BARYON DISTRIBUTION AMPLITUDES FROM LATTICE QCD

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Physics issues

- **Electromagnetic form factors**

$$\gamma^* + N \quad \to \quad N$$

$\hookrightarrow$ *Charge and current distributions*

- **Electroproduction of resonances**

$$\gamma^* + N \quad \to \quad N^*$$

$\hookrightarrow$ *Restoration of chiral symmetry? Are all resonances alike?*

- **Weak decays of heavy baryons**

$$\Lambda_b \quad \to \quad \Lambda, \Lambda^* + \ell^+\ell^-$$

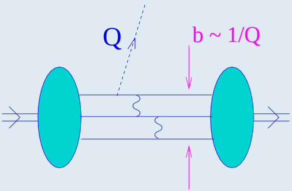
$\hookrightarrow$ *Helicity structure of new physics contributions*

Common: large momentum transfer from a point-like source to the final-state baryon

How to transfer a large momentum to a weekly bound system?

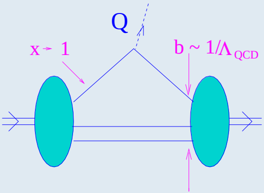
Heuristic picture:

- quarks can acquire large transverse momenta when they exchange gluons
- "hard" gluon exchanges can be separated from "soft" nonperturbative wave functions
- hard gluons can only be exchanged at small transverse separations



Hard rescattering:

Small b
Average $0 < x < 1$



Soft (Feynman):

Average b
Large $x \to 1$

In practice three-quark states indeed seem to dominate, however

- "Squeesing" to small transverse separations occurs very slowly
- Helicity selections rules do not work. Orbital angular momentum?
- $\Rightarrow$ More complicated nonperturbative input needed

Wave functions and Distribution amplitudes

- **Nucleon light-cone wave function** — Brodsky, Lepage

$$|P\uparrow\rangle^{\ell_z=0} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1x_2x_3}}\, \psi^{L=0}(x_i,\vec{k}_i) \times$$
$$\times\Big\{\Big|u^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)d^\uparrow(x_3,\vec{k}_3)\Big\rangle - \Big|u^\uparrow(x_1,\vec{k}_1)d^\downarrow(x_2,\vec{k}_2)u^\uparrow(x_3,\vec{k}_3)\Big\rangle\Big\}$$

- **Leading-twist-three distribution amplitude**

Brodsky, Lepage, Peskin, Chernyak, Zhitnitsky

$$\Phi_3(x_1,x_2,x_3;\mu) = 2\int^\mu [d^2\vec{k}]\ \psi^{L=0}(x_1,x_2,x_3;\vec{k}_1,\vec{k}_2,\vec{k}_3)$$

can be studied using the OPE

$$\begin{aligned}\Phi_3(x_i;\mu) = {}& 120 f_N x_1x_2x_3\Big\{1 + c_{10}(x_1-2x_2+x_3)L^{\frac{8}{3\beta_0}} \\ & + c_{11}(x_1-x_3)L^{\frac{20}{9\beta_0}} + c_{20}\left[1+7(x_2-2x_1x_3-2x_2^2)\right]L^{\frac{14}{3\beta_0}} \\ & + c_{21}\,(1-4x_2)\,(x_1-x_3)\,L^{\frac{40}{9\beta_0}} + c_{22}\left[3-9x_2+8x_2^2-12x_1x_3\right]L^{\frac{32}{9\beta_0}} + \ldots\Big\}\end{aligned}$$

- $f_N(\mu_0)$: **wave function at the origin**
- $c_{nk}(\mu_0)$: **shape parameters**

$L \equiv \alpha_s(\mu)/\alpha_s(\mu_0)$

Braun, Manashov, Rohwild

Wave functions and Distribution amplitudes (II)

- **Contributions of orbital angular momentum** — Ji, Ma, Yuan, '03

$$|P\uparrow\rangle^{\ell_z=1} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1x_2x_3}}\left[k_1^+\psi_1^{L=1}(x_i,\vec{k}_i)+k_2^+\psi_2^{L=1}(x_i,\vec{k}_i)\right]\times$$
$$\times\left\{\left|u^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)d^\downarrow(x_3,\vec{k}_3)\right\rangle-\left|d^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)u^\downarrow(x_3,\vec{k}_3)\right\rangle\right\}$$

are related to higher-twist-four distribution amplitudes — Belitsky, Ji, Yuan, '03

$$\Phi_4(x_2,x_1,x_3;\mu) = 2\int^\mu \frac{[d^2\vec{k}]}{m_N x_3}\,k_3^-\left[k_1^+\psi_1^{L=1}+k_2^+\psi_2^{L=1}\right](x_i,\vec{k}_i)$$
$$\Psi_4(x_1,x_2,x_3;\mu) = 2\int^\mu \frac{[d^2\vec{k}]}{m_N x_2}\,k_2^-\left[k_1^+\psi_1^{L=1}+k_2^+\psi_2^{L=1}\right](x_i,\vec{k}_i)$$

$$k^\pm = k_x \pm ik_y$$

and, again, can be studied using OPE — Braun, Fries, Mahnke, Stein '00

$$\Phi_4(x_i;\mu) = 12\lambda_1 x_1x_2 + 12f_N x_1x_2\left[1+\frac{2}{3}(1-5x_3)\right]+\ldots$$
$$\Psi_4(x_i;\mu) = 12\lambda_1 x_1x_3 + 12f_N x_1x_3\left[1+\frac{2}{3}(1-5x_2)\right]+\ldots$$

- **to this accuracy only one new nonperturbative constant $\lambda_1(\mu)$**

A large-scale long-term research project within QCDSF:

First principles calculation of lowest moments of baryon distribution amplitudes

with emphasize on the comparison of states with opposite parity

Lattice three-quark operators

Example:

$$\mathcal{O}^{\alpha\beta\gamma}_{\dot\alpha\dot\beta} \quad\simeq\quad \epsilon^{abc}(\psi^{\alpha})_a(\bar\chi_{\dot\alpha})_b(D^{\beta}{}_{\dot\beta}\psi^{\gamma})_c$$

- **Irreducible spinor representations of $H(4)$**

  Kaltenbrunner, Göckeler, Schäfer, Eur.Phys.J.C55(2008)387

	$d=9/2$ (0 derivatives)	$d=11/2$ (1 derivative)	$d=13/2$ (2 derivatives)
$\tau^{\frac{4}{1}}$	$\mathcal{B}^{(0)}_{1,i}$, $\mathcal{B}^{(0)}_{2,i}$, $\mathcal{B}^{(0)}_{3,i}$, $\mathcal{B}^{(0)}_{4,i}$, $\mathcal{B}^{(0)}_{5,i}$		$\mathcal{B}^{(2)}_{1,i}$, $\mathcal{B}^{(2)}_{2,i}$, $\mathcal{B}^{(2)}_{3,i}$
$\tau^{\frac{4}{2}}$			$\mathcal{B}^{(2)}_{4,i}$, $\mathcal{B}^{(2)}_{5,i}$, $\mathcal{B}^{(2)}_{6,i}$
$\tau^{\underline{8}}$	$\mathcal{B}^{(0)}_{6,i}$	$\mathcal{B}^{(1)}_{1,i}$	$\mathcal{B}^{(2)}_{7,i}$, $\mathcal{B}^{(2)}_{8,i}$, $\mathcal{B}^{(2)}_{9,i}$
$\tau^{\frac{12}{1}}$	$\mathcal{B}^{(0)}_{7,i}$, $\mathcal{B}^{(0)}_{8,i}$, $\mathcal{B}^{(0)}_{9,i}$	$\mathcal{B}^{(1)}_{2,i}$, $\mathcal{B}^{(1)}_{3,i}$, $\mathcal{B}^{(1)}_{4,i}$	$\mathcal{B}^{(2)}_{10,i}$, $\mathcal{B}^{(2)}_{11,i}$, $\mathcal{B}^{(2)}_{12,i}$, $\mathcal{B}^{(2)}_{13,i}$
$\tau^{\frac{12}{2}}$		$\mathcal{B}^{(1)}_{5,i}$, $\mathcal{B}^{(1)}_{6,i}$, $\mathcal{B}^{(1)}_{7,i}$, $\mathcal{B}^{(1)}_{8,i}$	$\mathcal{B}^{(2)}_{14,i}$, $\mathcal{B}^{(2)}_{15,i}$, $\mathcal{B}^{(2)}_{16,i}$, $\mathcal{B}^{(2)}_{17,i}$, $\mathcal{B}^{(2)}_{18,i}$

- **Nonperturbative renormalization and conversion RI-MOM → $\overline{\text{MS}}$**

  Göckeler et al., [QCDSF], Nucl. Phys. B **812** (2009) 205

- **A consistent subtraction scheme for three-quark operators in dim.reg.**

  Kränkl, Manashov, Phys. Lett. B **703** (2011) 519

- **Two-loop mathching between RI'-MOM and $\overline{\text{MS}}$ for three-quark operators**

  Gruber, work in progress

Lattice three-quark operators

Rarita-Schwinger basis (directly relalted to moments of DAs)

$$(\mathcal{V}^{\bar{l}\bar{m}\bar{n}})^{\rho}_{\tau}(0) = \epsilon^{abc}\left[i^l D^{\bar{l}} u(0)\right]^a_\alpha (C\gamma^\rho)_{\alpha\beta}\left[i^m D^{\bar{m}} u(0)\right]^b_\beta \left[i^n D^{\bar{n}} (\gamma_5 d(0))\right]^c_\tau$$

$$(\mathcal{A}^{\bar{l}\bar{m}\bar{n}})^{\rho}_{\tau}(0) = \epsilon^{abc}\left[i^l D^{\bar{l}} u(0)\right]^a_\alpha (C\gamma^\rho\gamma_5)_{\alpha\beta}\left[i^m D^{\bar{m}} u(0)\right]^b_\beta \left[i^n D^{\bar{n}} d(0)\right]^c_\tau$$

$$(\mathcal{T}^{\bar{l}\bar{m}\bar{n}})^{\rho}_{\tau}(0) = \epsilon^{abc}\left[i^l D^{\bar{l}} u(0)\right]^a_\alpha \left(C(-i\sigma^{\xi\rho})\right)_{\alpha\beta}\left[i^m D^{\bar{m}} u(0)\right]^b_\beta \left[i^n D^{\bar{n}} (\gamma_\xi\gamma_5 d(0))\right]^c_\tau$$

Relation to KGS operators:

$$\begin{pmatrix} -\mathcal{B}^{(0)}_{8,4} \\ \mathcal{B}^{(0)}_{8,3} \\ -\mathcal{B}^{(0)}_{8,10} \\ \mathcal{B}^{(0)}_{8,9} \end{pmatrix}_\tau = \frac{1}{4}\left(-\gamma_3(\mathcal{V}^3_\tau - \mathcal{A}^3_\tau) + \gamma_4(\mathcal{V}^4_\tau - \mathcal{A}^4_\tau)\right)$$

$$\begin{pmatrix} -\mathcal{B}^{(0)}_{9,4} \\ \mathcal{B}^{(0)}_{9,3} \\ -\mathcal{B}^{(0)}_{9,10} \\ \mathcal{B}^{(0)}_{9,9} \end{pmatrix}_\tau = \frac{1}{4}\left(-\gamma_3\mathcal{T}^3_\tau + \gamma_4\mathcal{T}^4_\tau\right) \qquad \text{etc.}$$

Isospin 1/2 operators:

$$\mathcal{K} = \mathcal{V} - \mathcal{A} - \mathcal{T}$$

Lattice three-quark operators

Lattice operators, no derivatives

$$\mathcal{O}_{B,0} = \frac{1}{3}\left[-\gamma_3\mathcal{K}^3 + \gamma_4\mathcal{K}^4\right]$$

$$\mathcal{O}_{C,0} = \frac{1}{3}\left[\gamma_1\mathcal{K}^1 + \gamma_2\mathcal{K}^2 + \gamma_3\mathcal{K}^3 + \gamma_4\mathcal{K}^4\right]$$

Lattice operators, one derivative

$$\mathcal{O}_{A,1} = \frac{1}{3}\left[-2\gamma_1\gamma_2\mathcal{K}^{\{12\}} + \gamma_1\gamma_3\mathcal{K}^{\{13\}} + \gamma_1\gamma_4\mathcal{K}^{\{14\}} - \gamma_2\gamma_3\mathcal{K}^{\{23\}} - \gamma_2\gamma_4\mathcal{K}^{\{24\}}\right]$$

$$\mathcal{O}_{B,1} = \frac{1}{3}\left[2\gamma_3\gamma_4\mathcal{K}^{\{34\}} + \gamma_1\gamma_3\mathcal{K}^{\{13\}} - \gamma_1\gamma_4\mathcal{K}^{\{14\}} + \gamma_2\gamma_3\mathcal{K}^{\{23\}} - \gamma_2\gamma_4\mathcal{K}^{\{24\}}\right]$$

$$\mathcal{O}_{C,1} = \frac{1}{3}\left[-\gamma_1\gamma_3\mathcal{K}^{\{13\}} + \gamma_1\gamma_4\mathcal{K}^{\{14\}} + \gamma_2\gamma_3\mathcal{K}^{\{23\}} - \gamma_2\gamma_4\mathcal{K}^{\{24\}}\right]$$

Lattice operators, two derivatives

$$\mathcal{O}_2 = \frac{1}{3}\left[\gamma_1\gamma_2\gamma_3\mathcal{K}^{\{123\}} - \gamma_1\gamma_2\gamma_4\mathcal{K}^{\{124\}} + \gamma_1\gamma_3\gamma_4\mathcal{K}^{\{134\}} - \gamma_2\gamma_3\gamma_4\mathcal{K}^{\{234\}}\right]$$

Parity separation at non-zero momentum

These operators are contracted with a smeared nucleon source $\mathcal{N}_\tau = \epsilon^{abc} u^a_\alpha (C\gamma_5)_{\alpha\beta} d^b_\beta u^c_\tau$

$$\langle (\gamma_4\gamma_1 \mathcal{O}^{lmn}_{A,1}(t,\vec{p}))_\tau (\overline{\mathcal{N}}(0,\vec{p}))_{\tau'} (P_+)_{\tau'\tau} \rangle =$$
$$= -f_N \phi^{lmn} \sqrt{Z_N}\, p_1 \frac{E(m_N + kE) + k(2p_2^2 - p_3^2)}{E} e^{-Et} + \dots$$
$$\langle (\gamma_2\gamma_3\gamma_4 \mathcal{O}^{lmn}_{2}(t,\vec{p}))_\tau (\overline{\mathcal{N}}(0,\vec{p}))_{\tau'} (P_+)_{\tau'\tau} \rangle =$$
$$= -f_N \phi^{lmn} \sqrt{Z_N}\, p_2 p_3 \frac{E(m_N + kE) + kp_1^2}{E} e^{-Et} + \dots$$

Parity projector

$$P_+ = 1 + k\gamma_4$$

- **We find that choosing $k = m_*/E_*$ (a la Lee-Leinweber) extracts the positive-parity state, and $k = -m_N/E_N$ extracts the negative parity state for specially chosen momenta combinations**

Parity separation at non-zero momentum

- **Lee-Leinweber parity projectors applicable for special momentum configurations after suitable additional "rotations" in Dirac space**

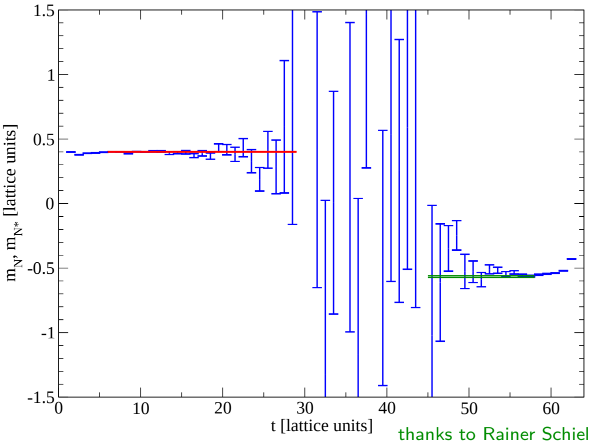
$32^3 \times 64$

$a = 0.0753$ fm
$a^{-1} = 2.620$ GeV

$m_\pi = 282(2)$ MeV

$m_\pi L = 3.44$

$$m_N = 1051(5)\ \text{MeV}$$
$$m_{N^*} = 1482(17)\ \text{MeV}$$



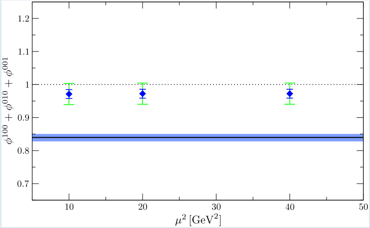
thanks to Rainer Schiel

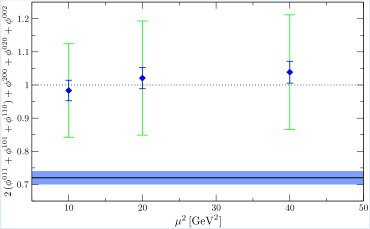
Nonperturbative renormalization of three-quark operators

Energy conservation on a lattice:

$$\partial(A \cdot B) = (\partial A) \cdot B + A \cdot (\partial B) + \mathcal{O}(a)$$

Braun *et al* [QCDSF] Phys. Rev. D **79**, 034504 (2009)





$$x_1 + x_2 + x_3 = 1 \qquad\qquad (x_1 + x_2 + x_3)^2 = 1$$

blue band: bare operators
blue dots: renormalized operators, stat. errors only
green dots: renormalized operators, with errors due to chiral extrapolation (old data)

Wilson gauge action, Wilson clover fermions

- **new $N_f = 2$ configurations:**

β	κ	m_π[GeV]	volume	a[fm]	L[fm]	$m_\pi L$
5.29	0.13632	0.270	$24^3 \times 48$	0.075	1.8	2.5
5.29	0.13632	0.270	$32^3 \times 64$	0.075	2.4	3.3
5.29	0.13632	0.270	$40^3 \times 64$	0.075	3.0	4.1

- **in progress $N_f = 2$:**

β	κ	m_π[GeV]	volume	a[fm]	L[fm]	$m_\pi L$
5.29	0.13640	0.170	$48^3 \times 64$	0.075	3.6	3.1

- **starting $N_f = 2 + 1$ (PRACE proposal):** $\Lambda(1116), \Lambda(1405)$

β	κ_l	m_π[GeV]	volume	a[fm]	L[fm]	$m_\pi L$
5.5	0.121095	0.290	$32^3 \times 64$	0.079	2.5	3.7
5.5	0.121145	0.241	$32^3 \times 64$	0.079	2.5	3.1
5.5	0.121193	0.180	$32^3 \times 64$	0.079	2.5	2.3
5.5	0.121193	0.180	$48^3 \times 96$	0.079	3.8	3.5

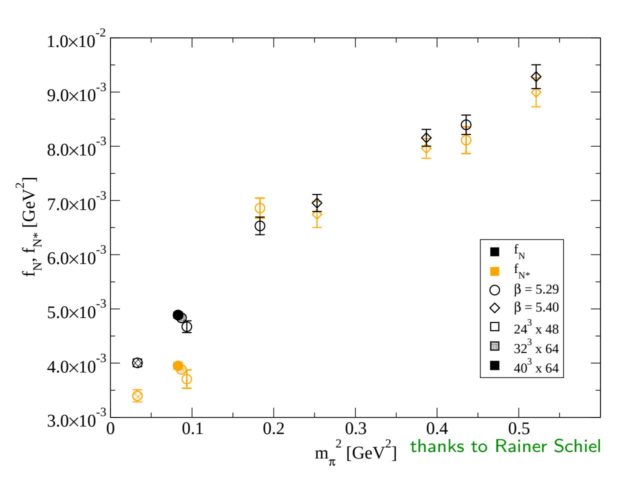
Wave functions at the origin, $L = 0$

new:

$$a = 0.075 \text{ fm}$$
$$m_\pi = 270 \text{ MeV}$$
$$m_\pi L = 2.5, 3.3, 4.1$$

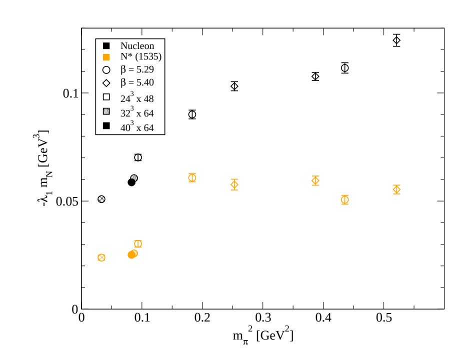
and

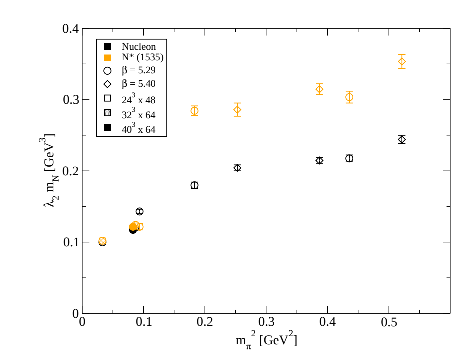
$$a = 0.075 \text{ fm}$$
$$m_\pi = 170 \text{ MeV}$$
$$m_\pi L = 3.1$$



- **All results preliminary, statistical errors only**

Orbital angular momentum $L = 1$

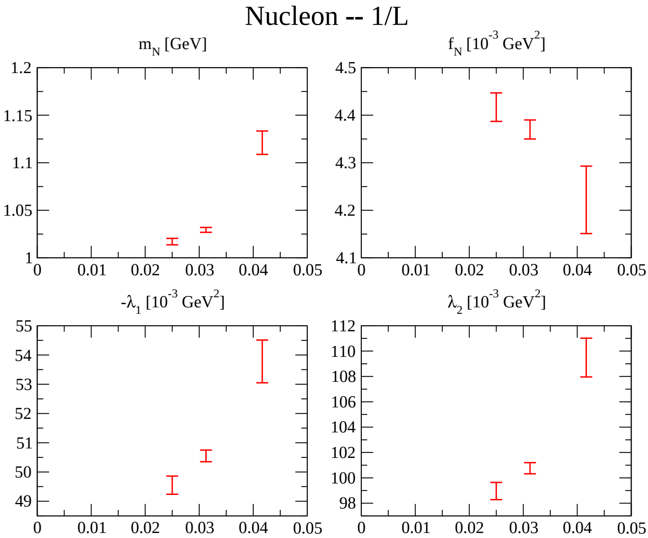




thanks to Rainer Schiel

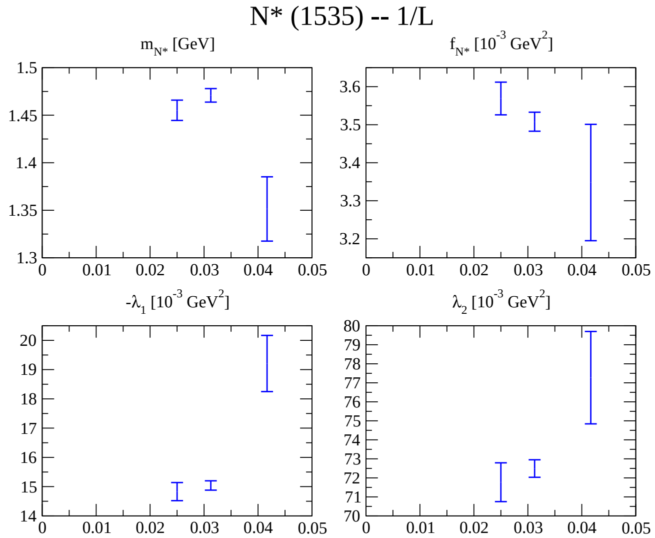
- **All results preliminary, statistical errors only**

Finite size effects



thanks to Rainer Schiel

Finite size effects



thanks to Rainer Schiel

Chiral extrapolation

$N_f = 2$ **covariant baryon chiral perturbation theory**

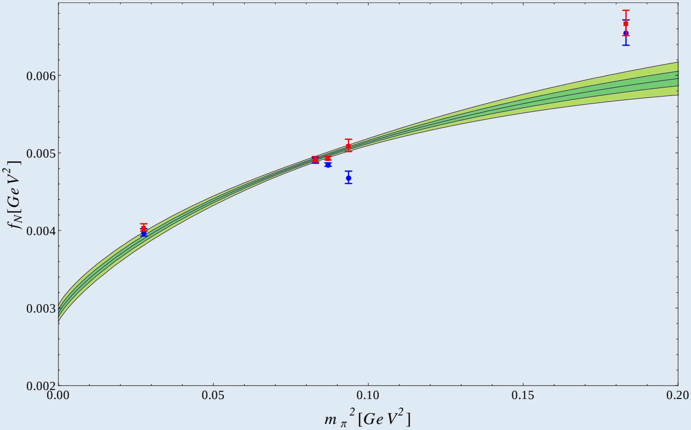
$$\begin{aligned}
(\lambda_1 m_N)(m_\pi) &= \alpha_1 \left[2 - \frac{m_\pi^2}{4(4\pi F_\pi)^2} \left(6g_A^2 + (3 + 9g_A^2) \ln \frac{m_\pi^2}{\mu^2} \right) \right] \\
&\quad + 4\alpha_{\text{sb}}^{(r)} m_\pi^2 + \mathcal{O}(m_\pi^3)
\end{aligned}$$

$$\begin{aligned}
(\lambda_2 m_N)(m_\pi) &= \beta_1 \left[2 - \frac{m_\pi^2}{4(4\pi F_\pi)^2} \left(6g_A^2 + (3 + 9g_A^2) \ln \frac{m_\pi^2}{\mu^2} \right) \right] \\
&\quad + 4\beta_{\text{sb}}^{(r)} m_\pi^2 + \mathcal{O}(m_\pi^3)
\end{aligned}$$

$$\begin{aligned}
f_N(m_\pi) &= \kappa_1 \left[2 - \frac{m_\pi^2}{4(4\pi F_\pi)^2} \left(6g_A^2 + (19 + 9g_A^2) \ln \frac{m_\pi^2}{\mu^2} \right) \right] \\
&\quad + 4\kappa_{\text{sb}}^{(r)} m_\pi^2 + \mathcal{O}(m_\pi^3)
\end{aligned}$$

P. Wein *et al.*, Eur. Phys. J. A **47** (2011) 149

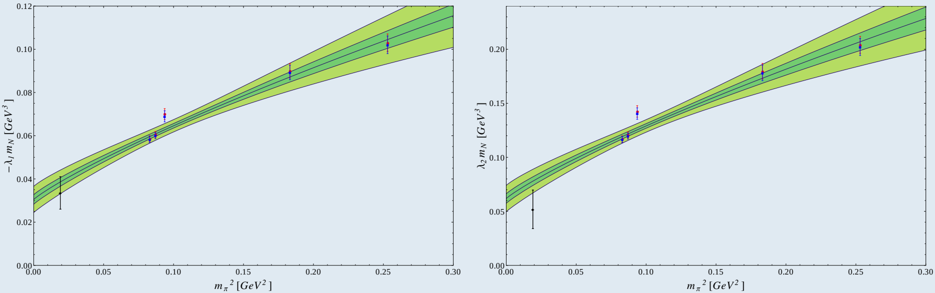
Chiral extrapolation



thanks to Philipp Wein

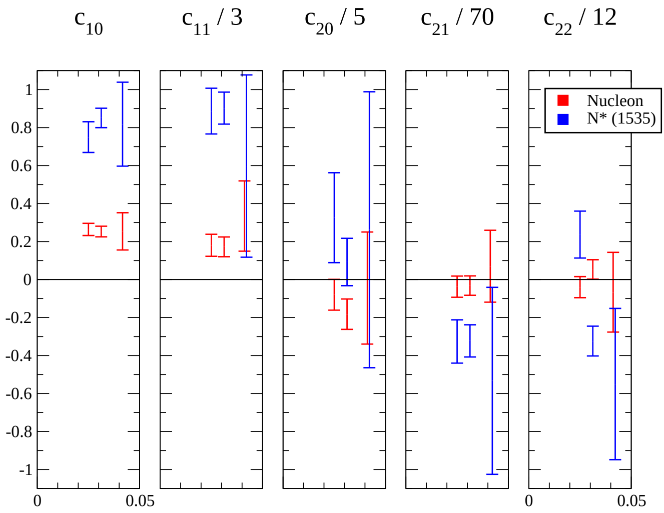
Chiral extrapolation

Higher-twist couplings (orbital angular momentum $L = 1$)



- **QCD sum rules results shown at physical pion mass**

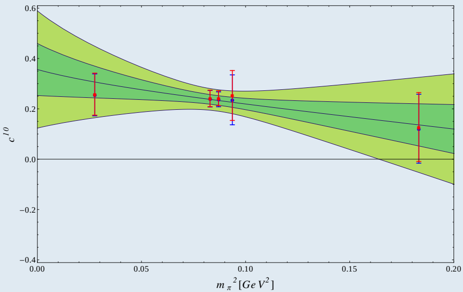
Shape parameters

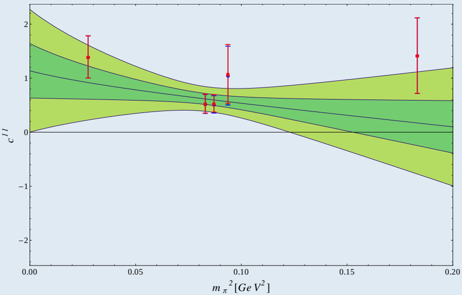


thanks to Rainer Schiel

Shape parameters

shape parameters of first order (orbital angular momentum $L = 1$)





- **work in progress**

Shape parameters

shape parameters for the nucleon DA

	our result	K-S	CZ	COZ	QCD-SR	BK	BLW
c_{10}	0.357 (82)	1.295	1.925	1.484	1.418 (248)	0.245	0.508
c_{11}	1.068 (393)	3.255	4.305	3.675	3.476 (700)	0.735	1.208
c_{20}	-0.925 (848)	-0.342	-0.006	-0.342	-1.312 (624)	-	-
c_{21}	-7.505 (10.15)	-0.315	1.155	2.205	-3.749 (3.76)	-	-
c_{22}	1.604 (1.65)	3.969	2.247	2.898	4.744 (1.58)	-	-

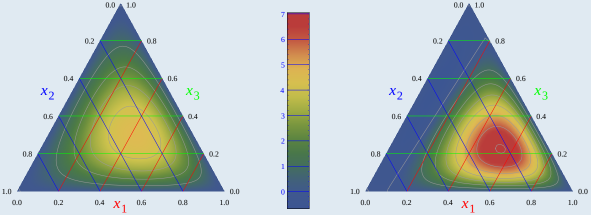
Table: Comparison of our results for the nucleon shape parameters to the models of King and Sachrajda (K-S), Chernyak and Zhitnitsky (CZ), Chernyak, Ogloblin and Zhitnitsky (COZ), direct QCD sum-rule calculations (QCD-SR), the Bolz and Kroll model (BK) and Braun, Lenz and Wittmann (BLW). The numbers correspond to a renormalization scale of 1 GeV^2.

- **Highly preliminary!**

Valence quark distributions

"Mandelstam plot": $s + t + u = 4m^2 \Rightarrow x_1 + x_2 + x_3 = 1$

$N(940)$ $N^*(1535)/N^*(1650)$



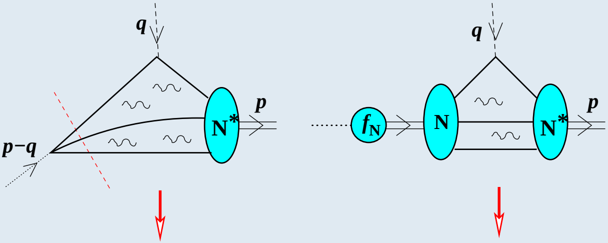
thanks to Rainer Schiel

Momentum fractions carried by valence quarks:

	N	N^*
$u^\uparrow$	$0.37 \pm (?)$	$0.49 \pm (?)$
$q^\downarrow$	$0.31 \pm (?)$	$0.26 \pm (?)$
$q^\uparrow$	$0.32 \pm (?)$	$0.25 \pm (?)$

Light-cone sum rules

- dispersion relations + duality ⇒ Light-Cone Sum Rules:



$$\frac{1}{\pi}\int_0^{s_0} ds\ \mathrm{Im}\ T(p,q) \quad \stackrel{duality}{=} \quad f_N\, F_{N\to N^*}(Q^2)$$

- $T(p,q)$ is calculated in terms of N^* distribution amplitudes

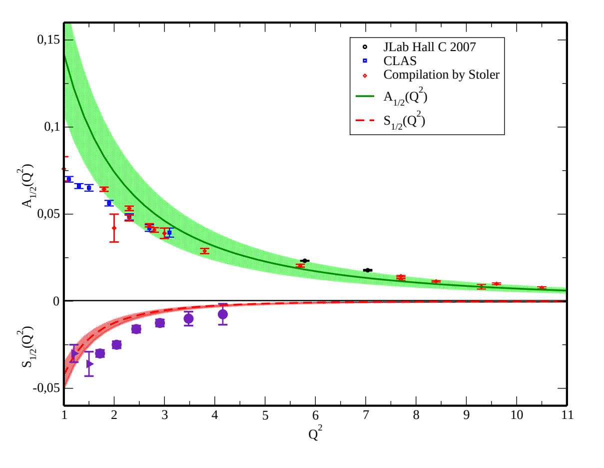
  Balitsky, Braun, Kolesnichenko, Nucl.Phys.B312:509-550,1989
  Braun, Halperin, Phys.Lett.B328:457-465,1994

- This is a Feynman (soft) contribution; hard terms can be added systematically and without double counting

$\gamma^* N \to N^*(1535)$: helicity amplitudes

- **A pilot project:** Braun *et al.* Phys.Rev.Lett.103:072001,2009

Electroproduction of $N^*(1535)$ with lattice-constrained N^* distribution amplitides



CLAS data: *I.G. Aznauryan et al., Phys.Rev.C80:055203,2009*

Outlook

- **First quantitative results on baryon DAs**
  - Nucleon and $N^*(1535)$
  - Wave functions at the origin
  - Momentum fractions carried by valence quarks
- **In 1-2 years from now:**
  - $N_f = 2 + 1$ dynamic fermions
  - Λ and $\Lambda^*(1405)$
- **Possible but not included in planned simulations:**
  - full $\frac{1}{2}^{\pm}$ octets
  - Δ ...
- **Long term:**
  - custom-made interpolating operators for resonances
  - continuum limit