Analytic and numerical modeling of BS scattering

Ulrich Sperhake

T Damour, T Jain [arXiv:2512.00945]

DAMTP, University of Cambridge

GW:UK@Nottingham
University of Nottingham
15 Jan 2026

Science & Technology Facilities Council

LSC

NSF

The idea of boson stars

- “Gravitational-electromagnetic entities” or Geons
- Wheeler 1955

PHYSICAL REVIEW VOLUME 97, NUMBER 2 JANUARY 15, 1955

Geons*

JOHN ARCHIBALD WHEELER
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received September 8, 1954)

Associated with an electromagnetic disturbance is a mass, the gravitational attraction of which under appropriate circumstances is capable of holding the disturbance together for a time long in comparison with the characteristic periods of the system. Such

of equations of self-consistent geon; mass and radius values. 4. Transformations and interactions of electromagnetic geons; evaluation of refractive index barrier penetration integral for spherical geon; photon-photon collision processes as additional

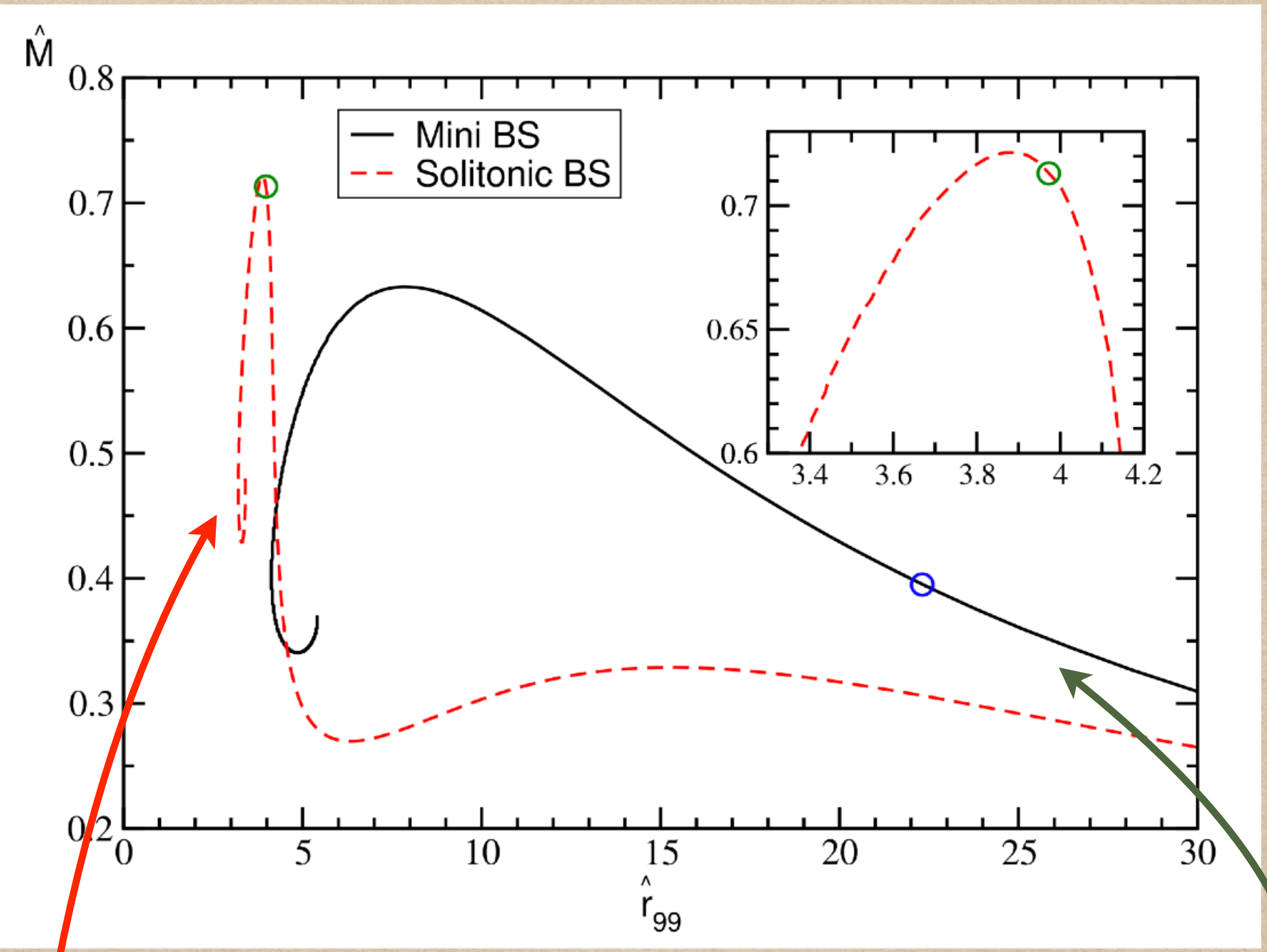
- Energy = mass gravitates $\rightarrow$ Compact (equilibrium?) objects
- Geons are not equilibrium configurations
- Dark matter candidates: QCD axions, ALPs, dark photons,…
- Complex fields (scalar, vector,…)

  $\rightarrow$ Genuine equilibrium states; $T_{\alpha\beta}$ stationary!
- First shown for scalar fields $\rightarrow$ “Boson stars”
- Feinblum & McKinley PR 168 (1968), Kaup PR 172 (1968),

  Ruffini & Bonazzola PR 187 (1969)

Basic features

- Mass-Radius curves similar to Tolman-Oppenheimer-Volkoff stars



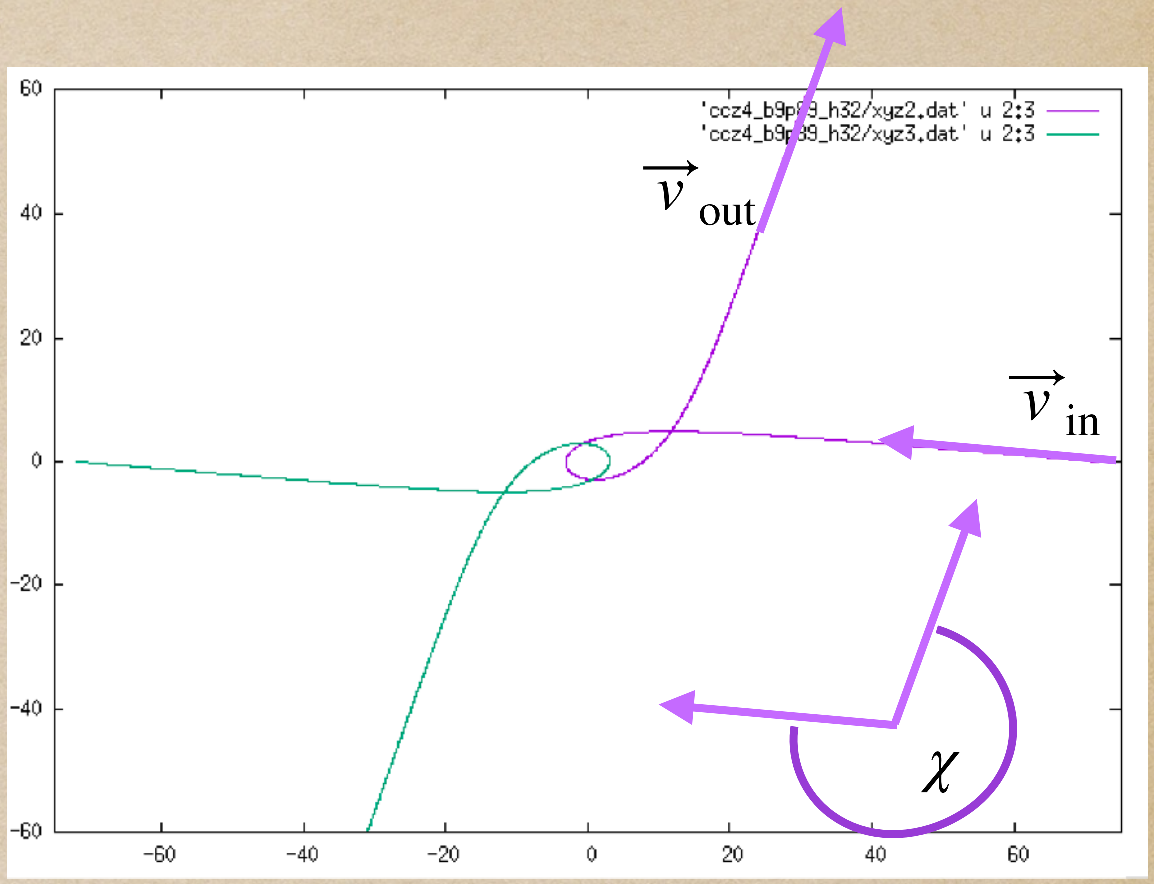
unstable

stable

- Radial stability similar to TOV stars Marks 2508.11757

Analytic Modeling of BS Binaries

Damour, Jain & US   2512.00945



- Consider 2 scattering BSs
- NR simulations with Lean;

  error $\Delta\chi \approx 1.6^{\circ}$
- 3 contributions:
  - Point particle BH
  - Tidal “tid”
  - Scalar interaction “scl”
- PM scattering angle: $\chi(\gamma, j) = \chi^{\mathrm{BH}}(\gamma, j) + \chi^{\mathrm{tid}}(\gamma, j) + \chi^{\mathrm{scl}}(\gamma, j)$

  EOB potential: $w(\gamma, \bar{r}) = w^{\mathrm{BH}}(\gamma, \bar{r}) + w^{\mathrm{tid}}(\gamma, \bar{r}) + w^{\mathrm{scl}}(\gamma, \bar{r})$
- EOB map: $$\pi + \chi(\gamma, j) = 2j \int_0^{\bar{u}_{\max}(\gamma, j)} \frac{\mathrm{d}\bar{u}}{\sqrt{\gamma^2 - 1 + w(\gamma, \bar{u}) - j^2 \bar{u}^2}}, \quad \bar{u} = \frac{1}{\bar{r}}$$

Damour 1710.10599

The analytic potential

- BH:
  - χ^{BH} known up to 4PM Kälin, Liu & Porto 2008.06047

    $\to$ w^{BH} up to 4PM Damour & Rettegno 2211.01399
  - Numerically fitted 5PM term Rettegno et al 2307.06999
- Tidal:
  - at 6PM and 7PM using scattering angles from

    Kälin, Liu & Porto 2008.06047, Bini, Damour & Geralico 2001.00352,

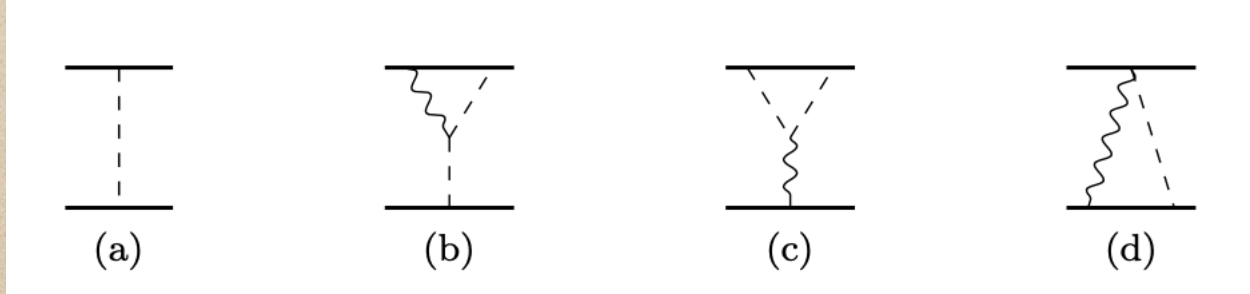
    Cheung & Solon 2006.06665
- Scalar: Previously unknown…

The scalar potential contribution

- EFT action for 2BSs as world lines $\mathbf{z}_A(\tau_A)\,,\; A = 1, 2\,,$ with source

$$s_A(\tau_A) = c_A e^{\mathrm{i}\epsilon_A \omega_A \tau_A + \mathrm{i}\Phi_A}\,, \qquad c_A = \text{ scalar charges}$$

- Perturbative analysis $\rightarrow$ Feynman diagrams



- Leading Order from (a): $\chi^{\mathrm{scl}}_{\mathrm{LO}}(\gamma, j) = \dfrac{8\pi M h \tilde{\mu}}{m_1 m_2} \dfrac{c_1 c_2}{\gamma^2 - 1} K_1(\tilde{\mu}\,|\,b\,|) \cos(\Phi_2 - \Phi_1)$,

where $h = \sqrt{1 + 2\nu(\gamma - 1)}\,, \quad \tilde{\mu}^2 = \mu^2 - \dfrac{2\omega_1\omega_2\gamma - \omega_1^2 - \omega_2^2}{\gamma^2 - 1}$

plus a negligible nonconservative part for $\Phi_1 \neq \Phi_2$

$$\rightarrow \qquad w^{\mathrm{scl}}_{\mathrm{LO}}(\gamma, \bar{r}) = \frac{8\pi c_1 c_2}{G m_1 m_2} \frac{e^{-G M h \tilde{\mu} \bar{r}}}{\bar{r}} \cos(\Phi_2 - \Phi_1)$$

… continued

- Higher-order contribution $w_{\rm HO}^{\rm scl}$ not fully known;

  we assume a factorized form

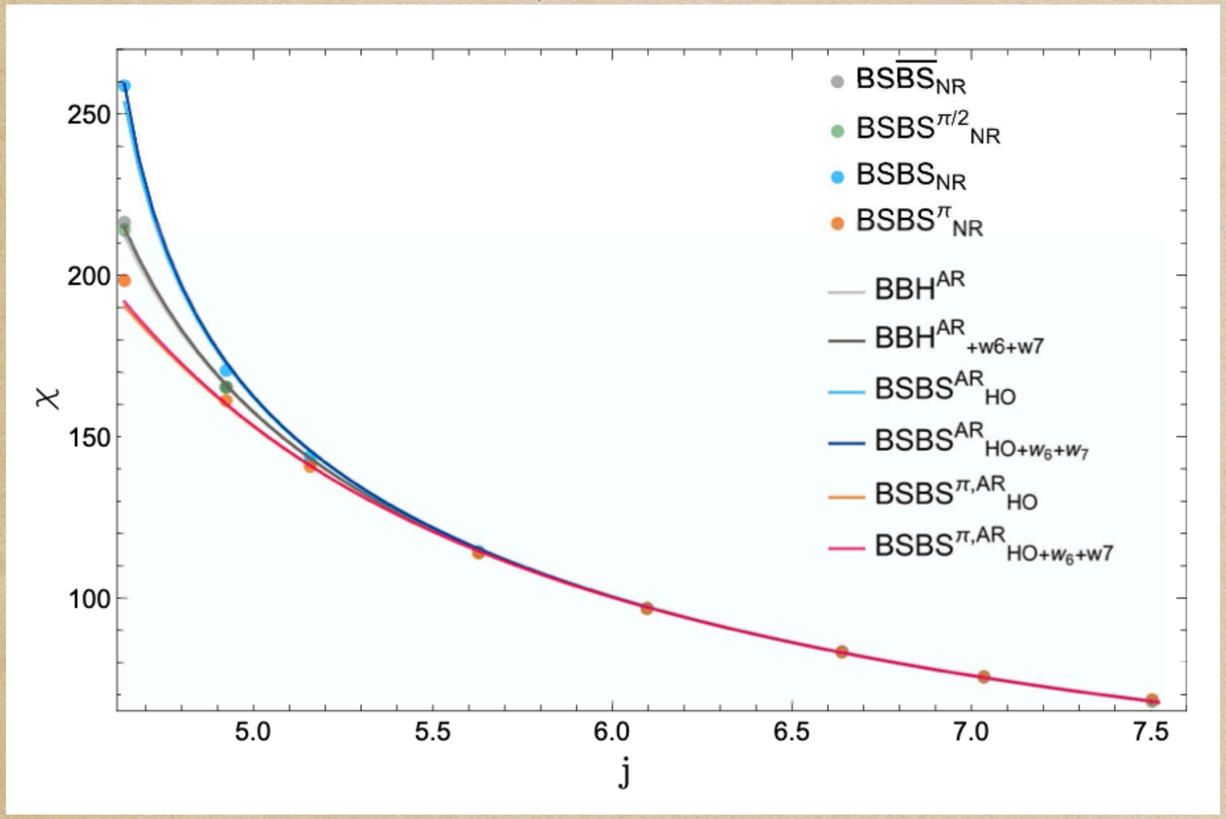
$$w_{\rm HO}^{\rm scl} = w_{\rm g-dressing}^{\rm scl} \times w_{\rm LO}^{\rm scl}\,, \qquad w_{\rm g-dressing}^{\rm scl} = e^{GM((2\omega^2-\mu^2)/\tilde{\mu})\,\log(2\gamma_{\rm E}GM\tilde{\mu}\tilde{r})}$$

- Input needed:
  - total energy E
  - total angular momentum $J = 1.006\,J_{\rm NR}$
  - tidal deformability parameters $\Lambda_1 = \Lambda_2 \approx 7$

    Sennett et al 1704.08651
  - scalar charges c_1, c_2 from BS asymptotics in isotropic radius

Results

- Equal-mass BS binaries with $\sigma_0 = 0.2,\ |\varphi_c| = 0.17$
- Four binaries: $\mathrm{BSBS},\ \mathrm{BS\overline{BS}},\ \Phi_2 - \Phi_1 = \frac{\pi}{2},\ \Phi_2 - \Phi_1 = \pi$



- Scalar corrections dominant
- Tidal effects still important for small impact parameter
- Scalar contribution attractive (repulsive) for

$$\Phi_2 - \Phi_1 = 0 \quad (\Phi_2 - \Phi_1 = \pi)$$

- $\mathrm{BS\overline{BS}}$ close to BH binaries

Bonus Material

A boson-star précis

- BS = Classical limit of a quantum field

  Ground state = Boson-Einstein condensate
- Only known scalar field: Higgs, $\mu = 125\,\mathrm{GeV}$
- Units $G = c = \hbar = 1$, scale determined by μ [inverse length]

  E.g. $\mu = 1.33 \times 10^{-10}\,\mathrm{eV} \;\Rightarrow\; \mu^{-1} = M_\odot = 1.48\,\mathrm{km}$
- Astrophysical BSs need ultralight fields → Beyond SM

  axion like, dark matter candidates: $10^{-11} \ldots 10^{-20}\,\mathrm{eV}$
- Properties: Compactness 0 to > NSs, any Mass
- BSs can have light rings → Black-hole mimickers; stable???

Reviews: Liebling & Palenzuela LRR 2023, Visinelli 2109.05481,

Mielke Fund.Theor.Phys. 2016, Schunck & Mielke GRG 1999, …

Formalism and basic features

- GR + minimally coupled complex scalar field φ

$$S = \int \sqrt{-g} \left\{ \frac{1}{16\pi G} R - \frac{1}{2} [g^{\mu\nu} \nabla_\mu \bar{\varphi} \nabla_\nu \varphi + V(\varphi)] \right\} \mathrm{d}x^4$$

$$T_{\alpha\beta} = \partial_{(\alpha} \bar{\varphi} \, \partial_{\beta)} \varphi - \frac{1}{2} g_{\alpha\beta} [g^{\mu\nu} \partial_\mu \bar{\varphi} \, \partial_\nu \varphi + V(\varphi)]$$

- Potential; analogous to EOS:

$$V_{\mathrm{mas}}(\varphi) = \mu^2 |\varphi|^2 + \lambda |\varphi|^4, \quad V_{\mathrm{soli}}(\varphi) = \mu^2 |\varphi|^2 \left(1 - 2 \frac{|\varphi|^2}{\sigma_0^2} \right)^2$$

- Mini Boson Stars: $\lambda \to 0, \quad \sigma_0 \to \infty$

- Ansatz $\varphi(t,r) = A(r) e^{\mathrm{i}\epsilon\omega t + \delta\Phi}, \quad \varphi(t,r,\theta,\phi) = A(r,\theta) e^{\mathrm{i}(\epsilon\omega t + m\phi + \delta\Phi)}$

  $\delta\Phi$ = phase offset, ϵ = BS vs anti-BS

- Regular solutions only for countably infinite values

  $\omega_0 < \omega_1 < \omega_2 < \dots$ (ground state, excited states)

Theory and Numerical Modelling

- Massive complex scalar field + GR

$$S = \int \frac{\sqrt{-2}}{2} \left\{ \frac{R}{8\pi G} - [g^{\mu\nu} \nabla_\mu \bar{\varphi} \nabla_\nu \varphi + V(\varphi)] \right\} \mathrm{d}^4 x$$

$\Rightarrow$ Einstein-Klein-Gordon equations

- Space-time formulations: GHG, CCZ4, BSSNOK
- Techniques like for BH binaries. We use:

GRChombo Radia et al CQG 2021

Lean US PRD 2006

- Typical grid setups:

$\mathrm{d}x = \frac{M}{48} \dots \frac{M}{32}$, domain size $\sim 1024\, M$, 8 refinement levels

- Improve binary initial data: Constant Volume Element (CVE)

Helfer et al PRD 2019, CQG 2021, Evstafyeva et al CQG 2023,

Atteneder et al PRD 2024