EXCLUSIVE $b \to s\,\ell^+\ell^-$:

BENEFITS OF LOW RECOIL

AND

GLOBAL $\Delta B = 1$ FITS

Christoph Bobeth

TU Munich – Excellence Cluster Universe

ECT-Workshop

“Beautiful Mesons and Baryons on the Lattice”

I) Introduction to $b \to s \ell^+ \ell^-$

- A) Motivation, Observables and Experimental status
- B) Effective theory (EFT) of $\Delta B = 1$ decays: $b \to s + (\gamma, \ell^+ \ell^-)$

II) Phenomenological benefits of low recoil

- A) Form factor relations and optimized observables
- B) Summary: The SM operator basis
- C) Beyond the SM: SM', (S + P), (T + T5)
- D) OPE of 4-quark contributions

III) Global fits of $\Delta B = 1$ fits of $b \to s \gamma$ and $b \to s \ell^+ \ell^-$

– Introduction –

Motivation

Experimental status

EFT of $b \to s + (\gamma,\ \ell^+\ell^-)$

Flavour changes in SM – only via $W^\pm$ exchange

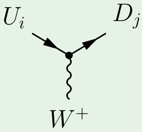
$U_i = \{u, c, t\}$:

$Q_U = +2/3$

$D_j = \{d, s, b\}$:

$Q_D = -1/3$

$$\mathcal{L}_{\rm CC} = \frac{g_2}{\sqrt{2}} (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \gamma^\mu P_L \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^+$$



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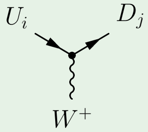

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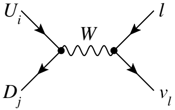

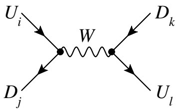

$H \to \ell \nu_\ell$

$H_1 \to H_2 + \ell \nu_\ell$

$H_1 \to H_2 H_3$

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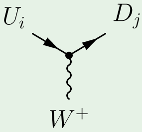

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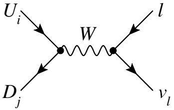

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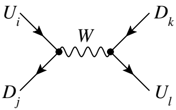

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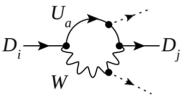

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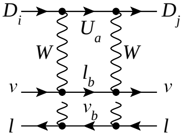

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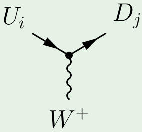

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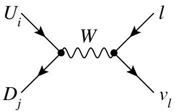

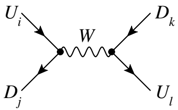

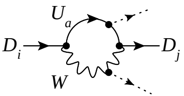

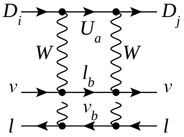

$H \to \ell\nu_\ell$ $H_1 \to H_2 + \ell\nu_\ell$	$H_1 \to H_2 H_3$	$H_1 \to H_2 + \{\gamma, Z, g\}$ $\{\gamma, Z, g\} \to \{\gamma, \bar{\ell}\ell, H_3\}$	$H_1 \to \bar{\ell}\ell$ $H_1 \to H_2 + \{\bar{\ell}\ell, \bar{\nu}\nu\}$
$\mathcal{A} \sim G_F V_{ij}$	$\sim G_F V_{ij} V^*_{lk}$	$\sim G_F g \sum_a V_{ai} V^*_{aj} f(m_a)$	$\sim G_F g^2 \sum_{a,b} V_{ai} V^*_{aj} f(m_{a,b})$

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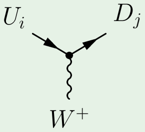

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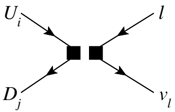

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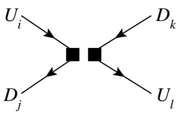

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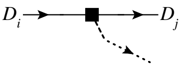

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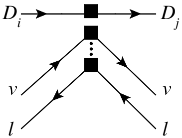

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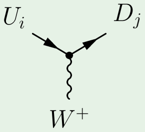


In the SM: FCNC-decays w.r.t. tree-decays are ...

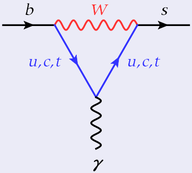
quantum fluctuations = loop-suppressed

$\Rightarrow$ no suppression of contributions beyond SM (BSM) wrt SM itself

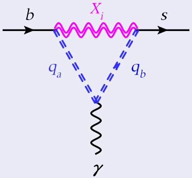
$\Rightarrow$ indirect search for BSM signals

BUT requires high precision, experimentally and theoretically !!!

SM



BSM



Rich phenomenology ...

$b \to s + \gamma$

$B \to K^*\gamma \quad (B_s \to \phi\gamma)$

– Br

– time-dep. CP asy's: S, C, H

– iso-spin asymmetry Δ_{0-}

$B \to X_s\gamma$

– Br, dBr/dE_γ

– $A_{\rm CP}$ in $B \to X_s\gamma$ and $B \to X_{s+d}\gamma$

$B_s \to \gamma\gamma$

– Br ($A_{\rm CP}$)

$b \to s + \ell^+\ell^-$

$B_s \to \bar{\ell}\ell$

– Br

$B \to K + \bar{\ell}\ell$

– $d^2Br/dq^2\, d\cos\theta_l \to dBr/dq^2$, $A_{\rm FB}$, F_H

$B \to K^*(\to K\pi) + \bar{\ell}\ell \quad (B_s \to \phi(\to \bar{K}K) + \bar{\ell}\ell)$

– $d^4Br/dq^2 d\cos\theta_l d\cos\theta_{K^*} d\phi$

$\to$ angular observables $I_{1,\ldots,9}^{(s,c)}(q^2)$

$\to dBr/dq^2$, $A_{\rm FB}$, F_L, $A_T^{(2,3,4,{\rm re},{\rm im})}$, $H_T^{(1,2,3,4,5)}$, ...

$B \to X_s + \bar{\ell}\ell$

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Rich phenomenology . . .

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. . . in $b \to s + \{\gamma, \gamma\gamma, \bar\ell\ell\}$ FCNC's to test short-distance flavour physics:

C_i for $i = 7, 7'$

C_i for $i = 7, 7', 9, 9', 10, 10', \ldots$

BUT need non-perturbative hadronic input:

Decay constants and LCDA's for $B_{d,s}$, K, K^*, ϕ, . . .

Form factors: $(B \to K) \to f_{+,T,0}$ and $(B \to K^*,\ B_s \to \phi) \to V, A_{0,1,2}, T_{1,2,3}$

Experimental data: $b \to s \ell^+ \ell^-$ – number of events

# of evts	BaBar 2012 471 M $\bar{B}B$	Belle 2009 605 fb^{-1}	CDF 2011 6.8 fb^{-1}	LHCb 2011 1 fb^{-1}
$B^0 \to K^{*0} \bar{\ell}\ell$	$137 \pm 44^\dagger$	$247 \pm 54^\dagger$	164 ± 15	900 ± 34
$B^+ \to K^{*+} \bar{\ell}\ell$			20 ± 6	
$B^+ \to K^+ \bar{\ell}\ell$	$153 \pm 41^\dagger$	$162 \pm 38^\dagger$	234 ± 19	
$B^0 \to K_S^0 \bar{\ell}\ell$			28 ± 9	
$B_s \to \phi \bar{\ell}\ell$			49 ± 7	
$\Lambda_b \to \Lambda \bar{\ell}\ell$			24 ± 5	

- CP-averaged results
- vetoed q^2 region around J/ψ and ψ' regions
- † unknown mixture of B^0 and $B^\pm$

Babar Moriond EW 2012

Belle arXiv:0904.0770

CDF arXiv:1107.3753 + 1108.0695

LHCb LHCb-CONF-2012-008

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Outlook / Prospects

Belle reprocessed all data 711 fb^{-1} $\to$ final analysis ?

CDF recorded about 10 fb^{-1} $\to$ final analysis ?

LHCb recorded now 1 fb^{-1} $\to$ still about 1000 evts $B \to K\bar{\mu}\mu$ to analyse
(extrapolated from (35 ± 7) evts in 37 pb^{-1}) $\to$ about 2.5 fb^{-1} by end of 2012

ATLAS / CMS pursue also analysis of $B \to K^*\bar{\mu}\mu$ and $B \to K\bar{\mu}\mu$

Belle II / SuperB expects about (10-15) K events $B \to K^*\bar{\ell}\ell$ ($\gtrsim$ 2020) [A.J.Bevan arXiv:1110.3901]

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$B \to K^* [\to K\pi] + \ell^+\ell^-$:

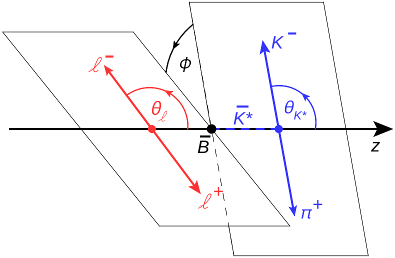
4-body decay with intermediate on-shell K^* (vector)

1) $q^2 = m^2_{\bar{\ell}\ell} = (p_{\bar{\ell}} + p_\ell)^2 = (p_B - p_{K*})^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle(\vec{p}_B, \vec{p}_{\bar{\ell}})$ in $(\bar{\ell}\ell)$ – c.m. system

3) $\cos\theta_K$ with $\theta_K \angle(\vec{p}_B, \vec{p}_K)$ in $(K\pi)$ – c.m. system

4) $\phi \angle(\vec{p}_K \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



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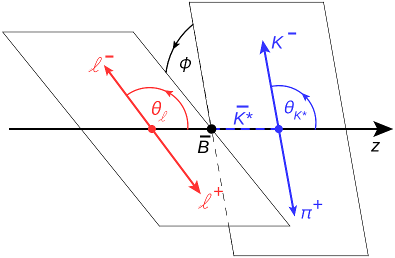
4) ϕ $\angle(\vec{p}_K \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



$J_i(q^2)$ = "Angular Observables"

$$\frac{32\pi}{9} \frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_\ell$$
$$+ J_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + J_5 \sin 2\theta_K \sin\theta_\ell \cos\phi$$
$$+ (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_\ell + J_7 \sin 2\theta_K \sin\theta_\ell \sin\phi$$
$$+ J_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi$$

$B \to K^* [\to K\pi] + \ell^+\ell^-$:

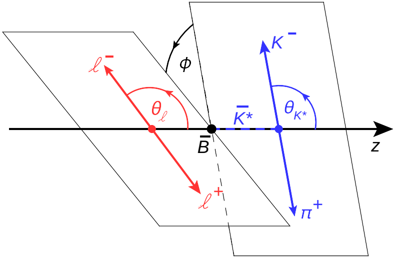
4-body decay with intermediate on-shell K^* (vector)

1) $q^2 = m^2_{\bar{\ell}\ell} = (p_{\bar{\ell}} + p_\ell)^2 = (p_B - p_{K*})^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle(\vec{p}_B, \vec{p}_{\bar{\ell}})$ in $(\bar{\ell}\ell)$ – c.m. system

3) $\cos\theta_K$ with $\theta_K \angle(\vec{p}_B, \vec{p}_K)$ in $(K\pi)$ – c.m. system

4) $\phi \ \angle(\vec{p}_K \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



$J_i(q^2)$ = "ANGULAR OBSERVABLES"

$$\frac{32\pi}{9} \frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_\ell$$
$$+ J_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + J_5 \sin 2\theta_K \sin\theta_\ell \cos\phi$$
$$+ (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_\ell + J_7 \sin 2\theta_K \sin\theta_\ell \sin\phi$$
$$+ J_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi$$

$\Rightarrow$ "$2 \times (12 + 12) = 48$" if measured separately: A) decay + CP-conj and B) for $\ell = e, \mu$

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4-body decay with intermediate on-shell K^* (vector)

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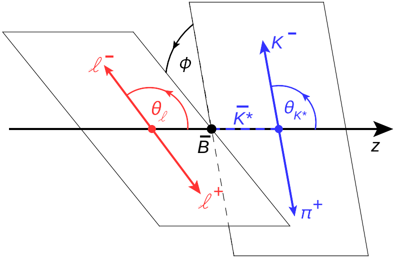

CP-conj. decay $B^0 \to K^{*0}(\to K^+\pi^-) + \bar{\ell}\ell$: $d^4\bar{\Gamma}$ from $d^4\Gamma$ by replacing

$$\text{CP-even} \quad : \quad J_{1,2,3,4,7} \quad \longrightarrow \quad +\bar{J}_{1,2,3,4,7}[\delta_W \to -\delta_W]$$

$$\text{CP-odd} \quad : \quad J_{5,6,8,9} \quad \longrightarrow \quad -\bar{J}_{5,6,8,9}[\delta_W \to -\delta_W]$$

with $\ell \leftrightarrow \bar{\ell} \Rightarrow \theta_\ell \to \theta_\ell - \pi$ and $\phi \to -\phi$ and weak phases δ_W conjugated

$B \to K^* [\to K\pi] + \ell^+\ell^-$:

4-body decay with intermediate on-shell K^* (vector)

1) $q^2 = m^2_{\bar\ell\ell} = (p_{\bar\ell} + p_\ell)^2 = (p_B - p_{K*})^2$

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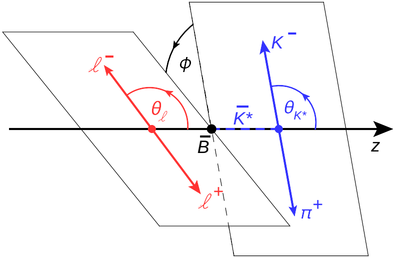

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with $\ell \leftrightarrow \bar\ell \Rightarrow \theta_\ell \to \theta_\ell - \pi$ and $\phi \to -\phi$ and weak phases δ_W conjugated

1) CP-odd : $A_{\rm CP} \sim (J_i - \bar{J}_i) \sim d^4(\Gamma + \bar\Gamma)$ = flavour-untagged B samples

2) T-odd $J_{7,8,9}$: $A_{\rm CP} \sim \cos\delta_s \sin\delta_W$ not suppressed by small strong phases δ_s

[CB/Hiller/Piranishvili arXiv:0805.2525, Altmannshofer et al. arXiv:0811.1214]

Data for $B \to K^* + \ell^+\ell^-$: data in 6 q^2-bins for $\langle Br\rangle$, $\langle A_{\rm FB}\rangle$, $\langle F_L\rangle$

angular analysis in each q^2-bin in θ_ℓ, θ_K

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_K} = \frac{3}{2} F_L \cos^2\theta_K + \frac{3}{4}(1 - F_L)\sin^2\theta_K$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} = \frac{3}{4} F_L \sin^2\theta_\ell + \frac{3}{8}(1 - F_L)(1 + \cos^2\theta_\ell) + A_{\rm FB}\cos\theta_\ell$$

$\Rightarrow$ fitted F_L and $A_{\rm FB}$

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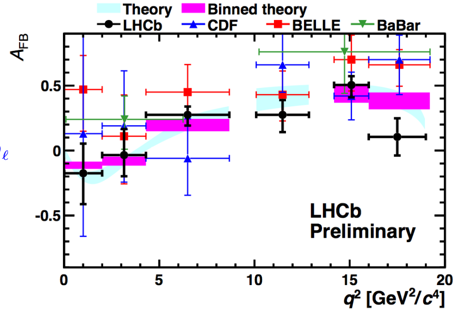
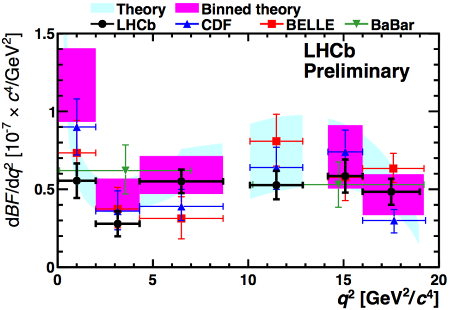
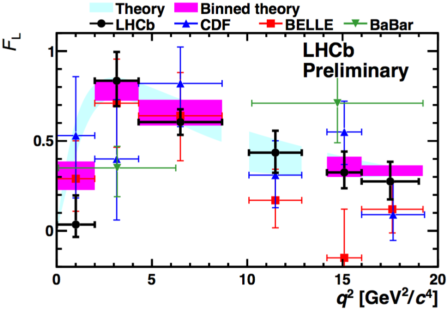
$\Rightarrow$ fitted F_L and $A_{\rm FB}$

[SM-predictions: CB/Hiller/van Dyk arXiv:1006.5013]







- additional measurement of S_3 (or $A_T^{(2)}$) and A_{im} from CDF and LHCb

$$\frac{2\pi}{(\Gamma + \bar{\Gamma})} \frac{d(\Gamma + \bar{\Gamma})}{d\phi} = 1 + S_3 \cos 2\phi + A_{im} \sin 2\phi$$

with

$$S_3 = \frac{J_3 + \bar{J}_3}{\Gamma + \bar{\Gamma}} = \frac{1}{2}(1 - F_L)\, A_T^{(2)}$$

$$A_{im} = \frac{J_9 - \bar{J}_9}{\Gamma + \bar{\Gamma}}$$

(since J_9 CP-odd, the CP-asymmetry $\sim (J_9 - \bar{J}_9)$ from untagged B-sample)

- new data from BaBar and LHCb from march 2012 not included in previous plots

  → LHCb: see talk of Thomas Blake (tomorrow = tuesday)

$B \to K + \ell^+\ell^-$: 3-body decay → 2 kinematic variables

$$\frac{1}{(d\Gamma/dq^2)} \frac{d^2\Gamma}{dq^2\, d\cos\theta_\ell} = \frac{3}{4}\left[1 - F_H\right]\sin^2\theta_\ell + \frac{1}{2}F_H + A_{\rm FB}\cos\theta_\ell$$

3 observables × CP-conj: dBr/dq^2, $A_{\rm FB}(q^2)$, $F_H(q^2)$

$B \to K + \ell^+\ell^-$: 3-body decay → 2 kinematic variables

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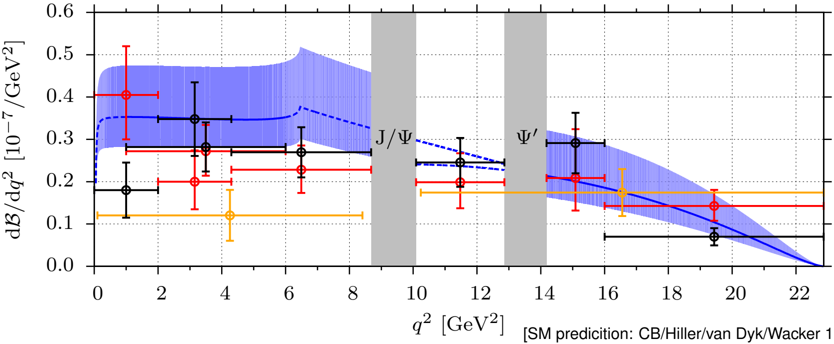
3 observables × CP-conj: dBr/dq^2, $A_{\rm FB}(q^2)$, $F_H(q^2)$

data in 6 q^2-bins for: $\langle Br \rangle$ from [BaBar] [Belle] [CDF]



[SM predicition: CB/Hiller/van Dyk/Wacker 1111.2558]

$B \to K + \ell^+\ell^-$: 3-body decay → 2 kinematic variables

$$\frac{1}{(\mathrm{d}\Gamma/\mathrm{d}q^2)}\frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell} = \frac{3}{4}\left[1 - F_H\right]\sin^2\theta_\ell + \frac{1}{2}F_H + A_{\rm FB}\cos\theta_\ell$$

3 observables × CP-conj: $\mathrm{d}Br/\mathrm{d}q^2$, $A_{\rm FB}(q^2)$, $F_H(q^2)$

also measured:

- 6 q^2-bins lepton forward-backward asymmetry: $\langle A_{\rm FB}\rangle$
- 6 q^2-bins isospin asymmetry:

$$\langle A_I\rangle = \frac{(\tau_{B^\pm}/\tau_{B^0})\langle Br[B^0 \to K^0\bar{\ell}\ell]\rangle - \langle Br[B^\pm \to K^\pm\bar{\ell}\ell]\rangle}{(\tau_{B^\pm}/\tau_{B^0})\langle Br[B^0 \to K^0\bar{\ell}\ell]\rangle - \langle Br[B^\pm \to K^\pm\bar{\ell}\ell]\rangle}$$

$B \to K + \ell^+\ell^-$: 3-body decay → 2 kinematic variables

$$\frac{1}{(\mathrm{d}\Gamma/\mathrm{d}q^2)} \frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell} = \frac{3}{4}\left[1 - F_H\right]\sin^2\theta_\ell + \frac{1}{2}F_H + A_{\rm FB}\cos\theta_\ell$$

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... and improved measurements of

- exclusive $b \to s\gamma$: $B \to K^*\gamma$, $B_s \to \phi\gamma$ (LHCb)
- leptonic $B_s \to \mu^+\mu^-$ and related (LHCb, CMS, ATLAS)
- inclusive $b \to s\gamma$ and $b \to s\,\ell^+\ell^-$ (Belle II, SuperB)

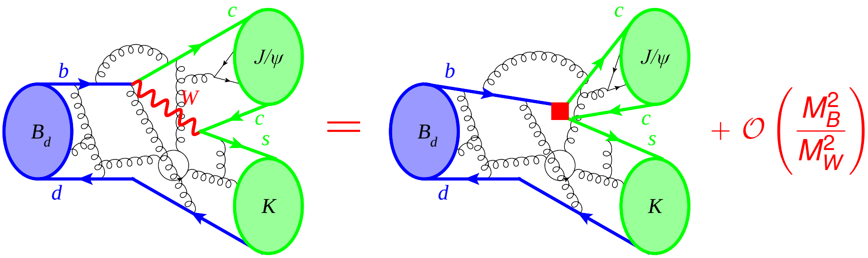
B-Hadron decays are a Multi-scale problem . . .

. . . Typical interaction (IA) scales

electroweak IA	$\gg$	hadron in restframe, external momenta	$\gg$	QCD-bound state effects
$M_W \approx 80$ GeV $M_Z \approx 91$ GeV $m_t \approx 172$ GeV		$M_B \approx 5$ GeV		$\Lambda_{\mathrm{QCD}} \approx 0.5$ GeV

electroweak IA is "short-distance = local" compared to QCD IA

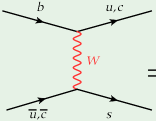
$\Rightarrow$ Effective theory (EFT) of electroweak IA = separation of scales



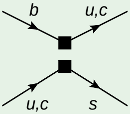
MATCHING "FULL" THEORY ON EFT

decoupling (OPE) of heavy particles $(W, Z, t, \ldots)$ @ EW scale $\mu_0 \gtrsim M_W$ factorisation into ...

- ... short-distance = effective couplings $\qquad C_i = C_i^{(0)} + \frac{\alpha_s}{4\pi} C_i^{(1)} + \ldots$
- ... long-distance = flavour-changing operators $\qquad \mathcal{O}_i \; (dim > 4)$



$\stackrel{!}{=} \quad C^{(0)}(\mu_0, M_W)$



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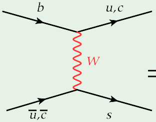
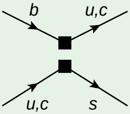
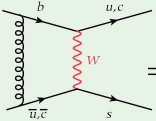
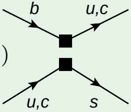


$\overset{!}{=}$ $C^{(0)}(\mu_0, M_W)$

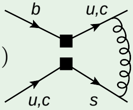




$\overset{!}{=}$ $\frac{\alpha_s}{4\pi} C^{(1)}(\mu_0, M_W)$



$+ \; C^{(0)}(\mu_0, M_W)$



Matching "full" theory on EFT

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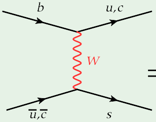
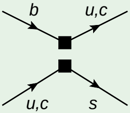
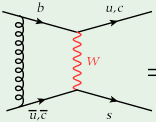
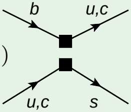


$\stackrel{!}{=} \quad C^{(0)}(\mu_0, M_W)$

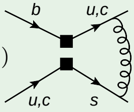




$\stackrel{!}{=} \quad \frac{\alpha_s}{4\pi} C^{(1)}(\mu_0, M_W)$



$+ \; C^{(0)}(\mu_0, M_W)$



Renormalisation group from $\mu_0 \sim M_W$ to $\mu_b \sim m_b$

$\rightarrow$ resums large logarithm's: $[\alpha_s \ln(\mu_b/\mu_0)]^n$

LO: $\alpha_s^n \ln^n(\mu_b/\mu_0)$, NLO: $\alpha_s^n \ln^{n-1}(\mu_b/\mu_0)$, NNLO: $\alpha_s^n \ln^{n-2}(\mu_b/\mu_0)$, [+ QED LO, QED NLO]

MATCHING "FULL" THEORY ON EFT

decoupling (OPE) of heavy particles ($W, Z, t, \ldots$) @ EW scale $\mu_0 \gtrsim M_W$ factorisation into …

… short-distance = effective couplings $\qquad C_i = C_i^{(0)} + \frac{\alpha_s}{4\pi} C_i^{(1)} + \ldots$

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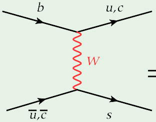
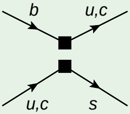
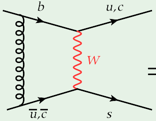
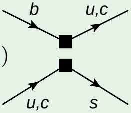


$\stackrel{!}{=}$ $C^{(0)}(\mu_0, M_W)$

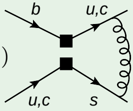




$\stackrel{!}{=}$ $\frac{\alpha_s}{4\pi} C^{(1)}(\mu_0, M_W)$



$+\ C^{(0)}(\mu_0, M_W)$



RENORMALISATION GROUP FROM $\mu_0 \sim M_W$ TO $\mu_b \sim m_b$

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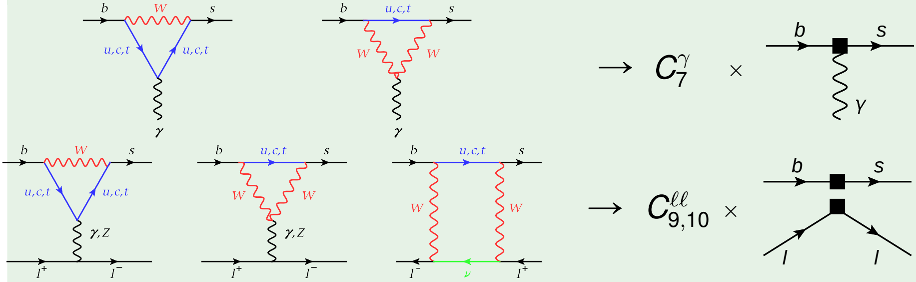
LO: $\alpha_s^n \ln^n(\mu_b/\mu_0)$, NLO: $\alpha_s^n \ln^{n-1}(\mu_b/\mu_0)$, NNLO: $\alpha_s^n \ln^{n-2}(\mu_b/\mu_0)$, [+ QED LO, QED NLO]

$\Rightarrow$ $\mathcal{L}_{\rm eff}^{\rm NNLO}(\mu_b)$ @ $\mu_b \approx m_b$ includes EW up to $\mathcal{O}(m_b^2/M_W^2)$ and QCD $\times$ QED corr.

$\Delta B = 1$ EFT in the SM for $b \to s$

$b \to s + \gamma$ AND $b \to s + \ell^+ \ell^-$



$$\mathcal{O}_7 = m_b [\bar{s}\, \sigma^{\mu\nu} P_R\, b] F_{\mu\nu} \qquad \mathcal{O}_{9,10} = [\bar{s}\, \gamma^{\mu} P_L\, b][\bar{\ell}\, \gamma_{\mu} (1,\ \gamma_5)\, \ell]$$

$\Delta B = 1$ EFT in the SM for $b \to s$

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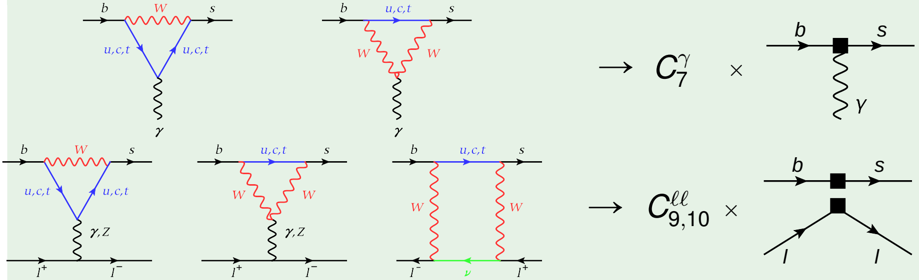
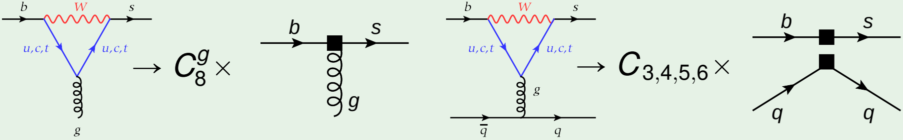


$\mathcal{O}_7 = m_b[\bar{s}\,\sigma^{\mu\nu} P_R\, b] F_{\mu\nu}$ $\mathcal{O}_{9,10} = [\bar{s}\,\gamma^\mu P_L\, b][\bar{\ell}\,\gamma_\mu(1,\ \gamma_5)\,\ell]$

$b \to s + gluon$ AND $b \to s + \bar{q}q$



$\mathcal{O}_8 = m_b[\bar{s}\,\sigma^{\mu\nu} P_R T^a\, b] G^a_{\mu\nu}$ $\mathcal{O}_{3,4} = [\bar{s}\,\gamma^\mu(1,\ T^a) P_L\, b] \sum_q [\bar{q}\,\gamma_\mu(1,\ T^a)\, q]$

Extension of EFT beyond the SM . . .

$$\mathcal{L}_{\text{eff}}(\mu_b) = \mathcal{L}_{\text{QED}\times\text{QCD}}(u, d, s, c, b,\ e, \mu, \tau,\ ???)$$

$$+ \frac{4G_F}{\sqrt{2}} V_{\text{CKM}} \sum_{\text{SM}} (C_i + \Delta C_i)\mathcal{O}_i + \sum_{\text{NP}} C_j \mathcal{O}_j(???)$$

$\Rightarrow$ ΔC_i . . . NP contributions to SM C_i

$\Rightarrow$ $\sum_{\text{NP}} C_j \mathcal{O}_j$ . . . NP operators (e.g. $C'_{7,9,10}$, $C^{(\prime)}_{S,P}$, . . .)

$\Rightarrow$??? . . . additional light degrees of freedom ($\Leftarrow$ not pursued in the following)

Extension of EFT beyond the SM ...

$$\mathcal{L}_{\text{eff}}(\mu_b) = \mathcal{L}_{\text{QED}\times\text{QCD}}(u, d, s, c, b,\ e, \mu, \tau,\ ???)$$

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MODEL-DEP. 1) decoupling of new heavy particles @ NP scale: $\mu_{NP} \gtrsim M_W$

2) RG-running to lower scale $\mu_b \sim m_b$ (potentially tower of EFT's)

C_i are correlated, depend on fundamental parameters

MODEL-INDEP. extending SM EFT-Lagrangian $\rightarrow$ new C_j

C_j are UN-correlated free parameters

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Towards Observables

$\Rightarrow$ EFT universal starting point for calculation of observables

!!! Non-perturbative input required when evaluating QCD matrix elements

Inclusive decays: $B \to X_{s,d}\gamma$, $B \to X_{s,d}\bar{\ell}\ell$, $B \to X_{s,d}\bar{\nu}\nu$, $B \to X_{\not{c}}$

- Heavy Quark Expansion: only few universal non-perturbative input parameters from $B \to X_{u,c}\ell\bar{\nu}_\ell$ and $B \to X_s\gamma$ photon spectrum
- only @ $e^+e^- -$ B-factories

Exclusive decays

- $K \to \pi\bar{\nu}\nu$: isospin symmetries to $K \to \pi e\bar{\nu}_e$
- $B_{s,d} \to \bar{\ell}\ell$: decay constants from QCD-Lattice calculation
- hadronic 2-body B-decays: QCD factorisation (QCDF) or Soft-Collinear ET (SCET)
- $B \to \{K, K^*\} + \bar{\ell}\ell$:
  - @ low dilepton inv. mass: QCDF
  - @ high dilepton inv. mass: OPE
  - form factors from LCSR or QCD-Lattice calculations
- $B \to \{K, K^*\} + \bar{\nu}\nu$: form factors from LCSR or QCD-Lattice calculations

... other decays, $\Delta F = 2$, ...

– Low Recoil –

Form factor relations and optimised observables

Summary: SM operator basis

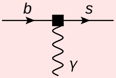
Beyond the SM basis

OPE of 4-quark contribution at low recoil

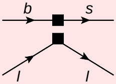
FCNC short-distance contributions in EFT

Hadronic amplitude $B \to K^*(\to K\pi)\,\bar{\ell}\ell$

$$\mathcal{M} = \langle K\pi| \; C_7^{\gamma} \times$$



$$+ \; C_{9,10}^{\ell\ell} \times$$



$$|B\rangle$$

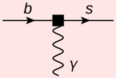
1) narrow width approximation

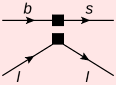
2) neglecting 4-quark operators (next couple of slides)

3) SM operator basis (next couple of slides)

FCNC short-distance contributions in EFT

HADRONIC AMPLITUDE $B \to K^*(\to K\pi)\,\bar\ell\ell$

$\mathcal{M} = \langle K\pi|\; C_7^\gamma \times$ 

$+ \; C_{9,10}^{\ell\ell} \times$ 

$|B\rangle$

DECOMPOSITION OF DECAY AMPLITUDE USING

- K^* on-shell: 3 polarisations $\epsilon_{K*}(m = +, -, 0)$
- V^* off-shell: 4 polarisations $\epsilon_{V*}(n = +, -, 0, t)$ (t=time-like $\sim m_\ell$)

$$\mathcal{M}^{L,R}[B \to K^* + V^*(\to \bar\ell\ell)] \sim \sum_{m,n,n'} \epsilon^{*\mu}_{K*}(m)\, \epsilon^{*\nu}_{V*}(n)\, \mathcal{M}_{\mu\nu}\, \epsilon^{\alpha}_{V*}(n')\, g_{nn'}\, [\bar\ell\,\gamma_\alpha P_{L,R}\,\ell]$$

Helicity amplitudes $\longrightarrow$ Transversity amplitudes

$$A^{L,R}_{\perp,\parallel} = \frac{1}{\sqrt{2}}\left[\mathcal{M}^{L,R}_{(+,+)} \mp \mathcal{M}^{L,R}_{(-,-)}\right], \qquad A_0^{L,R} = \mathcal{M}^{L,R}_{(0,0)}$$

Transversity amplitudes ($m_\ell = 0$)

$B \to K^*$ form factors V, $A_{1,2}$, $T_{1,2,3}$

$$A_\perp^{L,R} \sim \sqrt{2\,\lambda}\left[(C_9 \mp C_{10})\frac{V}{M_B + M_{K*}} + \frac{2\,m_b}{q^2}C_7\,T_1\right],$$

$$A_\parallel^{L,R} \sim -\sqrt{2}\,(M_B^2 - M_{K*}^2)\left[(C_9 \mp C_{10})\frac{A_1}{M_B - M_{K*}} + \frac{2\,m_b}{q^2}C_7\,T_2\right],$$

$$A_0^{L,R} \sim -\frac{1}{2\,M_{K*}\sqrt{q^2}}\left\{(C_9 \mp C_{10})\Big[\ldots A_1 + \ldots A_2\Big] + 2\,m_b C_7\Big[\ldots T_2 + \ldots T_3\Big]\right\}$$

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Heavy-to-light ($B \to K^*$) form factor relations

at low hadronic recoil limit: $E_{K*} \sim M_{K*} \sim \Lambda_{\rm QCD}$ [Isgur/Wise PLB232 (1989) 113, PLB237 (1990) 527]

$$T_1 \approx V, \qquad T_2 \approx A_1, \qquad T_3 \approx A_2 \frac{M_B^2}{q^2}$$

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$$T_1 \approx V, \qquad T_2 \approx A_1, \qquad T_3 \approx A_2\frac{M_B^2}{q^2}$$

at large hadronic recoil limit: $E_{K*} \sim M_B$ [Charles et al. hep-ph/9812358, Beneke/Feldmann hep-ph/0008255]

$$\xi_\perp \equiv \frac{M_B}{M_B + M_{K*}}V \approx \frac{M_B + M_{K*}}{2E_{K*}}A_1 \approx T_1 \approx \frac{M_B}{2E_{K*}}T_2$$

$$\xi_\parallel \equiv \frac{M_B + M_{K*}}{2E_{K*}}A_1 - \frac{M_B - M_{K*}}{M_{K*}}A_2 \approx \frac{M_B}{2E_{K*}}T_2 - T_3$$

Transversity amplitudes ($m_\ell = 0$)

Low hadronic recoil

$$A_i^{L,R} \sim C^{L,R} \times f_i \qquad , \qquad C^{L,R} = (C_9 \mp C_{10}) + \frac{2 m_b M_B}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \parallel, 0$)

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1 + \hat{M}_{K*}} V, \quad f_\parallel = \sqrt{2}\,(1 + \hat{M}_{K*})\, A_1, \quad f_0 = \frac{(1 - \hat{s} - \hat{M}_{K*}^2)(1 + \hat{M}_{K*})^2 A_1 - \hat{\lambda}\, A_2}{2\,\hat{M}_{K*}\,(1 + \hat{M}_{K*})\sqrt{\hat{s}}}$$

("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

Transversity amplitudes ($m_\ell = 0$)

Low hadronic recoil

FF symmetry breaking corrections

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7\, \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}, \alpha_s\right), \qquad C^{L,R} = (C_9 \mp C_{10}) + \frac{2 m_b M_B}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \parallel, 0$) $\quad C_7^{\text{SM}} \approx -0.3,\ C_9^{\text{SM}} \approx 4.2,\ C_{10}^{\text{SM}} \approx -4.2$

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Large hadronic recoil

$$A_{\perp,\parallel}^{L,R} \sim \pm C_\perp^{L,R} \times \xi_\perp + \mathcal{O}\left(\alpha_s, \frac{\Lambda_{\rm QCD}}{m_b}\right), \qquad A_0^{L,R} \sim C_\parallel^{L,R} \times \xi_\parallel + \mathcal{O}\left(\alpha_s, \frac{\Lambda_{\rm QCD}}{m_b}\right)$$

2 SD-coefficients $C_{\perp,\parallel}^{L,R}$ and 2 FF's $\xi_{\perp,\parallel}$

$$C_\perp^{L,R} = (C_9 \mp C_{10}) + \frac{2 m_b M_B}{q^2} C_7, \qquad C_\parallel^{L,R} = (C_9 \mp C_{10}) + \frac{2\, m_b}{M_B} C_7,$$

Angular observables ($m_\ell = 0$) – in terms of transversity amplitudes

$$4\,J_{2s} = |A_\perp^L|^2 + |A_\parallel^L|^2 + (L \to R), \qquad -J_{2c} = |A_0^L|^2 + |A_0^R|^2,$$

$$2\,J_3 = |A_\perp^L|^2 - |A_\parallel^L|^2 + (L \to R), \qquad \sqrt{2}\,J_4 = \mathrm{Re}\Big[A_0^L A_\parallel^{L*} + (L \to R)\Big],$$

$$\frac{J_5}{\sqrt{2}} = \mathrm{Re}\Big[A_0^L A_\perp^{L*} - (L \to R)\Big], \qquad \frac{J_{6s}}{2} = \mathrm{Re}\Big[A_\parallel^L A_\perp^{L*} - (L \to R)\Big],$$

$$\frac{J_7}{\sqrt{2}} = \mathrm{Im}\Big[A_0^L A_\parallel^{L*} - (L \to R)\Big], \qquad \sqrt{2}\,J_8 = \mathrm{Im}\Big[A_0^L A_\perp^{L*} + (L \to R)\Big],$$

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Within SM-basis and $m_\ell = 0 \to$ out of 12 J_i only 8 independent

$$J_{1s} = 3\,J_{2s}, \qquad J_{1c} = -J_{2c}, \qquad J_{6c} = 0,$$

and a 4th (not so trivial) relation

[Egede/Hurth/Matias/Ramon/Reece arXiv:1005.0571]

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$$J_9 = \mathrm{Im}\Big[A_\perp^L A_\parallel^{L*} + (L \to R)\Big]$$

For example at Large Recoil: $J_{2s},\ J_3,\ J_{6s},\ J_9\ \sim\ \xi_\perp$ $\Rightarrow$ ratios have reduced hadronic uncertainty

$$A_T^{(2)} = \frac{J_3}{2\,J_{2s}}, \qquad A_T^{(\mathrm{re})} = \frac{J_{6s}}{4\,J_{2s}}, \qquad A_T^{(\mathrm{im})} = \frac{J_9}{2\,J_{2s}}$$

[Krüger/Matias hep-ph/0502060, Becirevic/Schneider, arXiv:1106.3283]

“Transversity” Observables @ Large Recoil . . . ($m_\ell = 0$)

. . . “designed” from transversity amplitudes, in order to have reduced FF-uncertainty

$$A_T^{(2)} = \frac{|A_\perp^L|^2 + |A_\perp^R|^2 - |A_\parallel^L|^2 - |A_\parallel^R|^2}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2}, \qquad A_T^{(3)} = \frac{|A_0^L A_\parallel^{L*} + A_0^{R*} A_\parallel^R|}{\sqrt{\left(|A_0^L|^2 + |A_0^R|^2\right)\left(|A_\perp^L|^2 + |A_\perp^R|^2\right)}},$$

$$A_T^{(4)} = \frac{|A_0^L A_\perp^{L*} - A_0^{R*} A_\perp^R|}{|A_0^{L*} A_\parallel^L + A_0^R A_\parallel^{R*}|}, \qquad A_T^{(5)} = \frac{|A_\parallel^L A_\perp^{R*} + A_\perp^L A_\parallel^{R*}|}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2},$$

$$A_T^{(\mathrm{re})} = \frac{2\,\mathrm{Re}\left[A_\parallel^L A_\perp^{L*} - A_\parallel^R A_\perp^{R*}\right]}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2}, \qquad A_T^{(\mathrm{im})} = \frac{2\,\mathrm{Im}\left[A_\parallel^L A_\perp^{L*} + A_\parallel^R A_\perp^{R*}\right]}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2}$$

[Krüger/Matias hep-ph/0502060, Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589 + 1005.0571]

"Transversity" Observables @ Large Recoil ... ($m_\ell = 0$)

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$$A_T^{(2)} = \frac{J_3}{2\, J_{2s}}, \qquad A_T^{(3)} = \sqrt{\frac{(2\, J_4)^2 + J_7^2}{-2\, J_{2c}\,(2\, J_{2s} + J_3)}},$$

$$A_T^{(4)} = \sqrt{\frac{J_5^2 + (2\, J_8)^2}{(2\, J_4)^2 + J_7^2}}, \qquad A_T^{(5)} = \frac{\sqrt{16\, J_{1s}^2 - 9\, J_{6s}^2 - 36\,(J_3^2 + J_9^2)}}{8\, J_{1s}},$$

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[Krüger/Matias hep-ph/0502060, Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589 + 1005.0571]

$$1 = \left(4\,A_T^{(5)}\right)^2 + \left(A_T^{(2)}\right)^2 + \left(A_T^{(\mathrm{re})}\right)^2 + \left(A_T^{(\mathrm{im})}\right)^2$$

[Becirevic/Schneider, arXiv:1106.3283]

1st measurement of $A_T^{(2)}$ and $S_3 = J_3/\Gamma$ available from CDF and LHCb in 6 q^2 bins

[CDF arXiv:1108.0695, LHCb-CONF-2012-008]

Angular observables @ Low Recoil using FF relations

[CB/Hiller/van Dyk arXiv:1006.5013]

$$\frac{4}{3}(2\,J_{2s} + J_3) = 2\,\rho_1\,f_\perp^2, \qquad -\frac{4}{3}J_{2c} = 2\,\rho_1\,f_0^2, \qquad \frac{2\sqrt{2}}{3}J_5 = 4\,\rho_2\,f_0 f_\perp,$$

$$\frac{4}{3}(2\,J_{2s} - J_3) = 2\,\rho_1\,f_\parallel^2, \qquad \frac{4\sqrt{2}}{3}\,J_4 = 2\,\rho_1\,f_0 f_\parallel, \qquad \frac{2}{3}J_{6s} = 4\,\rho_2\,f_\parallel f_\perp,$$

$$J_7 = J_8 = J_9 = 0, \qquad f_{\perp,\parallel,0} = \text{form factors}$$

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$$J_7 = J_8 = J_9 = 0, \qquad f_{\perp,\parallel,0} = \text{form factors}$$

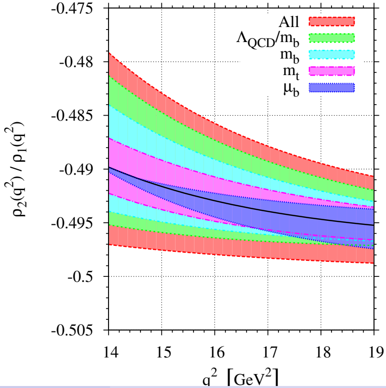
ρ_1 and ρ_2 are largely μ_b-scale independent

$$\rho_1(q^2) \equiv \left| C_9^{\rm eff} + \kappa \frac{2m_b^2}{q^2} C_7^{\rm eff} \right|^2 + |C_{10}|^2\,,$$

$$\rho_2(q^2) \equiv \mathrm{Re}\left[\left(C_9^{\rm eff} + \kappa \frac{2m_b^2}{q^2} C_7^{\rm eff}\right) C_{10}^*\right]$$

$\kappa(\mu_b)$ radiative QCD-correction to matching of FF relations between QCD and HQET

⇒ accounts for μ_b-dependence of tensor form factors $T_{1,2,3}$



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[CB/Hiller/van Dyk arXiv:1006.5013]

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$$\frac{4}{3}(2\,J_{2s} - J_3) = 2\,\rho_1\, f_\parallel^2, \qquad \frac{4\sqrt{2}}{3}\,J_4 = 2\,\rho_1\, f_0 f_\parallel, \qquad \frac{2}{3}J_{6s} = 4\,\rho_2\, f_\parallel f_\perp,$$

$$J_7 = J_8 = J_9 = 0,$$

$f_{\perp,\parallel,0}$ = form factors

$$\frac{d\Gamma}{dq^2} = 2\,\rho_1 \times (f_0^2 + f_\perp^2 + f_\parallel^2), \qquad A_{\rm FB} = 3\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp f_\parallel}{(f_0^2 + f_\perp^2 + f_\parallel^2)},$$

$$F_L = \frac{f_0^2}{f_0^2 + f_\perp^2 + f_\parallel^2}, \qquad A_T^{(2)} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}, \qquad A_T^{(3)} = \frac{f_\parallel}{f_\perp}, \qquad A_T^{(4)} = 2\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp}{f_\parallel}$$

at low recoil: F_L, $A_T^{(2)}$, $A_T^{(3)}$ are short-distance independent, contrary to large recoil

$\Rightarrow$ could be used to fit form factor shape

Angular observables @ Low Recoil using FF relations

[CB/Hiller/van Dyk arXiv:1006.5013]

$$\frac{4}{3}(2\,J_{2s} + J_3) = 2\,\rho_1\, f_\perp^2, \qquad -\frac{4}{3}J_{2c} = 2\,\rho_1\, f_0^2, \qquad \frac{2\sqrt{2}}{3}J_5 = 4\,\rho_2\, f_0 f_\perp,$$

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at low recoil: F_L, $A_T^{(2)}$, $A_T^{(3)}$ are short-distance independent, contrary to large recoil

$\Rightarrow$ could be used to fit form factor shape

All relations valid up to sub-leading corrections in $C_7/C_9 \times \Lambda_{\rm QCD}/m_b$ due to FF relations.
(Later: OPE of 4-quark contributions yield also additional $(\Lambda_{\rm QCD}/m_b)^2$)

Optimised Observables @ Low Recoil ...

"short- & long-distance free"

$$H_T^{(1)} = \frac{\sqrt{2}J_4}{\sqrt{-J_{2c}\left(2J_{2s} - J_3\right)}} = \mathrm{sgn}(f_0)$$

[CB/Hiller/van Dyk arXiv:1006.5013]

Optimised Observables @ Low Recoil . . .

"short- & long-distance free"

$$H_T^{(1)} = \frac{\sqrt{2} J_4}{\sqrt{-J_{2c}\left(2J_{2s} - J_3\right)}} = \mathrm{sgn}(f_0)$$

"long-distance free" SD combinations: $\rho_1 = (|C^R|^2 + |C^L|^2)/2, \qquad \rho_2 = (|C^R|^2 - |C^L|^2)/4$

$$H_T^{(2)} = \frac{J_5}{\sqrt{-2J_{2c}\left(2J_{2s} + J_3\right)}} = 2\,\frac{\rho_2}{\rho_1}, \qquad H_T^{(3)} = \frac{J_{6s}}{2\sqrt{(2J_{2s})^2 - J_3^2}} = 2\,\frac{\rho_2}{\rho_1}$$

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Optimised Observables @ Low Recoil . . .

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“short-distance free” → measure form factors $f_{0,\parallel,\perp}$ (SM-operator basis only)

$$\frac{f_0}{f_\parallel} = \frac{\sqrt{2}J_5}{J_{6s}} = \frac{-J_{2c}}{\sqrt{2}J_4} = \frac{\sqrt{2}J_4}{2J_{2s} - J_3} = \sqrt{\frac{-J_{2c}}{2J_{2s} - J_3}} = \frac{\sqrt{2}J_8}{-J_9},$$

$$\frac{f_\perp}{f_\parallel} = \sqrt{\frac{2J_{2s} + J_3}{2J_{2s} - J_3}} = \frac{\sqrt{-J_{2c}\left(2J_{2s} + J_3\right)}}{\sqrt{2}J_4}, \qquad \frac{f_0}{f_\perp} = \sqrt{\frac{-J_{2c}}{2J_{2s} + J_3}}$$

[CB/Hiller/van Dyk arXiv:1006.5013]

Optimised Observables @ Low Recoil . . . only in SM operator basis

"short- & long-distance free"

$$H_T^{(1)} = \frac{\sqrt{2} J_4}{\sqrt{-J_{2c}\left(2J_{2s} - J_3\right)}} = \mathrm{sgn}(f_0)$$

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[CB/Hiller/van Dyk arXiv:1006.5013]

FF-free CP-asymmetries @ low recoil

[CB/Hiller/van Dyk arXiv:1105.0376]

$$a_{\rm CP}^{(1)} = \frac{\rho_1 - \bar\rho_1}{\rho_1 + \bar\rho_1}\,, \qquad a_{\rm CP}^{(2)} = \frac{\frac{\rho_2}{\rho_1} - \frac{\bar\rho_2}{\bar\rho_1}}{\frac{\rho_2}{\rho_1} + \frac{\bar\rho_2}{\bar\rho_1}}\,, \qquad a_{\rm CP}^{(3)} = 2\,\frac{\rho_2 - \bar\rho_2}{\rho_1 + \bar\rho_1}$$

- NLO QCD corrections large $\Rightarrow$ decrease CP-asymmetries
- still, theoretical uncertainties large: dominated by renorm. scale μ_b

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$B_s \to \phi(\to K^+K^-) + \ell^+\ell^-$

- time-integrated $a_{\rm CP}^{mix}$ in $B_s \to \phi(\to K^+K^-) + \bar\ell\ell$ is CP-odd = untagged
- $a_{\rm CP}^{mix}$ depends only on $(\Delta\Gamma_s/\Gamma_s)^2$ $\Rightarrow$ no sensitivity to sign of $\Delta\Gamma_s$
- since $(\Delta\Gamma_s/\Gamma_s)^2 \ll 1$ no significant sensitivity to B_s mixing parameters
  $\Rightarrow$ comparable to $a_{\rm CP}^{(3)}[B \to K^*\ell^+\ell^-]$

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$B \to K\,\ell^+\ell^-$

- @ high-q^2: $A_{\rm CP}[B \to K\,\ell^+\ell^-] = a_{\rm CP}^{(1)}[B \to K^*\ell^+\ell^-]$ in SM operator basis

BSM operator list

FREQUENTLY CONSIDERED IN MODEL-(IN)DEPENDENT SEARCHES: $b \to s \ell^+ \ell^-$

SM' = χ-flipped SM analogues

$$\mathcal{O}_{7'}^{\gamma} = \frac{e}{(4\pi)^2} m_b [\bar{s}\, \sigma_{\mu\nu} P_R\, b] F^{\mu\nu}, \qquad \mathcal{O}_{9',10'}^{\ell\ell} = \frac{\alpha_e}{4\pi} [\bar{s}\, \gamma^{\mu} P_R\, b][\bar{\ell}\, (\gamma^{\mu}, \gamma^{\mu}\gamma_5)\, \ell]$$

S + P = scalar + pseudoscalar

$$\mathcal{O}_{S,S'}^{\ell\ell} = \frac{\alpha_e}{4\pi} [\bar{s}\, P_{R,L}\, b][\bar{\ell}\, \ell], \qquad \mathcal{O}_{P,P'}^{\ell\ell} = \frac{\alpha_e}{4\pi} [\bar{s}\, P_{R,L}\, b][\bar{\ell}\, \gamma_5\, \ell]$$

T + T5 = tensor

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new Dirac-structures beyond SM:

- SM' : right-handed currents
- S + P : higgs-exchange & box-type diagrams
- T + T5 : box-type diagrams, Fierzed scalar tree exchange

BSM operator list

Frequently considered in model-(in)dependent searches: $b \to s \ell^+ \ell^-$

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??? Will there be changes beyond the SM basis to:

$$|H_T^{(1)}| = 1, \qquad H_T^{(2)} = H_T^{(3)}, \qquad J_7 = J_8 = J_9 = 0$$

$$f_0/f_{||}, \qquad f_\perp/f_{||}, \qquad f_0/f_\perp$$

$$\ldots$$

SM' = $\mathcal{O}_{7',9',10'}$ @ low recoil

[work in progress CB/Hiller/van Dyk]

transversity amplitudes : $A^{L,R}_{0,\|} = -C^{L,R}_{-} f_{0,\|}, \qquad A^{L,R}_{\perp} = +C^{L,R}_{+} f_{\perp}$

with short-distance coefficients $C^{L,R} \to C^{L,R}_{\pm}$

$$C^{L,R}_{-} = \left[(C_9^{\rm eff} - C_{9'}^{\rm eff}) + \kappa \frac{2 m_b^2}{q^2} (C_7^{\rm eff} - C_{7'}^{\rm eff}) \right] \mp (C_{10} - C_{10'}),$$

$$C^{L,R}_{+} = \left[(C_9^{\rm eff} + C_{9'}^{\rm eff}) + \kappa \frac{2 m_b^2}{q^2} (C_7^{\rm eff} + C_{7'}^{\rm eff}) \right] \mp (C_{10} + C_{10'})$$

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$$C^{L,R}_{+} = \left[(C^{\rm eff}_9 + C^{\rm eff}_{9'}) + \kappa \frac{2m_b^2}{q^2} (C^{\rm eff}_7 + C^{\rm eff}_{7'}) \right] \mp (C_{10} + C_{10'})$$

Now the angular observables J_i ($m_\ell = 0$) read

$$\frac{4}{3}(2J_{2s} + J_3) = 2\,\rho_1^+ f_\perp^2, \qquad \frac{4\sqrt{2}}{3} J_4 = 2\,\rho_1^- f_0 f_\|, \qquad J_7 = 0,$$

$$\frac{4}{3}(2J_{2s} - J_3) = 2\,\rho_1^- f_\|^2, \qquad \frac{2\sqrt{2}}{3} J_5 = 4\,{\rm Re}(\rho_2) f_0 f_\perp, \qquad \frac{4\sqrt{2}}{3} J_8 = 4\,{\rm Im}(\rho_2) f_0 f_\perp,$$

$$-\frac{4}{3} J_{2c} = 2\,\rho_1^- f_0^2, \qquad \frac{2}{3} J_{6s} = 4\,{\rm Re}(\rho_2) f_\| f_\perp, \qquad -\frac{4}{3} J_9 = 4\,{\rm Im}(\rho_2) f_\| f_\perp$$

where ρ_1 and ρ_2 have to be generalised

$$\rho_1^\pm = \frac{1}{2}\left(|C^R_\pm|^2 + |C^L_\pm|^2 \right), \qquad \rho_2 = \frac{1}{4}\left(C^R_+ C^{R*}_- - C^L_- C^{L*}_+ \right)$$

SM' = $\mathcal{O}_{7',9',10'}$ @ low recoil

[work in progress CB/Hiller/van Dyk]

- extension to $\rho_1 \to \rho_1^\pm$
- still have: $H_T^{(1)} = \mathrm{sgn}(f_0) \quad \Rightarrow$ deviations test OPE
- $J_7 = 0$, but $J_{8,9} \neq 0$
- generalisation: $H_T^{(2)} = H_T^{(3)} = \dfrac{2\,\mathrm{Re}(\rho_2)}{\sqrt{\rho_1^- \cdot \rho_1^+}}$
- 2 new FF-free ratios

$$H_T^{(4)} = \frac{\sqrt{2} J_8}{\sqrt{-J_{2c}\,(2J_{2s} + J_3)}} = \frac{2\,\mathrm{Im}(\rho_2)}{\sqrt{\rho_1^- \cdot \rho_1^+}}, \qquad H_T^{(5)} = \frac{-J_9}{\sqrt{(2J_{2s})^2 - J_3^2}} = \frac{2\,\mathrm{Im}(\rho_2)}{\sqrt{\rho_1^- \cdot \rho_1^+}}$$

- $a_{\mathrm{CP}}^{(1)} \to a_{\mathrm{CP}}^{(1,\pm)}$ and $a_{\mathrm{CP}}^{(2)} \to a_{\mathrm{CP}}^{(2,\pm)}$
- generalisation of $a_{\mathrm{CP}}^{(3)}$ and additional

$$a_{\mathrm{CP}}^{(3)} = \frac{2\,\mathrm{Re}(\rho_2 - \bar{\rho}_2)}{\sqrt{(\rho_1^+ - \bar{\rho}_1^+) \cdot (\rho_1^- - \bar{\rho}_1^-)}}, \qquad a_{\mathrm{CP}}^{(4)} = \frac{2\,\mathrm{Im}(\rho_2 - \bar{\rho}_2)}{\sqrt{(\rho_1^+ - \bar{\rho}_1^+) \cdot (\rho_1^- - \bar{\rho}_1^-)}}$$

- less “short-distance free” ratios: only $f_0/f_\parallel = \sqrt{2} J_8/J_9$ remains

Scalar (S + P) operators @ low recoil . . . and no tensor op's

- New form factor A_0 needed !!!
- No contribution to $J_{1s,2s,2c,3,4,6s,8,9}$

$\Rightarrow$ still $\quad |H_T^{(1)}| = 1, \quad J_{1s} = 3\, J_{2s}, \quad H_T^{(4)} = H_T^{(5)}$

- unsuppressed contributions to J_{1c}

$$J_{1c} = -J_{2c} + \mathcal{O}\left(\frac{m_\ell^2}{q^2}\right) \quad \Rightarrow \quad J_{1c} = -J_{2c} + \ldots (C_{S,P} - C'_{S,P}) A_0 + \mathcal{O}\left(\frac{m_\ell^2}{q^2}\right)$$

since $F_L = (J_{1c} - J_{2c}/3)/\Gamma$, relation does not hold:

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} \neq \frac{3}{4} F_L \sin^2\theta_\ell + \frac{3}{8} F_T (1 + \cos^2\theta_\ell) + A_{\rm FB} \cos\theta_\ell$$

- (SM + SM') $\times$ S to $J_{5,6c,7}$ suppressed by $\sim m_\ell/\sqrt{q^2}$

$\Rightarrow H_T^{(2)}$ modified, only $\quad H_T^{(2)} + \mathcal{O}(m_\ell/\sqrt{q^2}) \approx H_T^{(3)}$
$\Rightarrow J_{6c} \neq 0$ modified
$\Rightarrow J_7 \neq 0$ if CPV beyond SM, since $J_7 \sim \mathrm{Im}[\ldots (C_S - C'_S)]$

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$$J_{1c} = -J_{2c} + \mathcal{O}\left(\frac{m_\ell^2}{q^2}\right) \quad \Rightarrow \quad J_{1c} = -J_{2c} + \ldots (C_{S,P} - C'_{S,P}) A_0 + \mathcal{O}\left(\frac{m_\ell^2}{q^2}\right)$$

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$\Rightarrow J_7 \neq 0$ if CPV beyond SM, since $J_7 \sim \mathrm{Im}[\ldots (C_S - C'_S)]$

Tensor (T + T5) operators @ low recoil

- 2 new short-distance coefficients

$$\rho_1^T \equiv 16\,\kappa^2 \frac{M_B^2 - M_{K*}^2}{q^2} \left(|C_T|^2 + |C_{T5}|^2\right), \qquad \rho_2^T \equiv 16\,\kappa^2 \frac{M_B^2}{q^2}\, C_T C_{T5}^*$$

- not $m_\ell/\sqrt{q^2}$ suppressed contributions to

  $\Rightarrow J_{1s,1c,2s,2c,3,4,8,9}$

  $\Rightarrow$ (S $\times$ T5) and (P $\times$ T) in $J_{5,6c}$ and (S $\times$ T) and (P $\times$ T) in J_7

- $\sim m_\ell/\sqrt{q^2}$-suppressed interference (SM + SM') $\times$ (T + T5) in $J_{1s,1c,5,6s,6c,7}$

- now $|H_T^{(1)}| = 1 + \mathcal{O}(M_{K*}^2/M_B^2)$

$$H_T^{(1)} \approx 1 + \frac{M_{K*}^2}{M_B^2} \times F(\rho_1^-, \rho_1^T) + \mathcal{O}\left(\frac{M_{K*}^4}{M_B^4}\right)$$

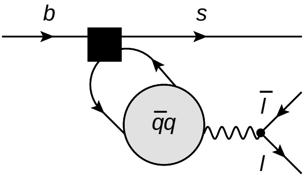
In BSM scenarios without tensor operators, deviations $|H_T^{(1)}| \neq 1$ can signal large long-distance effects

4-Quark operators

⇒ Have to be included because of operator mixing

⇒ Background contribution $B \to K^{(*)}(\bar{q}q) \to K^{(*)}\bar{\ell}\ell$ from 4-quark operators $b \to s\bar{q}q$

- up-type CC-operators suppressed by $V_{ub}V_{us}^*$
- QCD-penguin operators suppressed by small $C_{3,4,5,6}$



mainly charm CC-operators induce large peaking background around $q^2 = (M_{J/\psi})^2, (M_{\psi'})^2$:

$\longrightarrow$ vetoed in experiment

q^2 - REGIONS IN $b \to s + \bar{\ell}\ell$ $K^{(*)}$-ENERGY IN B-REST FRAME: $E_{K^{(*)}} = (M_B^2 + M_{K^{(*)}}^2 - q^2)/(2\, M_B)$

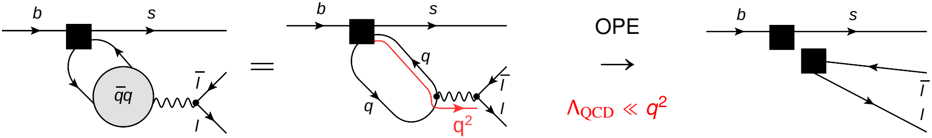
q^2-region	low-q^2: $q^2 \ll M_B^2$	high-q^2: $q^2 \sim M_B^2$
$K^{(*)}$-recoil	large recoil: $E_{K^{(*)}} \sim M_B/2$	low recoil: $E_{K^{(*)}} \sim M_{K^{(*)}} + \Lambda_{\rm QCD}$
theory method	QCDF, SCET: $q^2 \in [1, 6]$ GeV^2	OPE + HQET: $q^2 \gtrsim (14 \dots 15)$ GeV^2

[QCDF: Beneke/Feldmann/Seidel hep-ph/0106067, hep-ph/0412400]

[OPE: Grinstein/Pirjol hep-ph/0404250; Beylich/Buchalla/Feldmann arXiv:1101.5118]

High-q^2 = Low Recoil

Hard momentum transfer $(q^2 \sim M_B^2)$ through $(\bar{q}q) \to \bar{\ell}\ell$ allows local OPE



$$\mathcal{M}[\bar{B} \to \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi^2}{q^2} i \int d^4x\, e^{iq\cdot x} \langle \bar{K}^* | T\{\mathcal{L}^{\rm eff}(0), j_\mu^{\rm em}(x)\} | \bar{B}\rangle\, [\bar{\ell}\gamma^\mu \ell]$$

$$= \left(\sum_a \mathcal{C}_{3a} \mathcal{Q}_{3a}^\mu + \sum_b \mathcal{C}_{5b} \mathcal{Q}_{5b}^\mu + \sum_c \mathcal{C}_{6c} \mathcal{Q}_{6c}^\mu + \mathcal{O}(dim > 6) \right) [\bar{\ell}\gamma_\mu \ell]$$

Buchalla/Isidori hep-ph/9801456, Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann arXiv:1101.5118

Leading $dim = 3$ operators: $\langle \bar{K}^* | \mathcal{Q}_{3,a} | \bar{B}\rangle \sim$ usual $B \to K^*$ form factors $V, A_{0,1,2}, T_{1,2,3}$

$$\mathcal{Q}_{3,1}^\mu = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) [\bar{s}\, \gamma_\nu (1 - \gamma_5)\, b] \quad \to \quad C_9 \to C_9^{\rm eff}, \quad (V, A_{0,1,2})$$

$$\mathcal{Q}_{3,2}^\mu = \frac{i m_b}{q^2} q_\nu\, [\bar{s}\, \sigma_{\nu\mu} (1 + \gamma_5)\, b] \quad \to \quad C_7 \to C_7^{\rm eff}, \quad (T_{1,2,3})$$

High-q^2 = Low Recoil

[Beylich/Buchalla/Feldmann arXiv:1101.5118]

$dim = 3$	α_s matching corrections are also known
$m_s \neq 0$	2 additional $dim = 3$ operators, suppressed with $\alpha_s m_s/m_b \sim 0.5$ %, NO new form factors
$dim = 4$	absent
$dim = 5$	suppressed by $(\Lambda_{\rm QCD}/m_b)^2 \sim 2$ %, explicite estimate @ $q^2 = 15$ GeV^2: < 1%
$dim = 6$	suppressed by $(\Lambda_{\rm QCD}/m_b)^3 \sim 0.2$ % and small QCD-penguin's: $C_{3,4,5,6}$ spectator quark effects: from weak annihilation
BEYOND OPE	duality violating effects

- based on Shifman model for c-quark correlator + fit to recent BES data
- ± 2 % for integrated rate $q^2 > 15$ GeV^2

$\Rightarrow$ OPE of exclusive $B \to K^{(*)}\ell^+\ell^-$ predicts small sub-leading contributions !!!

BUT, still missing $B \to K^{(*)}$ form factors @ high-q^2

for predictions of angular observables J_i

FF-FREE RATIOS !!! TEST SD FLAVOUR COUPLINGS VERSUS EXP. DATA + OPE

$$H_T^{(1)} = \frac{\sqrt{2} J_4}{\sqrt{-J_{2c}\,(2J_{2s} - J_3)}} = \text{sgn}(f_0)$$

$$H_T^{(2)} = \frac{J_5}{\sqrt{-2J_{2c}\,(2J_{2s} + J_3)}} = 2\,\frac{\rho_2}{\rho_1}, \qquad H_T^{(3)} = \frac{J_6}{2\sqrt{(2J_{2s})^2 - J_3^2}} = 2\,\frac{\rho_2}{\rho_1}$$

SM predictions integrated $q^2 \in [14,\ 19.2]$ GeV^2 [CB/Hiller/van Dyk arXiv:1006.5013]

$$\langle H_T^{(1)} \rangle = +0.997 \pm 0.002\Big|_{\rm FF}\ {}^{+0.000}_{-0.001}\Big|_{\rm IWR},$$

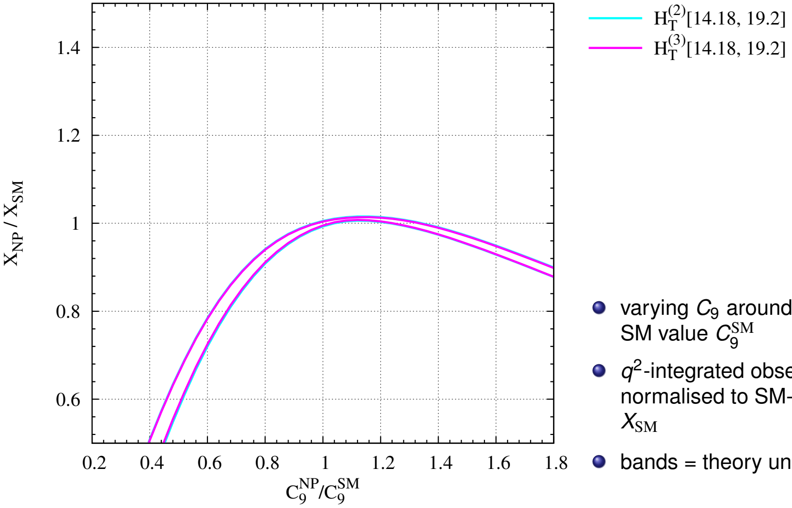
$$\langle H_T^{(2)} \rangle = -0.972\ {}^{+0.004}_{-0.003}\Big|_{\rm FF} \pm 0.001\Big|_{\rm SL}\ {}^{+0.008}_{-0.005}\Big|_{\rm IWR}\ {}^{+0.003}_{-0.004}\Big|_{\rm SD},$$

$$\langle H_T^{(3)} \rangle = -0.958 \pm 0.001\Big|_{\rm SL}\ {}^{+0.008}_{-0.006}\Big|_{\rm IWR}\ {}^{+0.003}_{-0.004}\Big|_{\rm SD}$$

⇒ Assuming validity of LCSR extrapolation Ball/Zwicky [hep-ph/0412079] of $V, A_{1,2}(q^2)$ to $q^2 > 14$ GeV^2 based form factor parametrisation using dipole formula

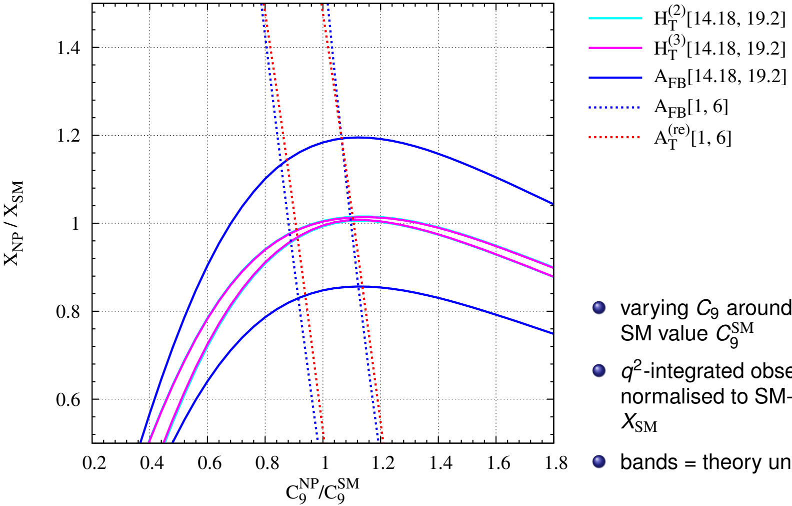
⇒ $\langle \ldots \rangle$ = q^2-integration performed in analogy to experimental measurement for each $I_i^{(k)}$ before taking ratio and $\sqrt{\ldots}$

Sensitivity of $H_T^{(2,3)}$ – example: real C_9



- varying C_9 around SM value C_9^{SM}
- q^2-integrated observables X_{NP} normalised to SM-prediction X_{SM}
- bands = theory uncertainties

Sensitivity of $H_T^{(2,3)}$ – example: real C_9



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At high-q^2 $H_T^{(2,3)}$ are the “better” A_{FB} !!!

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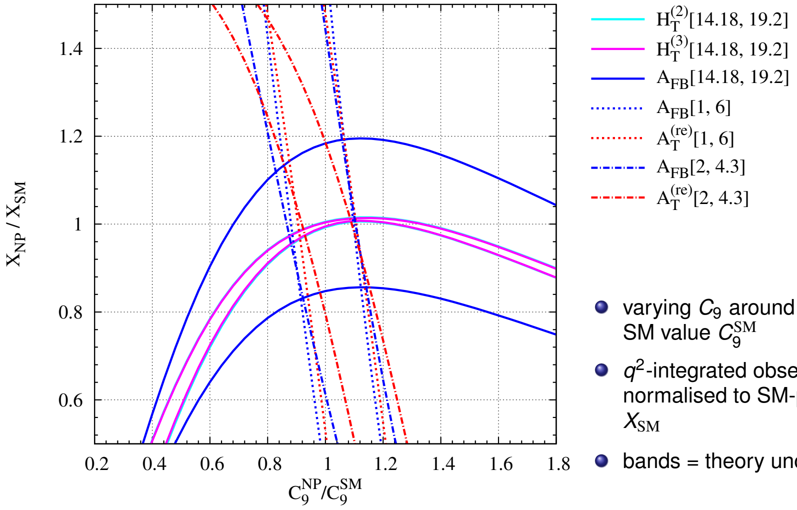

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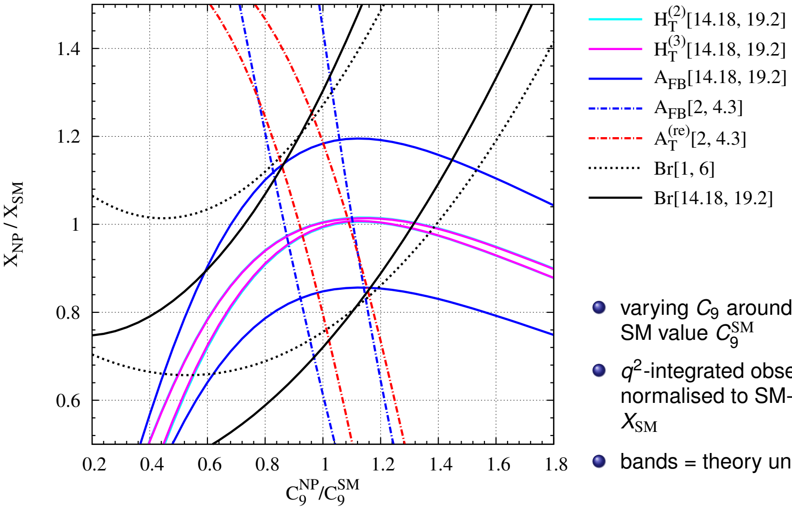

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- q^2-integrated observables X_{NP} normalised to SM-prediction X_{SM}
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At high-q^2 $H_T^{(2,3)}$ are the "better" A_{FB} !!!

Open Issues

- $B \to K$ and $B \to K^*$ form factors at high-q^2 (from Lattice)
  preliminary results without final uncertainty estimate:
  [Liu/Meinel/Hart/Horgan/Müller/Wingate arXiv:0911.2370, 1101.2726]

- better understanding of sub-leading contributions
  1) QCD factorization at low-q^2
  2) OPE at high-q^2 - known up to sub-leading form factors (Lattice?)
  [Grinstein/Pirjol hep-ph/0404250; Beylich/Buchalla/Feldmann arXiv:1101.5118]

- inclusion of $\bar{c}c$-tails at low-q^2
  [Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]

- beyond narrow width approximation for K^* in: $B \to K^* \to K\pi$???

- non-P wave $K\pi$ background to $K\pi$ pairs from K^* at high experimental statistics ???

- ...

Towards a global analysis of rare $\Delta B = 1$ decays

“Global” Fit of complex C_9 and C_{10} – (a) simple strategy

- scan parameter space $C_i = |C_i| \exp(i\phi_i)$ (i = 7, 9, 10) on grid
- evaluate theory uncertainties at each grid point:
  FF's, sub-leading Λ/m_b based on power counting,
  μ_b, CKM, m_c, m_b, m_t
- calculate likelihood and obtain 68 & 95 % regions

$B \to X_s \bar{\ell}\ell$:

Br: $\in [1, 6]$ GeV^2

[Babar/Belle]

$B \to K^* \bar{\ell}\ell$:

Br, A_{FB}, F_L:

$q^2 \in [1, 6]$ GeV^2

Br, A_{FB}:

$[14, 16]$, $[> 16]$ GeV^2

[Belle/CDF/LHCb]

$B \to K \bar{\ell}\ell$:

Br: $q^2 \in [1, 6]$,

$[14, 16]$, $[> 16]$ GeV^2

[Belle/CDF]

green square = SM

contour
= without $B \to K\bar{\ell}\ell$

"Global" Fit of complex C_9 and C_{10} – (a) simple strategy

- scan parameter space $C_i = |C_i| \exp(i\phi_i)$ (i = 7, 9, 10) on grid
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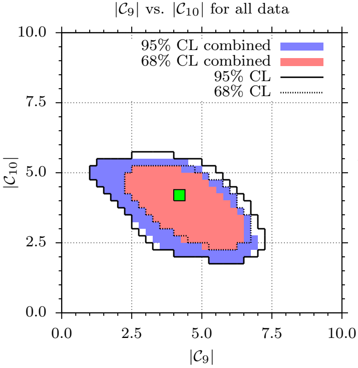
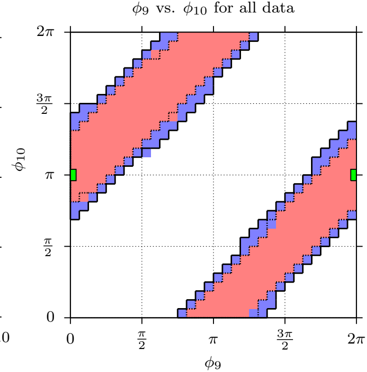




[CB/Hiller/van Dyk/Wacker arXiv:1111.2558]

$B \to X_s \bar{\ell}\ell$:

Br: $\in [1, 6]$ GeV^2 [Babar/Belle]

$B \to K^* \bar{\ell}\ell$:

Br, $A_{\rm FB}$, F_L: $q^2 \in [1, 6]$ GeV^2

Br, $A_{\rm FB}$: $[14, 16], [> 16]$ GeV^2 [Belle/CDF/LHCb]

$B \to K \bar{\ell}\ell$:

Br: $q^2 \in [1, 6]$, $[14, 16], [> 16]$ GeV^2 [Belle/CDF]

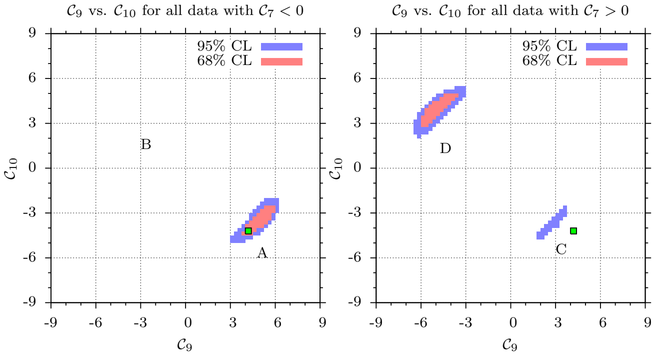
green square = SM

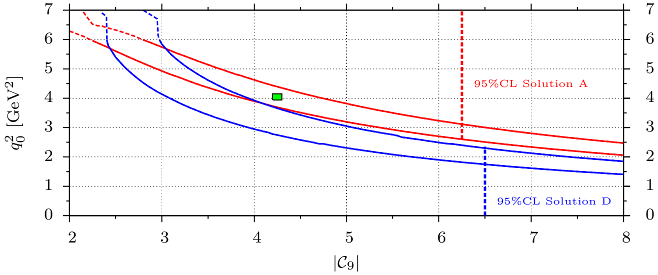
contour = without $B \to K\bar{\ell}\ell$

Case of real C_9 & C_{10}

green square = SM

$|C_7|$ fixed to SM value





$\Rightarrow$ zero crossing of $A_{\rm FB}$

solution A:

$q_0^2 > 2.6$ GeV^2

solution D:

$q_0^2 > 1.7$ GeV^2

Descotes-Genon/Ghosh/Matias/Ramon
arXiv:1104.3342

included data (before summer)

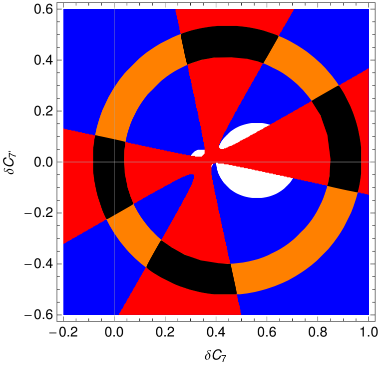
- $B \to X_s\gamma$ (Br),
  $B \to K^*\gamma$ (S, A_I)
- $B \to X_s\bar{\ell}\ell$ (low-q^2: Br),
  $B \to K^*\bar{\ell}\ell$ (only low-q^2: $A_{\rm FB}$, F_L)
- upper bound on $Br(B_s \to \bar{\mu}\mu)$

and NP in real Wilson coefficients

- $C_{7,7'}$
- + $C_{9,10}$
- + $C_{9',10'}$

⇒ sign-flipped solution of C_7 excluded at 1.6σ

⇒ predictions for $A_T^{(2)}$ which is sensitive to chirality-flipped operators $C_{7',9',10'}$



$C_7 - C_{7'}$ plane

$Br[B \to X_s\gamma]$

$S[B \to K^*\gamma]$

$A_I[B \to K^*\gamma]$

at 2σ
exp + th
uncertainty

Descotes-Genon/Ghosh/Matias/Ramon
arXiv:1104.3342

included data (before summer)

- $B \to X_s\gamma$ (Br),
  $B \to K^*\gamma$ (S, A_I)
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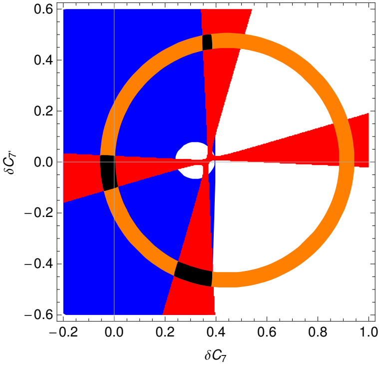


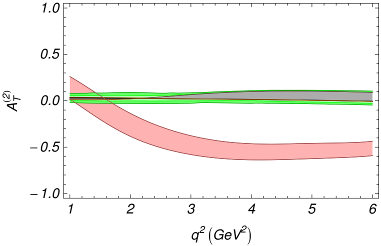
$C_7 - C_{7'}$ plane

$Br[B \to X_s\gamma]$

$S[B \to K^*\gamma]$

$A_I[B \to K^*\gamma]$

at 1σ
exp + th
uncertainty



$A_T^{(2)}$ for
NP in $C_{7,7'}$
at 1σ

neg solution
for $\delta C_{7'} \approx -0.4$

green = SM

Altmannshofer/Paradisi/Straub
arXiv:1111.1257

included data (up to date)

- $B \to X_s \gamma$ (*Br*), $B \to K^* \gamma$ (*S*)
- $B \to X_s \bar{\ell}\ell$ (lo + hi-q^2: Br),

  $B \to K^* \bar{\ell}\ell$ (lo + hi-q^2: *Br*, $A_{\rm FB}$, F_L)

NP in real and complex

- $C_{7,7', 9,9', 10,10'}$ (in varying stages)
- Z-penguin + $C_{7,7'}$

  $\Rightarrow$ relates $b \to s\,\bar{\ell}\ell$ and $b \to s\,\bar{\nu}\nu$

→ based on MCMC

→ constraints in 2 parameter Scenarios

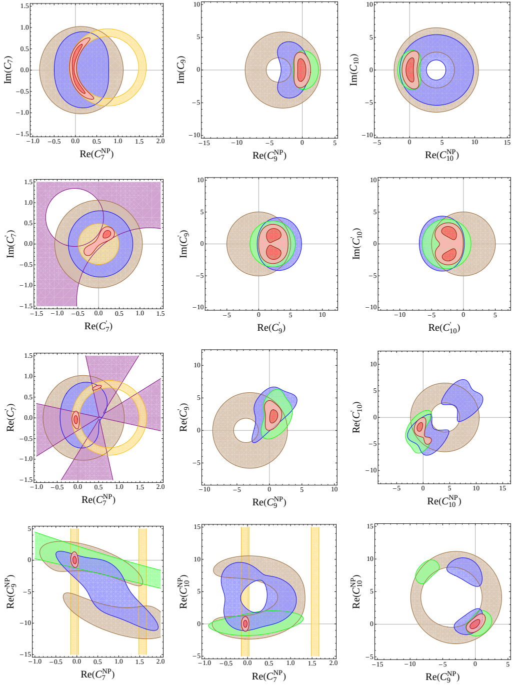
individual constraints at 95% CL

$S[B \to K^* \gamma]$

$Br[B \to X_s \gamma]$ lo+hi-q^2 $Br[B \to X_s \bar{\ell}\ell]$

lo-q^2 $B \to K^* \bar{\ell}\ell$ hi-q^2 $B \to K^* \bar{\ell}\ell$

combined constraints: 68% CL (95 % CL)



Beaujean/CB/van Dyk/Wacker (work in progress)

- theory uncertainties = nuisance parameters $\Rightarrow$ include them in the fit and profit from "short-distance" free observables @ low recoil
- use Bayes theorem = Bayesian inference
- based on Population MC (PMC) [Kilbinger et al. arXiv:0912.1614, 1101.0950]
  1) to avoid problems of Markov chains in presence of multi-modal posterior
  2) allows for parallelized evaluation of likelihood
- Flavour tool "EOS": observables for http://project.het.physik.tu-dortmund.de/eos/

  $$B \to (K^*, X_s)\gamma,\ B \to K\,\ell^+\ell^-,\ B \to K^*\ell^+\ell^-,\ B_s \to \mu^+\mu^-$$
- Priors

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$$B \to (K^*, X_s)\gamma,\ B \to K\,\ell^+\ell^-,\ B \to K^*\ell^+\ell^-,\ B_s \to \mu^+\mu^-$$

- Priors
  1) flat priors for Wilson coefficients
  2) gaussian (symmetric) / LogGamma (asymmetric) priors for
     - CKM and quark-mass input
     - form factor results from LCSR at low-q^2, only extrapolation to high-q^2 [Ball/Zwicky hep-ph/0412079, Khodjamirian et al. arXiv:1006.4945]
     - parametrization of lacking sub-leading contributions @ low- and high-q^2

⇒ about $\mathcal{O}(30)$ nuisance parameters

⇒ test prior dependence

SUMMARY 1/2

- rare decays ($\Delta F = 1, 2$) are suppressed in the SM → indirect search of New Physics
  - provide strong constraints on generic extensions of flavour sector
- 2nd generation exp's: LHCb, Belle II and SuperB will provide new $b \to s\{\gamma, \bar{\ell}\ell\}$ data: with high statistics through next decade
- angular observables J_i in exclusive $B \to K^*(\to K\pi)\bar{\ell}\ell$ provide @ low- and high-q^2 combinations with small hadronic uncertainties
- OPE of 4-quark contributions @ low recoil yields relations ($|H_T^{(1)}| = 1$, $|H_T^{(2)}| = |H_T^{(3)}|$, etc.) depending on BSM type, which can be tested with data

  ⇒ BUT violations can also indicate size of long-distance corrections
- SM test and BSM search require extension of CKM-fit strategy:

  "combine all data and overconstrain scenarios"

EOS = new Flavour tool @ TU Dortmund by Danny van Dyk et al.
Download @ http://project.het.physik.tu-dortmund.de/eos/

We need $B \to K$ and $B \to K^*$ form factors in order to be able to constrain new physics scenarios beyond the SM with exclusive $b \to s\,\ell^+\ell^-$ decays.

We would like to have a q^2-parametrization of form factors, including uncertainties (as well as the correlations among the parameters of that parametrization).

Backup Slides

q^2-INTEGRATED OBSERVABLES

Experimental measurements of observables P always imply binning in kinematical variables x, i.e.

$$\langle P \rangle_{[x_{min},\, x_{max}]} \equiv \int_{x_{min}}^{x_{max}} \mathrm{d}x\, P(x)$$

Assume, that angular observables $J_i(q^2)$ are measured in experiment for certain q^2 binning (omitting q^2-interval boundaries)

$$\langle J_i \rangle = \int_{q^2_{min}}^{q^2_{max}} \mathrm{d}q^2\, J_i(q^2)$$

and “transversity observables” are then determined as follows (for example)

$$\langle A_T^{(3)} \rangle = \sqrt{\frac{4\langle J_4 \rangle^2 + \langle J_7 \rangle^2}{-2\langle J_{2c} \rangle \langle 2J_{2s} + J_3 \rangle}}$$

$\longrightarrow$ This has to accounted for in theoretical predictions !!!

MEASURING ANGULAR OBSERVABLES

likely that exp. results only in some q^2-integrated bins: $\langle \ldots \rangle = \int_{q^2_{min}}^{q^2_{max}} dq^2 \ldots$,
then use some (quasi-) single-diff. distributions in θ_ℓ, θ_{K*}, ϕ

- $$\frac{d\langle \Gamma \rangle}{d\phi} = \frac{1}{2\pi} \left\{ \langle \Gamma \rangle + \langle J_3 \rangle \cos 2\phi + \langle J_9 \rangle \sin 2\phi \right\}$$

- 2 bins in $\cos\theta_{K*}$

$$\frac{d\langle A_{\theta_{K*}} \rangle}{d\phi} \equiv \int_{-1}^{1} d\cos\theta_\ell \left[\int_0^1 - \int_{-1}^0 \right] d\cos\theta_{K*} \frac{d^3\langle \Gamma \rangle}{d\cos\theta_{K*}\, d\cos\theta_\ell\, d\phi}$$
$$= \frac{3}{16} \left\{ \langle J_5 \rangle \cos\phi + \langle J_7 \rangle \sin\phi \right\}$$

- (2 bins in $\cos\theta_{K*}$) + (2 bins in $\cos\theta_\ell$)

$$\frac{d\langle A_{\theta_{K*},\theta_\ell} \rangle}{d\phi} \equiv \left[\int_0^1 - \int_{-1}^0 \right] d\cos\theta_\ell \frac{d^2\langle A_{\theta_{K*}} \rangle}{d\cos\theta_\ell\, d\phi} = \frac{1}{2\pi} \left\{ \langle J_4 \rangle \cos\phi + \langle J_8 \rangle \sin\phi \right\}$$

HIGH-q^2: OPE + HQET

Framework developed by Grinstein/Pirjol hep-ph/0404250

1) OPE in $\Lambda_{\rm QCD}/Q$ with $Q = \{m_b, \sqrt{q^2}\}$ + matching on HQET + expansion in m_c

$$\mathcal{M}[\bar{B} \to \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi}{q^2} \sum_{i=1}^{6} \mathcal{C}_i(\mu)\, \mathcal{T}_\alpha^{(i)}(q^2, \mu)\, [\bar{\ell}\gamma^\alpha \ell]$$

$$\mathcal{T}_\alpha^{(i)}(q^2, \mu) = i \int d^4x\, e^{iq\cdot x} \langle \bar{K}^* | T\{\mathcal{O}_i(0), j_\alpha^{\rm em}(x)\} | \bar{B}\rangle$$

$$= \sum_{k \geqslant -2} \sum_j C_{i,j}^{(k)} \langle \mathcal{Q}_{j,\alpha}^{(k)} \rangle$$

$\mathcal{Q}_{j,\alpha}^{(k)}$	power	$\mathcal{O}(\alpha_s)$
$\mathcal{Q}_{1,2}^{(-2)}$	1	$\alpha_s^0(Q)$
$\mathcal{Q}_{1-5}^{(-1)}$	$\Lambda_{\rm QCD}/Q$	$\alpha_s^1(Q)$
$\mathcal{Q}_{1,2}^{(0)}$	m_c^2/Q^2	$\alpha_s^0(Q)$
$\mathcal{Q}_{j>3}^{(0)}$	$\Lambda_{\rm QCD}{}^2/Q^2$	$\alpha_s^0(Q)$
$\mathcal{Q}_i^{(2)}$	m_c^4/Q^4	$\alpha_s^0(Q)$

included,
unc. estimate by naive pwr cont.

2) HQET FF-relations at sub-leading order + α_s corrections in leading order

$$T_1(q^2) = \kappa\, V(q^2), \qquad T_2(q^2) = \kappa\, A_1(q^2), \qquad T_3(q^2) = \kappa\, A_2(q^2) \frac{M_B^2}{q^2},$$

$$\kappa = \left(1 + \frac{2D_0^{(v)}(\mu)}{C_0^{(v)}(\mu)}\right) \frac{m_b(\mu)}{M_B}$$

can express everything in terms of QCD FF's $V, A_{1,2}$ @ $\mathcal{O}(\alpha_s \Lambda_{\rm QCD}/Q)$!!!

HIGH-q^2 – TRANSVERSITY AMPLITUDES

$$A_\perp^{L,R} = +\left[C^{L,R} + \tilde{r}_a\right] f_\perp\,, \qquad A_\parallel^{L,R} = -\left[C^{L,R} + \tilde{r}_b\right] f_\parallel\,,$$

$$A_0^{L,R} = -C^{L,R} f_0 - N M_B \frac{(1-\hat{s}-\hat{M}_{K*}^2)(1+\hat{M}_{K*})^2 \tilde{r}_b A_1 - \hat{\lambda}\,\tilde{r}_c A_2}{2\,\hat{M}_{K*}(1+\hat{M}_{K*})\sqrt{\hat{s}}}$$

⇒ Universal short-distance coefficients: $C^{L,R} = C_9^{\text{eff}} + \kappa \frac{2 m_b M_B}{q^2} C_7^{\text{eff}} \mp C_{10}$

(SM: $C_9 \sim +4$, $C_{10} \sim -4$, $C_7 \sim -0.3$)

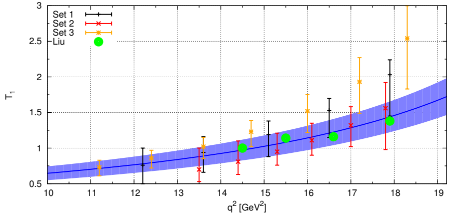
known structure of sub-leading corrections [Grinstein/Pirjol hep-ph/0404250]

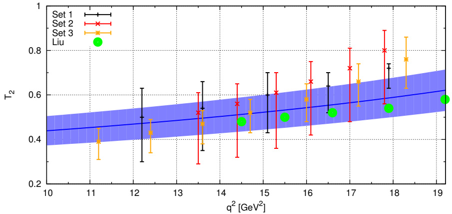
$$\tilde{r}_i \sim \pm \frac{\Lambda_{\text{QCD}}}{m_b} \left(C_7^{\text{eff}} + \alpha_s(\mu) e^{i\delta_i}\right)\,, \qquad i = a, b, c$$

form factors ("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1+\hat{M}_{K*}} V, \quad f_\parallel = \sqrt{2}\,(1+\hat{M}_{K*})\,A_1, \quad f_0 = \frac{(1-\hat{s}-\hat{M}_{K*}^2)(1+\hat{M}_{K*})^2 A_1 - \hat{\lambda}\,A_2}{2\,\hat{M}_{K*}(1+\hat{M}_{K*})\sqrt{\hat{s}}}$$

. . . only known from extrapolation of LCSR at low-q^2 $\Rightarrow$ Lattice results desirable





LCSR extrapolation (Ball/Zwicky hep-ph/0412079) of $T_1(q^2)$ and $T_2(q^2)$ to high-q^2 versus quenched Lattice (3 data sets from Becirevic/Lubicz/Mescia hep-ph/0611295)

new unquenched Lattice results to come $\rightarrow$ Liu/Meinel/Hart/Horgan/Müller/Wingate arXiv:0911.2370, arXiv:1101.2726
no final uncertainty estimate yet

Low-q^2 = Large Recoil

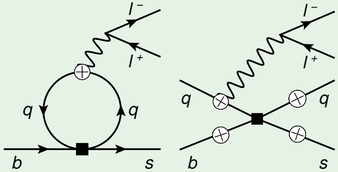
QCD Factorisation (QCDF)

[Beneke/Feldmann/Seidel hep-ph/0106067, hep-ph/0412400]

= (large recoil + heavy quark) limit [also Soft Collinear ET (SCET)]

$$\left\langle \bar{\ell}\ell\, K_a^* \,\middle|\, H_{\rm eff}^{(i)} \,\middle|\, B \right\rangle \sim$$

$$C_a^{(i)} \times \xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K*} + \mathcal{O}(\Lambda_{\rm QCD}/m_b)$$

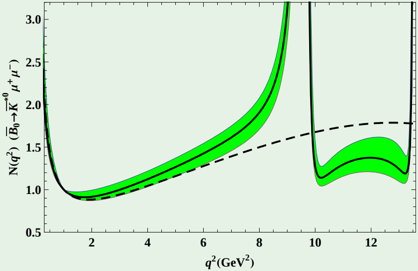


$C_a^{(i)}$, $T_a^{(i)}$: perturbative kernels in α_s ($a = \perp, \parallel$, $i = u, t$)

ϕ_B, $\phi_{a,K*}$: B– and K_a^*–distribution amplitudes

$c\bar{c}$-Contributions

[Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]



- OPE near light-cone incl. soft-gluon emission (non-local operator) for $q^2 \leqslant 4$ GeV^2 $\ll 4m_c^2$
- hadronic dispersion relation using measured $B \to K^{(*)}(\bar{c}c)$ amplitudes at $q^2 \geqslant 4$ GeV^2
- $B \to K^{(*)}$ form factors from LCSR
- up to (15-20) % in rate for $1 < q^2 < 6$ GeV^2