Relativistic heavy quarks on the lattice

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HPQCD collaboration

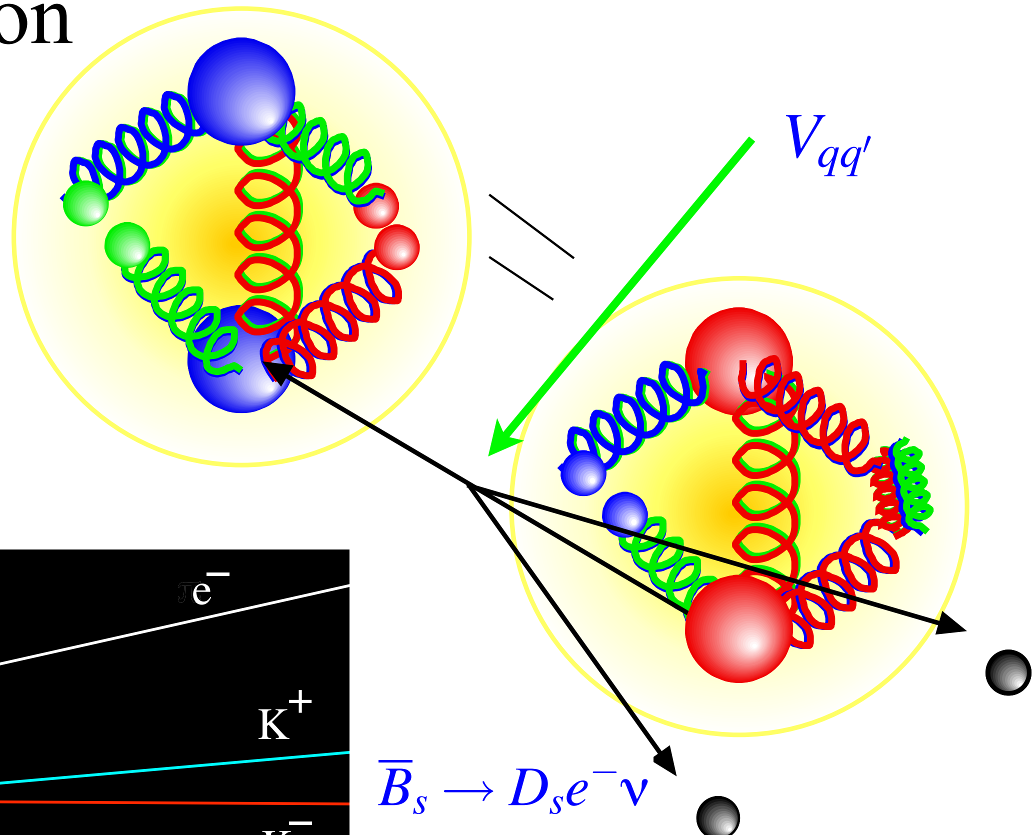
ECT* workshop
April 2012

Charm and bottom physics

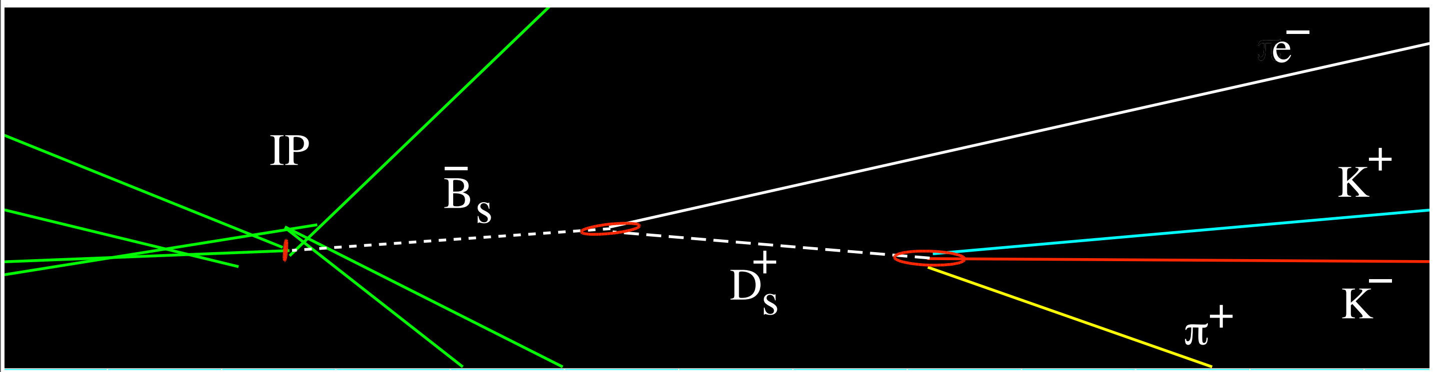
Lattice QCD calculations important because:

• simple hadronic weak decay matrix elements are key to Unitarity Triangle constraints

• spectrum has many gold-plated states for accurate tests/ predictions + m_Q determination



Precision is critical!
Need more tests of errors + more results using different methods.



$\bar{B}_s \to D_s e^- \nu$

ALEPH

Issues with handling 'heavy' quarks on the lattice:

$$L_q = \overline{\psi}(\not{D} + m)\psi \to \overline{\psi}(\gamma \cdot \Delta + ma)\psi$$

Δ is a finite difference on the lattice - leads to discretisation errors. What sets the scale for these?

For light hadrons the scale is Λ_{QCD}

For heavy hadrons the scale can be m_Q

$$E = E_{a=0}(1 + A(m_Q a)^2 + B(m_Q a)^3 + \ldots)$$

hadron energy assuming O(m$_Q$a) improved

$m_c a \approx 0.4, m_b a \approx 2$ for $a \approx 0.1 \mathrm{f}m$

➡ can use improved light quark action for c on fine lattices. Less clear for b - non rel. actions have $(\Lambda a)^n$ errors

➡ best approach to c and b not necessarily same

Charm quarks in lattice QCD - heavy or light?

Advantages of relativistic light quark method:

- $E_{sim} = m$

- PCAC relation (if enough chiral symmetry) gives $Z = 1$ and other currents can also be renormalised nonperturbatively.

- same action as for u, d, s, so cancellation in ratios

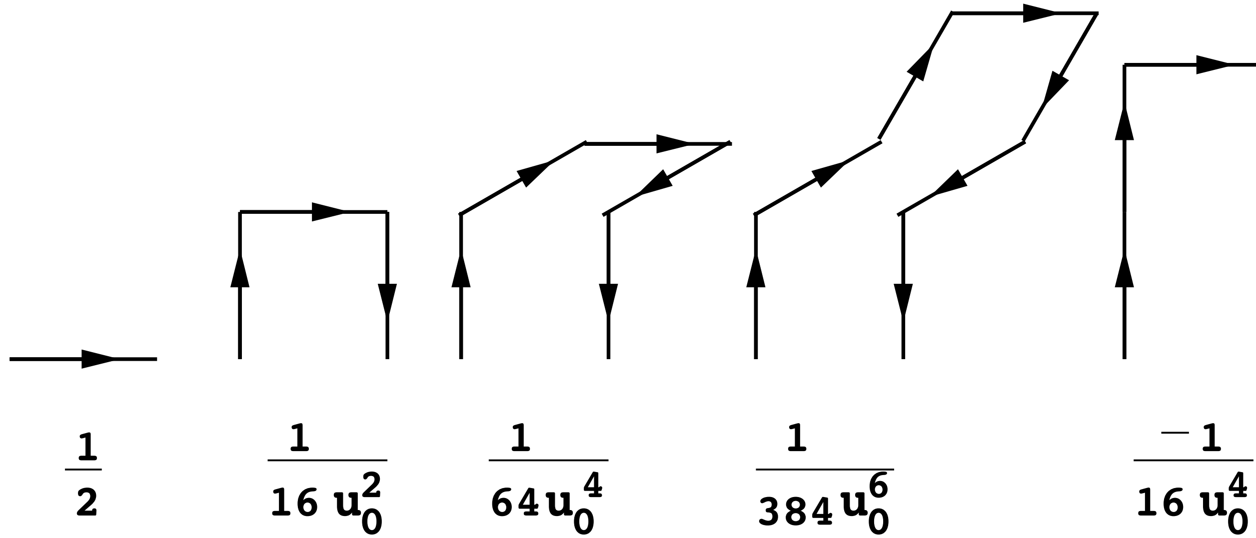
Best action to date:

Highly improved staggered quarks (HISQ)   HPQCD

Errors: $\alpha_s (am)^2, (am)^4$ + small taste-changing

HISQ based on 'asqtad' improved staggered quarks - smeared link reduces high-momentum gluon exchange taste errors + 'Naik' 3-link term improves derivative.

asqtad fat link:



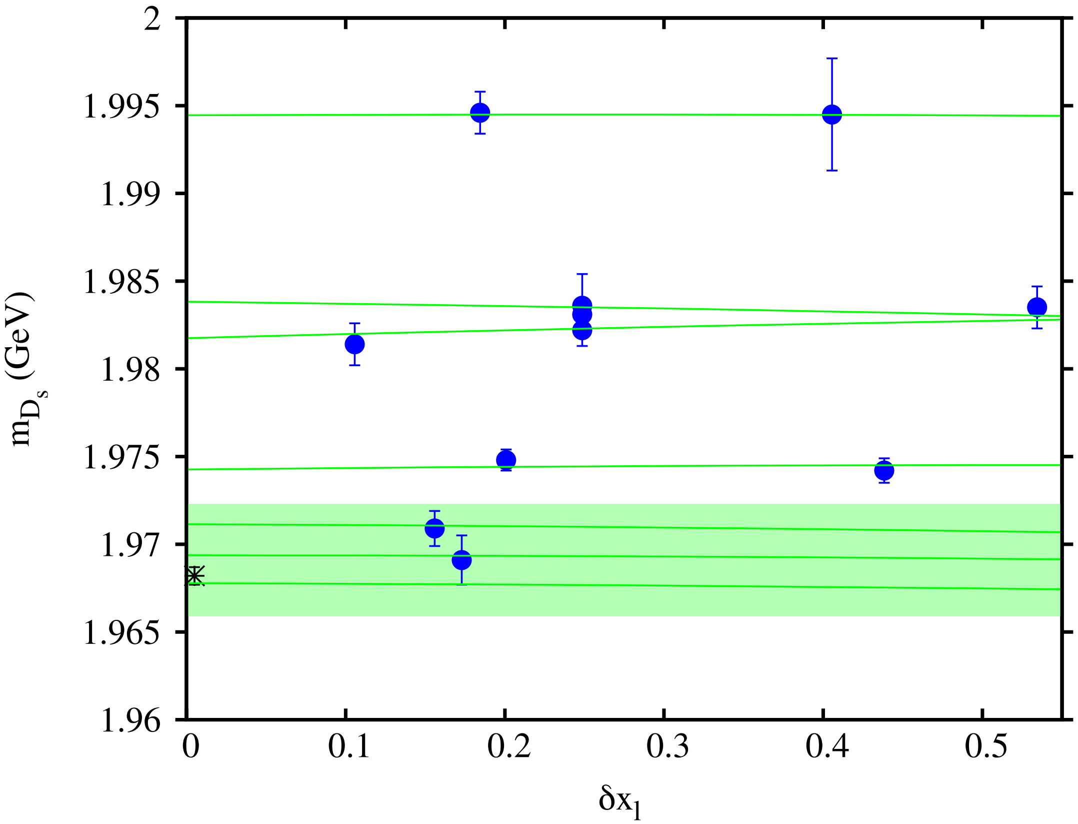
HISQ repeats fattening with (SU(3)/U(3)) reunitarisation (so NO tadpole-improvement). Coefficient of Naik term becomes $1+\epsilon$ to remove $(am)^4$ errors in speed of light.

$$\epsilon = -\frac{27}{40}(am)^2 + \ldots$$

HPQCD hep-lat/0610092

Spectrum Tests

D_s, η_c masses easy + accurate using relativistic methods. Test D_s , tuning m_c from η_c .



$\pm$3MeV

inc. em effects etc.

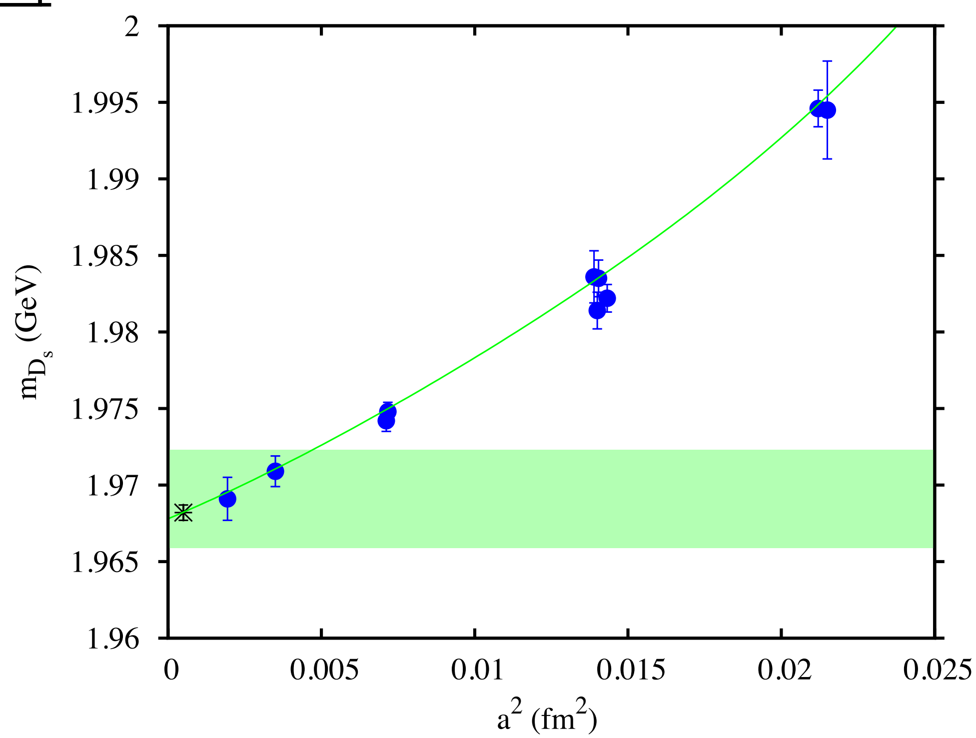
MILC 2+1 asqtad configs:

5 lattice spacing values:

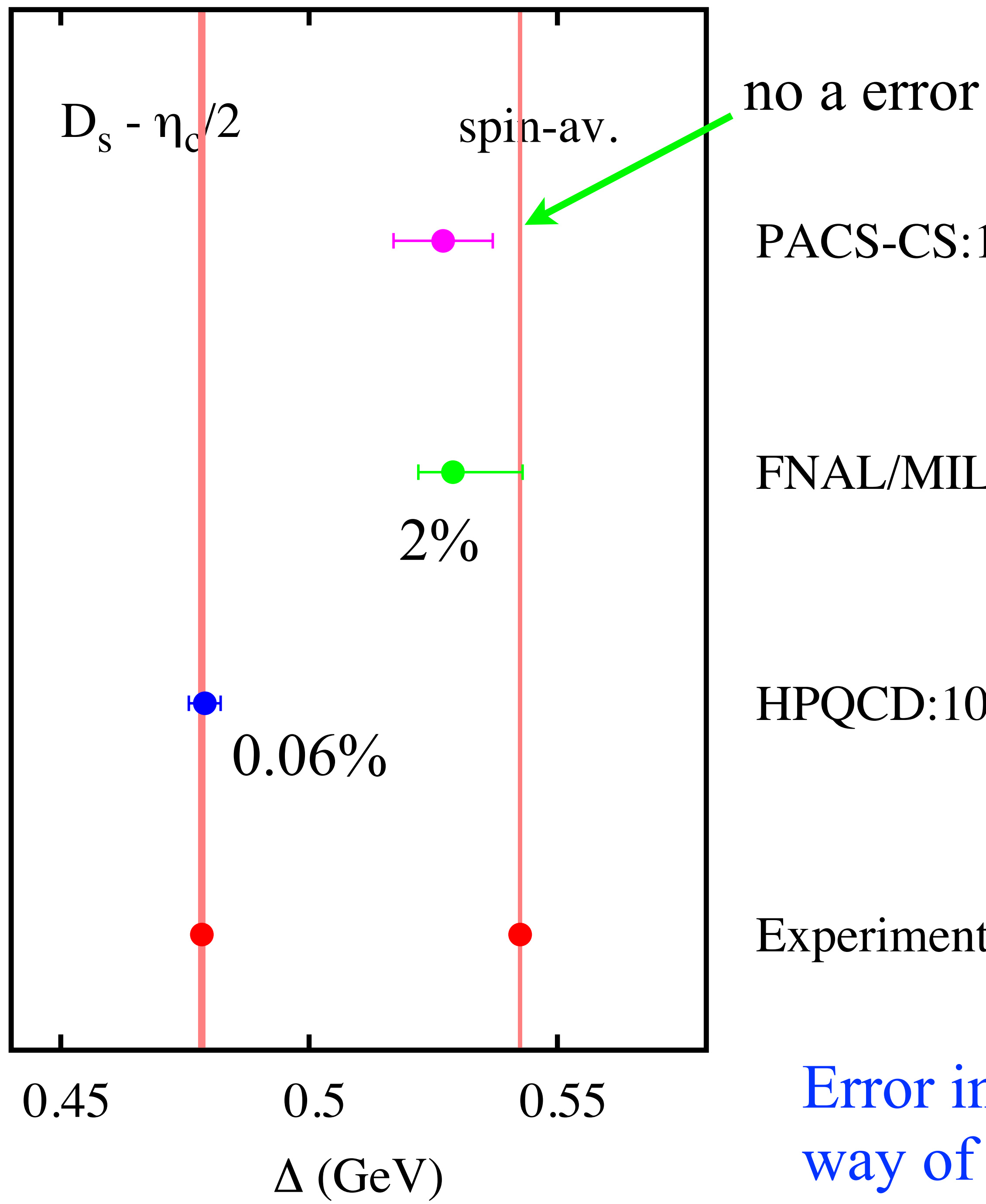
a=0.15-0.04fm

full error budget:

HPQCD 1008.4018



$N_f = 2+1$ tests of D_s, η_c mass differences



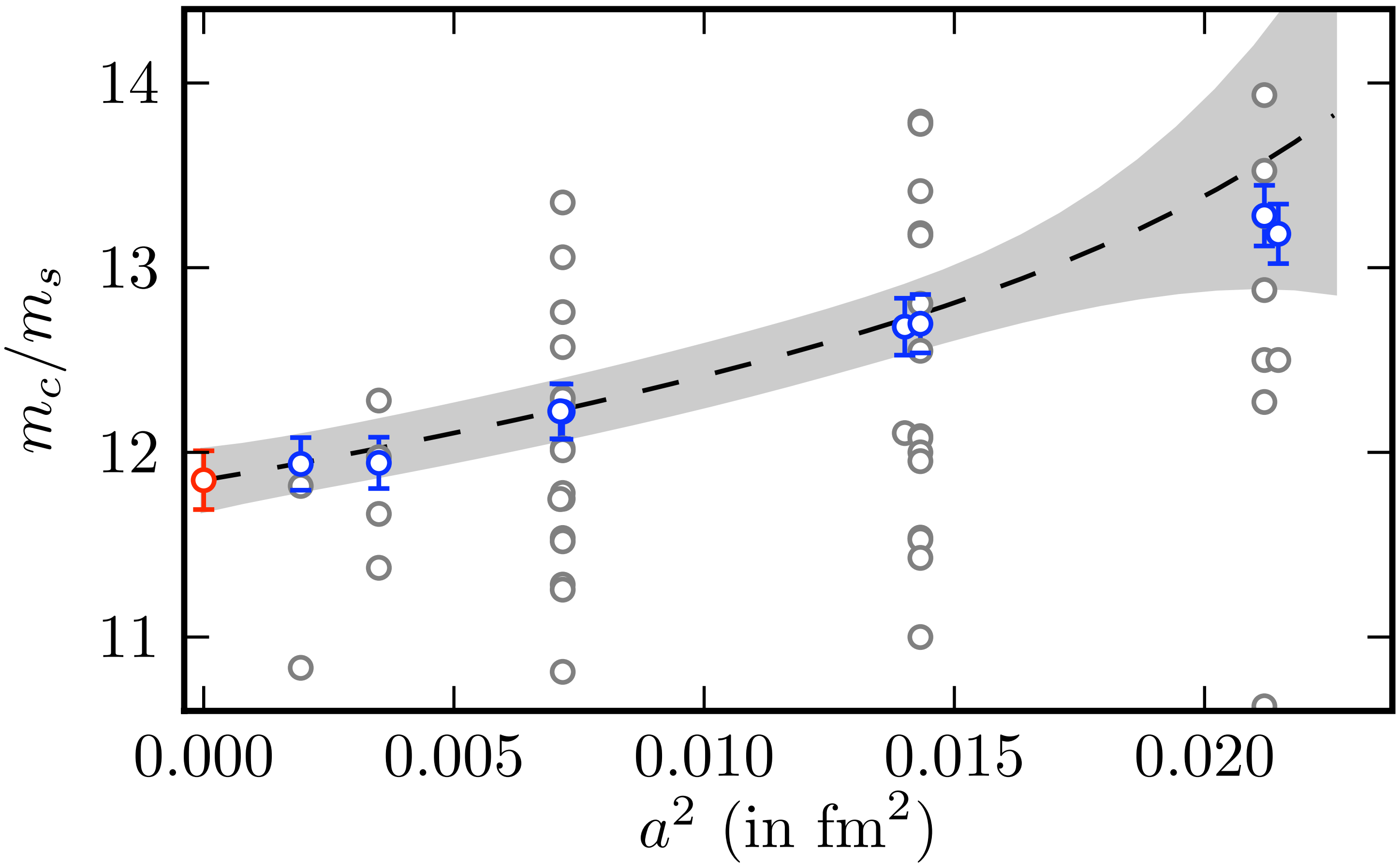
RHQ,
a=0.09fm
$m_{u,d}$ physical

Fermilab,
a=0.15,0.12,0.09fm
$m_{u,d}$ extrapoln

HISQ,
5 a values to 0.04fm
$m_{u,d}$ extrapoln

Error in binding energy diff. is one way of assessing later errors ..

Quark mass ratios from lattice QCD



$$\frac{\overline{m}_{q1}(\mu)}{\overline{m}_{q2}(\mu)} = \left(\frac{m_{q1,latt}}{m_{q2,latt}}\right)_{a=0}$$

Determine m_c/m_s using HISQ for both - allows connection from heavy to light for first time

$$\frac{m_c}{m_s} = 11.85(16)$$

use for accurate m_s if m_c known ....

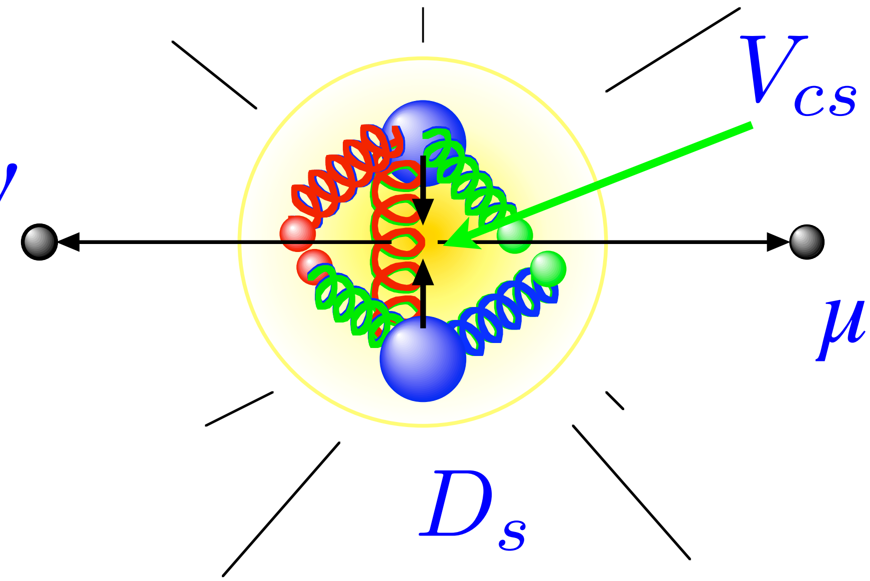
HPQCD, 0910.3102

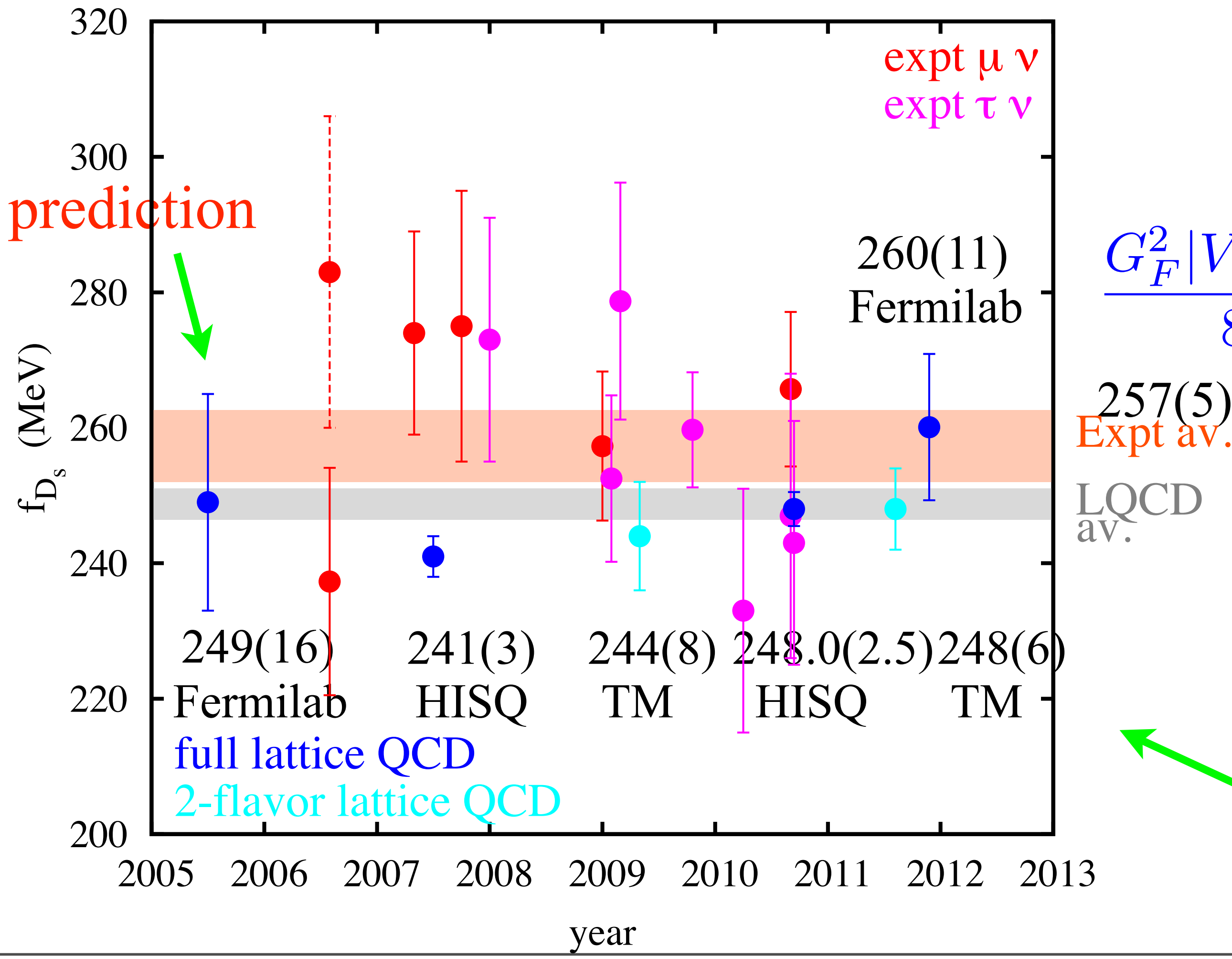
Decay constants (2pt amp.)

$$< 0|\bar{c}\gamma_0\gamma_5 s|D_s >= f_{D_s} M_{D_s}$$

or, in formalism with PCAC reln:

$$(m_s + m_c) < 0|\bar{c}\gamma_5 s|D_s >= f_{D_s} M_{D_s}^2$$





$$\mathcal{B}(D_s \to l\nu_l) = \frac{G_F^2 |V_{cs}|^2 \tau_{D_s}}{8\pi} f_{D_s}^2 m_{D_s} m_l^2 \left(1 - \frac{m_l^2}{m_{D_s}^2}\right)^2$$

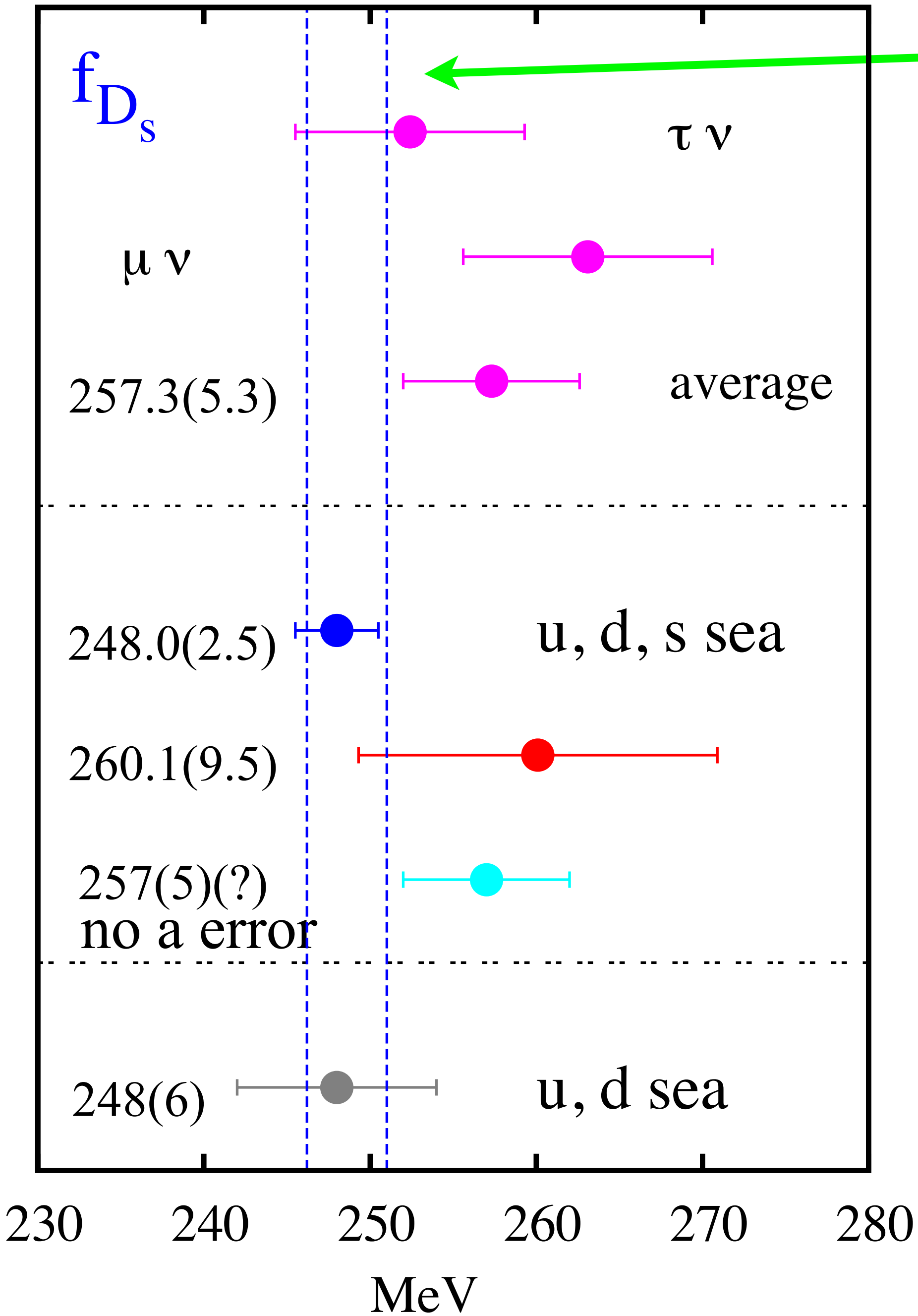
can extract from expt. if V_{cs} assumed

exciting history!

Decay constants - updates 2011

CD, LAT11, 1203.3862

av. of HPQCD Fermilab/MILC =248.6(2.4) MeV



HFAG, Jan.11

CLEO, BaBar - BES will improve ....

1.6σ

HPQCD HISQ 1008.4018

HISQ, 5 a to 0.04fm
$m_{u,d}$ extrapoln

FNAL/MILC 1112.3051

Fermilab, a=0.12,0.09fm
$m_{u,d}$ extrapoln

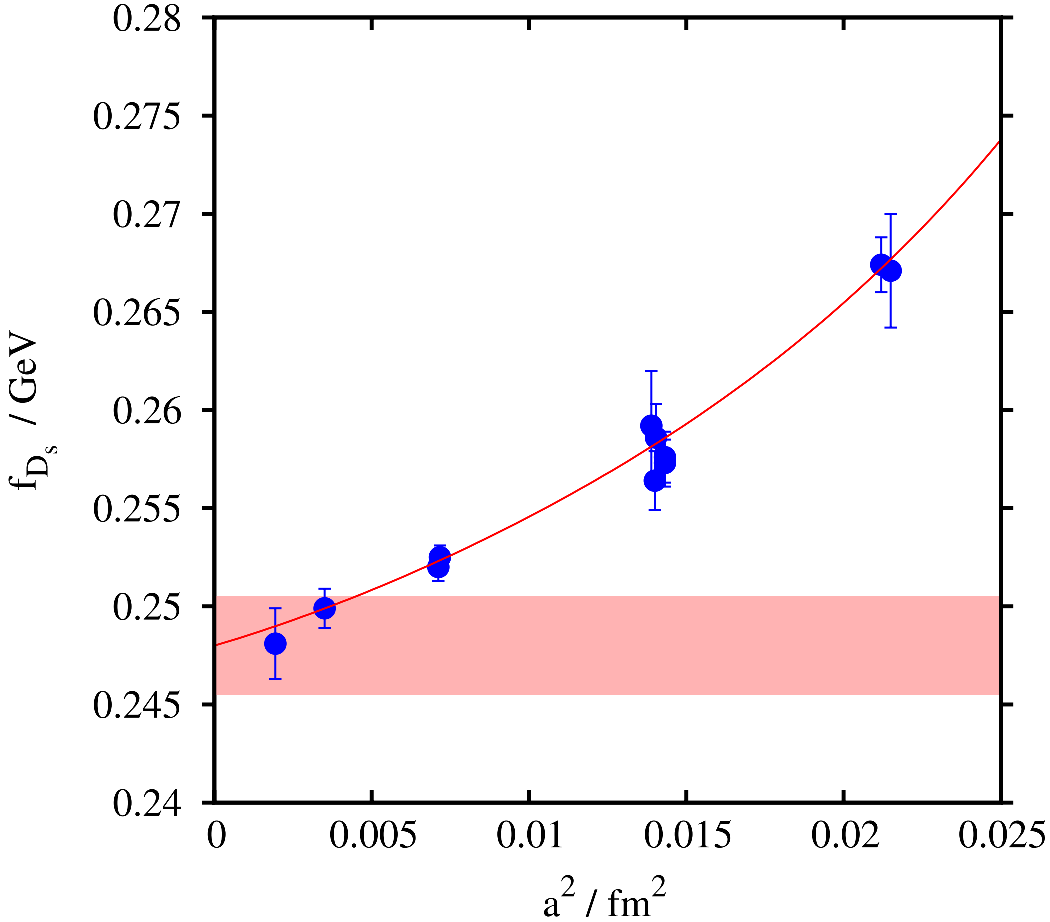
PACS-CS RHQ 1104.4600

RHQ, a=0.09fm
$m_{u,d}$ physical

ETMC 1107.1441

TM, 4 a to 0.05fm
$m_{u,d}$ extrapoln

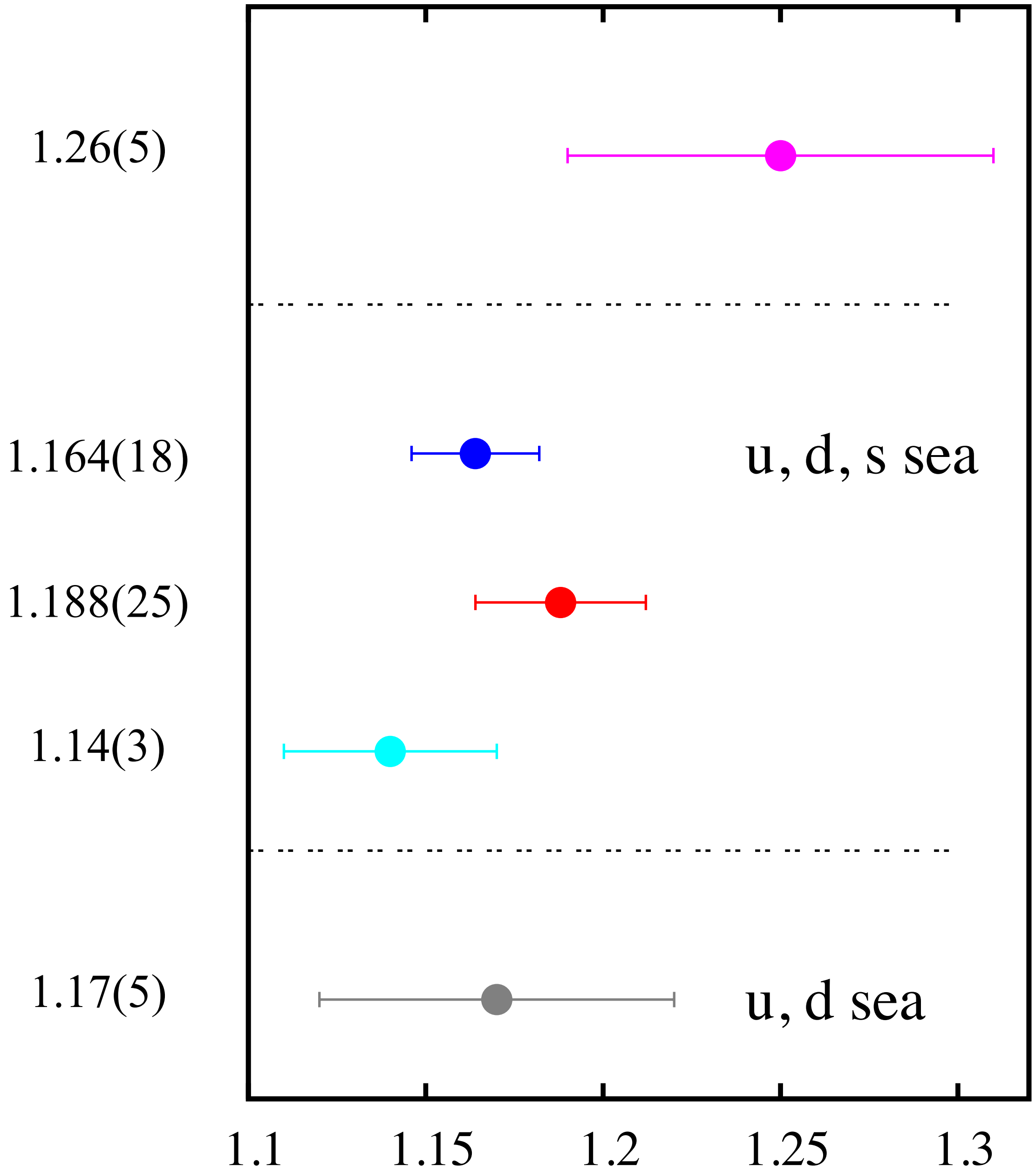
HPQCD/HISQ 1004.4018



Fermilab/
MILC
(coarse+fine
lattices)

Error	m_{D_s}	f_{D_s}
statistical/valence tuning	0.094%	$\boxed{0.57\%}$
r_1/a	0.025%	0.15%
r_1	0.051%	$\boxed{0.57\%}$
a^2 extrapoln	0.044%	0.40%
$m_{q,\mathrm{sea}}$ extrapoln	0.048%	0.34%
finite volume	0%	0.10%
m_{η_s}	0.056%	0.13%
em effects in D_s	0.036%	0.10%
em and annihln in m_{η_c}	0.076%	0.00%
em effects in η_c	-	-
missing c in sea	0.01%	0%
Total	0.16%	1.0%

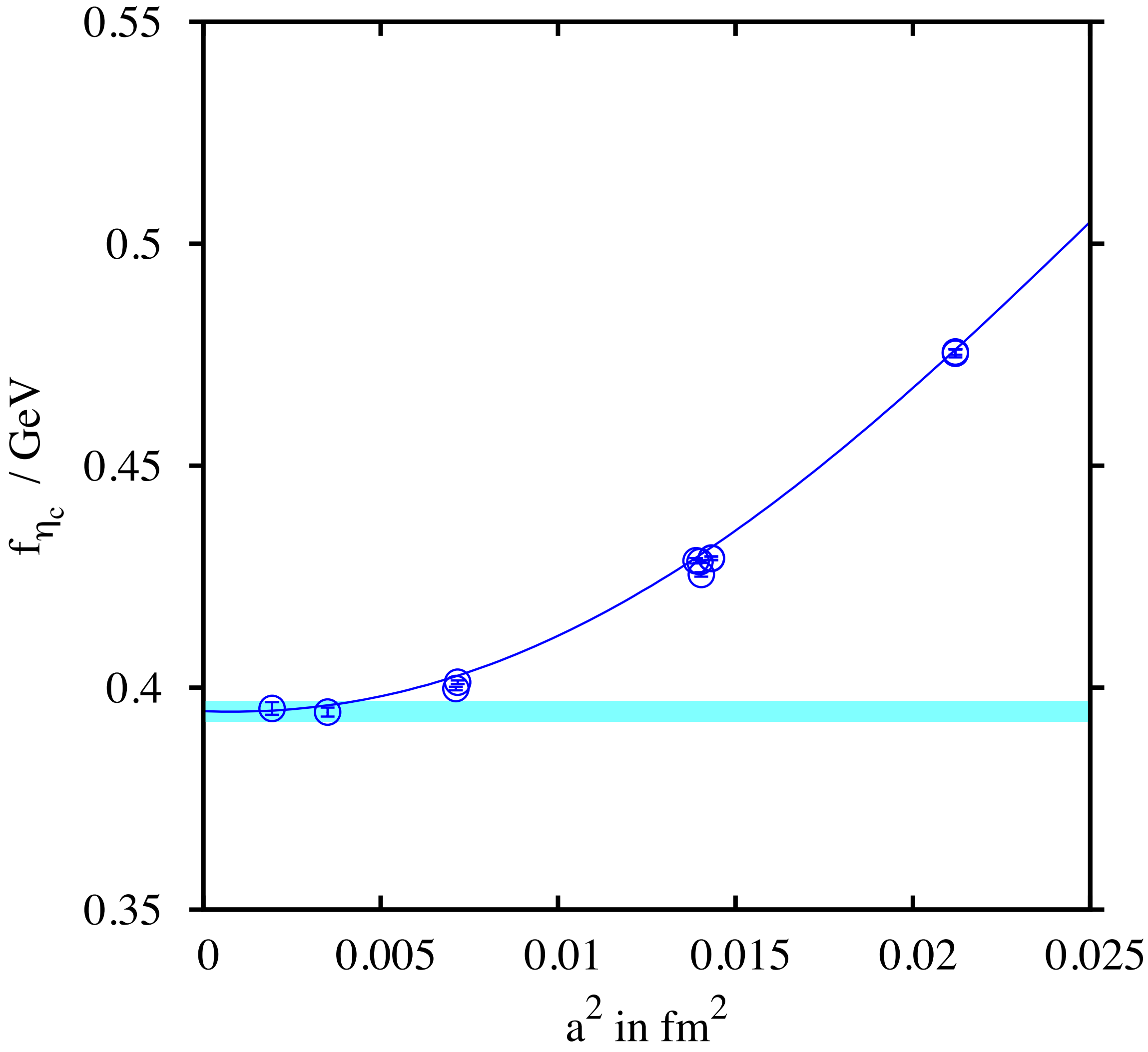
f_{D_s}/f_D comparison



cf $f_K/f_\pi = 1.193(5)$

improved results need more chiral (not finer) lattices e.g MILC 2+1+1 configs?

Decay constant of the η_c



Not directly accessible to experiment but good test of lattice QCD calculations.

HPQCD/HISQ 1004.4018 :

$$f_{\eta_c} = 394.7(2.4)\mathrm{MeV}$$

Surprisingly close, but *less than* $f_{J/\psi} = 407(5)\mathrm{MeV}$

Bottom quarks in lattice QCD - heavy or light?

increased noise in heavy-light over charm - an issue for all

Many options:

- Use HQET methods e.g. with step-scaling (Alpha)
- Use nonrelativistic method at b:
  Fermilab - same clover action as for c
  RHQ - coefficients ma-dependent
  NRQCD - disc. nonrel. expansion of L_q (now improved through $\alpha_s v_b^4$ and results on MILC 2+1+1 HISQ lattices)   R. Dowdall, HPQCD, later today
- Use relativistic method and extrapolate to b HISQ, TM

success for c makes this worth exploring

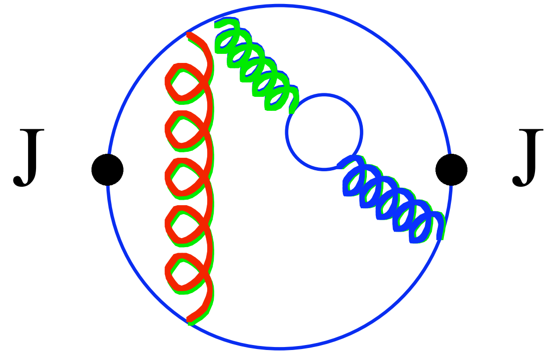
HISQ on MILC 2+1:
a=0.15 down to 0.05fm
Data below m_c to ~m_b

CD, C. McNeile, E. Follana, K. Hornbostel, P. Lepage

Determination of heavy quark masses

HPQCD + Chetyrkin et al, 0805.2999; HPQCD, 1004.4285

Current-current correlator method: match time-moments of heavy-heavy meson correlators to energy-derivative moments at $q^2 = 0$ of heavy quark vac. pol. calculated in continuum QCD pert. th. (thru α_s^3)



In continuum use J=V, but for lattice staggered quarks take J=PS (goldstone) since local and normalised from PCAC relation.

$$G(t) = a^6 \sum_{\vec{x}} (am_{0h})^2 \langle 0 | j_5(\vec{x}, t) j_5(0,0) | 0 \rangle$$

η_h correlator

$$G_n = \sum_{t=-T/2+1}^{t=T/2} (t/a)^n G(t)$$

$$R_{n,latt} = G_4/G_4^{(0)} \quad n = 4$$

$$= \frac{am_{\eta_h}}{2am_{0h}}(G_n/G_n^{(0)})^{1/(n-4)} \quad n = 6, 8, 10 \ldots$$

extrapolate to a=0 (and physical sea quark masses) and then compare to continuum:

$$R_{n,cont} = g_4/g_4^0 \quad n = 4$$

$$= \frac{m_{\eta_h}}{2\overline{m}_h(\mu)}(g_n/g_n^{(0)})^{1/(n-4)} \quad n = 6, 8, 10 \ldots$$

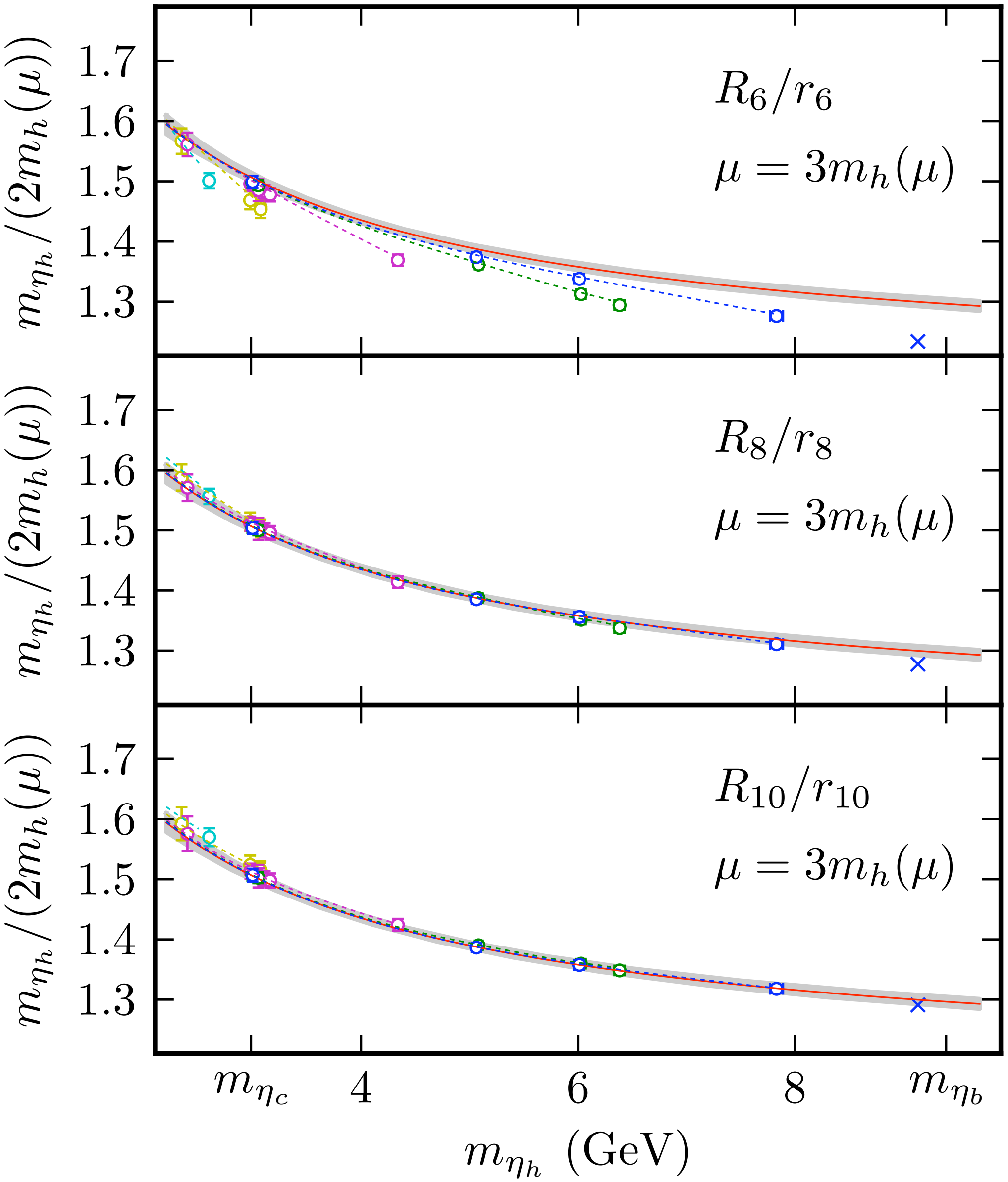
$z(\mu)$

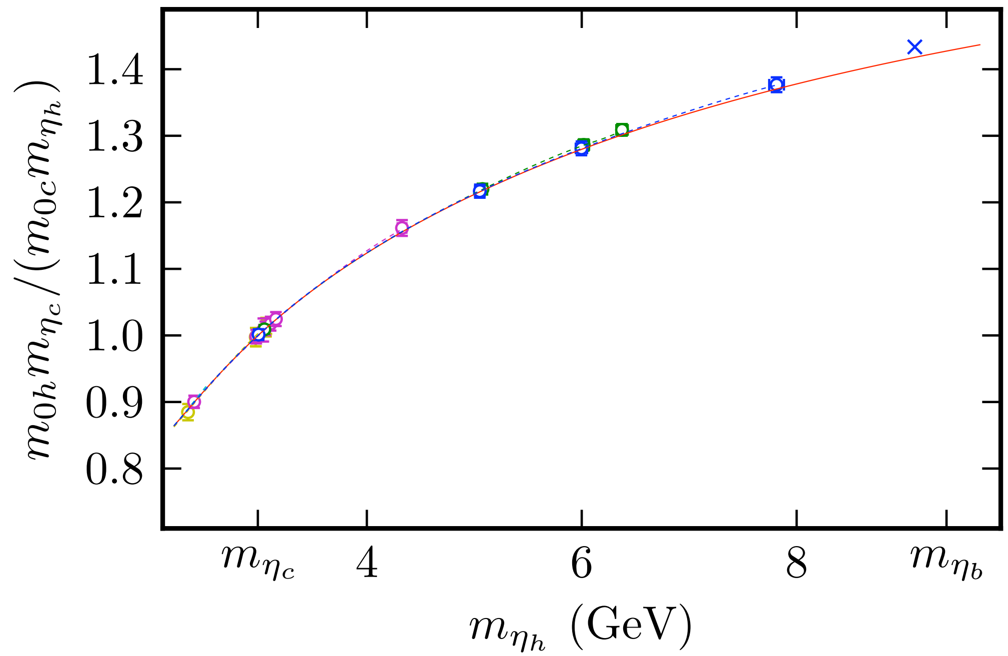
extraction allows determination of m at c and b and in between

$$g_n/g_n^0 = 1 + \sum_i c_i(\mu/\overline{m}(\mu))\alpha_{\overline{MS}}(\mu)^i$$

fit allows determination of α_s

Fit results at 5 values of a, allowing for disc. errors and higher orders in pert. th. Parameterise z as a function of m_{η_h}. Fix c and b from m_{η_c} and m_{η_b}.





completely nonpert. ratio:

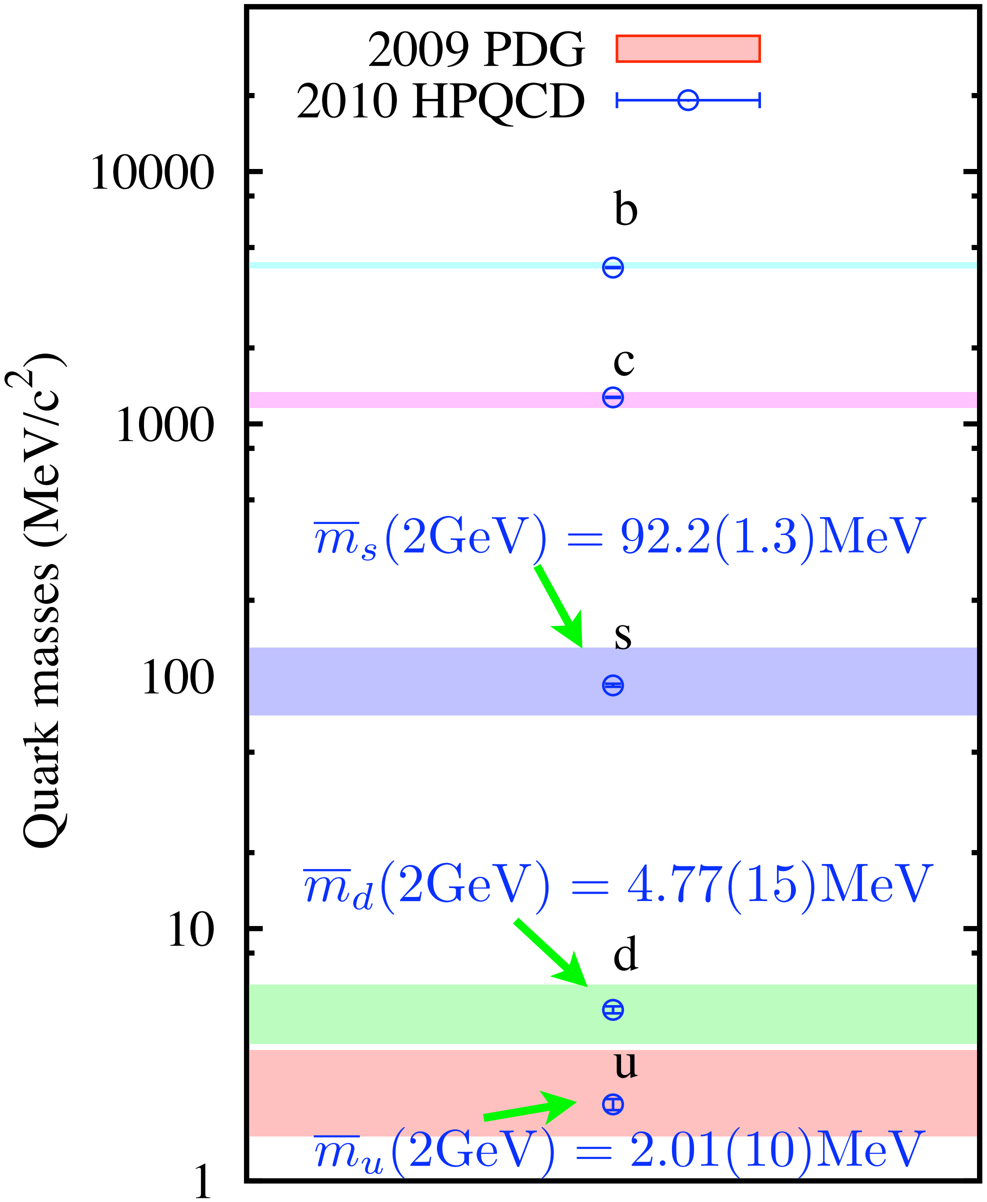
$$\frac{m_b}{m_c} = 4.51(4)$$

strong test of c.c. method.

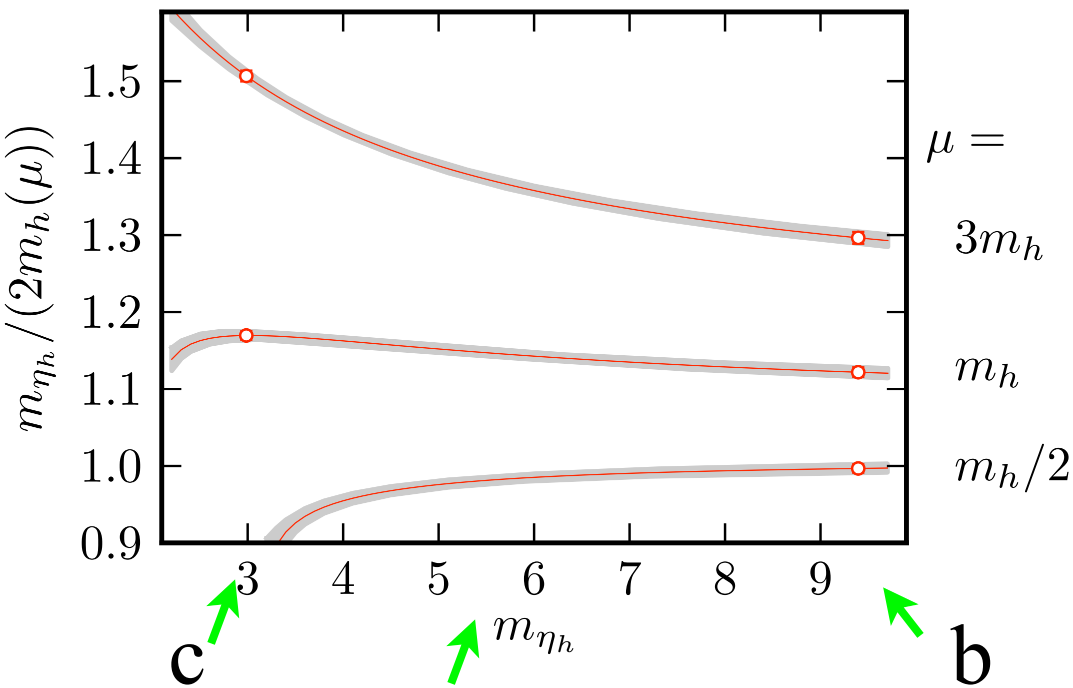
Leverage small light quark errors from heavy quark masses using ratios.

$m_c^{n_f=4}(3\text{GeV}) = 0.986(6)\text{GeV}$

$m_b^{n_f=5}(10\text{GeV}) = 3.618(25)\text{GeV}$

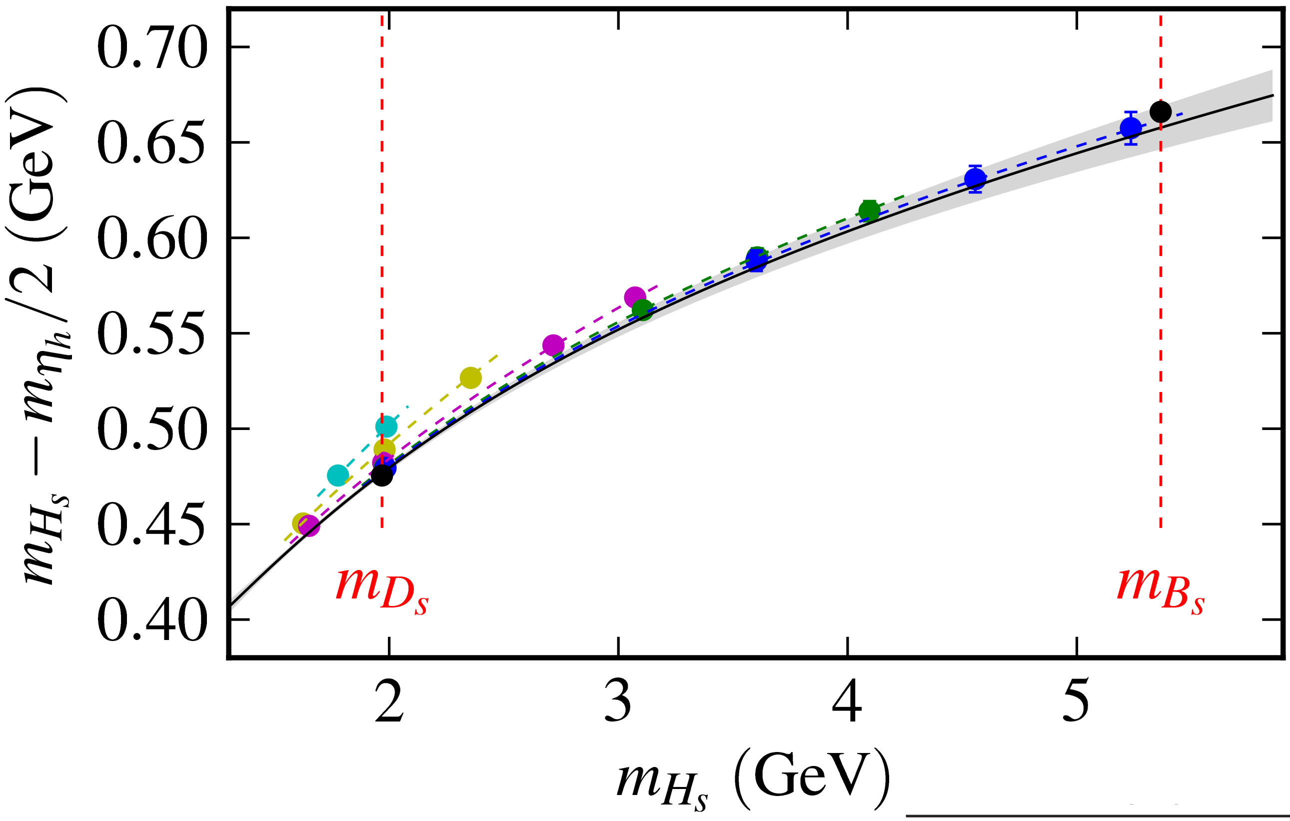


HPQCD, 1004.4285



Can define masses in between c and b also in terms of meson mass. Note how flat curve is for $\mu = m_h$

Heavy-strange masses



Difference between heavy-strange and heavyonium mapped out as a function of m_{H_s} (as proxy for quark mass). Sea mass dependence very weak.

$$m_{B_s} - m_{\eta_h}/2$$

$$= 0.658(11)\mathrm{GeV}$$

error a bit worse than NRQCD

	f_{B_s}	$m_{B_s} - m_{\eta_b}/2$
Monte Carlo statistics	1.30%	1.49%
$m_{H_s} \longrightarrow m_{B_s}$ extrapolation	0.81	0.05
r_1 uncertainty	0.74	0.33
$a^2 \longrightarrow 0$ extrapolation	0.63	0.76
$m_{\eta_s} \longrightarrow m_{\eta_s,\mathrm{phys}}$ extrapolation	0.13	0.18
r_1/a uncertainties	0.12	0.17
Total	1.82%	1.73%

$$\Delta = A m_{H_s} \sum D_i(a) \left(\frac{1\mathrm{GeV}}{m_{H_s}}\right)^i$$

$D_i(a)$ inc. even powers of

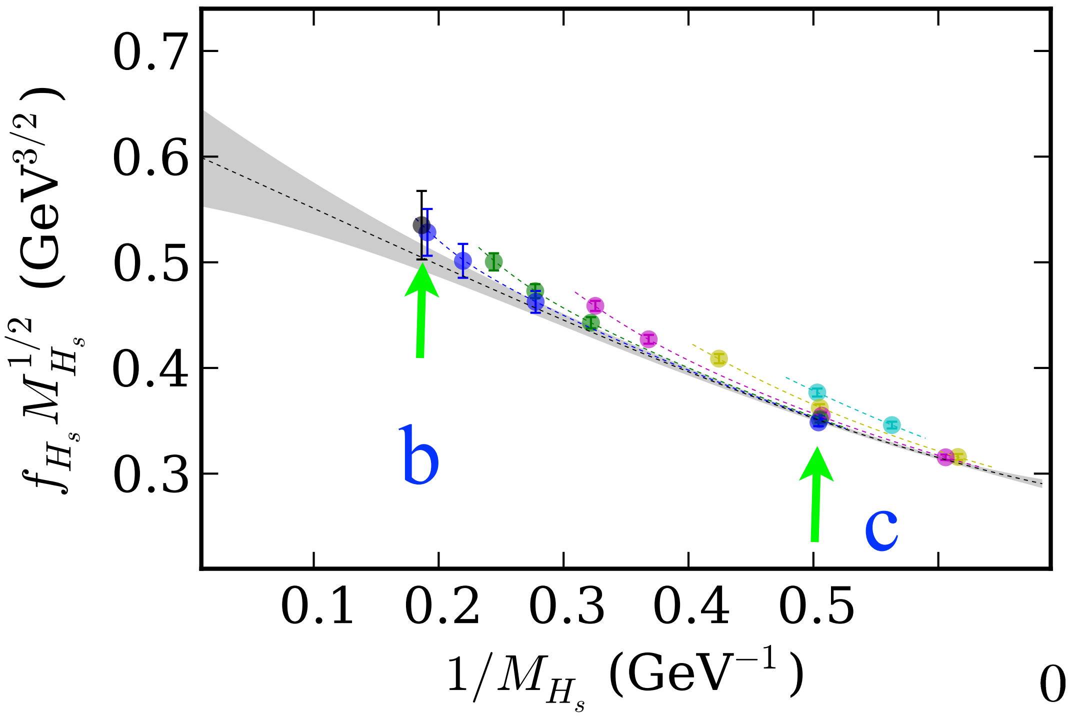
$$am_h, am_s, a\Lambda_{QCD}$$

allow for m_s mistuning

HPQCD, 1110.4510

Monday, 2 April 2012

Heavy-strange decay constants – extrapolating to b



Ditto decay constant - note no renormln needed.

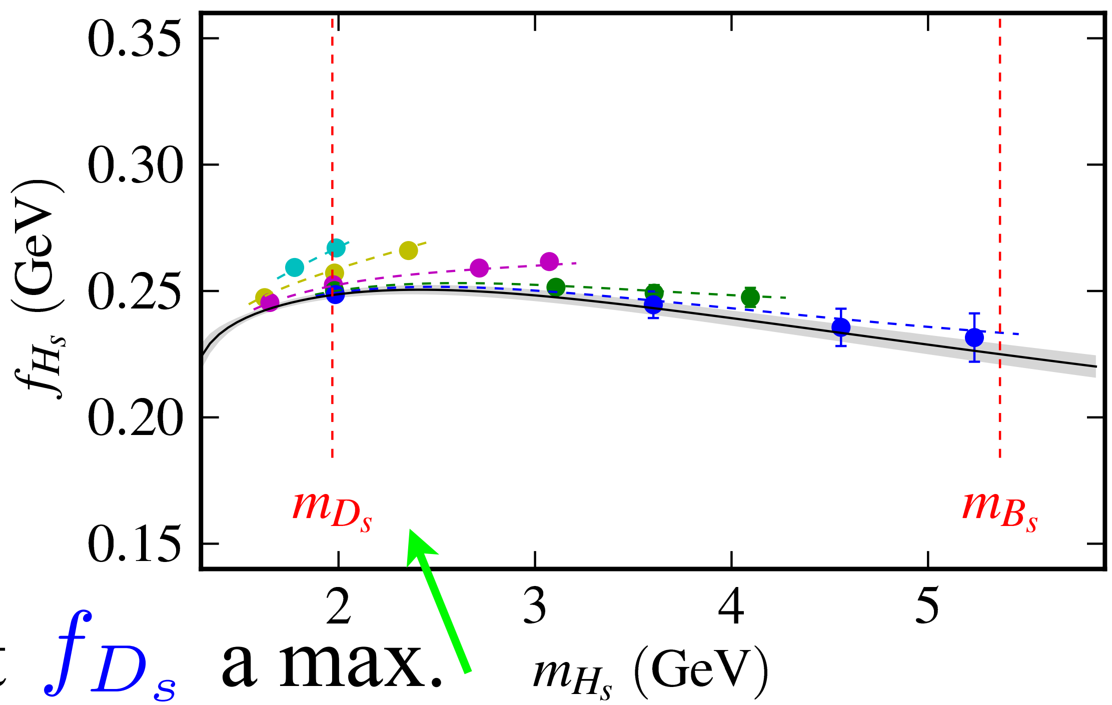
$$f_{B_s} = 225(4)\mathrm{MeV}$$

much better error than nonrel. methods

$$f_{H_s} = A(m_{H_s})^b \left(\frac{\alpha_V(m_{H_s})}{\alpha_V(m_{D_s})}\right)^{-2/\beta_0} \times$$

$$\sum C_i(a) \left(\frac{1\mathrm{GeV}}{m_{H_s}}\right)^i$$

float b - gives b=-0.51(13)



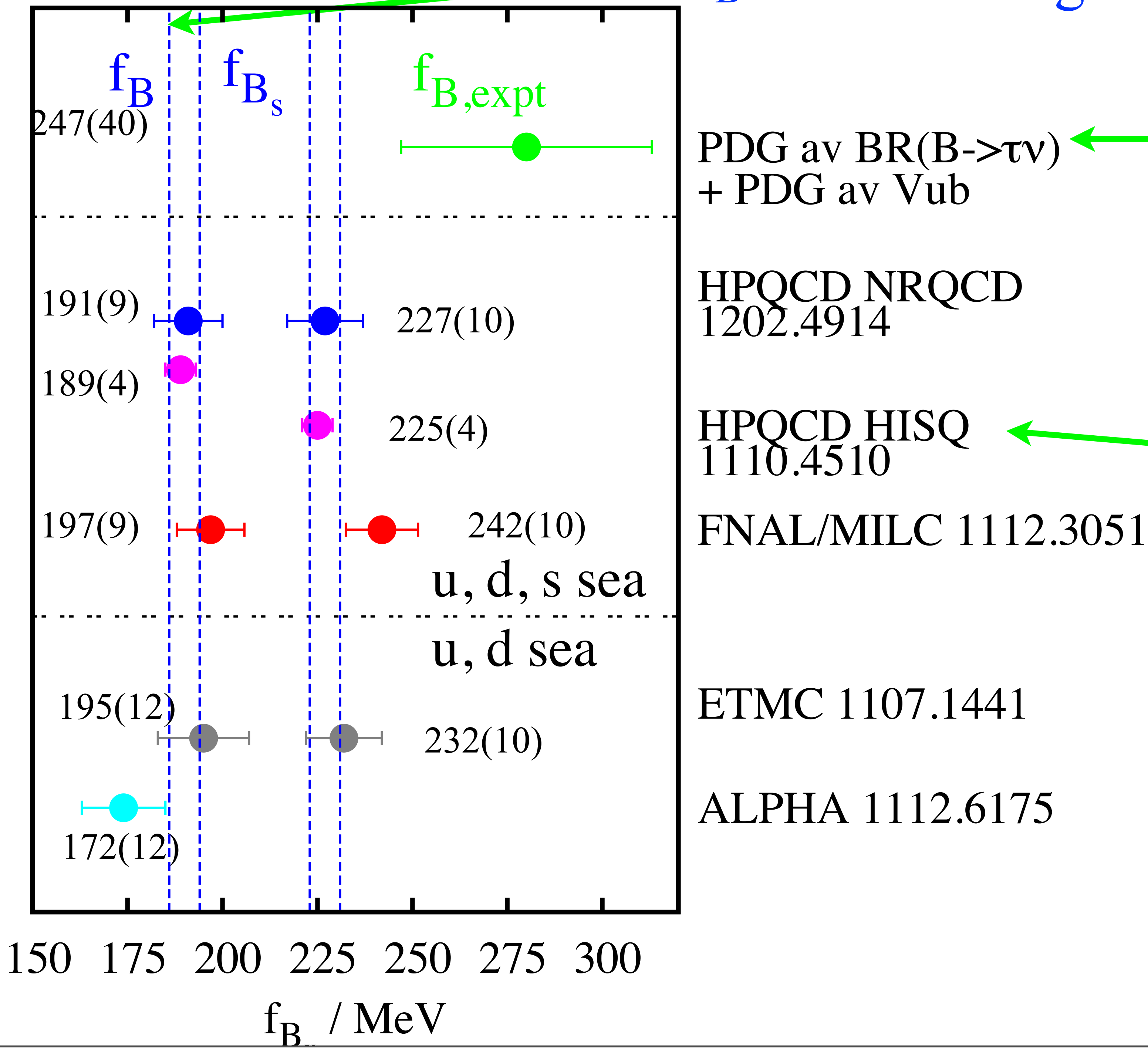
Note $f_{B_s} < f_{D_s}$ - in fact f_{D_s} a max.

Monday, 2 April 2012

f_{B_s}, f_B comparison

CD, LAT11, 1203.3862

f_B lattice average : 190(4) MeV



f_B expt

most accurate f_{B_s} available - most accurate f_B from combining with NRQCD ratio

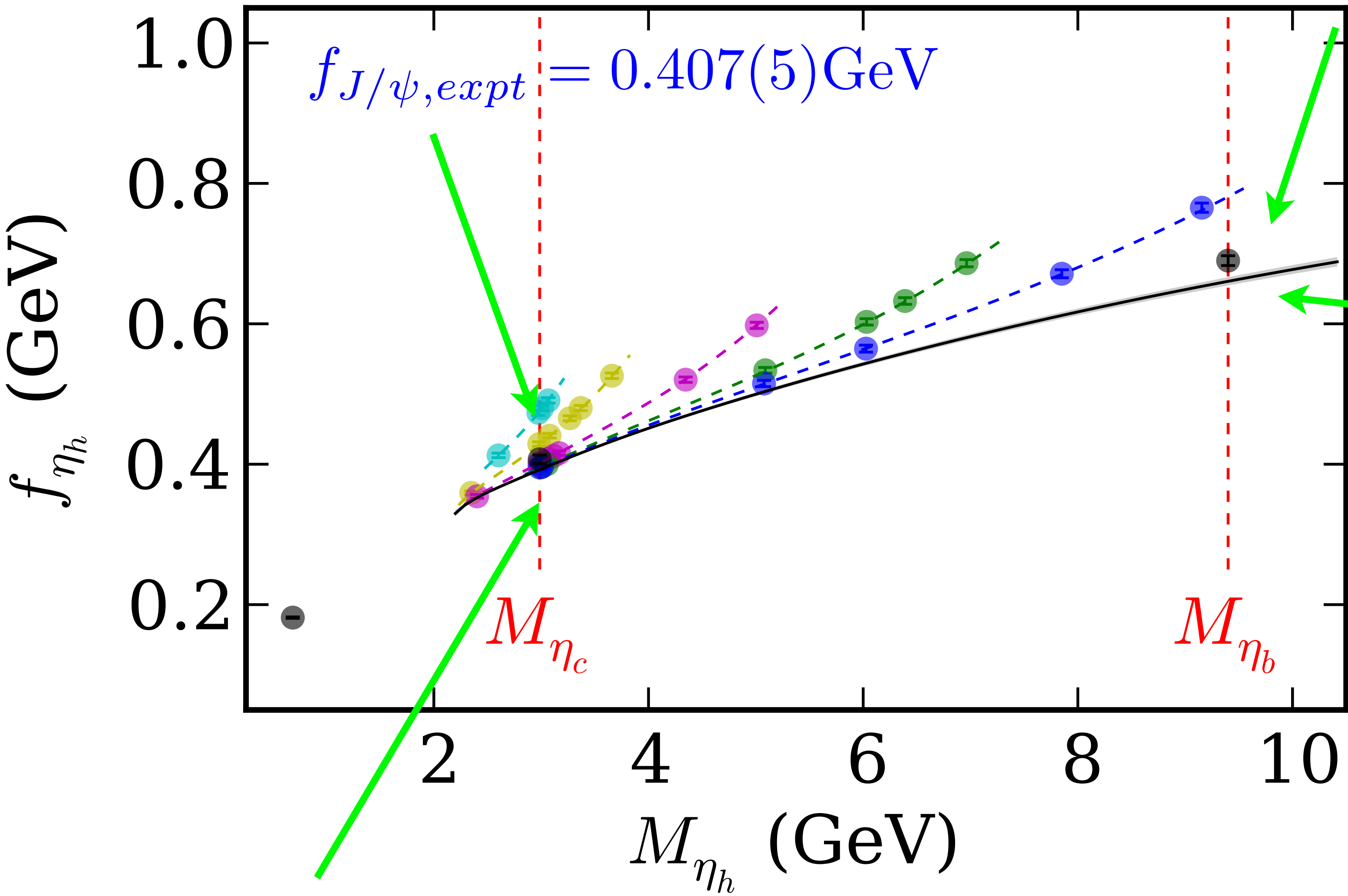
Extrapolating heavyonium to b

disc. errors much larger now ..

`HPQCD, in prep.`

$f_{\Upsilon,expt} = 0.689(5)\mathrm{GeV}$

$f_{J/\psi,expt} = 0.407(5)\mathrm{GeV}$



$f_{\eta_b} =$ $0.661(6)\mathrm{GeV}$

$$f_{\eta_h} = A\left(\frac{M_{\eta_h}}{2\mathrm{GeV}}\right)^b \times \sum_i D_i(a)\left(\frac{2\mathrm{GeV}}{M_{\eta_h}}\right)^i$$

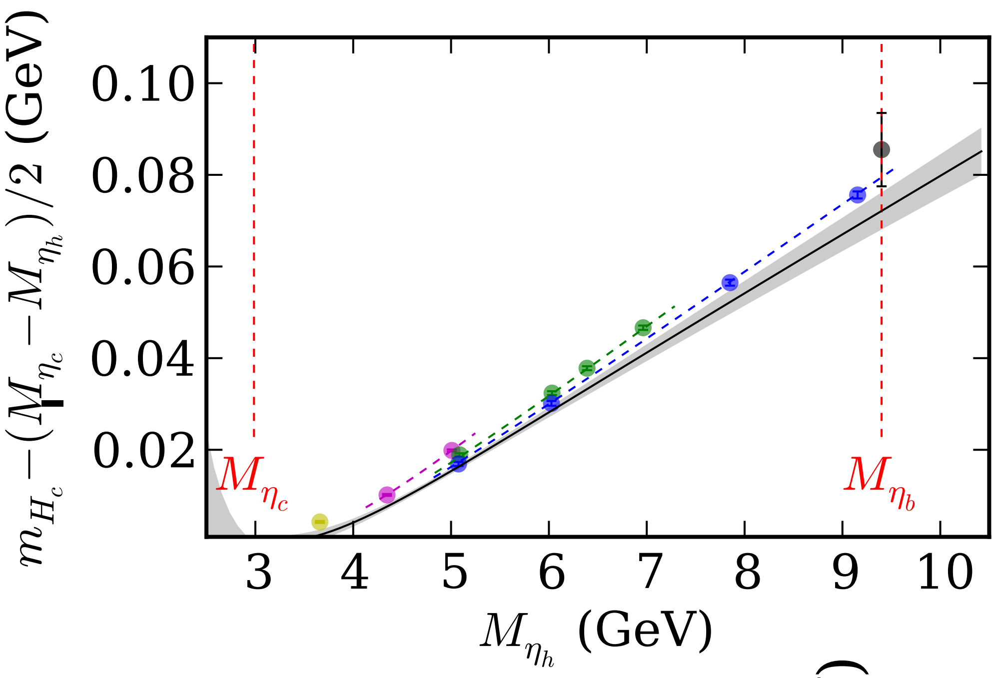
$f_{\eta_c} = 0.395(2)\mathrm{GeV}$

HPQCD 1008.4018

Find again f_{PS} 3-4 % below corresponding f_V, checking in NRQCD ...
Good check of Z factors?

Extrapolating heavy-charm to B_c

HPQCD, in prep.

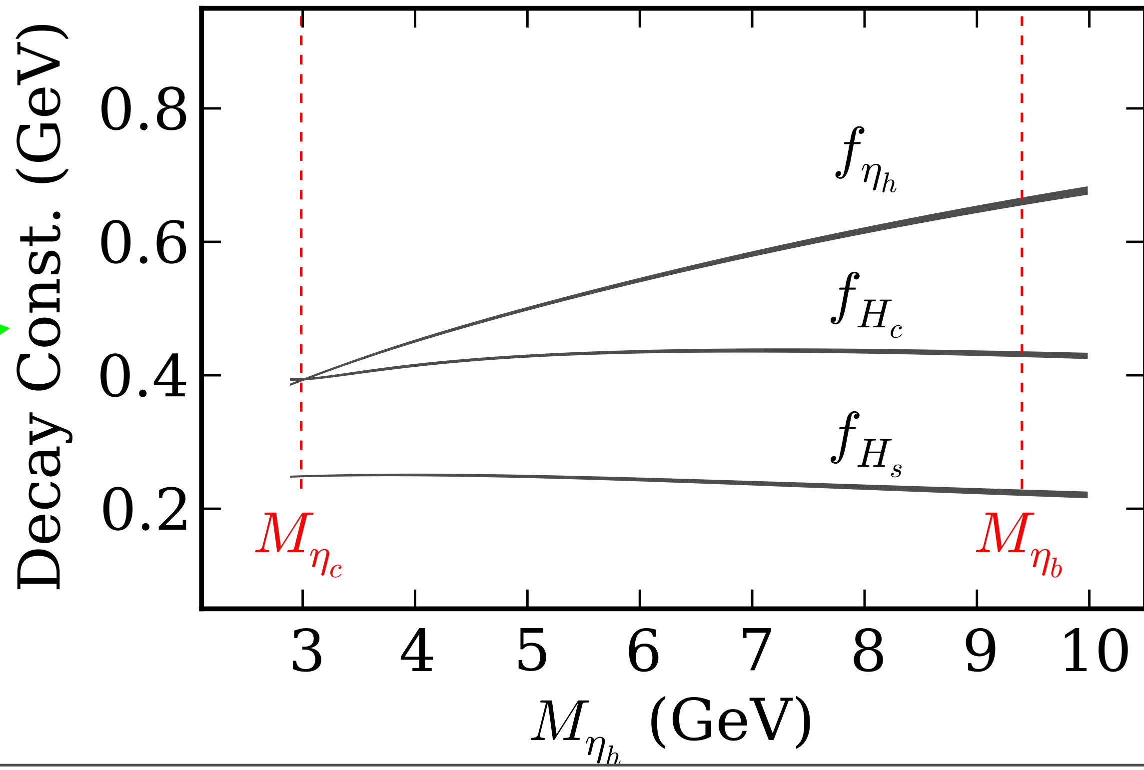


$$m_{B_c} - \frac{m_{\eta_b} + m_{\eta_c}}{2}$$

$$= 72(4)\mathrm{MeV}$$

expt $= 84(9)\mathrm{MeV}$

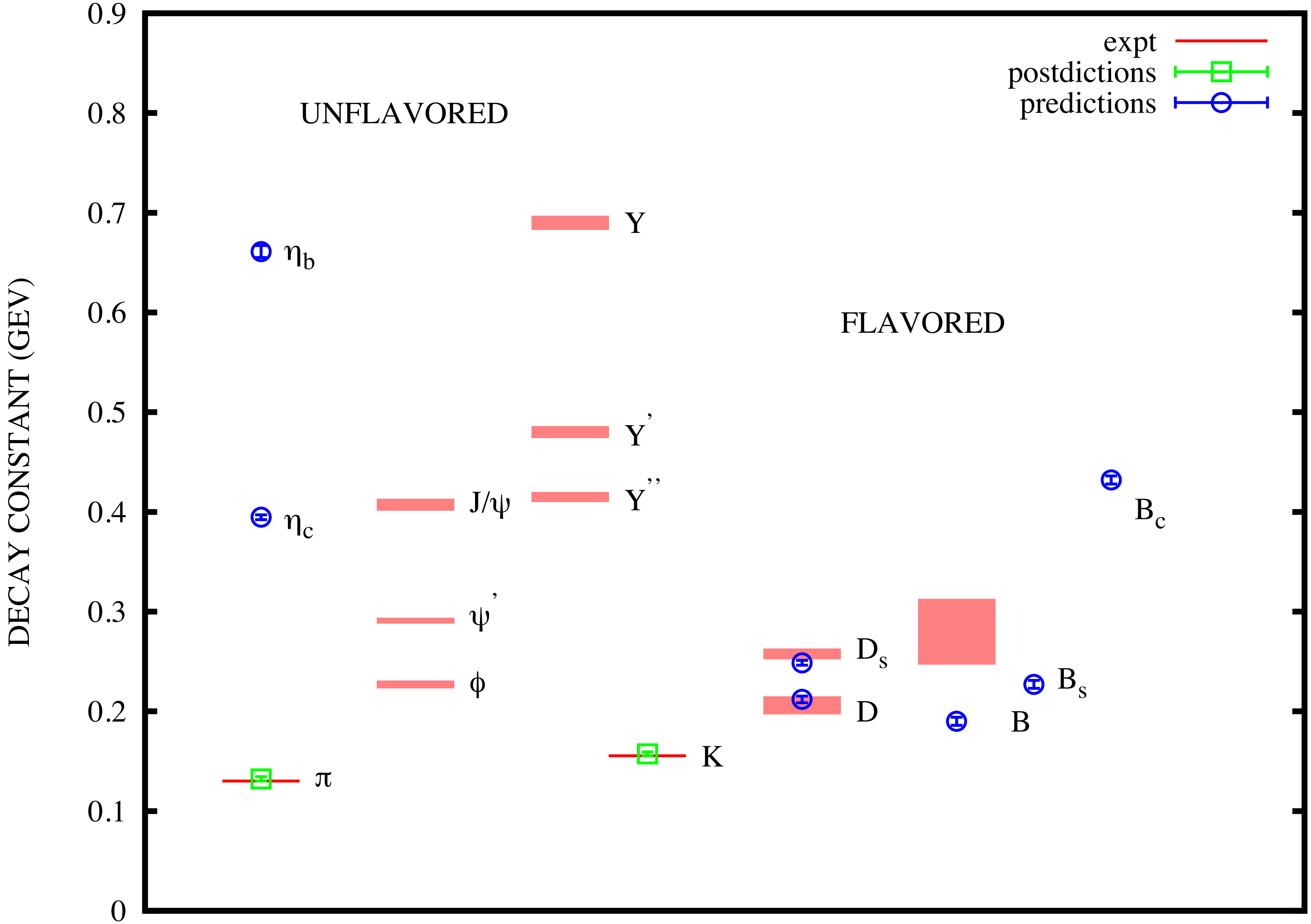
after adjusting for em/annihln



$f_{H_c} = 0.432(4)\mathrm{GeV}$

dependence on m is similar to f_{H_s} since is a heavy-light system as b mass becomes large.

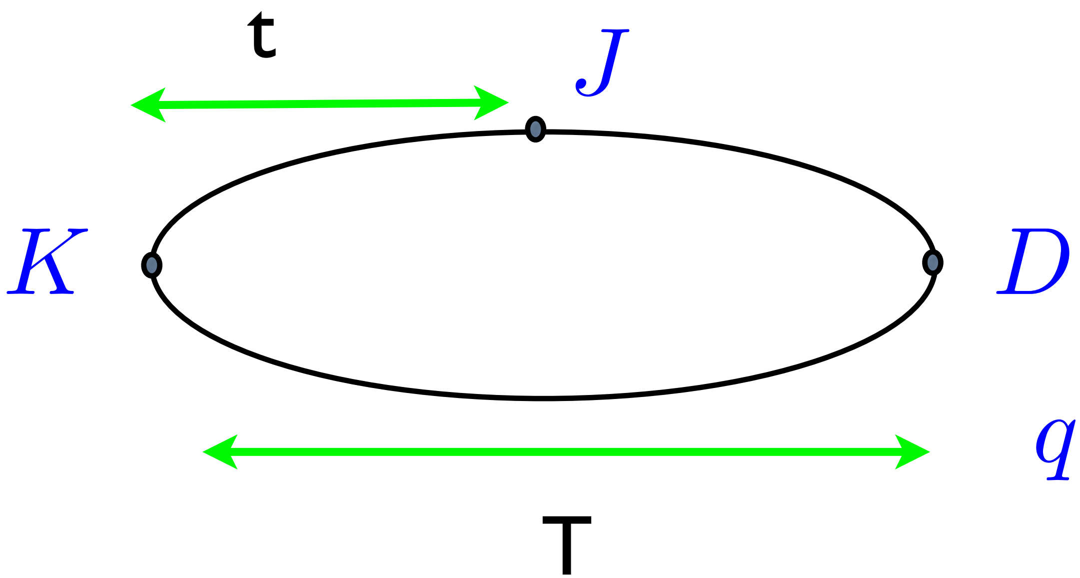
Summary plot for decay constants

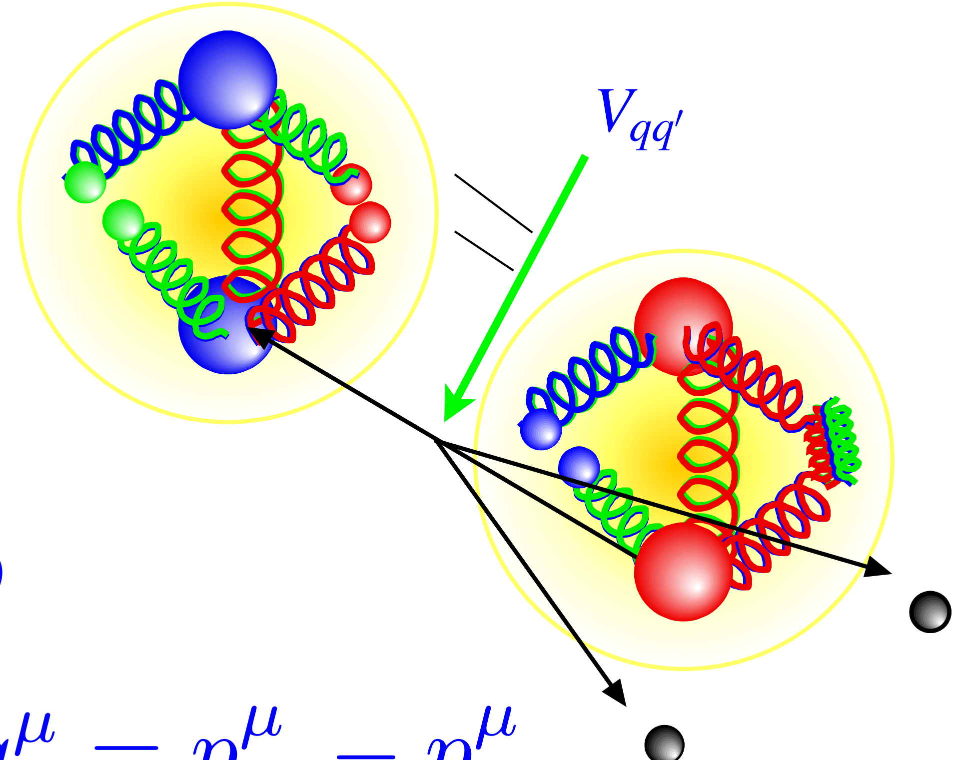


More work on vectors underway ....

Semileptonic form factors

3pt amp.





$$q^\mu = p^\mu_D - p^\mu_K$$

$$< K|V^\mu|D >= f_+(q^2)\left[p^\mu_D + p^\mu_K - \frac{M_D^2 - M_K^2}{q^2}q^\mu\right]$$

$$+f_0(q^2)\frac{M_D^2 - M_K^2}{q^2}q^\mu$$

$$f_0(0) = f_+(0)$$

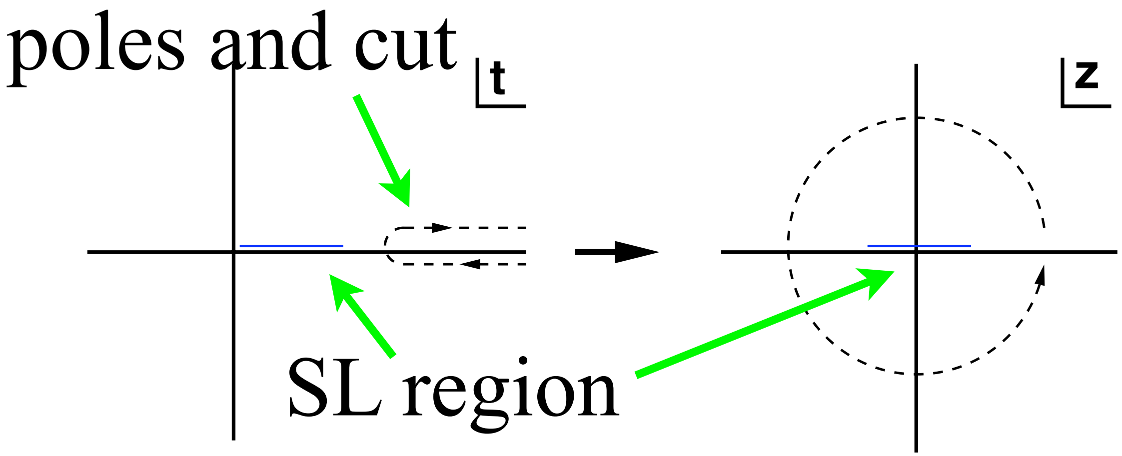
$$< K|S|D >= \frac{M_D^2 - M_K^2}{m_{0c} - m_{0s}}f_0(q^2)$$

abs. norm. for same c/s action HPQCD: 1008.4562

Experiment sees f_+ and maps in q^2 bins

$D \to K, D \to \pi$
CLEO: 0906.2983

$$\frac{d\Gamma^{D\to K l \nu_l}}{dq^2} = \frac{G_F^2 p_K^3}{24\pi^3} |V_{cs}|^2 |f_+(q^2)|^2$$



see e.g Bourrely et al, 0807.2722

Transforming q^2 to z

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

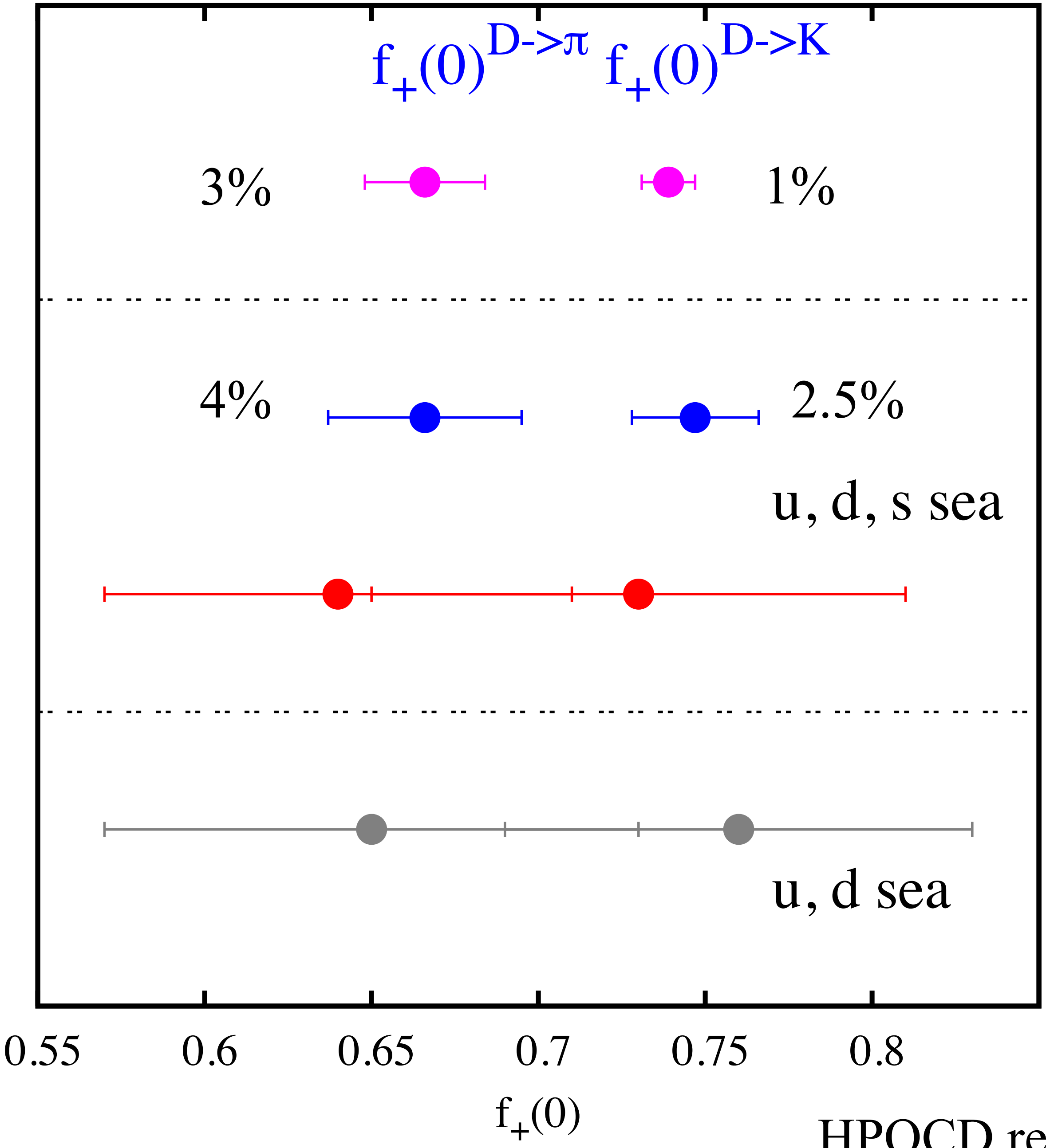
allows simple series parameterisation of shape - obtain $V_{cs}f_+(0)$

Lattice QCD : want shape $f_+(q^2)$, value $f_+(0) \equiv f_0(0)$

Techniques improving:

High statistics, random sources; multi-exponential fits to multiple T values; phase at boundary to tune q^2 (to 0);use of z expansion to fit and extrapolate shape to chiral/contnm limit

Updates on $f_+(0)$ 2011



using unitarity:

CLEO 0906.2983

$V_{cs} = 0.97345(16)$

$V_{cd} = 0.2252(7)$

HPQCD HISQ 1008.4562;1109.1501 — 2 a values; scalar

FNAL/MILC hep-ph/0408306+ — 1 a value; vector FNAL renorm.

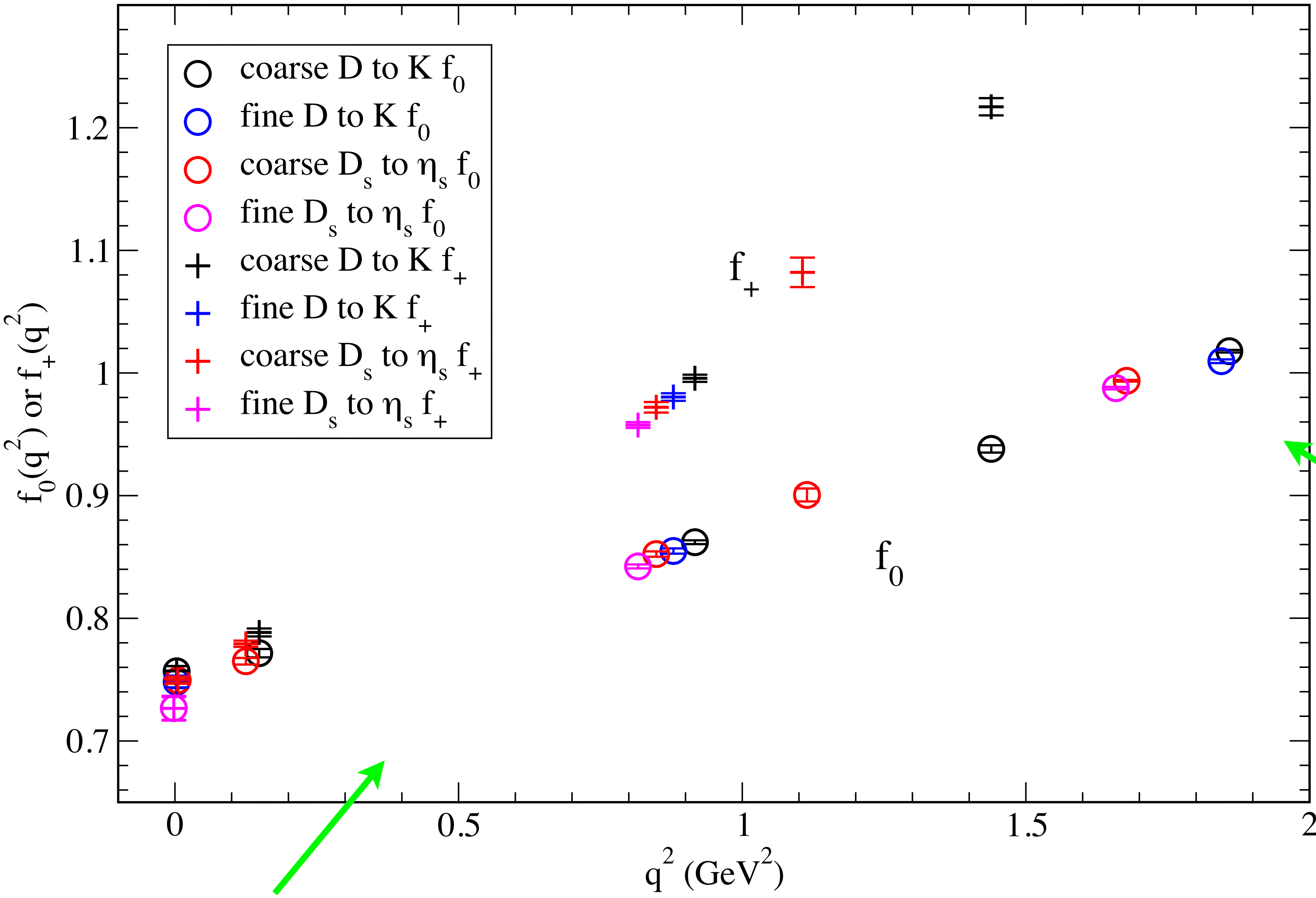
ETMC 1104.0869 — 3 a values; vector + double ratios to cancel Z

HPQCD results will improve further....

Updates on $f_+(q^2)$ 2011

HPQCD PRELIMINARY

J. Koponen, LAT11



Abs. norm. vector and scalar ops

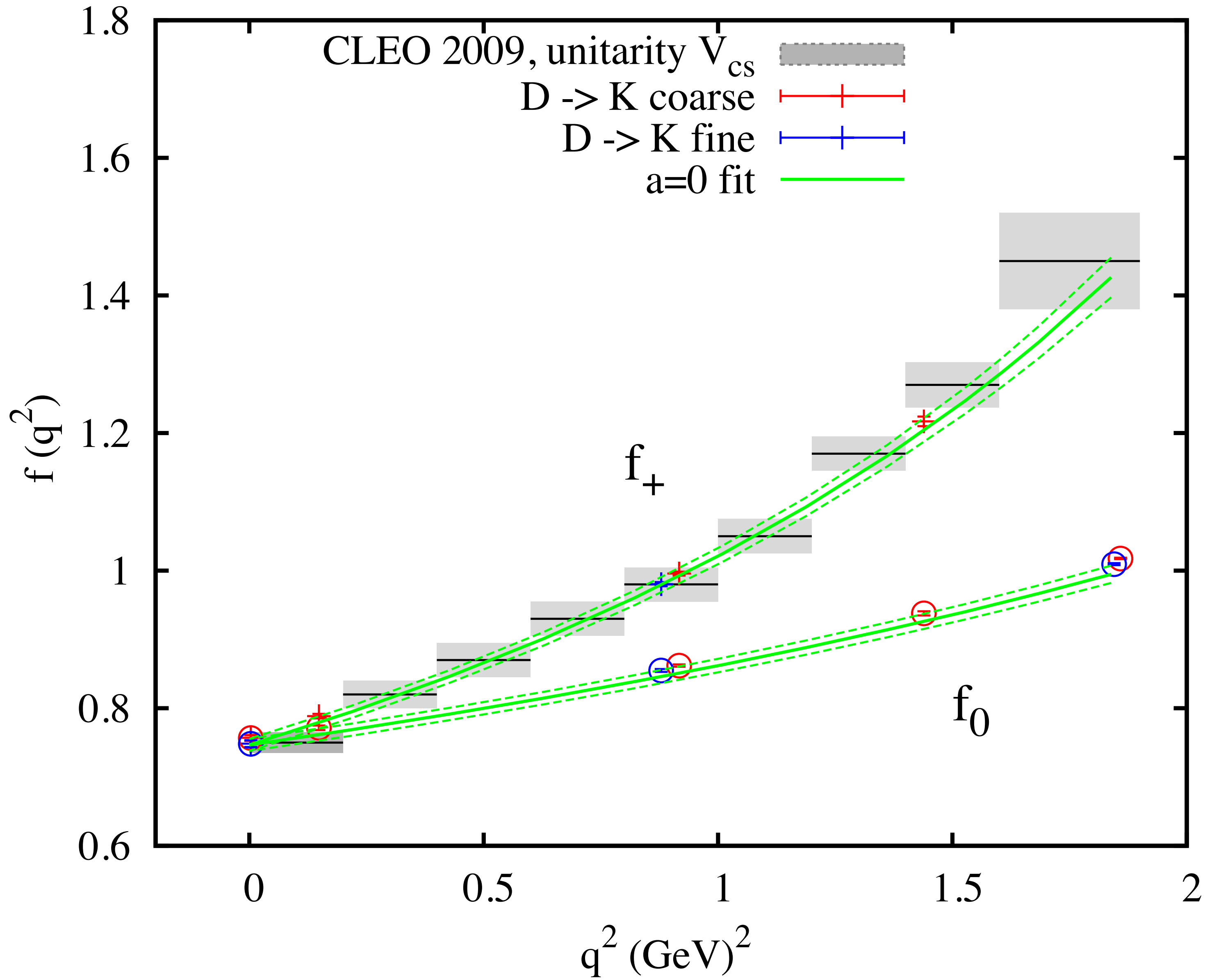
disc. effects very small

Very little dependence on spectator quark - implications for B/B_s form factors. (see Fermilab/MILC 1202.6346)

Updates on $f_+(q^2)$ 2011

$D \to K$

HPQCD PRELIMINARY



v. high stats - 16000 corrs x 4T

1-2% errors allow accurate comparison of shape to expt

J. Koponen, HPQCD in prep.

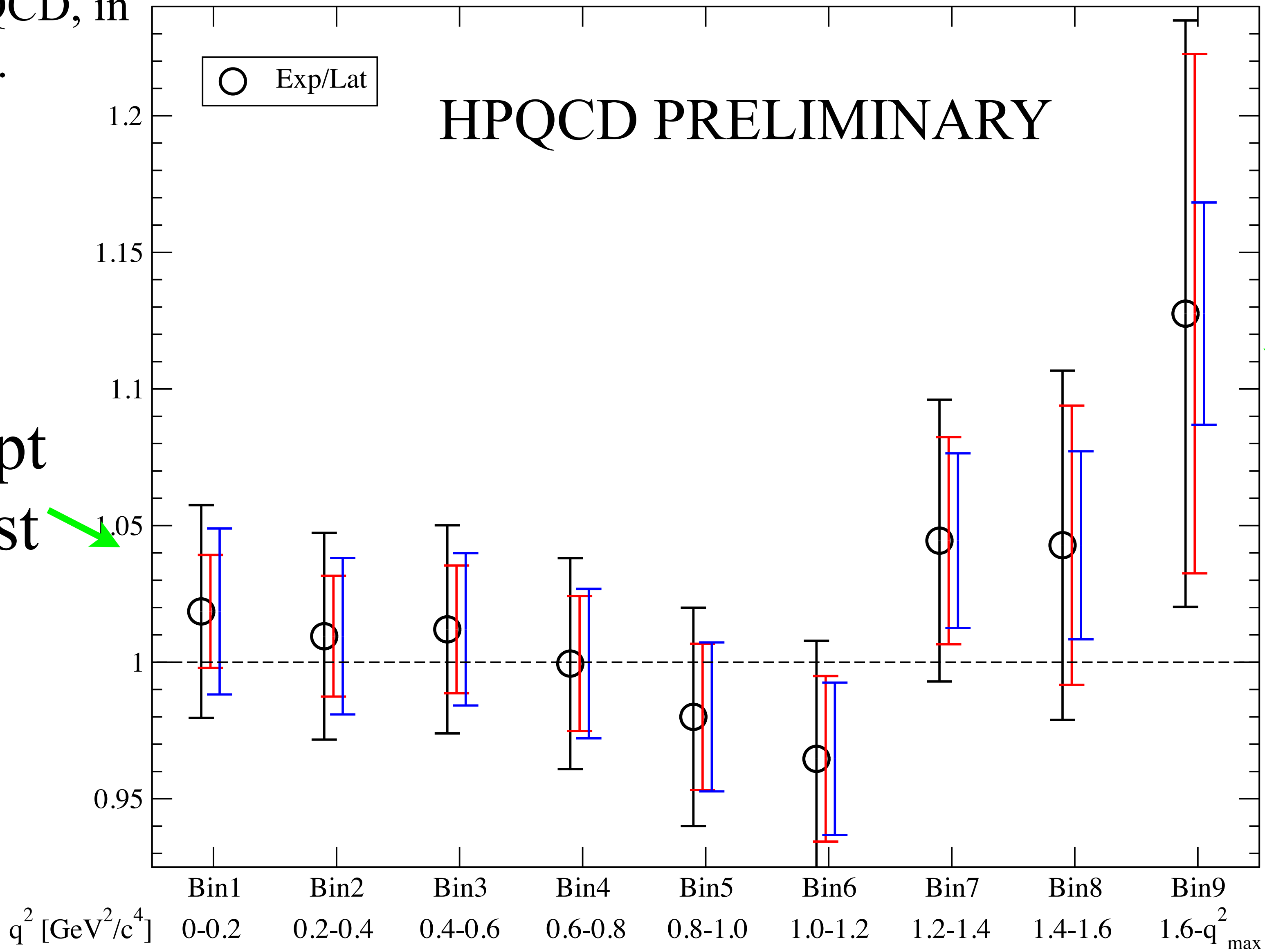
Need to look at more form factors ...vector and axial form factors for $D_s \to \phi l \nu$: G. Donald, HPQCD in prep.

Comparison of expt and lattice bin by bin

J. Koponen, HPQCD, in prep.

using unitarity V_{cs}

expt best

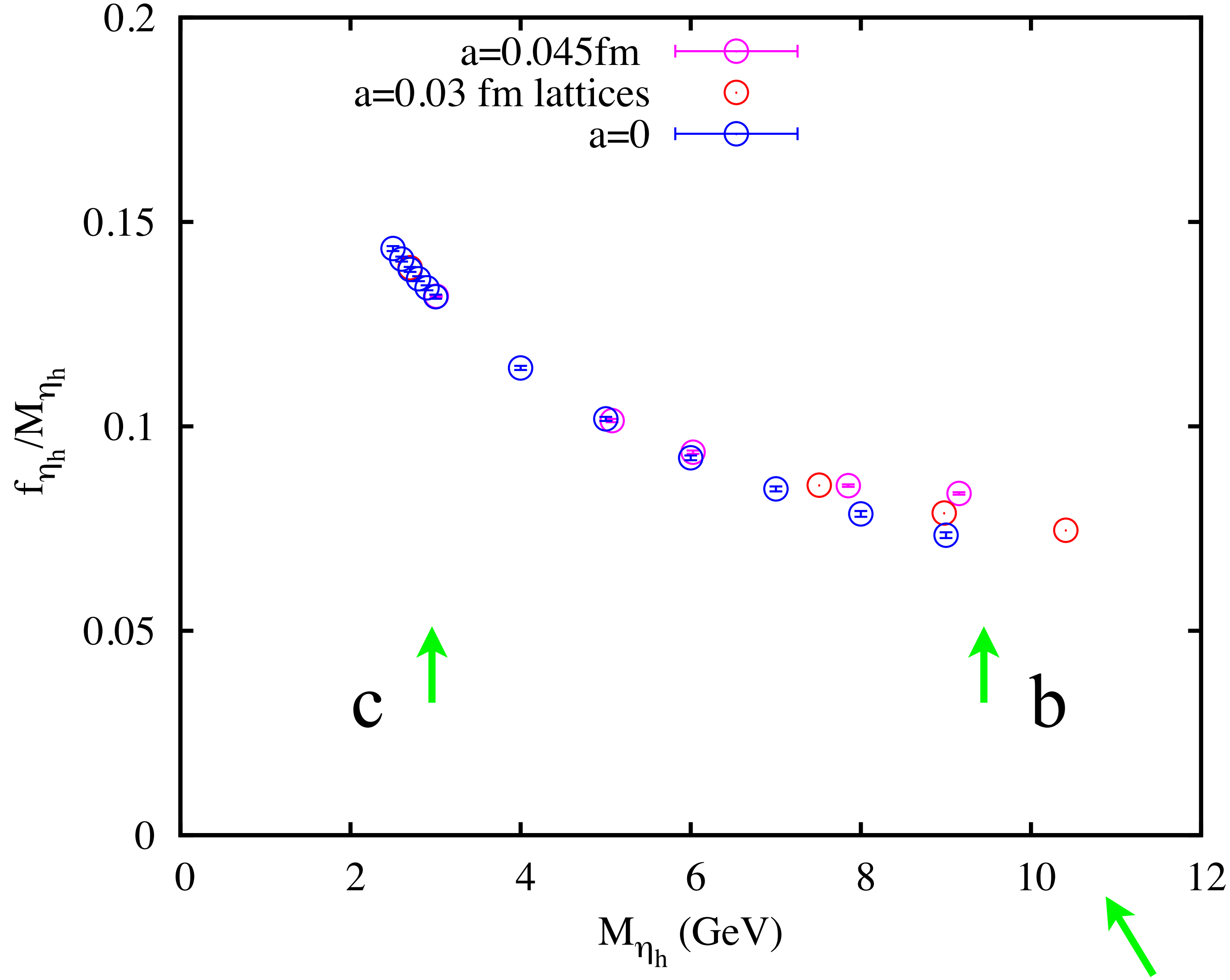


lattice best

Will allow 1.5% error in V_{cs}

move on to semileptonic analyses for heavier hisq quarks ..

Going forward ...



HPQCD making a=0.03 fm 72^3x192 lattices

Now

$m_c a \approx 0.1$

$m_b a \approx 0.5$

Preliminary results for heavyonium consistent with previous 'heavy HISQ' analysis.

Conclusions

- Charm physics in good shape. 1-2% precision possible with improved relativistic actions such as HISQ. Need more results with such formalisms .. e.g. TM
- Bottom physics still a problem.

NRQCD currently has sizeable renormln error
Fermilab/RHQ has heavy quark disc. error
HQET methods in practice hard?
Should get 1% accurate s/l ratios on lattices at chiral pt ..
Extrapolation to b from c with HISQ/TM. Promising for masses + decay constants. Now move to semileptonic ffs. Needs extremely fine lattices to do well and e.g. 4q operators harder ...

ALL – need more tests of errors using known quantities.
Look forward to results on N_f=2+1+1 lattices ... Dowdall 3pm