRL 2017
- Stable LRs = attractor for (massless) particles → Instability?

Keir CQG 2016, Cardoso et al PRD 2014, Cunha 2503.00117

- NR simulations of ultra compact boson stars inconclusive:
  - Migration or BH collapse: Cunha et al PRL 2023
  - Long-term stable BS evolutions: Marks et al PRL 2025,

Evstafyeva et al 2508.11527, Siemonsen PRL 2024, Staelens 2508.19460

# Ergoregion Instability

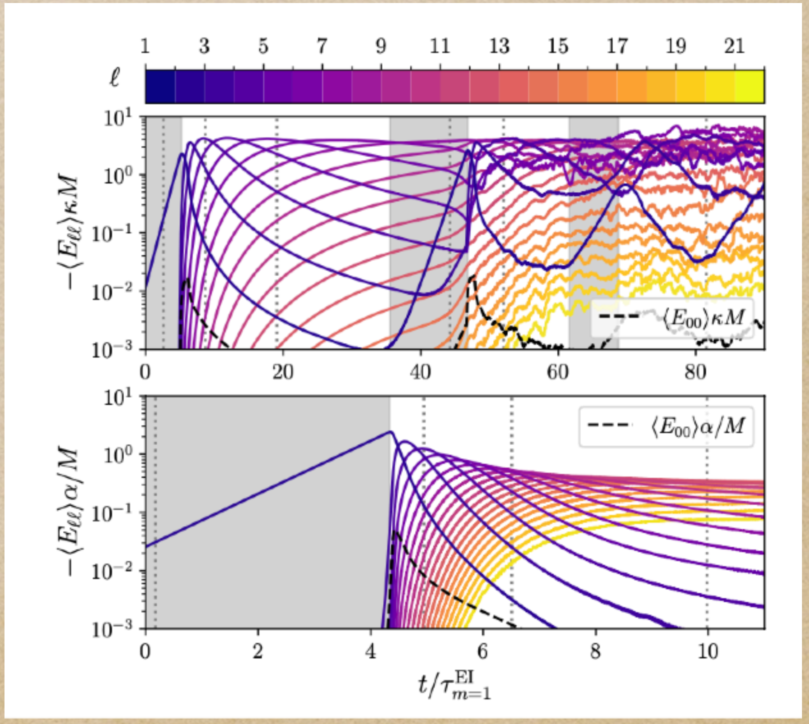
- Ergoregion = All dragged in rotation sense
- Horizonless compact objects with ergoregion

  → Ergoregion Instability: Exponential growth of negative energy modes through energy emission to $\infty$

  Friedmann C.Ma.Ph. 1978, Vilenkin Ph.Lett.B 1978, Moschidis C.Ma.Ph. 2018
- Cf. R-Mode Instability of Neutron stars, but unlike Superradiance
- NR simulations of rapidly spinning BSs and toy problem:
  - $e-$folding time can be $10^4\,M$
  - $l = m = 1$ retrograde mode grows
  - Then turbulent cascade
  - (short) NR runs: No instability

  Siemonsen 2510.07467, 2510.07468

  Siemonsen & East 2512.10526

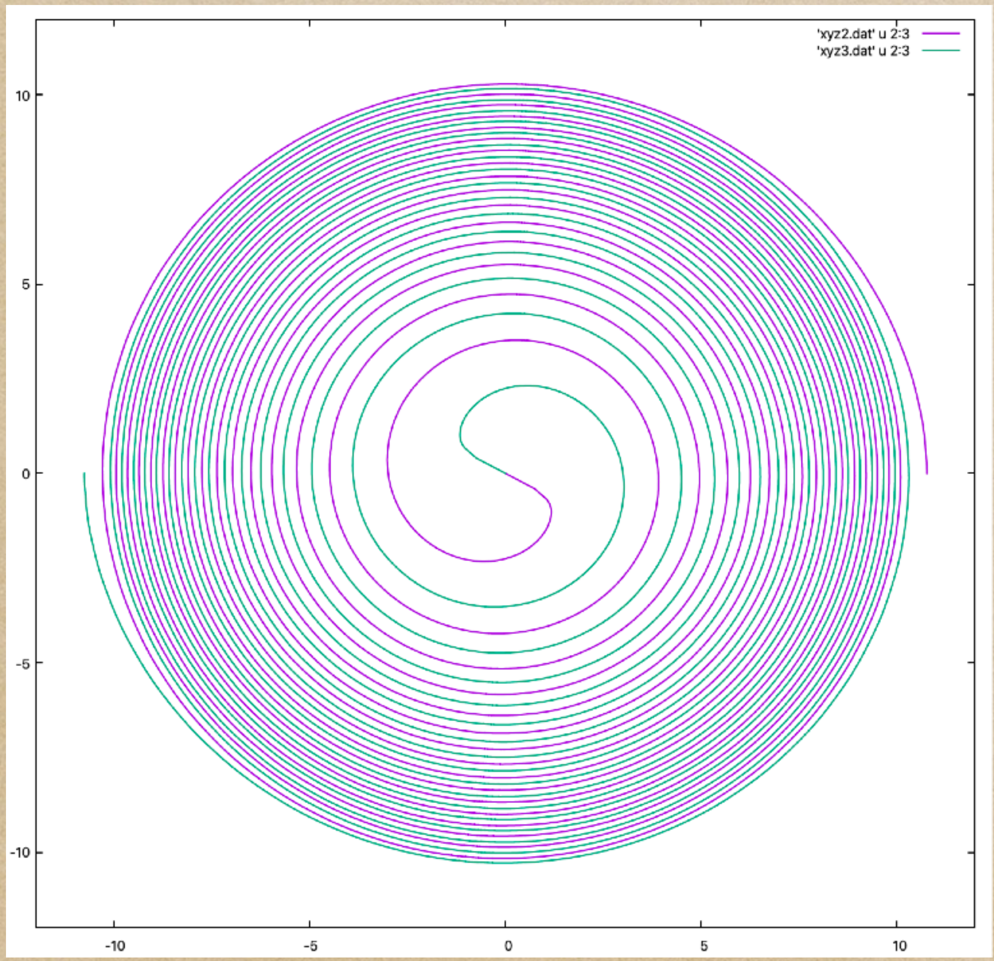


# BS binaries

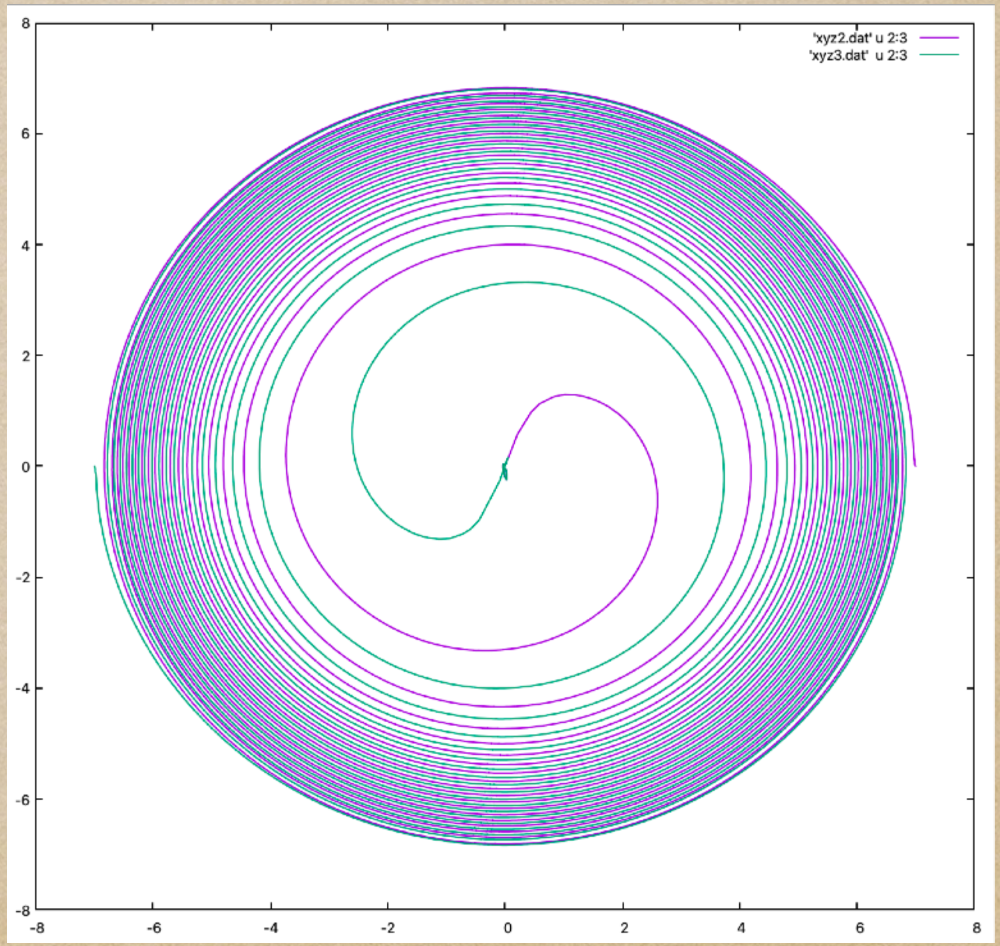
- Phase error $\approx 0.1 \ldots 0.2$
- Amplitude error $\lesssim 3\,\%$
- Eccentricity $\approx 0.002 \ldots 0.005$

Evstafyeva et al 2406.02715

compact BS binary: A17-d14

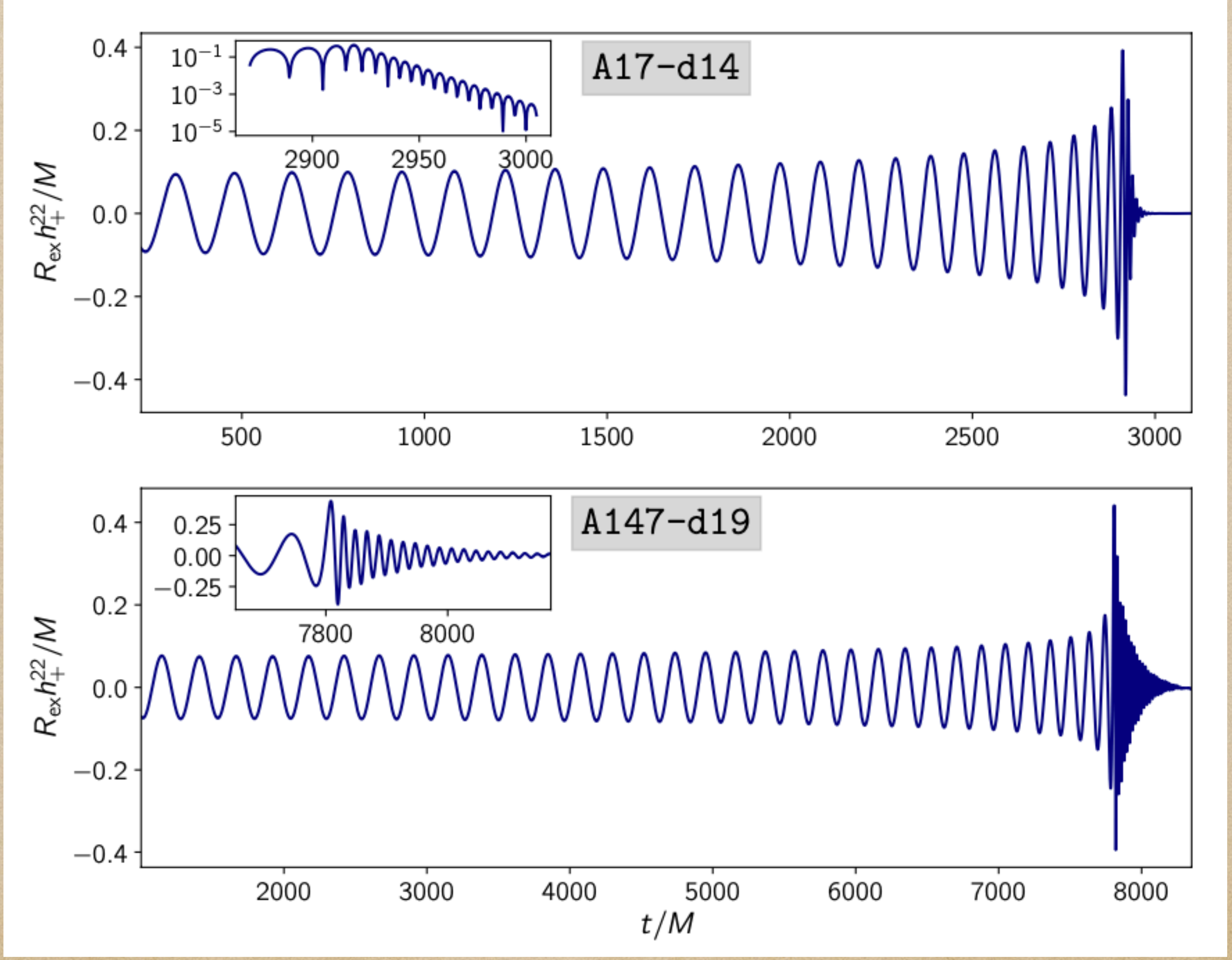



fluffy BS binary A147-d19



# BS binaries

- GW strain from compact and fluffy BSs

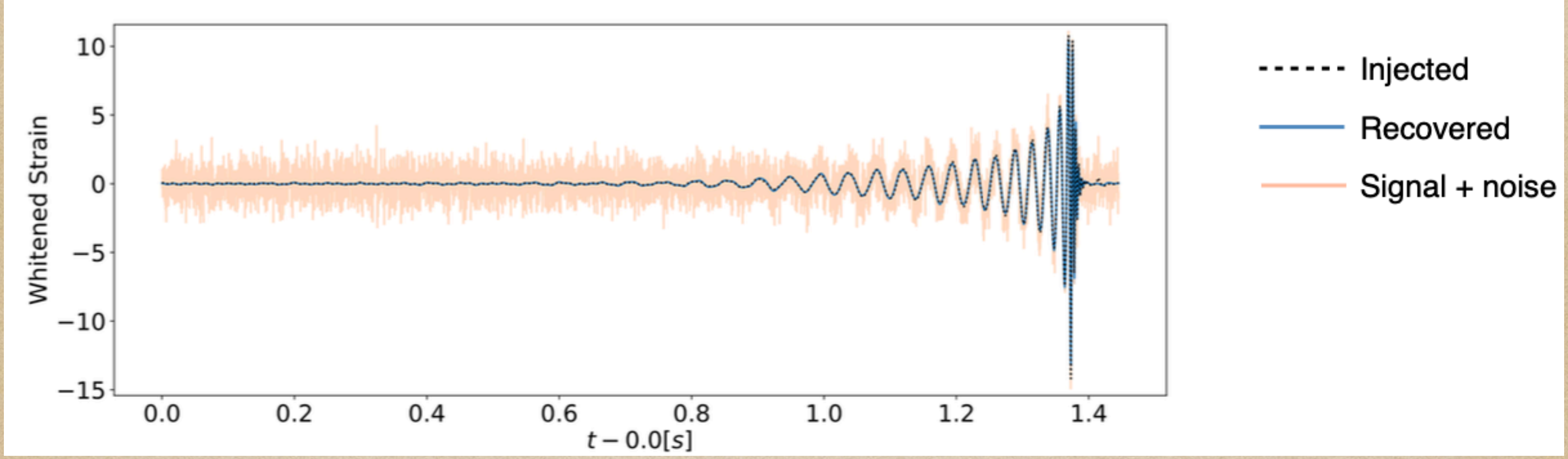


# Waveform approximants

- Parameter estimation performed with `Bilby` Ashton et al 2019
- `IMRPhenomXP`: Frequency domain Pratton et al 2021
  - Quasi-circular, spin-precessing black-hole binaries
  - Quadrupole modes
- `IMRPhenomPv2_NRTidal`: Frequency domain Dietrich et al 2017, 2019
  - Quasi-circular, spin-precessing neutron-star binaries
  - Tidal deformability parameters $\Lambda_{A,B}$
- We have tested more with similar results.

# Compact BSs using IMRPhenomXP

- Injections often recovered but with biased parameters!
- Example A17-d15 with $M_{\rm tot} = 77\,M_\odot$, $d_L = 200\,\mathrm{Mpc}$ in the analysis




- Recovered: $M_1 = 37.8 \pm 1.1\,M_\odot$, $M_2 = 25.4 \pm 1.2\,M_\odot$, $d_L = 236 \pm 20\,\mathrm{Mpc}$,

  $a_1 \approx 0.95$, $a_2 \approx 0.15$

  Recovered SNR $\approx$ injected SNR

  $\log \mathscr{B}^{\rm S}_{\rm N} = 5392$

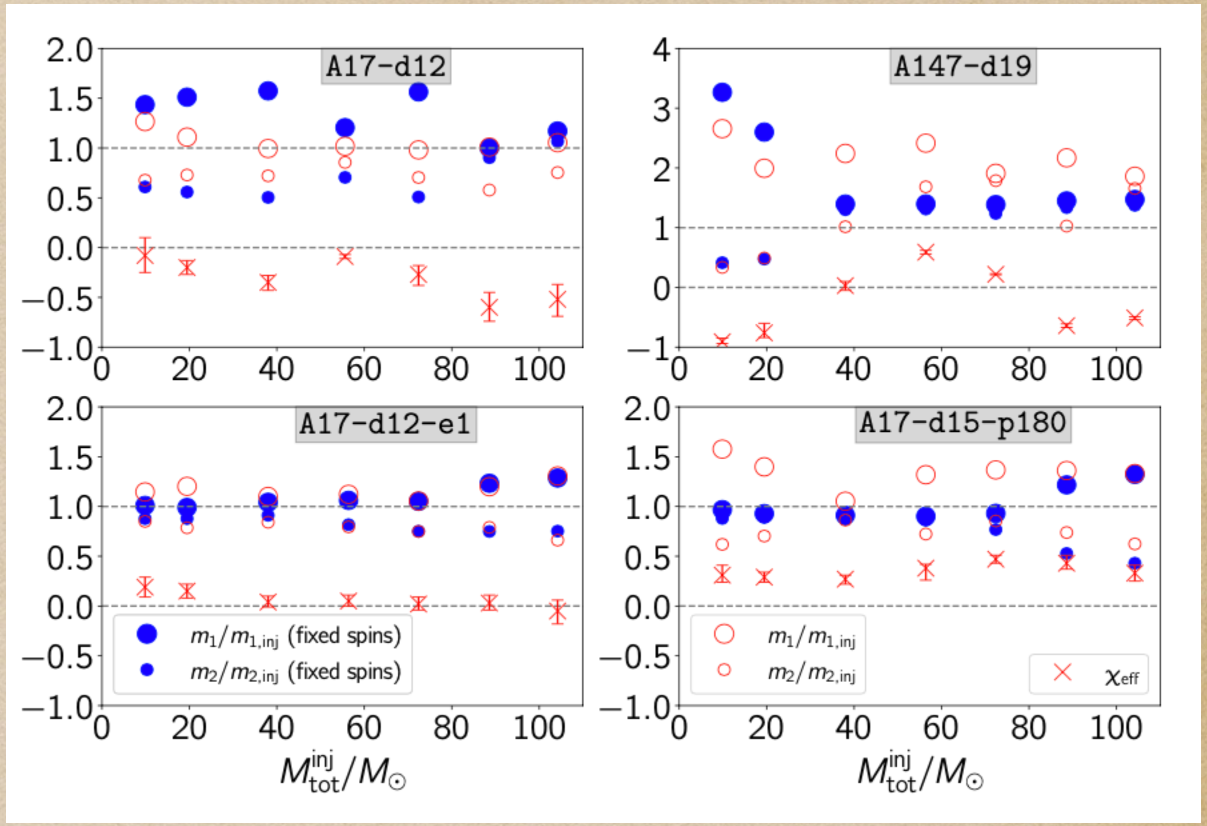
- Parameter bias not random!

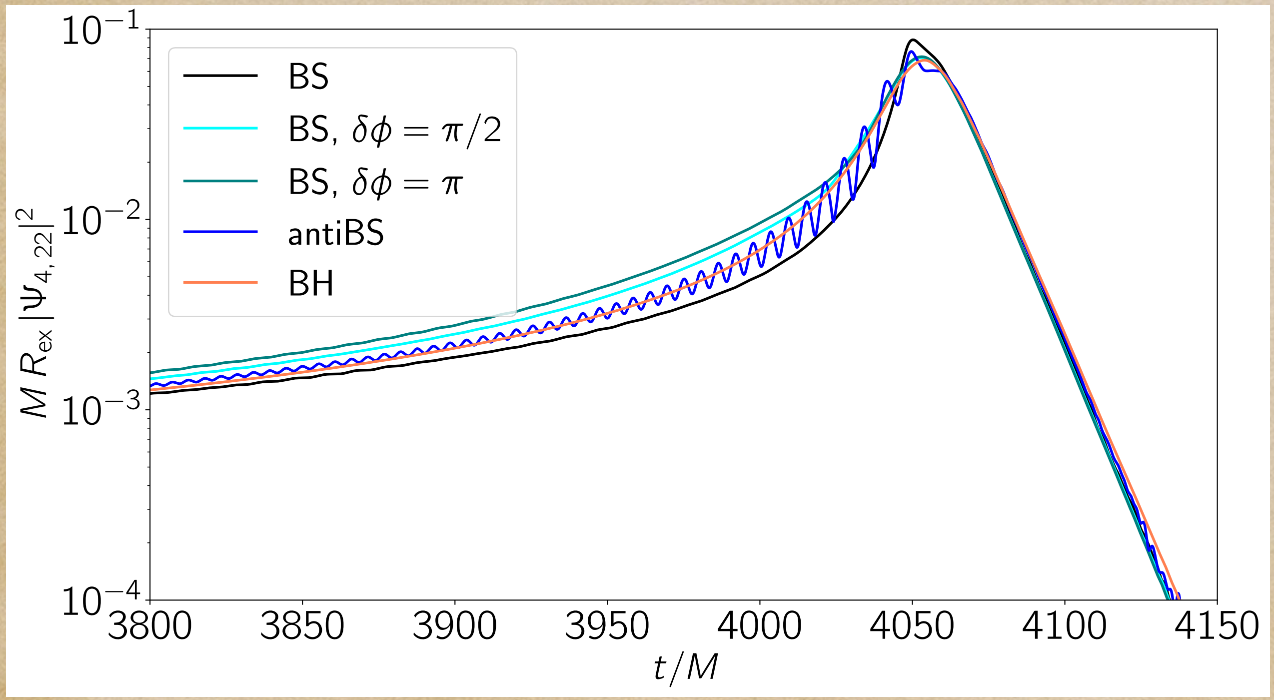
# Understanding the PE bias

- PE bias

vs

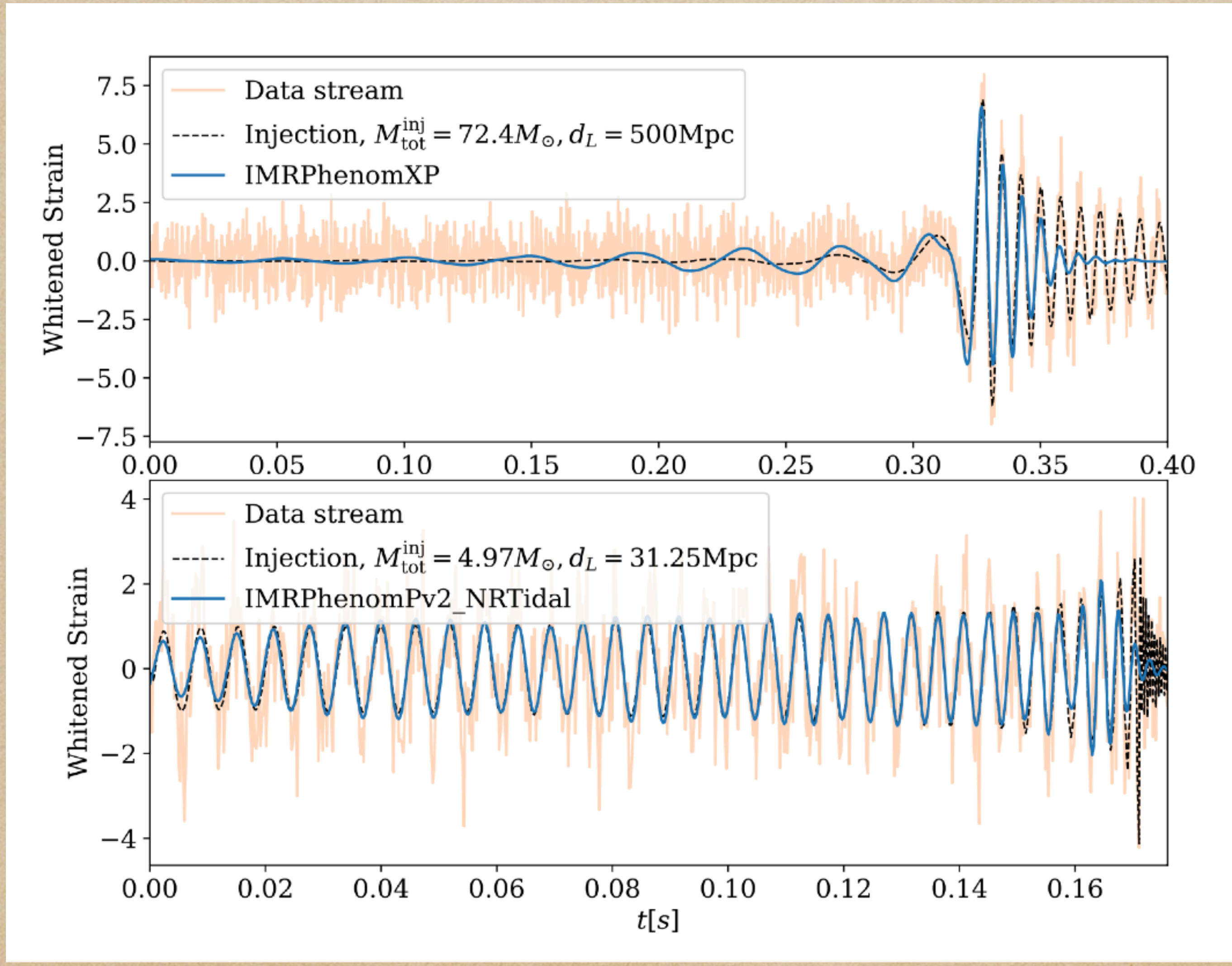
- Chirp steepness



# Recovery of "Fluffy" BS binaries

- Parameter estimation always erratic for *fluffy* BBS
- BH approximants may capture inspiral or merger but never both!
- Residual often not compatible with Gaussian noise




cf. also

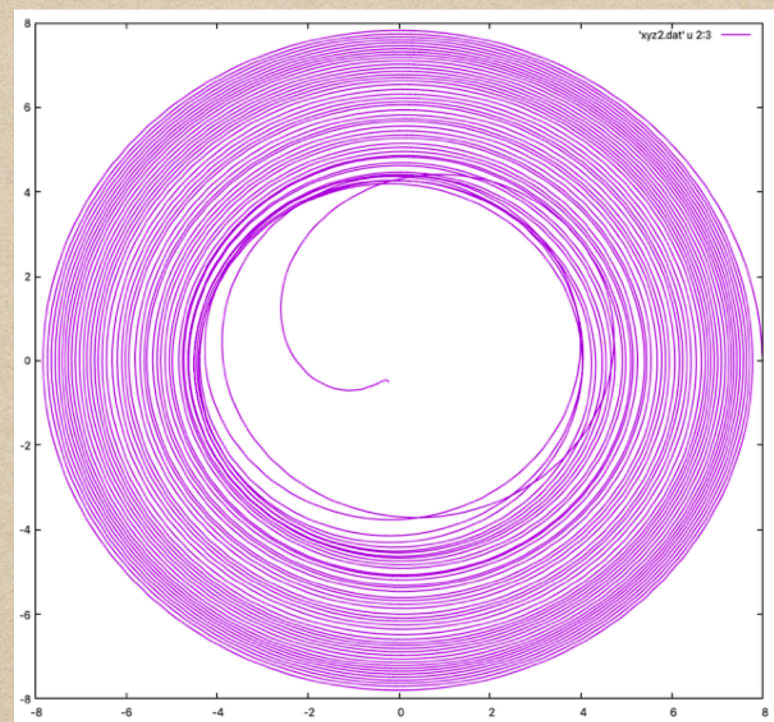
Krishnendu et al 2511.17341

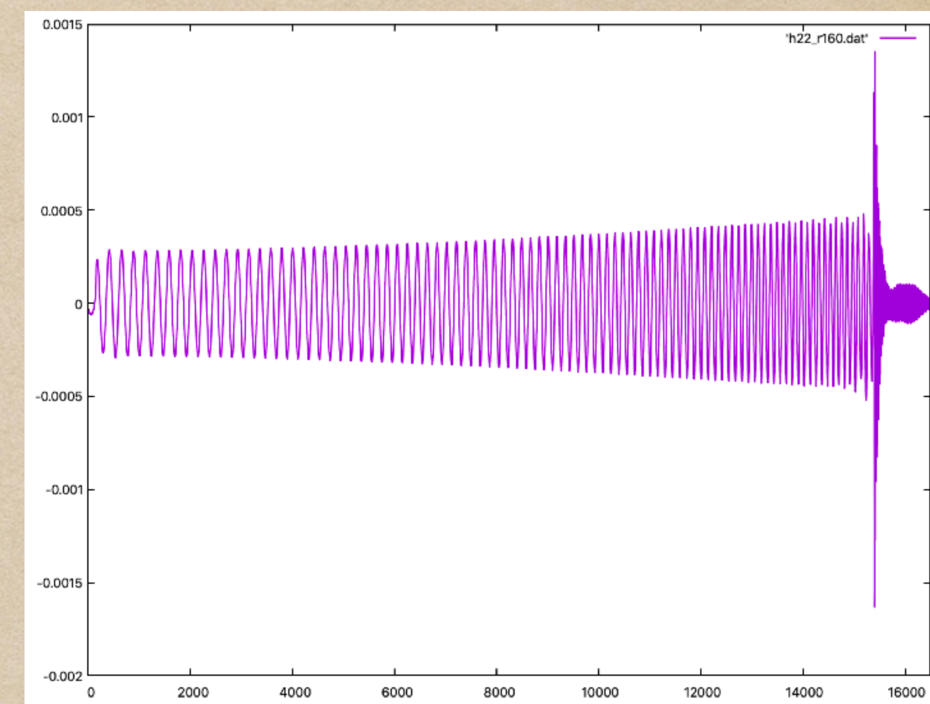
# Some “weird” waveform samples from fluffy BSs

$(x(t), y(t))$ | $h_{22}(t)$

$q = 1,$

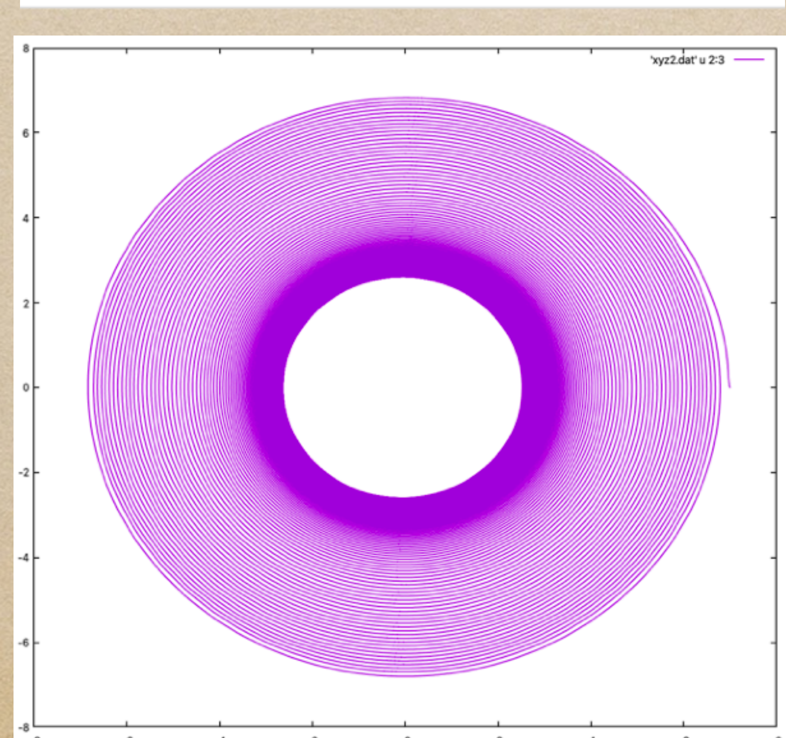
$\delta\Phi = 120^\circ$

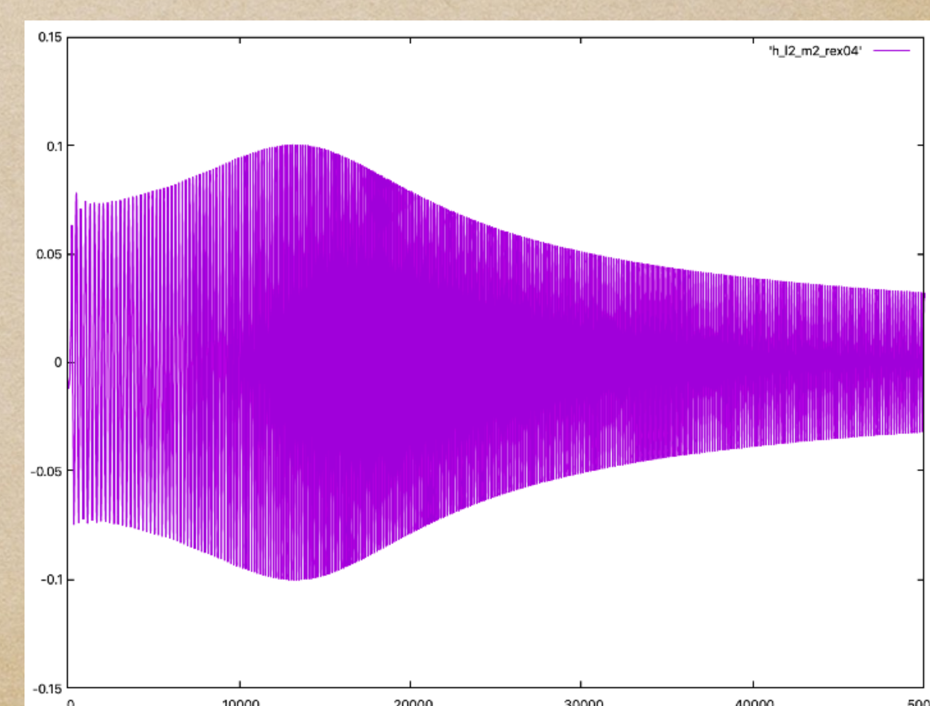



$q = 1,$

$\delta\Phi = 180^\circ$

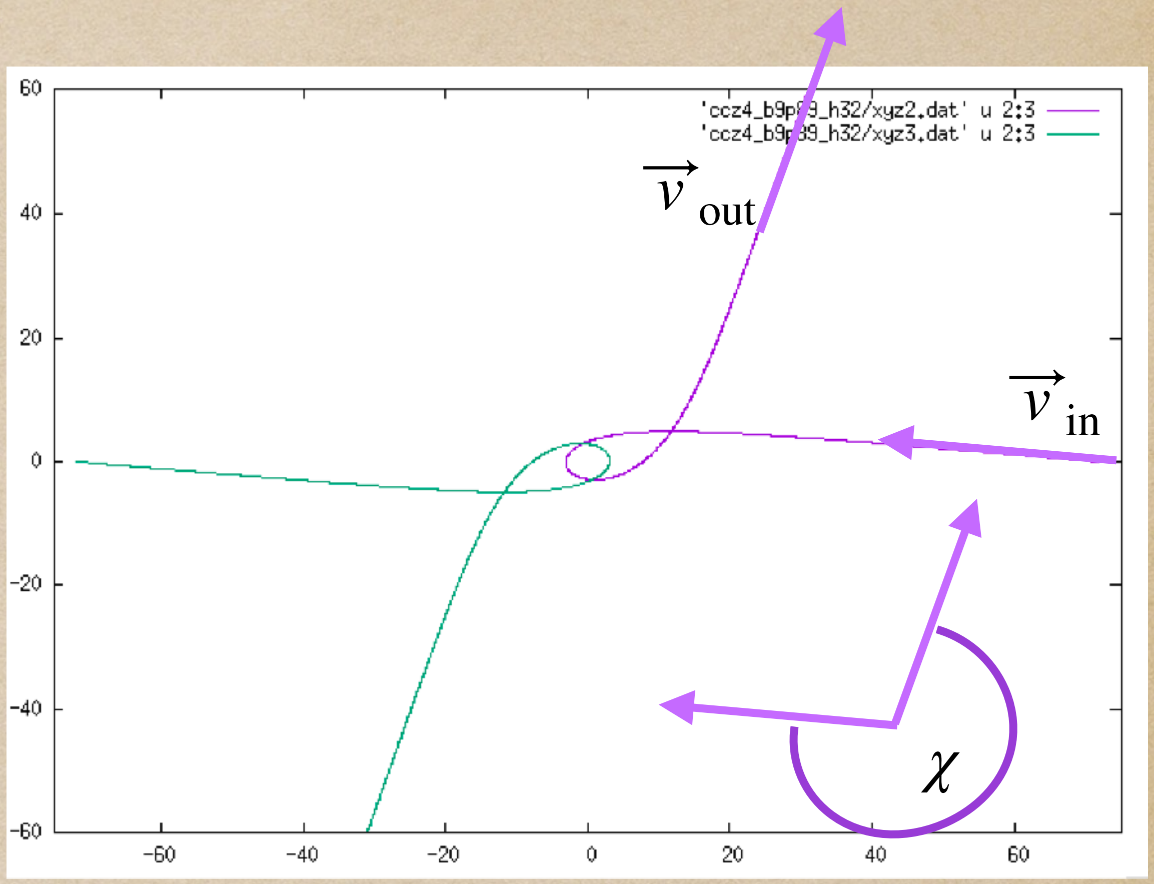


# 3. EOB modeling of BS scattering

# Analytic Modeling of BS Binaries

Damour, Jain & US   2512.00945




- Consider 2 scattering BSs
- NR simulations with Lean;

  error $\Delta\chi \approx 1.6^\circ$
- 3 contributions:
  - Point particle BH
  - Tidal "tid"
  - Scalar interaction "scl"
- PM scattering angle: $\chi(\gamma, j) = \chi^{\rm BH}(\gamma, j) + \chi^{\rm tid}(\gamma, j) + \chi^{\rm scl}(\gamma, j)$

  EOB potential: $w(\gamma, \bar{r}) = w^{\rm BH}(\gamma, \bar{r}) + w^{\rm tid}(\gamma, \bar{r}) + w^{\rm scl}(\gamma, \bar{r})$
- EOB map: $$\pi + \chi(\gamma, j) = 2j \int_0^{\bar{u}_{\rm max}(\gamma, j)} \frac{d\bar{u}}{\sqrt{\gamma^2 - 1 + w(\gamma, \bar{u}) - j^2 \bar{u}^2}}, \quad \bar{u} = \frac{1}{\bar{r}}$$

Damour 1710.10599

# The analytic potential

- BH:
  - $\chi^{\mathrm{BH}}$ known up to 4PM Kälin, Liu & Porto 2008.06047

    $\rightarrow$ $w^{\mathrm{BH}}$ up to 4PM Damour & Rettegno 2211.01399
  - Numerically fitted 5PM term Rettegno et al 2307.06999
- Tidal:
  - at 6PM and 7PM using scattering angles from

    Kälin, Liu & Porto 2008.06047, Bini, Damour & Geralico 2001.00352,

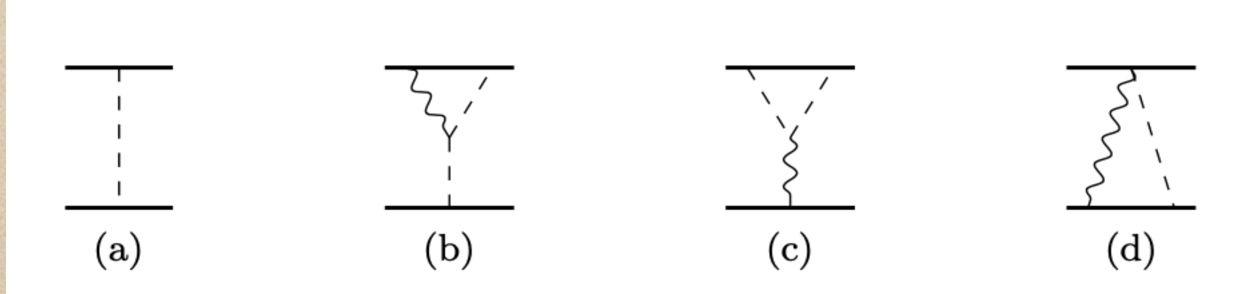
    Cheung & Solon 2006.06665
- Scalar: Previously unknown…

# The scalar potential contribution

- EFT action for 2BSs as world lines $\mathbf{z}_A(\tau_A)\,,\ A = 1, 2\,,$ with source

$$s_A(\tau_A) = c_A e^{\mathrm{i}\epsilon_A\omega_A\tau_A + \mathrm{i}\Phi_A}\,, \qquad c_A = \text{ scalar charges}$$

- Perturbative analysis $\rightarrow$ Feynman diagrams




- Leading Order from (a): $\chi^{\mathrm{scl}}_{\mathrm{LO}}(\gamma, j) = \dfrac{8\pi M h\tilde{\mu}}{m_1 m_2}\dfrac{c_1 c_2}{\gamma^2 - 1}K_1(\tilde{\mu}\,|\,b\,|)\,\cos(\Phi_2 - \Phi_1)$ ,

where $\quad h = \sqrt{1 + 2\nu(\gamma - 1)}\,, \quad \tilde{\mu}^2 = \mu^2 - \dfrac{2\omega_1\omega_2\gamma - \omega_1^2 - \omega_2^2}{\gamma^2 - 1}$

plus a negligible nonconservative part for $\quad \Phi_1 \neq \Phi_2$

$$\rightarrow \quad w^{\mathrm{scl}}_{\mathrm{LO}}(\gamma, \bar{r}) = \frac{8\pi c_1 c_2}{G m_1 m_2}\frac{e^{-GMh\tilde{\mu}\bar{r}}}{\bar{r}}\cos(\Phi_2 - \Phi_1)$$

# … continued

- Higher-order contribution $w_{\rm HO}^{\rm scl}$ not fully known;

  we assume a factorized form

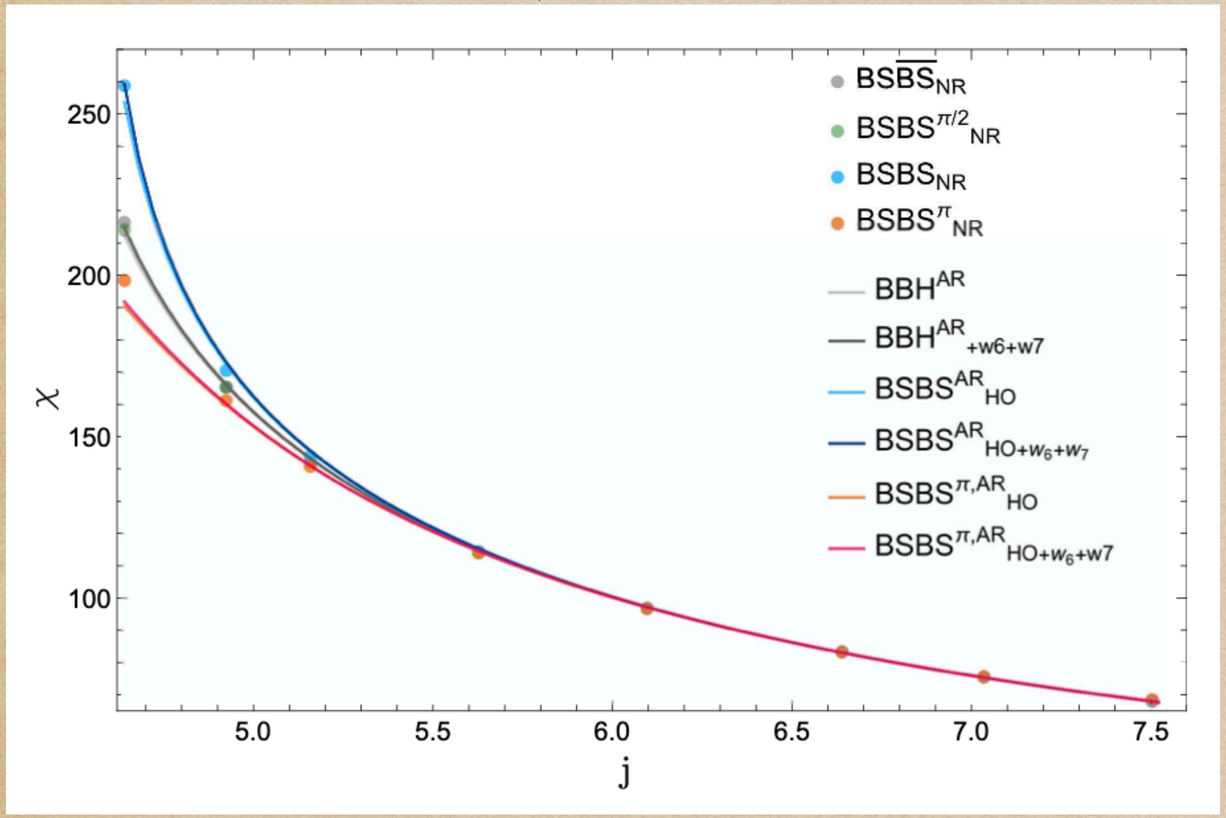
$$w_{\rm HO}^{\rm scl} = w_{\rm g-dressing}^{\rm scl} \times w_{\rm LO}^{\rm scl}\,, \qquad w_{\rm g-dressing}^{\rm scl} = e^{GM((2\omega^2-\mu^2)/\tilde{\mu})\,\log(2\gamma_{\rm E} GM\tilde{\mu}\tilde{r})}$$

- Input needed:
  - total energy $E$
  - total angular momentum $J = 1.006\, J_{\rm NR}$
  - tidal deformability parameters $\Lambda_1 = \Lambda_2 \approx 7$

    Sennett et al 1704.08651
  - scalar charges $c_1, c_2$ from BS asymptotics in isotropic radius

# Results

- Equal-mass BS binaries with $\sigma_0 = 0.2,\ |\varphi_c| = 0.17$
- Four binaries: $\mathrm{BSBS},\ \mathrm{BS}\overline{\mathrm{BS}},\ \Phi_2 - \Phi_1 = \dfrac{\pi}{2},\ \Phi_2 - \Phi_1 = \pi$




- Scalar corrections dominant
- Tidal effects still important for small impact parameter
- Scalar contribution attractive (repulsive) for

$$\Phi_2 - \Phi_1 = 0 \quad (\Phi_2 - \Phi_1 = \pi)$$

- $\mathrm{BS}\overline{\mathrm{BS}}$ close to BH binaries

# 5. Mixed binaries

# BH-BS Binaries

- "Piercing papers"

  Cardoso et al PRD 2022, Zhong et al PRD 2023

  BH passing through a fluffy mini or solitonic BS:

  $\gtrsim 90\,\%$ of BS mass accreted, even for light BHs (test particle limit)

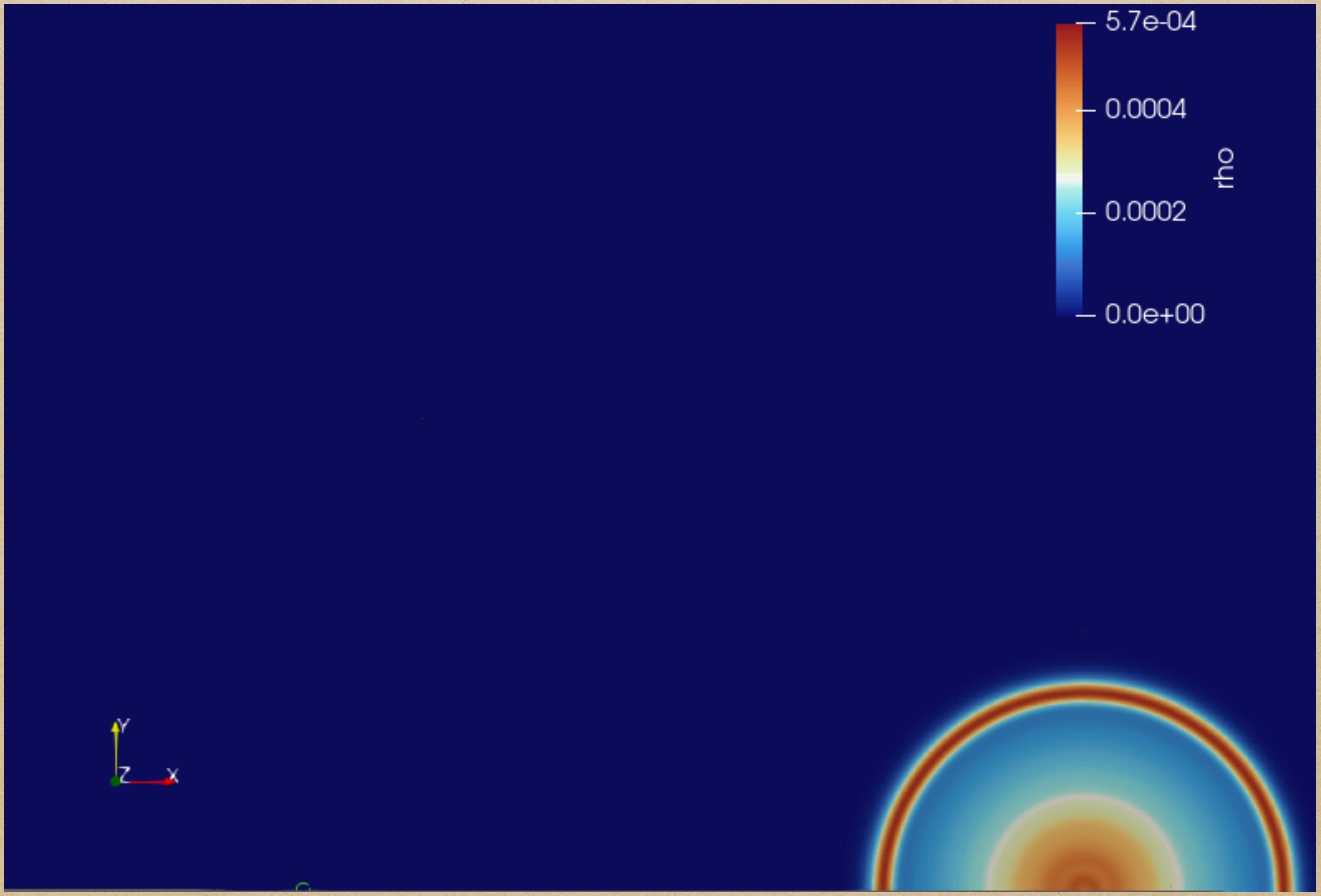
- Head-on collisions and orbital evolutions of BH-BS binaries

  Marks, Staelens & Sperhake in prep.

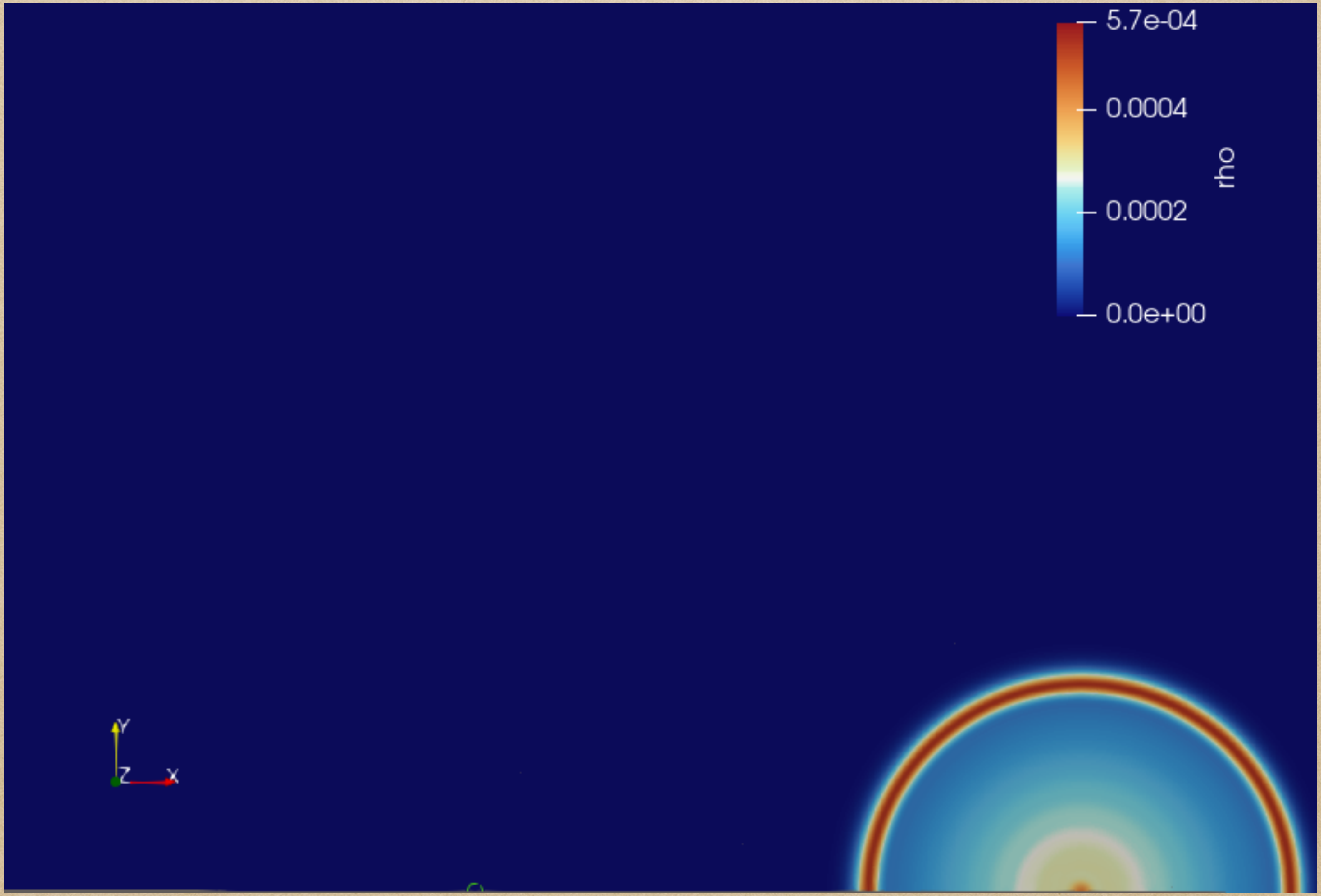
  - GRChombo code
  - CCZ4 formulation
  - Constant-Volume-Element initial data
  - Example: Thin-Shell BS Head-on collision with BH

  Credit to Gareth Marks!!!

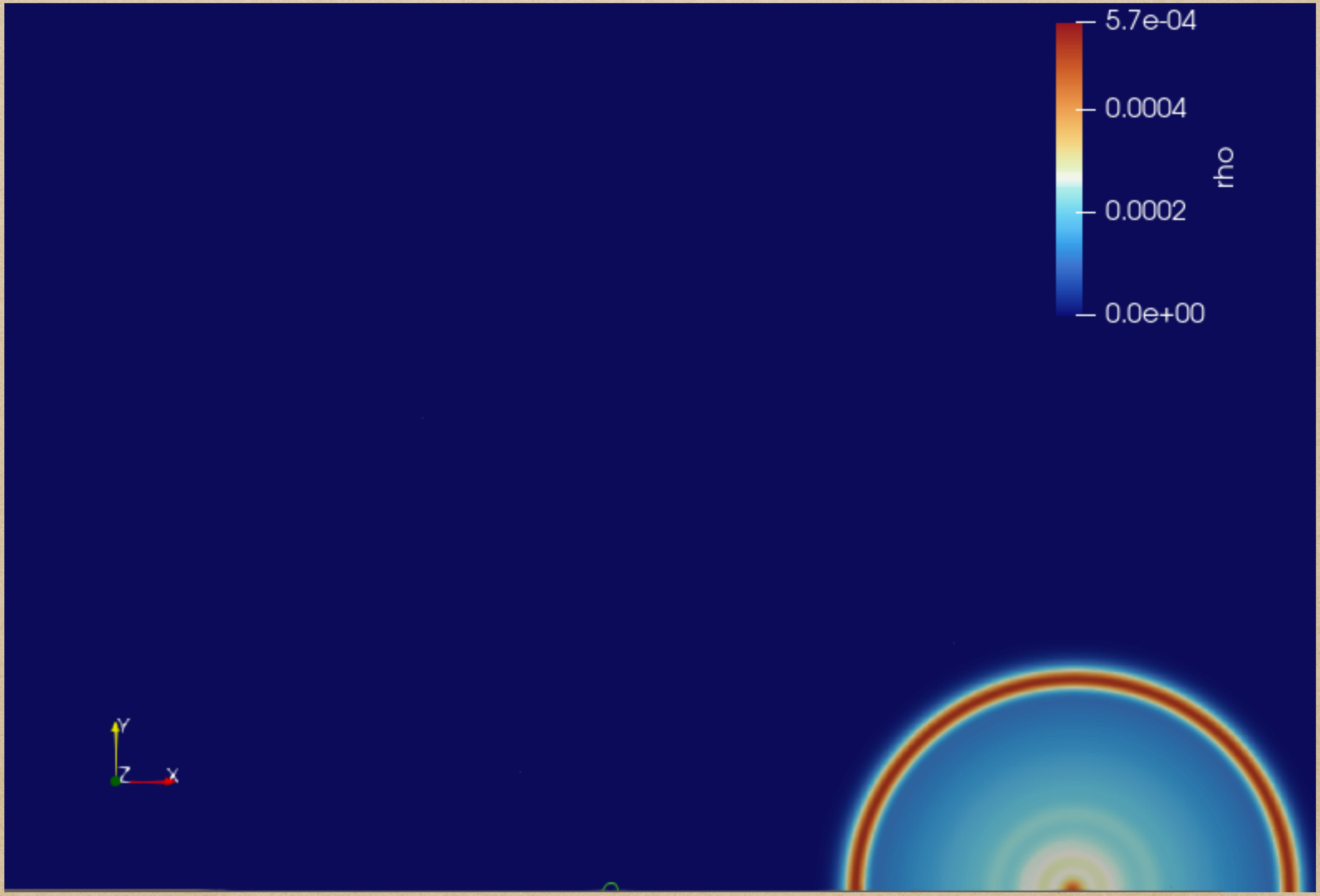
# BH-BS Binaries



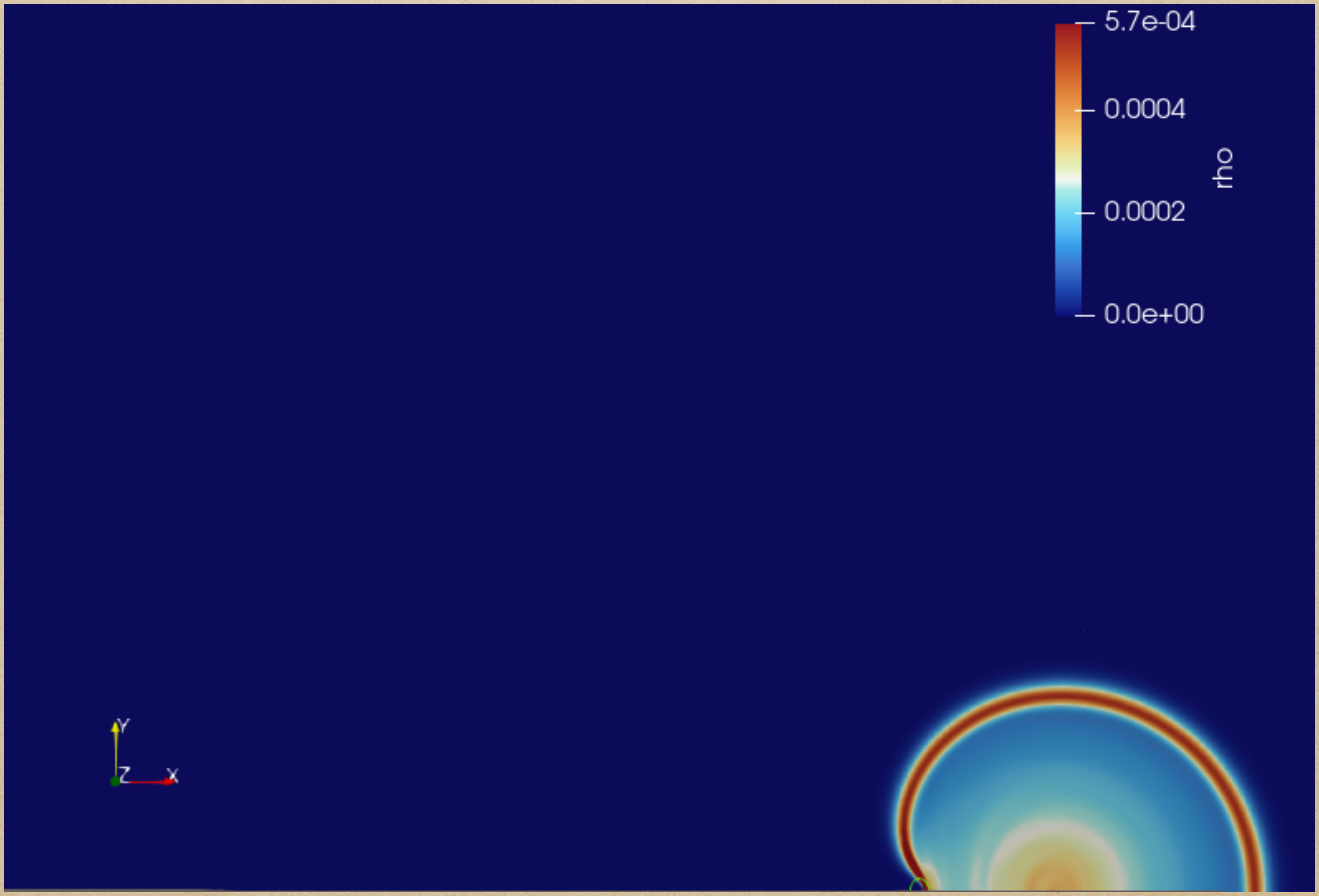
# BH-BS Binaries



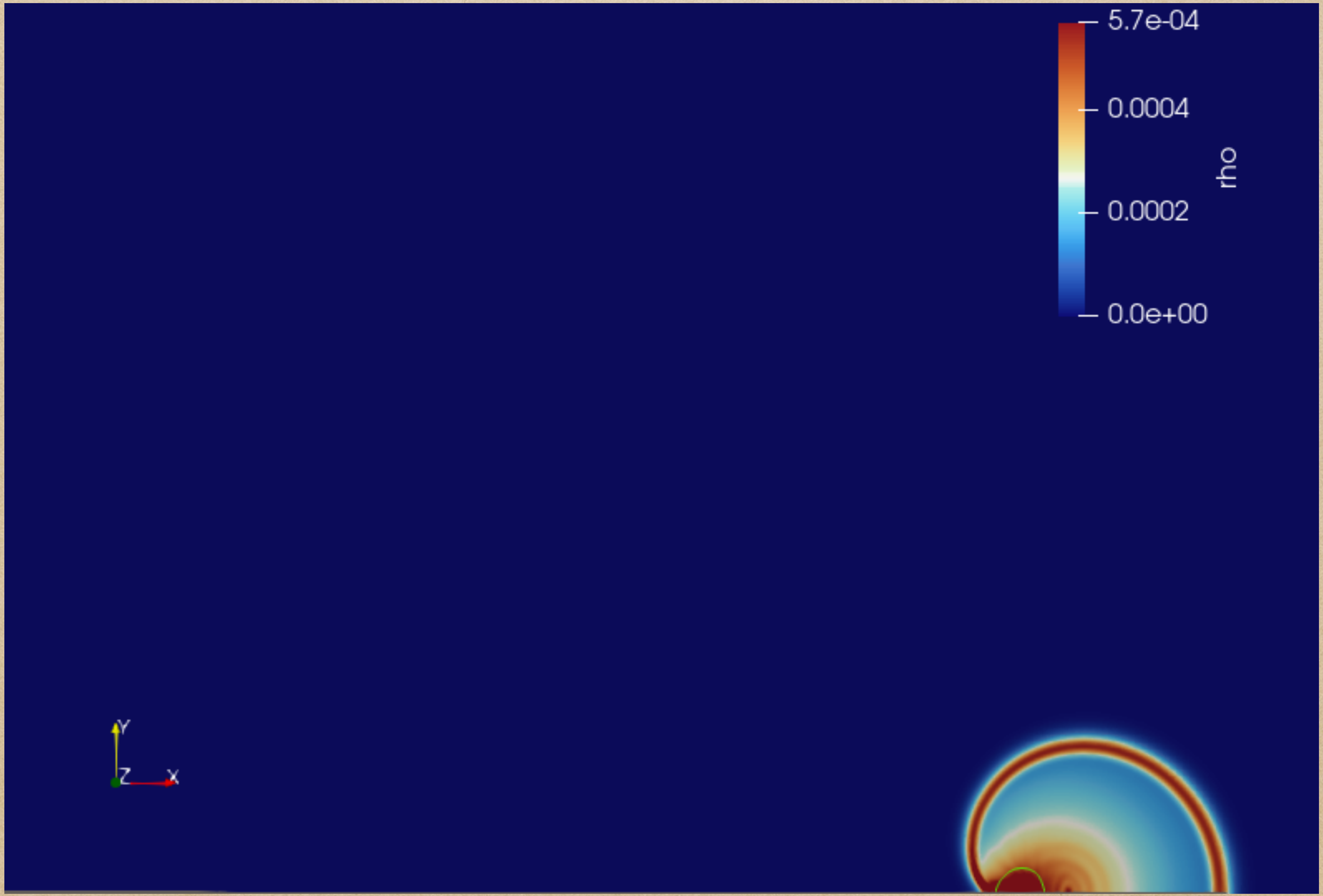
# BH-BS Binaries



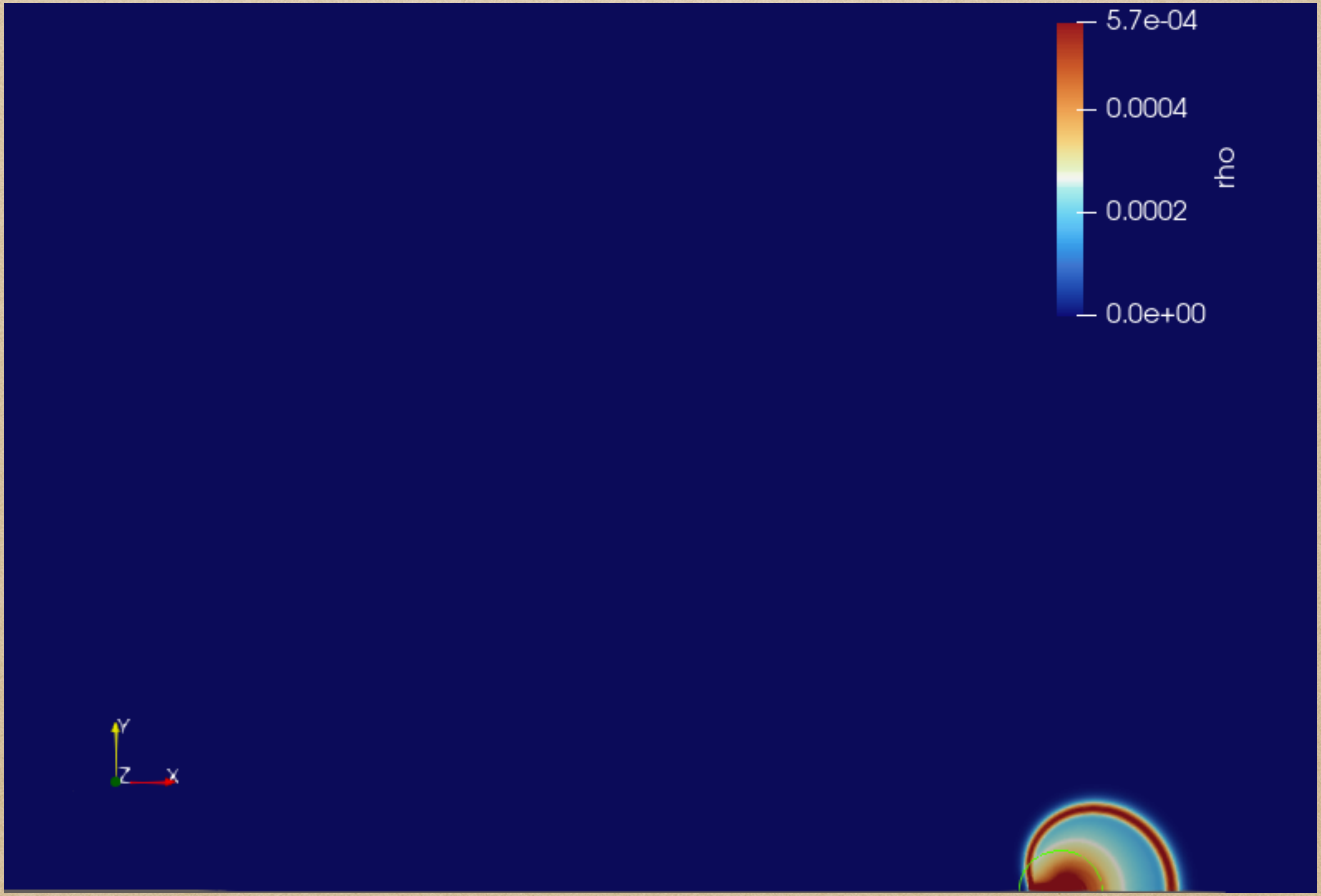
# BH-BS Binaries



# BH-BS Binaries



# BH-BS Binaries



# 6. Summary

# Conclusions

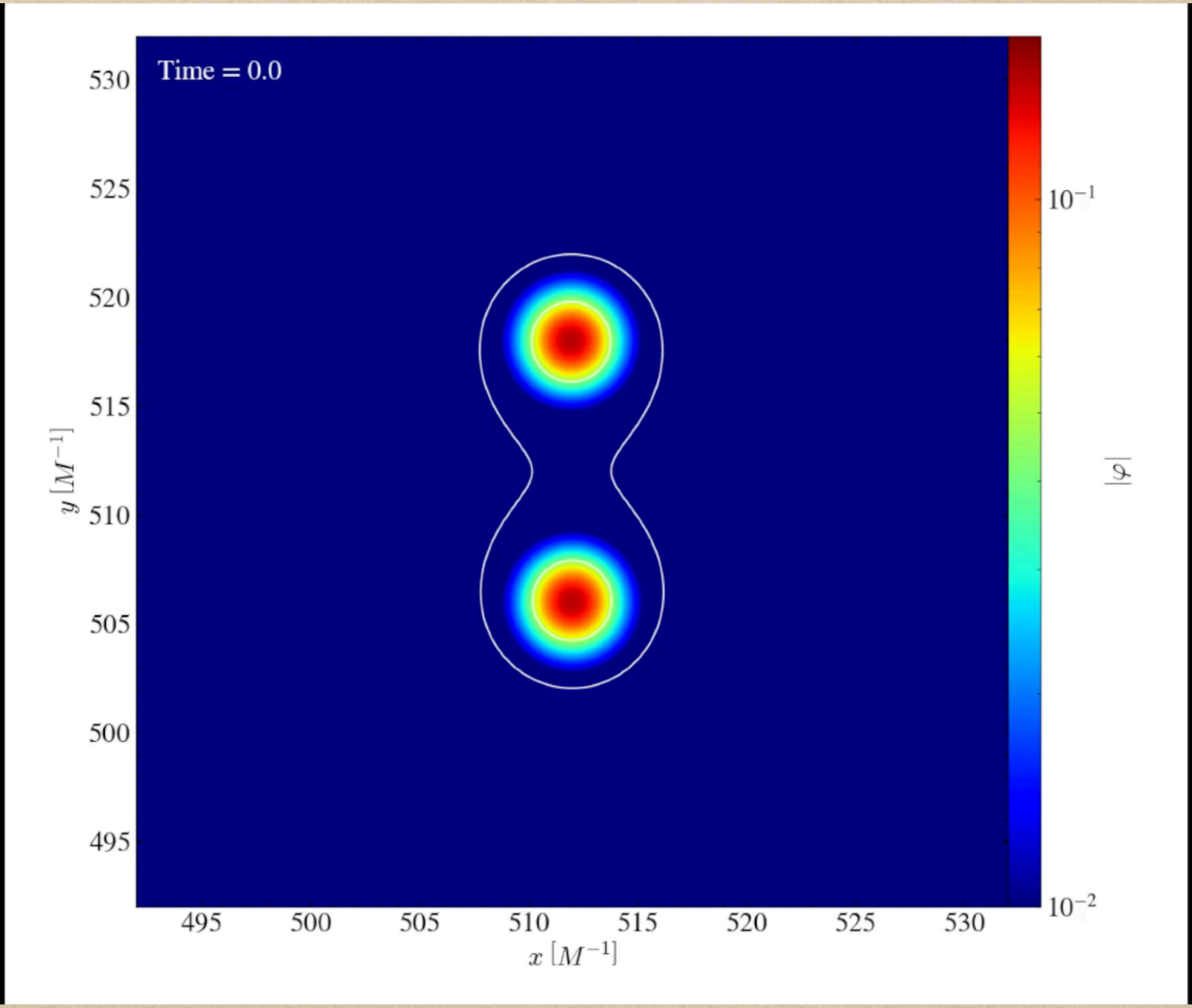
- BS highly amenable to NR! (Perhaps only ECO to be so…)
- Wide range of compactness and mass
- Stable and unstable branches similar to neutron stars
- Spin quantized; may form…
- Compact BBSs can look similar to BBHs → BS-BH degeneracy
- Three main contributions to GW forms: BH, Tidal, Field interaction
- Genuine BH Mimickers: Light ring + no horizon; seem to be stable
- Ergoregion instability

Next Challenges

- Need systematic NR waveforms
- Need for improved initial data!
- Need analytic waveform models
- Various stability questions: Spinning BSs, BH mimickers, ergoregion
- Mixed binary modeling in its infancy (foetus more like it…)

# 4. Extra slides

# Example boson-star inspiral



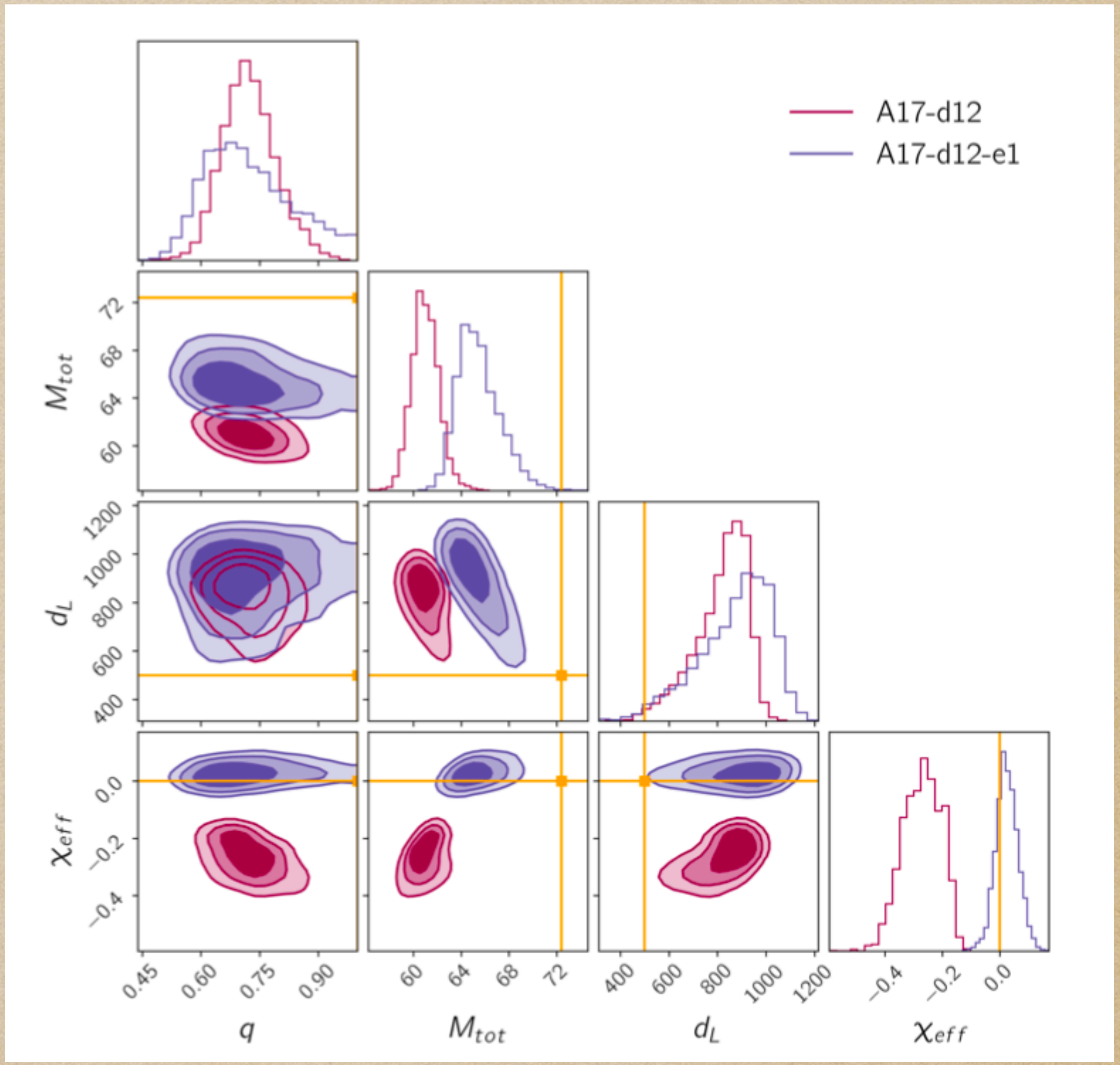

Courtesy of T Evstafyeva

# Results using IMRPhenomXP

- BBH approximants recover parameters best for *anti-BS* !!

  Corner plot:

  A17-d12-e1 vs. A17-d12

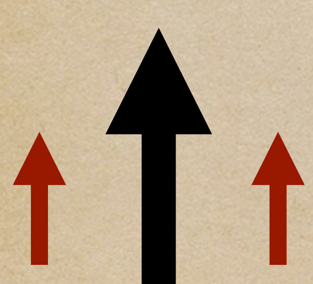
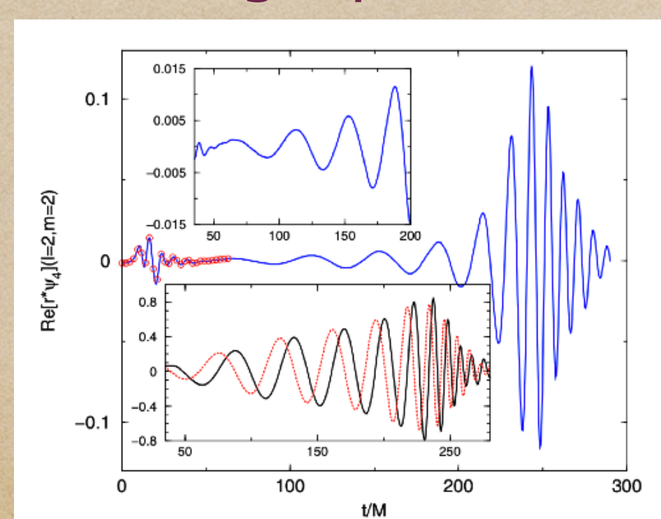


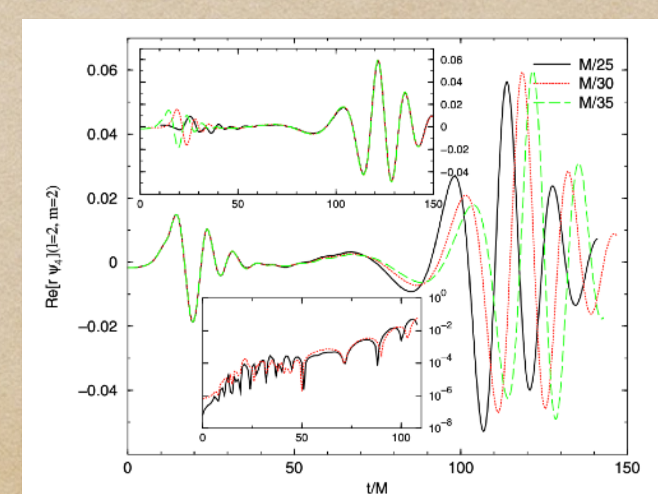

- These features can be explained with the chirp strength

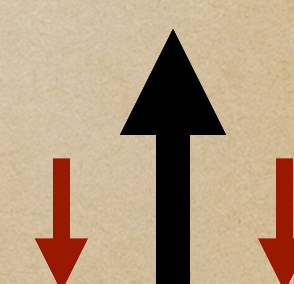
# Understanding the PE bias

- Main feature: Steepness of chirp
- For non-spinning BH binaries:
  - equal mass $\Rightarrow$ shallow chirp
  - unequal masses $\Rightarrow$ steep chirp (think of EMRIs)
- For spinning BH binaries:
  - aligned spins $\Rightarrow$ shallow chirp
  - anti-aligned spins $\Rightarrow$ steep chirp

The orbital ‘Hang-up’ effect Capanelli et al gr-qc/0601091





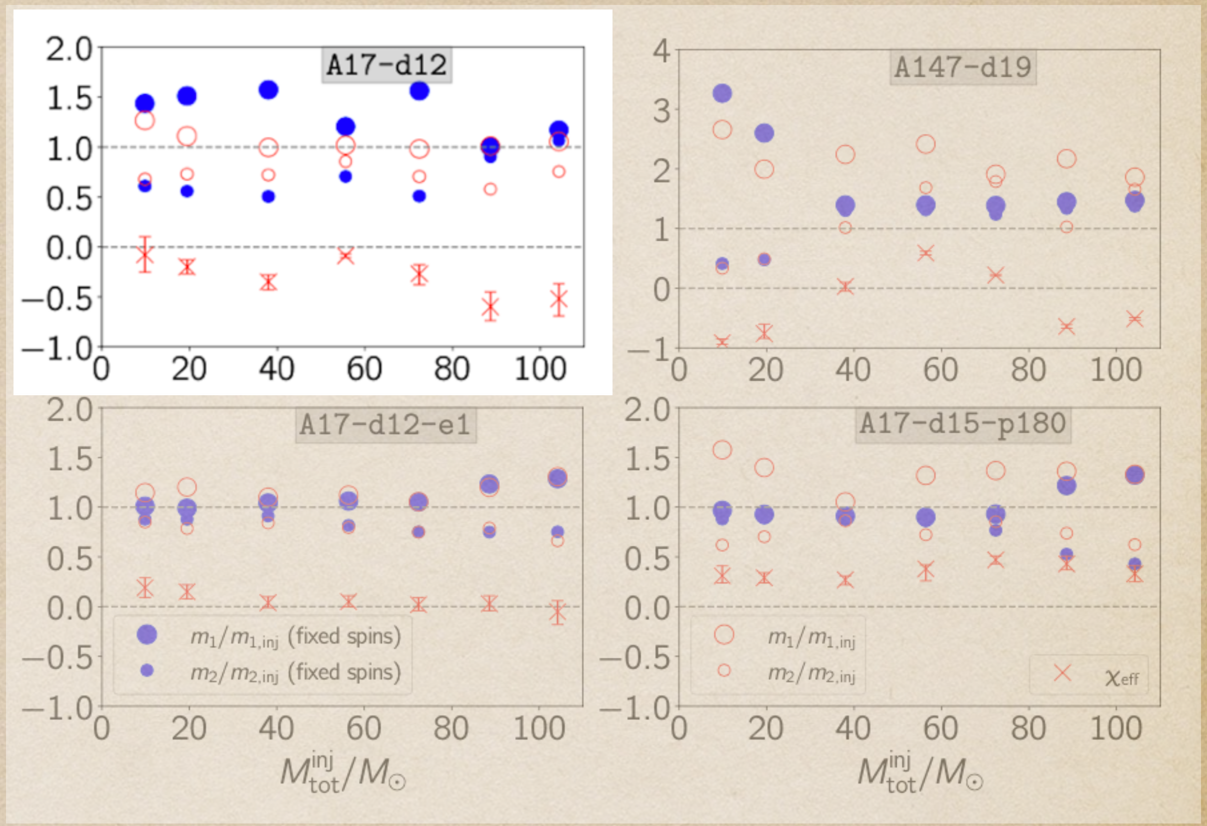
# Understanding the PE bias: Standard BS

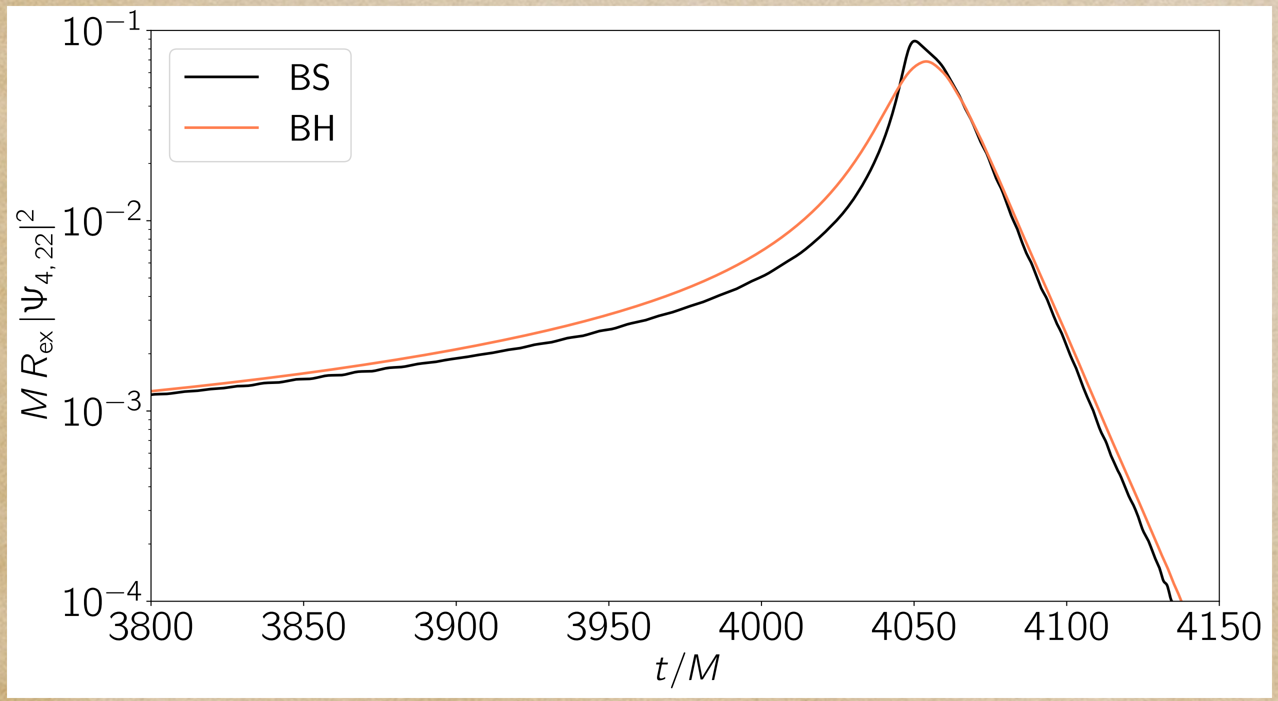
<u>Fixed spins</u>

BS chirp steeper

⇒ Like unequal-mass BHs

⇒ Bilby reports unequal masses




<u>Variable spins</u>

anti-aligned spins

⇒ Steeper chirp

⇒ Steep BS chirp also captured by anti-aligned spins

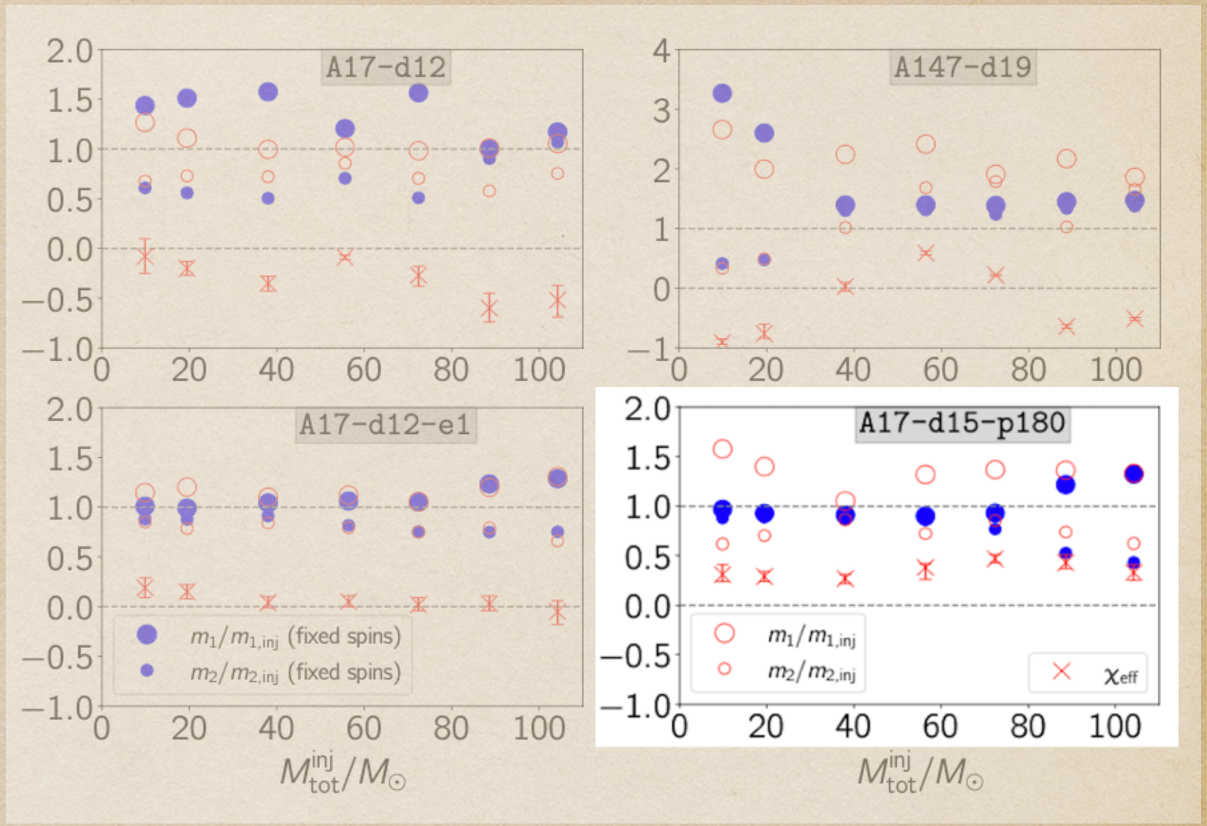
# Understanding the PE bias: anti-phase

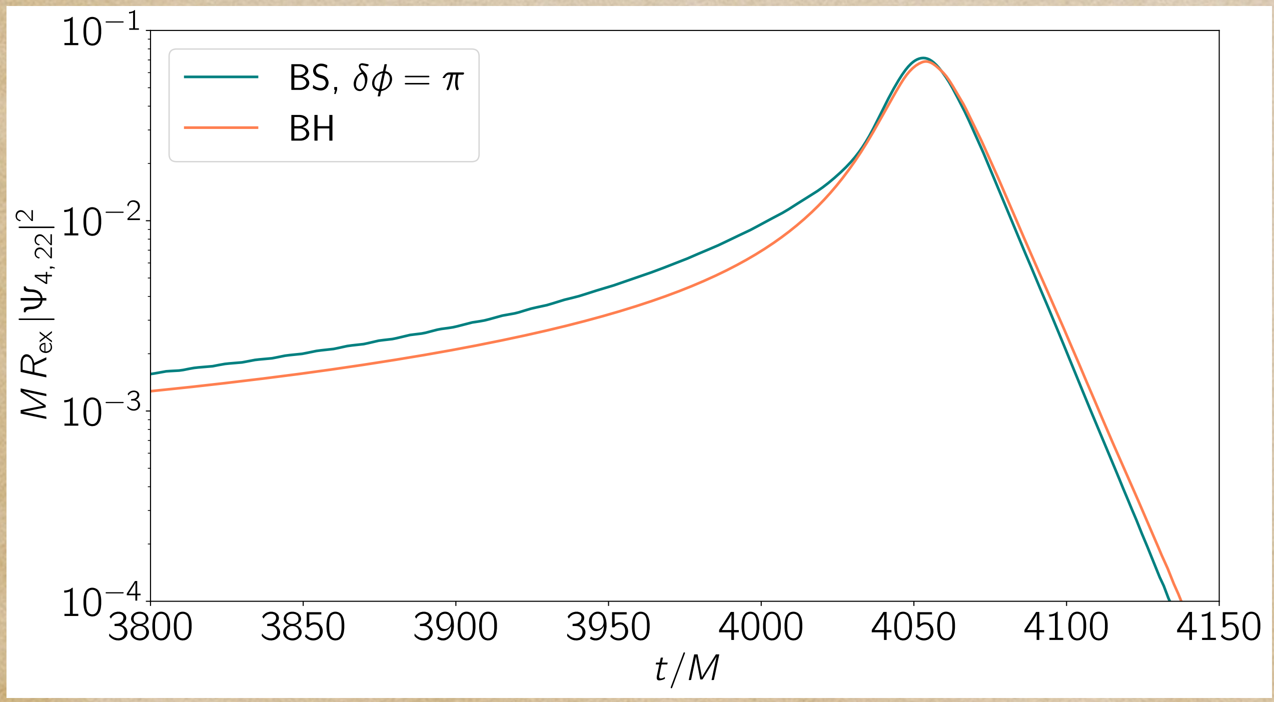
<u>Fixed spins</u>

BS chirp shallower

⇒ Best matched by ~equal mass BHs

⇒ Bilby reports ~equal masses




<u>Variable spins</u>

aligned spins

⇒ Shallow chirp

⇒ Bilby reports aligned spins and allows unequal masses

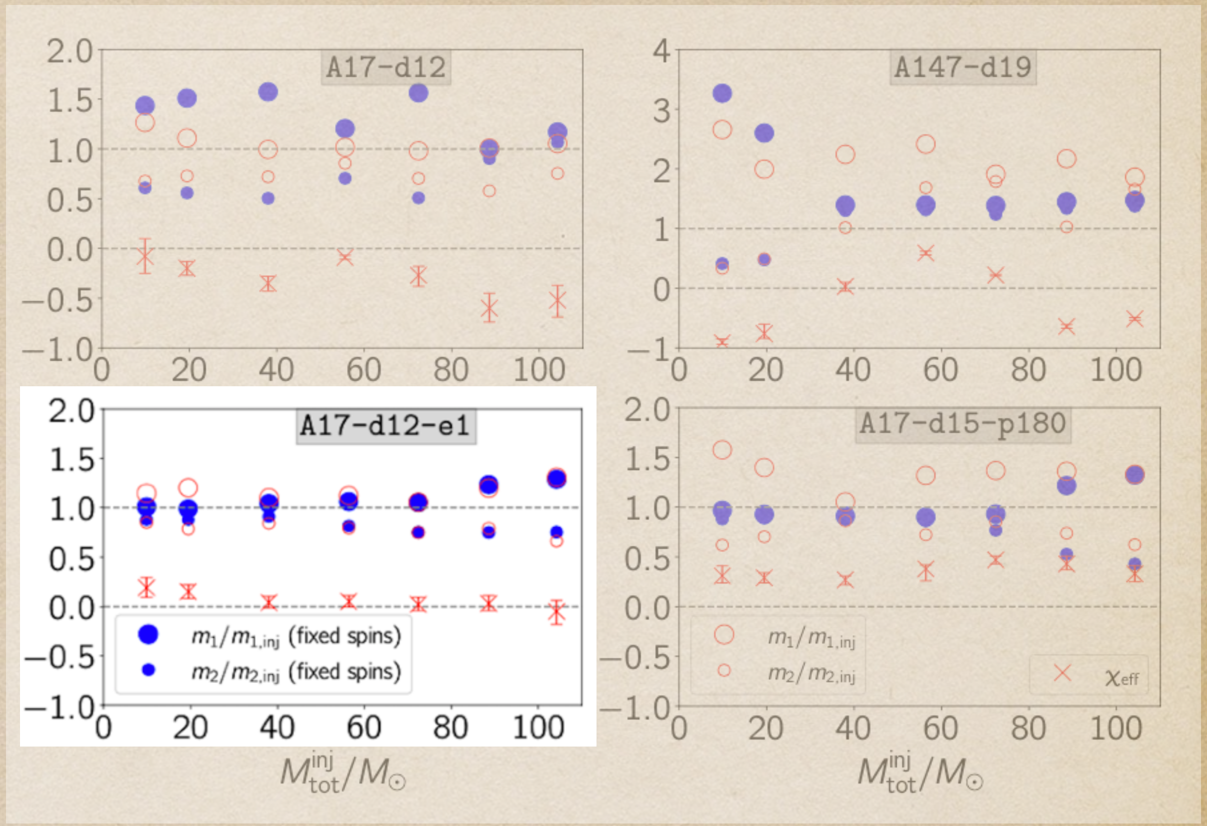
# Understanding the PE bias: anti-BS

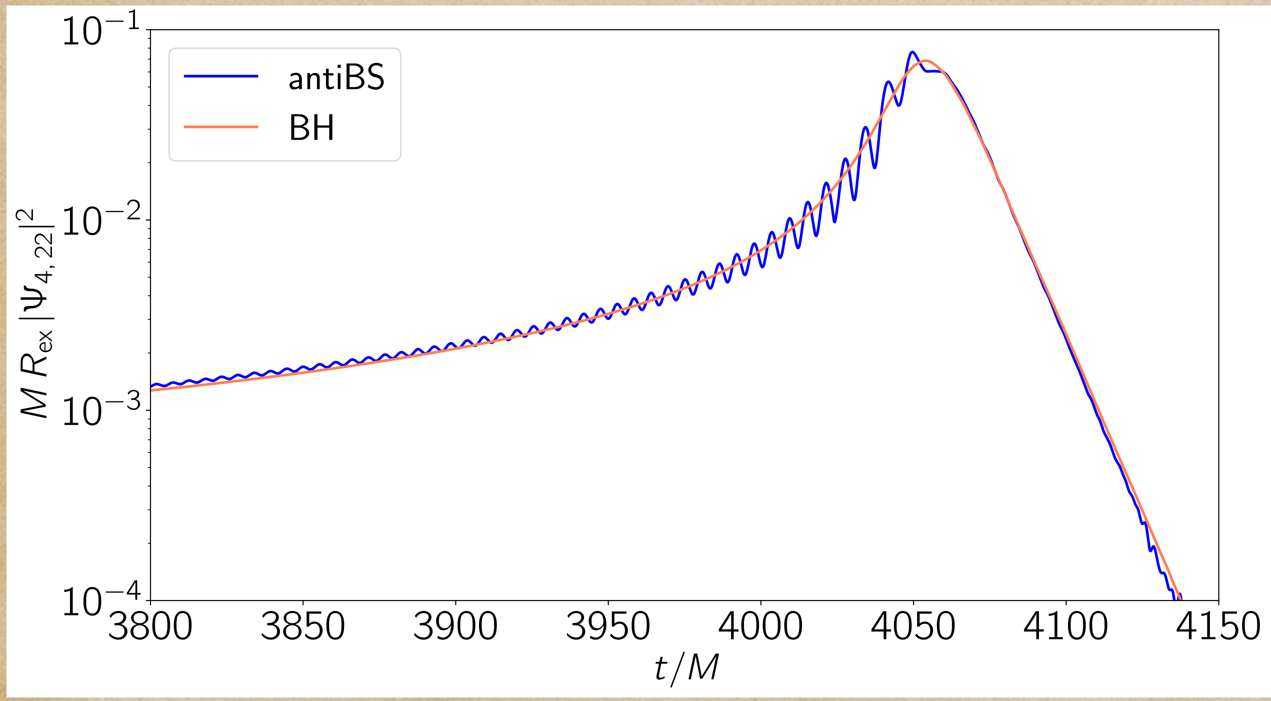
<u>Fixed or variable spins</u>

BS chirp similar to BHs

⇒ Comparable mass ratios

∧ Small spins




<u>High-mass regime</u>

LIGO mainly sees merger burst

⇒ Less reliable PE

# Convergence