



# Axial couplings of heavy hadrons

William Detmold

The College of William & Mary / Jefferson Lab

work in collaboration with [David Lin](#) & [Stefan Meinel](#)  
[arXiv:1109.2480 (to appear in PRL), arXiv:1203.3378]

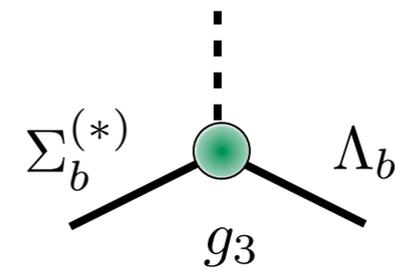
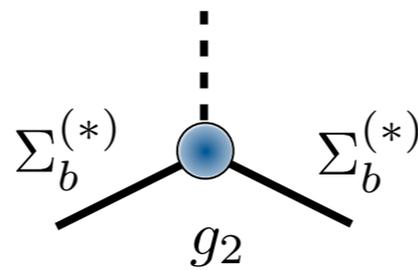
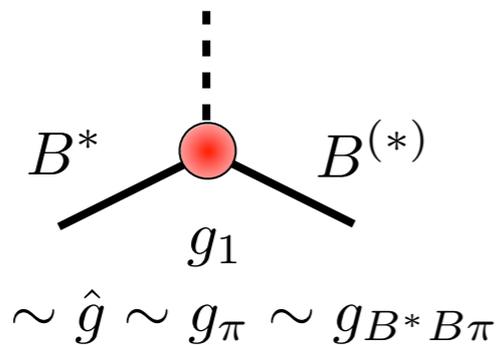
# Outline

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- Heavy-hadron axial couplings:  $g_{1,2,3}$
- Heavy hadron chiral perturbation theory  
[David Lin's talk on Tuesday]
- QCD calculation of axial couplings: details and results
- Combine LQCD and HH $\chi$ PT to control all systematic uncertainties
- Strong decay widths

# Chiral dynamics of heavy hadrons

- Axial couplings *defined* in static limit



$$B^* \quad \langle P^{*d}(0, s) | A_\mu^{-(\chi^{PT})}(0) | P^u(0) \rangle |_{\text{LO}} = -2 g_1 \varepsilon_\mu^*(s).$$

$$B \quad \langle S^{dd}(0, s) | A^{\mu-(\chi^{PT})}(0) | S^{du}(0, s') \rangle |_{\text{LO}} = -\frac{i}{\sqrt{2}} g_2 v_\lambda \varepsilon^{\lambda\mu\nu\rho} \bar{U}_\nu(s) U_\rho(s').$$

$$\langle S^{dd}(0, s) | A^{\mu-(\chi^{PT})}(0) | T^{du}(0, s') \rangle |_{\text{LO}} = -g_3 \bar{U}^\mu(s) \mathcal{U}(s').$$

$$\left( \begin{array}{cc} \Sigma_b^+ & \frac{1}{\sqrt{2}} \Sigma_b^0 \\ \frac{1}{\sqrt{2}} \Sigma_b^0 & \Sigma_b^- \end{array} \right)^{(*)} \quad \left( \begin{array}{cc} 0 & \Lambda_b \\ -\Lambda_b & 0 \end{array} \right)$$

- Heavy-light mesons and baryons: dynamics amenable to HQ and chiral expansions [Wise; Burdman & Donoghue; Cheng et al.]

# H-L hadrons in lattice QCD

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- Light quark mass dependence of H-L(L) observables controlled by pion loops, coupled through  $g_{1,2,3}$
- Very important for control of current lattice QCD calculations at unphysical quark masses  
Examples so far this week:
  - ETMC: chiral extrapolation of  $f_{B_s}/f_B$  is dominant systematic uncertainty
  - FNAL/MILC: B, D semi-leptonic FFs,  $g_\pi$  major systematic uncertainty
  - HPQCD: requires prior in B meson properties

# Current knowledge of $g_1$

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- Experimental extraction of  $g_1$  from  $D^* \rightarrow D\pi$ ,  $D^* \rightarrow D\gamma$ 
  - $g_1 = 0.5(?)$  [Arnesen et al.]
- Lattice calculations for  $g_1$

| Reference                             | $n_f$ , action | $[m_\pi^{(vv)}]^2$ (GeV <sup>2</sup> ) | $g_1$   |
|---------------------------------------|----------------|--|---|
| De Divitiis <i>et al.</i> , 1998 [14] | 0, clover      | 0.58 - 0.81                            | $0.42 \pm 0.04 \pm 0.08$                        |
| Abada <i>et al.</i> , 2004 [15]       | 0, clover      | 0.30 - 0.71                            | $0.48 \pm 0.03 \pm 0.11$                        |
| Negishi <i>et al.</i> , 2007 [16]     | 0, clover      | 0.43 - 0.72                            | $0.517 \pm 0.016$                               |
| Ohki <i>et al.</i> , 2008 [17]        | 2, clover      | 0.24 - 1.2                             | $0.516 \pm 0.005 \pm 0.033 \pm 0.028 \pm 0.028$ |
| Bećirević <i>et al.</i> , 2009 [18]   | 2, clover      | 0.16 - 1.2                             | $0.44 \pm 0.03^{+0.07}_{-0.00}$                 |
| Bulava <i>et al.</i> , 2010 [19]      | 2, clover      | 0.063 - 0.49                           | $0.51 \pm 0.02$                                 |

- Need fully quantified uncertainties

# Current knowledge of $g_{1,2,3}$

- Model estimates for  $g_{1,2,3}$  [Cho normalisation]

| Reference                           | Method   | $g_1$           | $g_2$                    | $g_3$                    |
|-------------------------------------|--|-----------------|--------------------------|--------------------------|
| Yan <i>et al.</i> , 1992 [5]        | Nonrelativistic quark model  | 1               | 2                        | $\sqrt{2}$               |
| Colangelo <i>et al.</i> , 1994 [45] | Relativistic quark model   | 1/3             | ...                      | ...                      |
| Bećirević, 1999 [46]                | Quark model with Dirac eq.   | $0.6 \pm 0.1$   | ...                      | ...                      |
| Guralnik <i>et al.</i> , 1992 [47]  | Skyrme model   | ...             | 1.6                      | 1.3                      |
| Colangelo <i>et al.</i> , 1994 [48] | Sum rules  | 0.15 - 0.55     | ...                      | ...                      |
| Belyaev <i>et al.</i> , 1994 [49]   | Sum rules  | $0.32 \pm 0.02$ | ...                      | ...                      |
| Dosch and Narison, 1995 [50]        | Sum rules  | $0.15 \pm 0.03$ | ...                      | ...                      |
| Colangelo and Fazio, 1997 [51]      | Sum rules  | 0.09 - 0.44     | ...                      | ...                      |
| Pirjol and Yan, 1997 [52]           | Sum rules  | ...             | $< \sqrt{6 - g_3^2}$     | $< \sqrt{2}$             |
| Zhu and Dai, 1998 [53]              | Sum rules  | ...             | $1.56 \pm 0.30 \pm 0.30$ | $0.94 \pm 0.06 \pm 0.20$ |
| Cho and Georgi, 1992 [54]           | $\mathcal{B}[D^* \rightarrow D \pi], \mathcal{B}[D^* \rightarrow D \gamma]$                                      | $0.34 \pm 0.48$ | ...                      | ...                      |
| Arnesen <i>et al.</i> , 2005 [57]   | $\mathcal{B}[D_{(s)}^* \rightarrow D_{(s)} \pi], \mathcal{B}[D_{(s)}^* \rightarrow D_{(s)} \gamma], \Gamma[D^*]$ | 0.51            | ...                      | ...                      |
| Li <i>et al.</i> , 2010 [58]        | $d\Gamma[B \rightarrow \pi l \nu]$   | $< 0.87$        | ...                      | ...                      |

- All over the place!
- Precise calculation needed

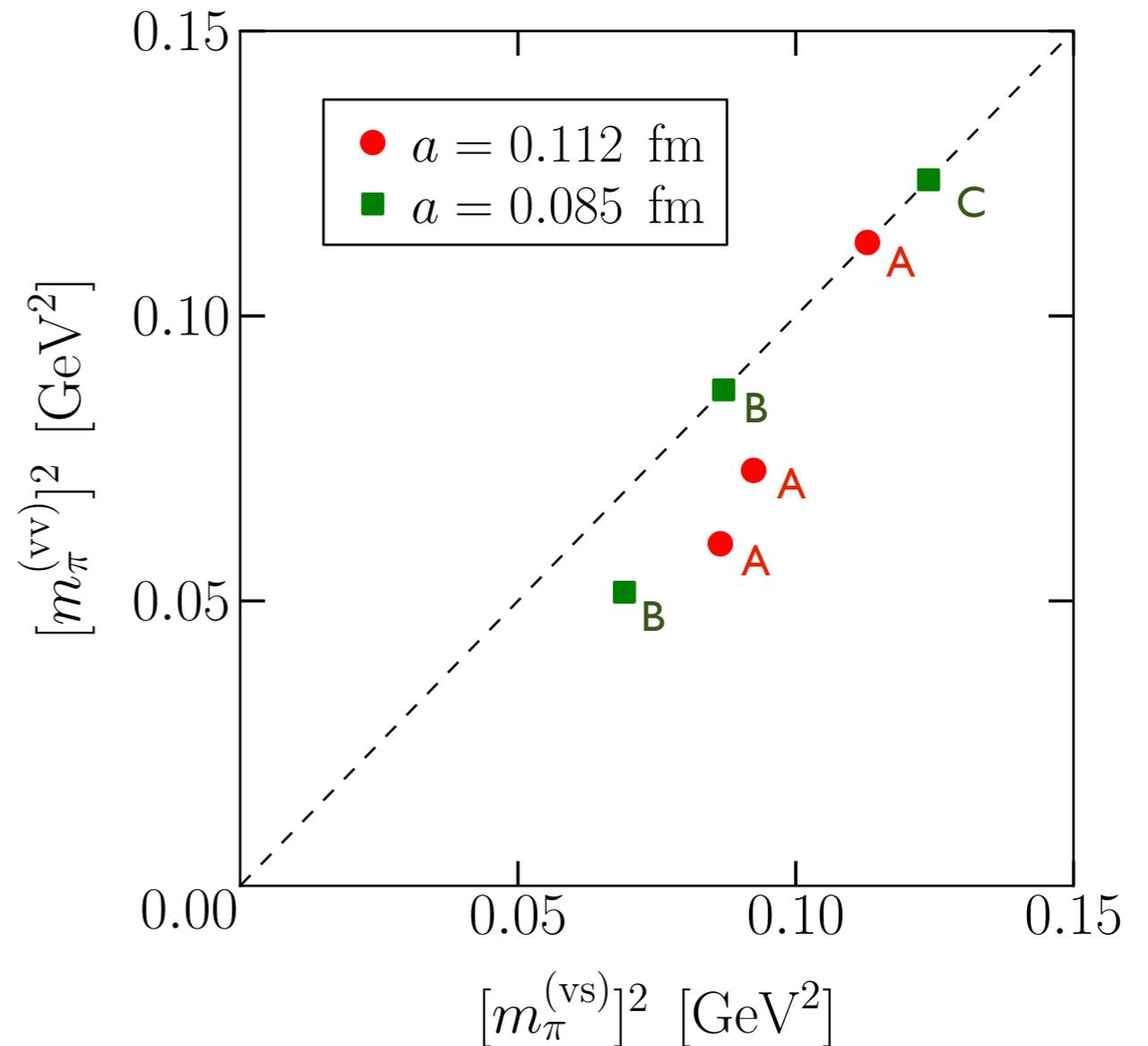
# Actions and ensembles

- Domain-wall light quarks [RBC/UKQCD]
- Lattice chiral symmetry
- Static heavy quarks with  $n_{\text{HYP}}=0,1,2,3,5,10$  levels of HYP smearing
- Two lattice spacings  $a = 0.085, 0.112$  fm
- Six valence quark masses  $m_{\pi} = 0.23\text{--}0.35$  GeV
- Single  $(2.5 \text{ fm})^3$  volume

| Ensemble | $a$ (fm)                  | $L^3 \times T$                | $am_{u,d}^{(\text{sea})}$     | $m_{\pi}^{(\text{ss})}$ (MeV) |
|----------|---------------------------|-------------------------------|-------------------------------|-------------------------------|
| A        | 0.1119(17)                | $24^3 \times 64$              | 0.005                         | 336(5)                        |
| B        | 0.0849(12)                | $32^3 \times 64$              | 0.004                         | 295(4)                        |
| C        | 0.0848(17)                | $32^3 \times 64$              | 0.006                         | 352(7)                        |
| Ensemble | $am_{u,d}^{(\text{val})}$ | $m_{\pi}^{(\text{vs})}$ (MeV) | $m_{\pi}^{(\text{vv})}$ (MeV) | $t/a$                         |
| A        | 0.001                     | 294(5)                        | 245(4)                        | 4, 5, ..., 10                 |
| A        | 0.002                     | 304(5)                        | 270(4)                        | 4, 5, ..., 10                 |
| A        | 0.005                     | 336(5)                        | 336(5)                        | 4, 5, ..., 10                 |
| B        | 0.002                     | 263(4)                        | 227(3)                        | 6, 9, 12                      |
| B        | 0.004                     | 295(4)                        | 295(4)                        | 6, 9, 12                      |
| C        | 0.006                     | 352(7)                        | 352(7)                        | 13                            |

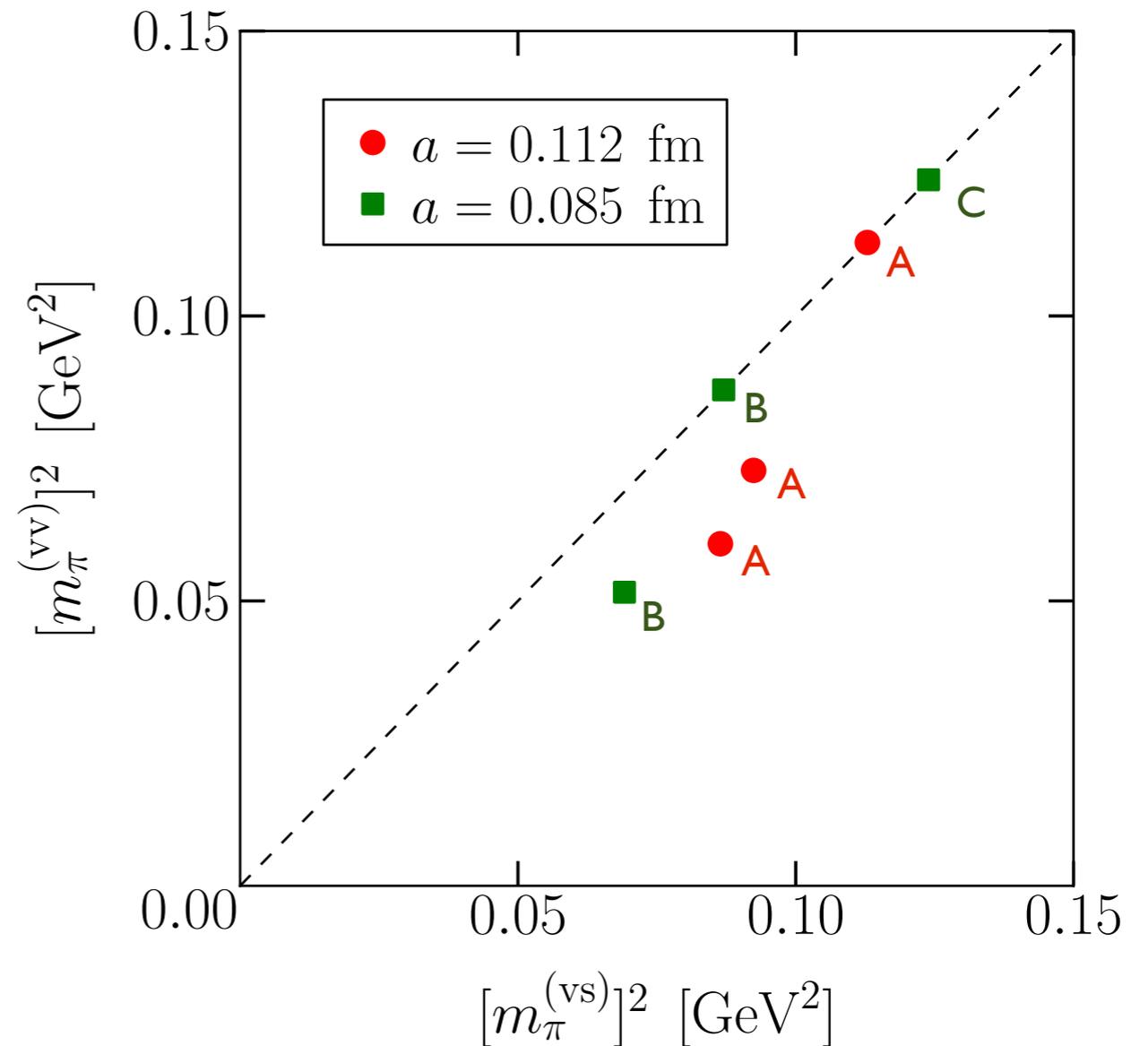
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- $O(a)$  improved\* axial current:

$$Z_A = \begin{cases} 0.7019(26) & \text{for } a = 0.112 \text{ fm,} \\ 0.7396(17) & \text{for } a = 0.085 \text{ fm.} \end{cases} \quad [\text{RBC}]$$

# Correlation functions

- Interpolating operators in static limit

$$P^i = \bar{Q}_{a\alpha} (\gamma_5)_{\alpha\beta} \tilde{q}_{a\beta}^i,$$

$$P_\mu^{*i} = \bar{Q}_{a\alpha} (\gamma_\mu)_{\alpha\beta} \tilde{q}_{a\beta}^i,$$

$$S_{\mu\alpha}^{ij} = \epsilon_{abc} (C\gamma_\mu)_{\beta\gamma} \tilde{q}_{a\beta}^i \tilde{q}_{b\gamma}^j Q_{c\alpha},$$

$$T_\alpha^{ij} = \epsilon_{abc} (C\gamma_5)_{\beta\gamma} \tilde{q}_{a\beta}^i \tilde{q}_{b\gamma}^j Q_{c\alpha}.$$

- Two point and three point correlation functions

$$C[P^u P_u^\dagger](t) = \sum_{\mathbf{x}} \langle P^u(\mathbf{x}, t) P_u^\dagger(0) \rangle,$$

$$C[P^{*d} P_d^{*\dagger}]^{\mu\nu}(t) = \sum_{\mathbf{x}} \langle P^{*d\mu}(\mathbf{x}, t) P_d^{*\nu\dagger}(0) \rangle,$$

$$C[S^{dd} \bar{S}_{dd}]_{\alpha\beta}^{\mu\nu}(t) = \sum_{\mathbf{x}} \langle S_\alpha^{dd\mu}(\mathbf{x}, t) \bar{S}_{dd\beta}^\nu(0) \rangle,$$

$$C[S^{du} \bar{S}_{du}]_{\alpha\beta}^{\mu\nu}(t) = \sum_{\mathbf{x}} \langle S_\alpha^{du\nu}(\mathbf{x}, t) \bar{S}_{du\beta}^\nu(0) \rangle,$$

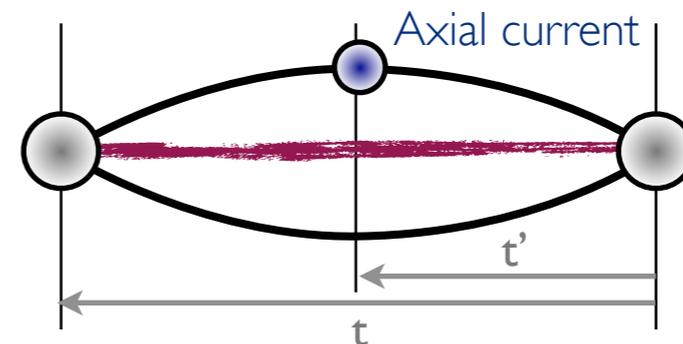
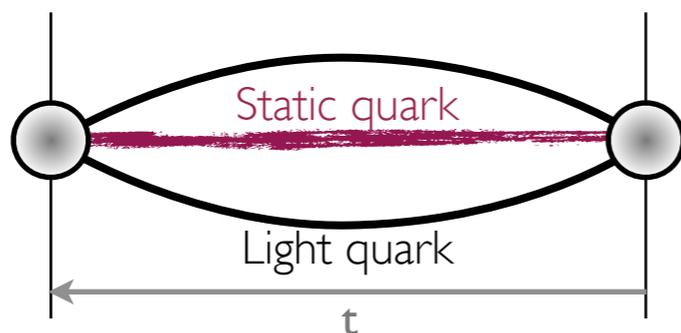
$$C[T^{du} \bar{T}_{du}]_{\alpha\beta}(t) = \sum_{\mathbf{x}} \langle T_\alpha^{du}(\mathbf{x}, t) \bar{T}_{du\beta}(0) \rangle.$$

$$C[P^{*d} A P_u^\dagger]^{\mu\nu}(t, t') = \sum_{\mathbf{x}} \sum_{\mathbf{x}'} \langle P^{*d\mu}(\mathbf{x}, t) A^\nu(\mathbf{x}', t') P_u^\dagger(0) \rangle,$$

$$C[S^{dd} A \bar{S}_{du}]_{\alpha\beta}^{\mu\nu\rho}(t, t') = \sum_{\mathbf{x}} \sum_{\mathbf{x}'} \langle S_\alpha^{dd\mu}(\mathbf{x}, t) A^\nu(\mathbf{x}', t') \bar{S}_{du\beta}^\rho(0) \rangle,$$

$$C[S^{dd} A \bar{T}_{du}]_{\alpha\beta}^{\mu\nu}(t, t') = \sum_{\mathbf{x}} \sum_{\mathbf{x}'} \langle S_\alpha^{dd\mu}(\mathbf{x}, t) A^\nu(\mathbf{x}', t') \bar{T}_{du\beta}(0) \rangle,$$

$$C[T^{du} A^\dagger \bar{S}_{dd}]_{\alpha\beta}^{\mu\nu}(t, t') = \sum_{\mathbf{x}} \sum_{\mathbf{x}'} \langle T_\alpha^{du}(\mathbf{x}, t) A^{\mu\dagger}(\mathbf{x}', t') \bar{S}_{dd\beta}^\nu(0) \rangle.$$



- Calculate with forward propagators from 2 sources

# Correlator ratios

- Ratios of 3pt to 2pt correlation functions give effective couplings

$$R_1(t, t') = -\frac{\frac{1}{3} \sum_{\mu=1}^3 C[P^{*d} A P_u^\dagger]^{\mu\mu}(t, t')}{C[P^u P_u^\dagger](t)} \xrightarrow{t, t' \rightarrow \infty} (g_1)_{\text{eff}}$$

$$R_2(t, t') = 2 \frac{\frac{i}{6} \sum_{\mu, \nu, \rho=1}^3 \epsilon_{0\mu\nu\rho} C[S^{dd} A \bar{S}_{du}]^{\mu\nu\rho}(t, t')}{\frac{1}{3} \sum_{\mu=1}^3 C[S^{dd} \bar{S}_{dd}]^{\mu\mu}(t)} \xrightarrow{t, t' \rightarrow \infty} (g_2)_{\text{eff}}$$

- For transition coupling, double ratio

$$R_3(t, t') = \sqrt{\frac{\left[ \frac{1}{3} \sum_{\mu=1}^3 C[S^{dd} A \bar{T}_{du}]^{\mu\mu}(t, t') \right] \left[ \frac{1}{3} \sum_{\mu=1}^3 C[T^{du} A^\dagger \bar{S}_{dd}]^{\mu\mu}(t, t') \right]}{\left[ \frac{1}{3} \sum_{\mu=1}^3 C[S^{dd} \bar{S}_{dd}]^{\mu\mu}(t) \right] [C[T^{du} \bar{T}_{du}](t)]}} \xrightarrow{t, t' \rightarrow \infty} (g_3)_{\text{eff}}$$

- Excited state contributions important for  $t, t' < \infty$

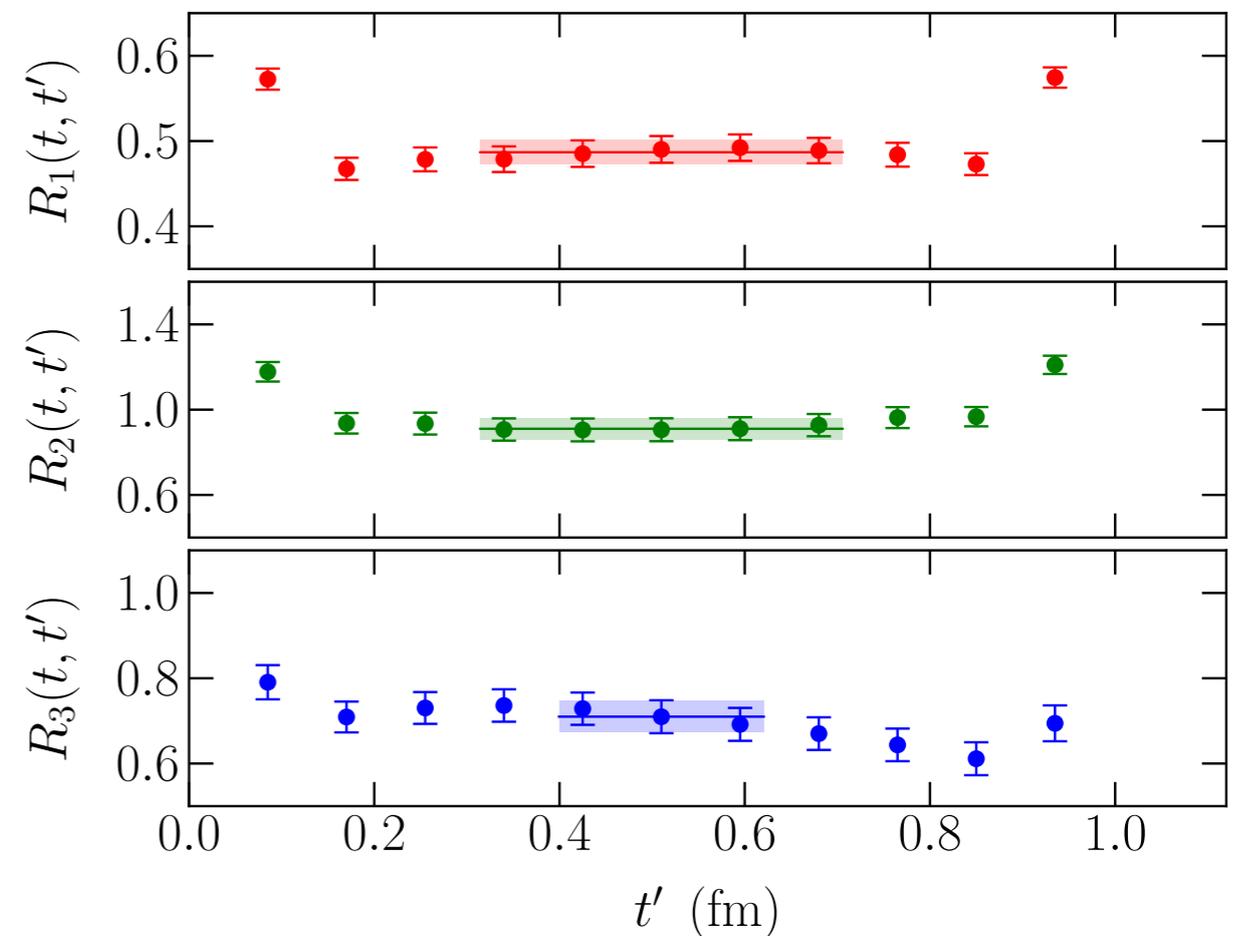
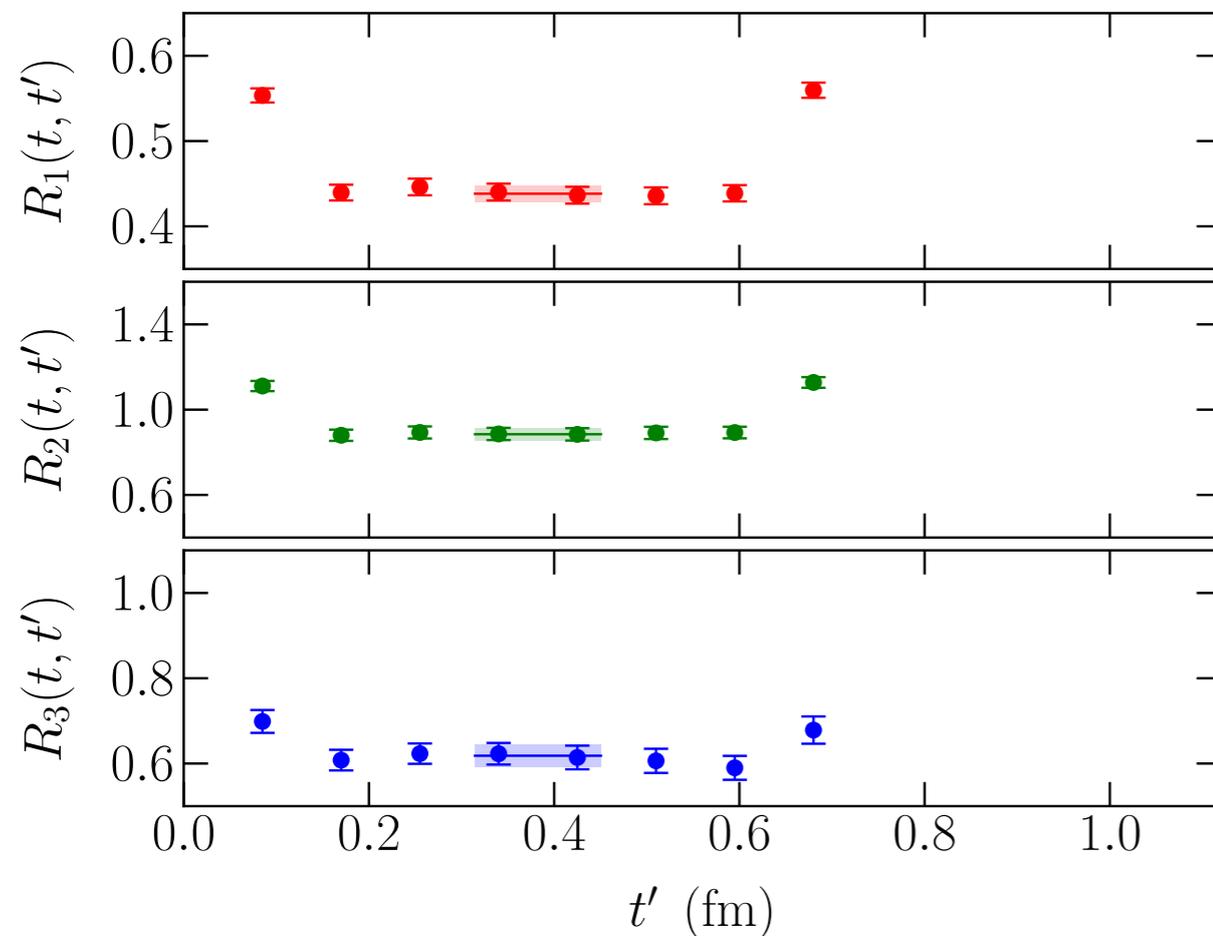
E.g.

$$R_2(t, t/2) = A_{11}^{(SS)} + \left| \frac{Z_{S,2}}{Z_{S,1}} \right|^2 (A_{22}^{(SS)} - A_{11}^{(SS)}) e^{-\delta_S t} + 2 \Re \left[ \frac{Z_{S,1} Z_{S,2}^*}{|Z_{S,1}|^2} A_{12}^{(SS)} \right] e^{-\frac{1}{2} \delta_S t} + \dots$$

with energy gap  $\delta_S = E_{S,2} - E_{S,1}$

# Correlator ratios

- Ratios for varying operator insertion time



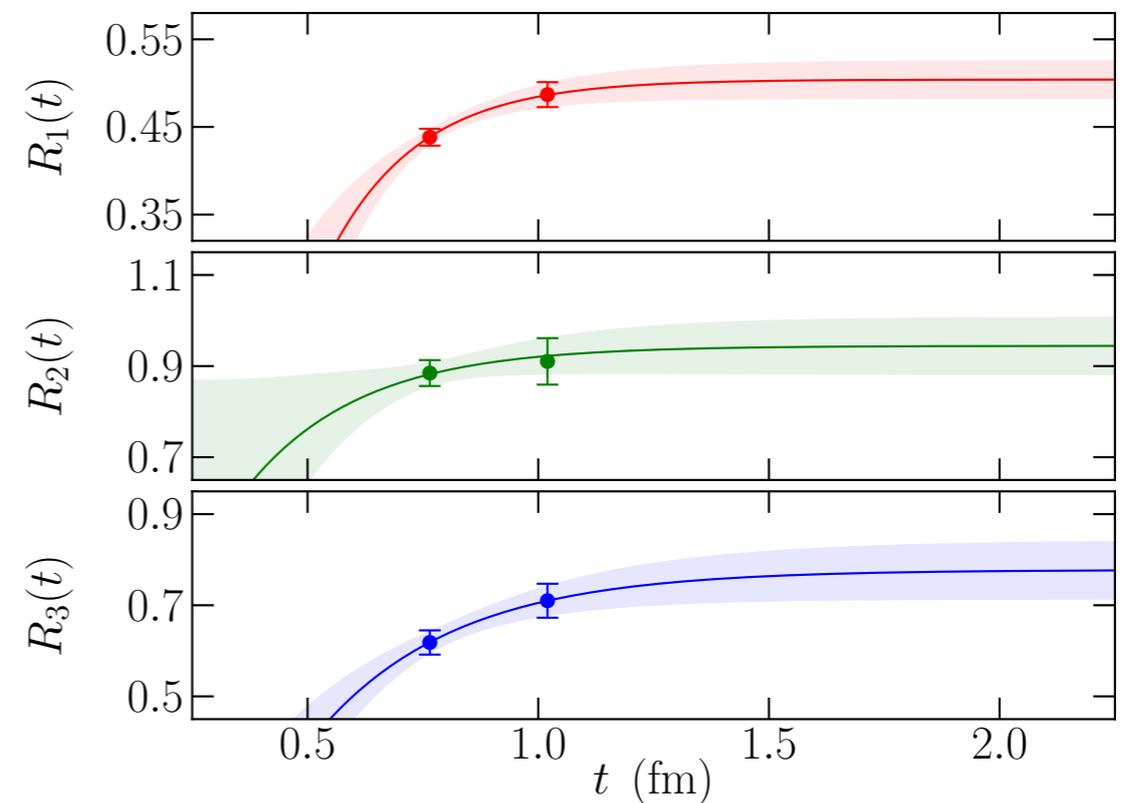
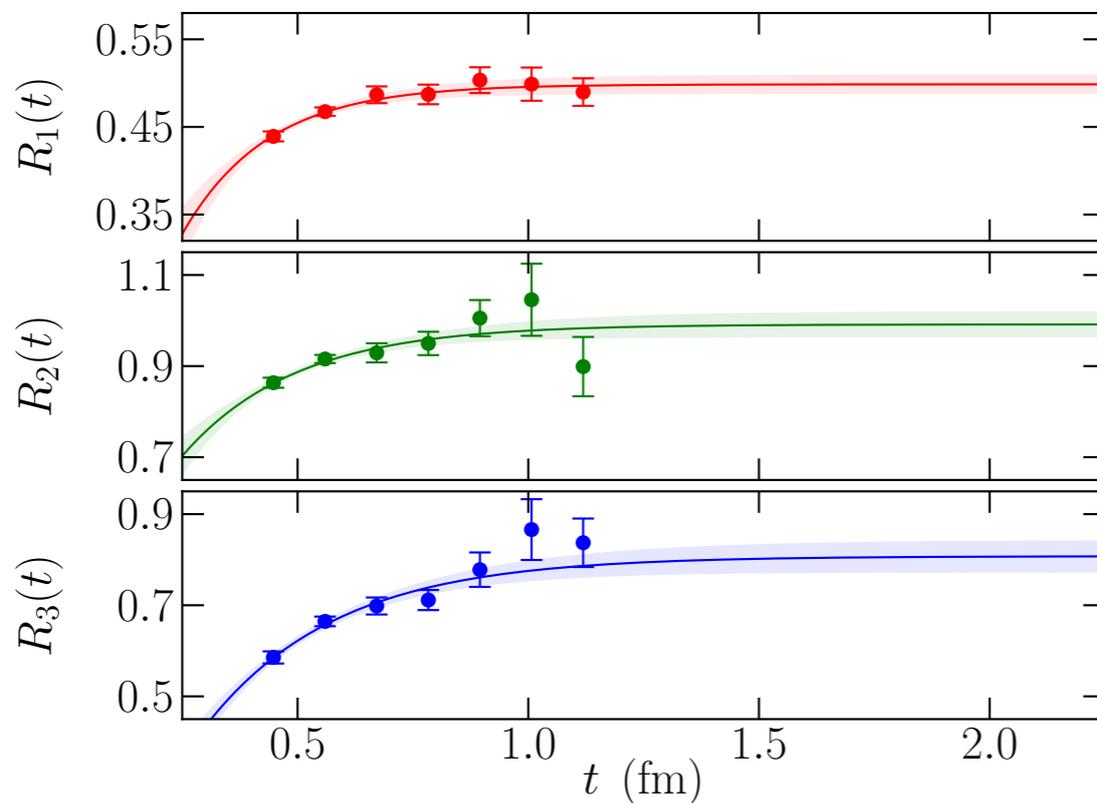
- Negligible  $t'$  dependence away from source/sink
- No evidence for transition matrix elements

# Source-sink separation

- Extract effective axial couplings  $(g_i)_{\text{eff}}$  from  $t$  extrapolation

$$R_i(t, a, m_\pi, n_{\text{HYP}}) = (g_i)_{\text{eff}}(a, m_\pi, n_{\text{HYP}}) - A_i(a, m_\pi, n_{\text{HYP}})e^{-\delta_i(a, m_\pi, n_{\text{HYP}})t}$$

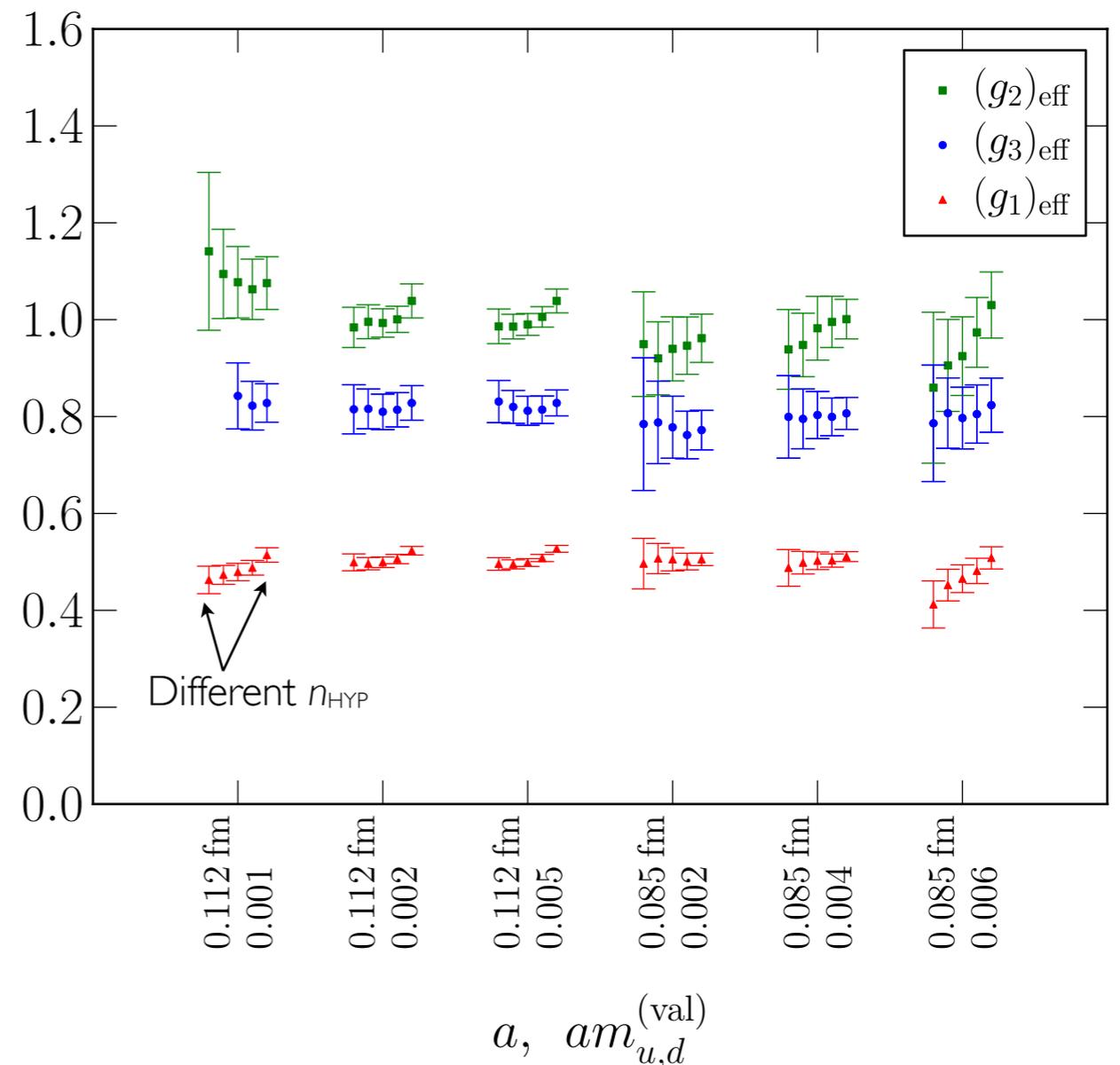
- Constrain  $\delta_i$  for  $a=0.086$  fm from  $\delta_i$  at  $a=0.112$  fm



- Fitted gaps:  $\delta_i \sim 0.7\text{--}1.0$  GeV

# Source-sink separation

- Extracted effective couplings  $(g_i)_{\text{eff}}(a, m_\pi, n_{\text{HYP}})$
- Estimate systematic uncertainty in extrapolation
- Remove 1 or 2 points
- Add second exp with Gaussian priors
- 2, 3, 5% for  $g_{1,2,3}$



# Chiral and continuum extrapolation

- Use NLO partially quenched SU(4|2) HH $\chi$ PT at finite volume and include polynomial discretisation effects

$$(g_1)_{\text{eff}}(a, m, n_{\text{HYP}}) = \textcircled{g_1} \left[ 1 - \frac{2}{f^2} \mathcal{I}(m_\pi^{(\text{vs})}) + \frac{\textcircled{g_1^2}}{f^2} \left\{ 4 \mathcal{H}(m_\pi^{(\text{vs})}, 0) - 4 \delta_{VS}^2 \mathcal{H}_{\eta'}(m_\pi^{(\text{vv})}, 0) \right\} + c_1^{(\text{vv})} [m_\pi^{(\text{vv})}]^2 + c_1^{(\text{vs})} [m_\pi^{(\text{vs})}]^2 + d_{1, n_{\text{HYP}}} a^2 \right].$$

Partial quenching

Loop functions

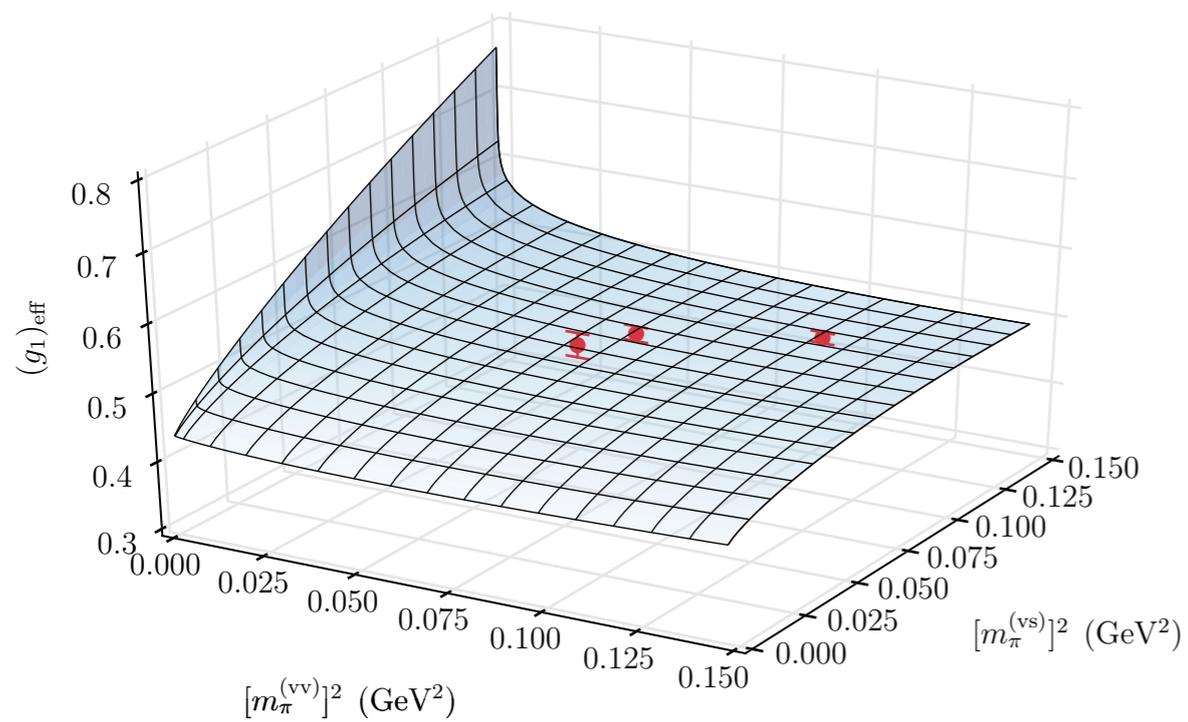
$$(g_2)_{\text{eff}}(a, m, n_{\text{HYP}}) = \textcircled{g_2} \left[ 1 - \frac{2}{f^2} \mathcal{I}(m_\pi^{(\text{vs})}) + \frac{\textcircled{g_2^2}}{f^2} \left\{ \frac{3}{2} \mathcal{H}(m_\pi^{(\text{vs})}, 0) - \delta_{VS}^2 \mathcal{H}_{\eta'}(m_\pi^{(\text{vv})}, 0) \right\} + \frac{\textcircled{g_3^2}}{f^2} \left\{ 2 \mathcal{H}(m_\pi^{(\text{vs})}, -\Delta) - \mathcal{H}(m_\pi^{(\text{vv})}, -\Delta) - 2 \mathcal{K}(m_\pi^{(\text{vs})}, -\Delta, 0) \right\} + c_2^{(\text{vv})} [m_\pi^{(\text{vv})}]^2 + c_2^{(\text{vs})} [m_\pi^{(\text{vs})}]^2 + d_{2, n_{\text{HYP}}} a^2 \right],$$

Lattice spacing effects depend on  $n_{\text{HYP}}$

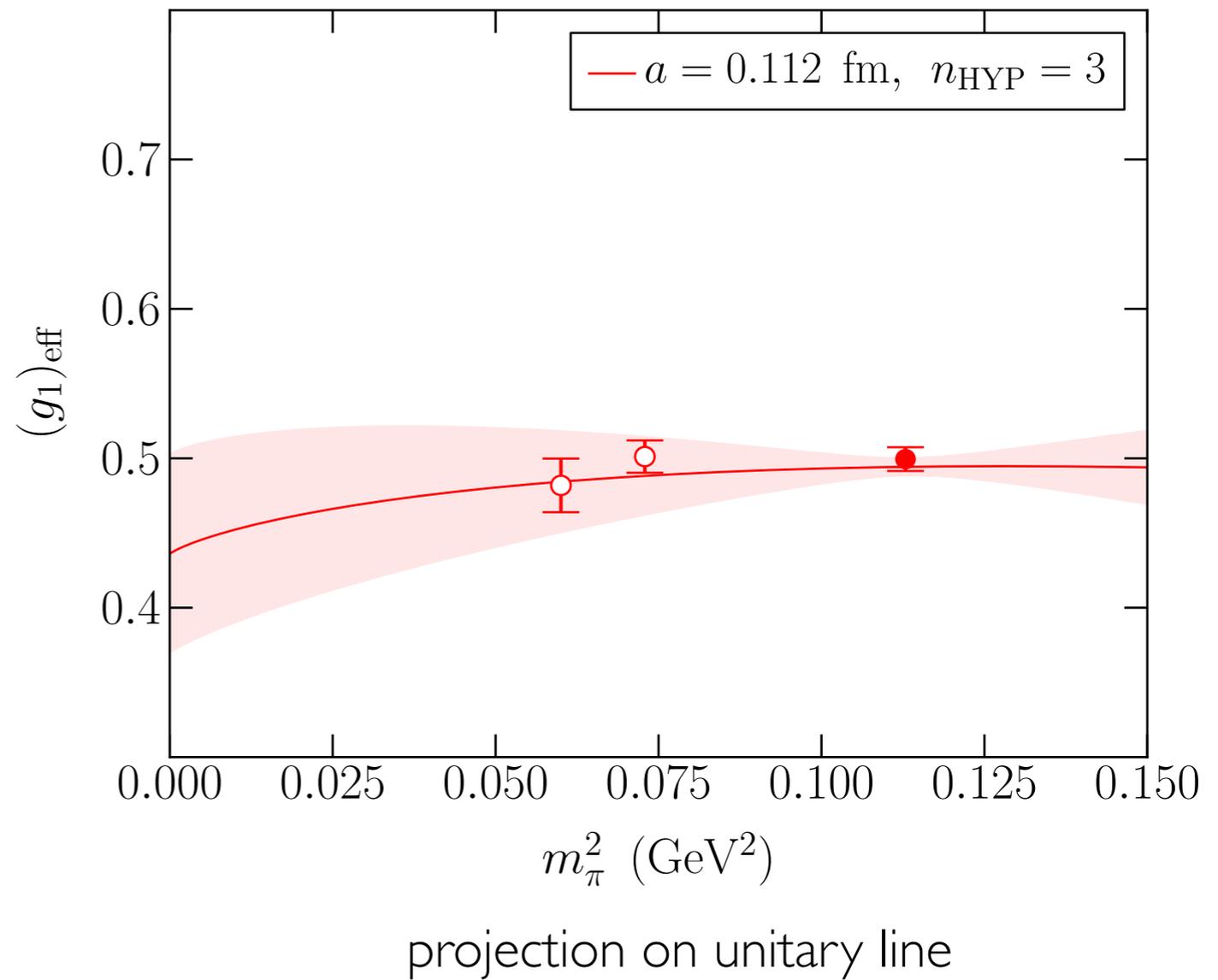
$$(g_3)_{\text{eff}}(a, m, n_{\text{HYP}}) = \textcircled{g_3} \left[ 1 - \frac{2}{f^2} \mathcal{I}(m_\pi^{(\text{vs})}) + \frac{\textcircled{g_3^2}}{f^2} \left\{ \mathcal{H}(m_\pi^{(\text{vs})}, -\Delta) - \frac{1}{2} \mathcal{H}(m_\pi^{(\text{vv})}, -\Delta) + \frac{3}{2} \mathcal{H}(m_\pi^{(\text{vv})}, \Delta) + 3 \mathcal{H}(m_\pi^{(\text{vs})}, \Delta) - \mathcal{K}(m_\pi^{(\text{vs})}, \Delta, 0) \right\} + \frac{\textcircled{g_2^2}}{f^2} \left\{ -\mathcal{H}(m_\pi^{(\text{vs})}, \Delta) - \mathcal{H}(m_\pi^{(\text{vv})}, \Delta) + \mathcal{H}(m_\pi^{(\text{vs})}, 0) - \delta_{VS}^2 \mathcal{H}_{\eta'}(m_\pi^{(\text{vv})}, 0) \right\} + c_3^{(\text{vv})} [m_\pi^{(\text{vv})}]^2 + c_3^{(\text{vs})} [m_\pi^{(\text{vs})}]^2 + d_{3, n_{\text{HYP}}} a^2 \right].$$

$g_{2,3}$  extrapolation is coupled

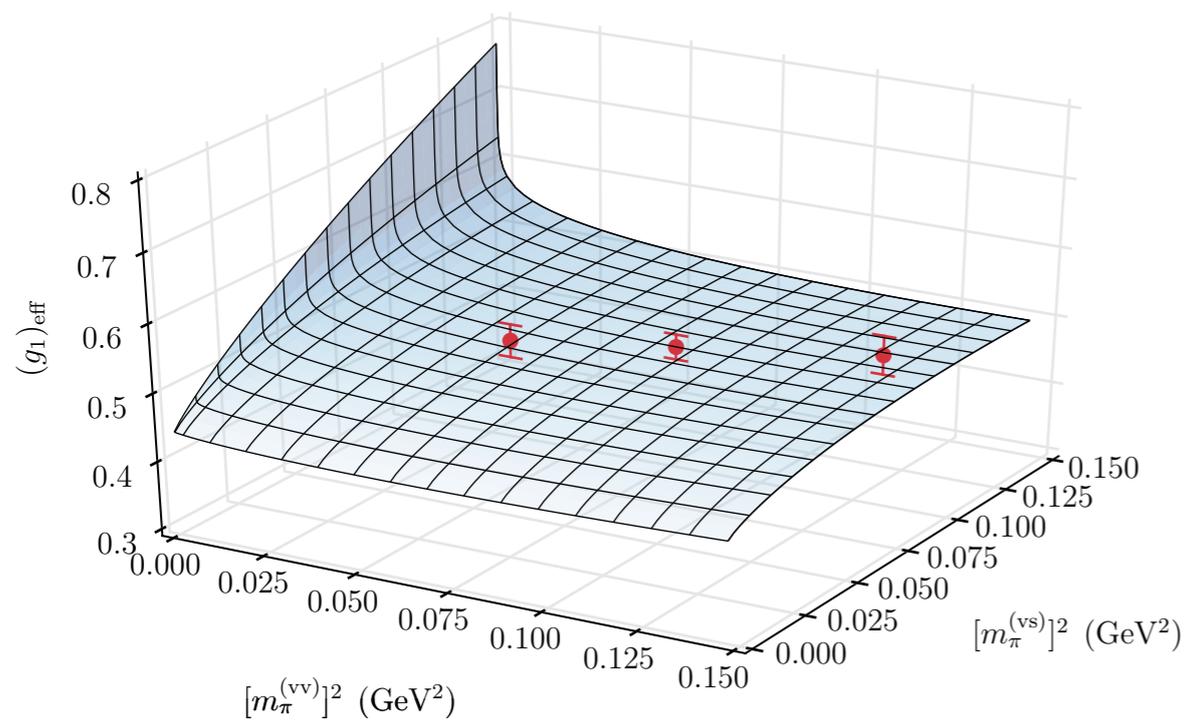
# Chiral and continuum extrapolation



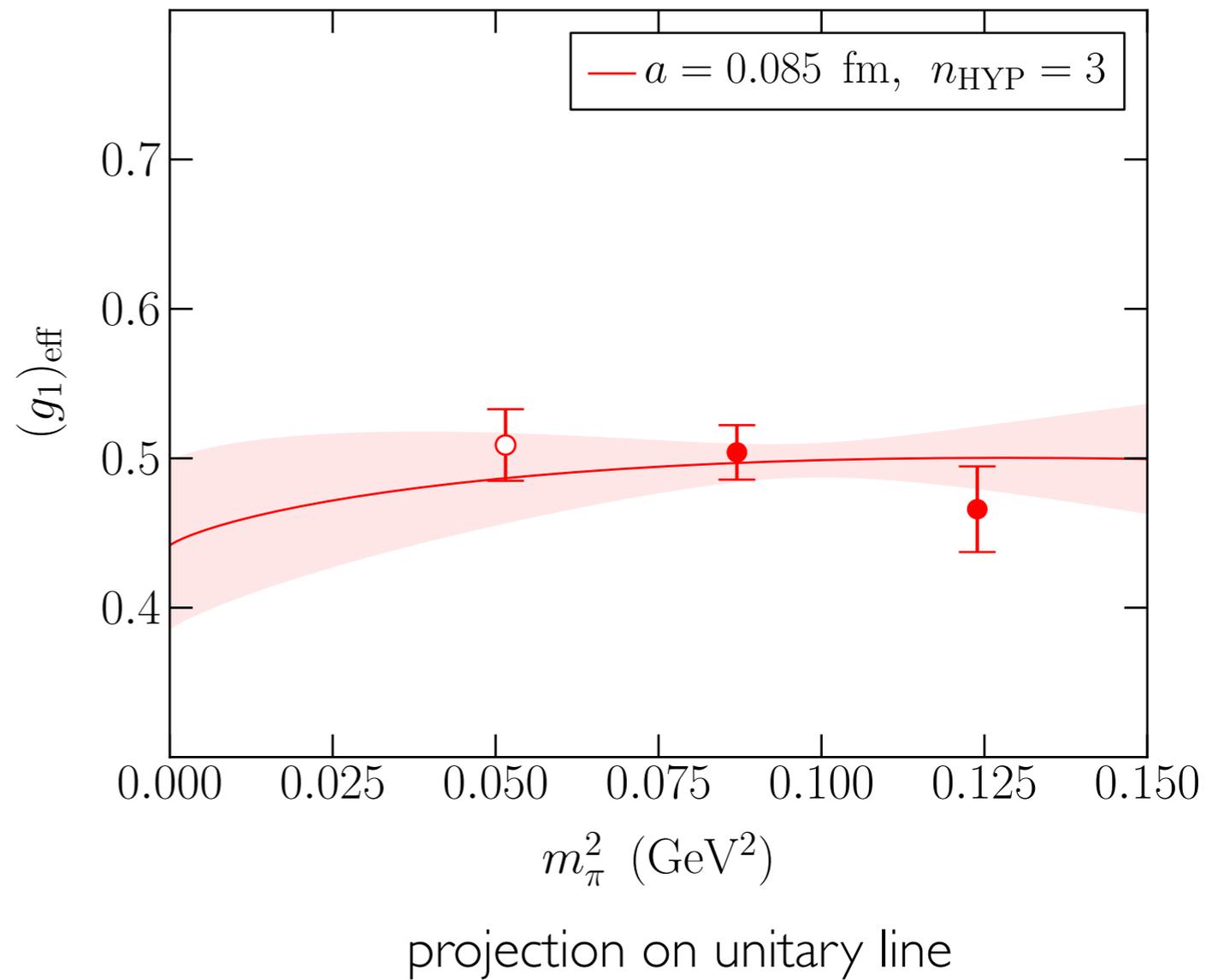
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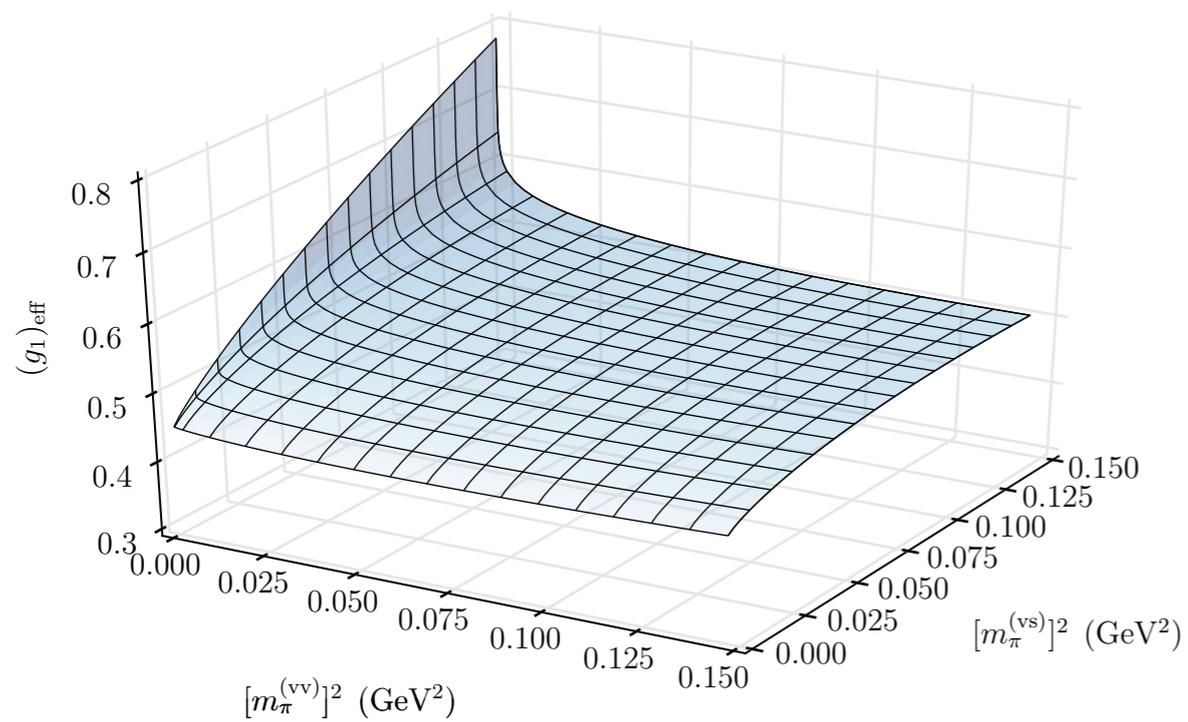
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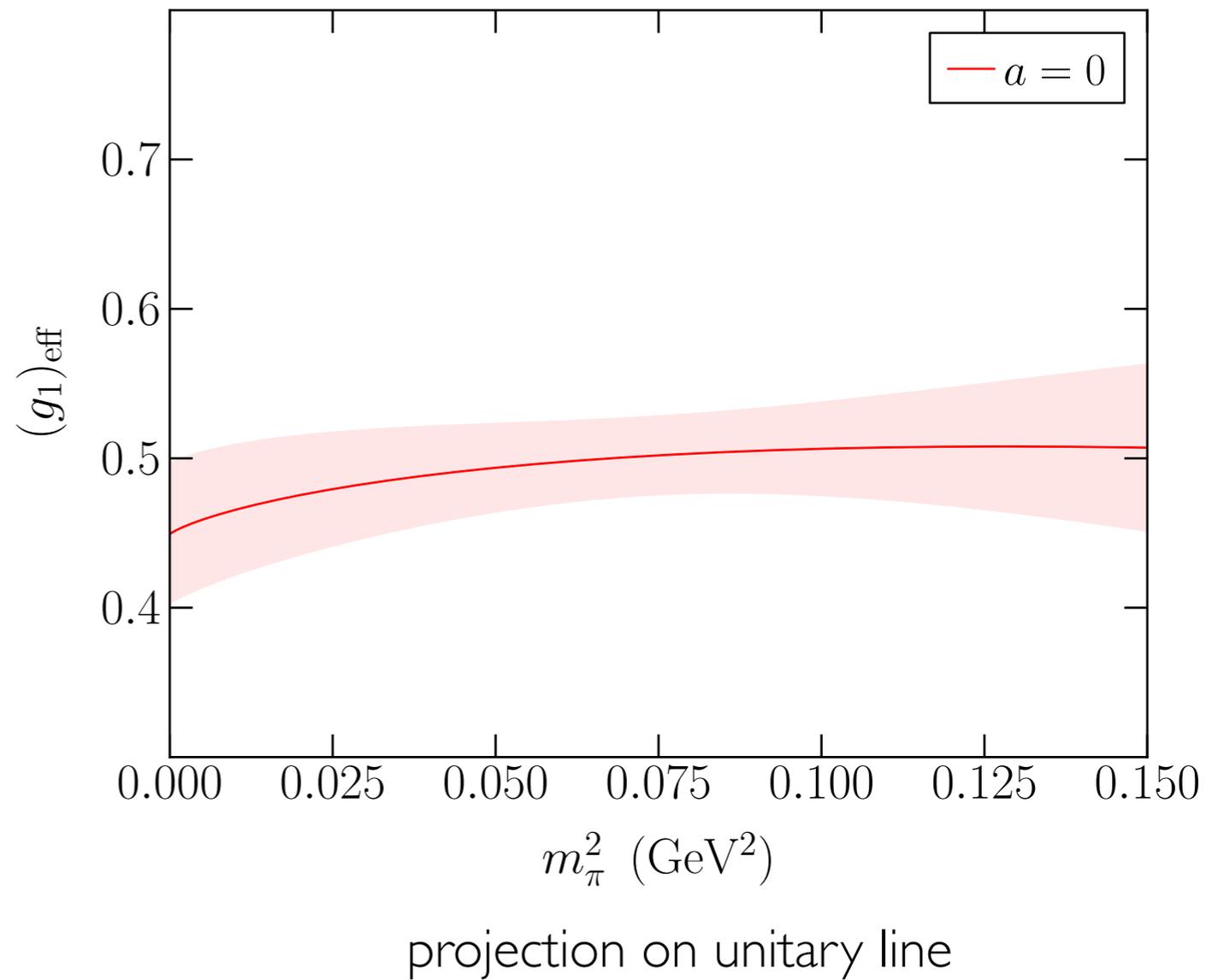
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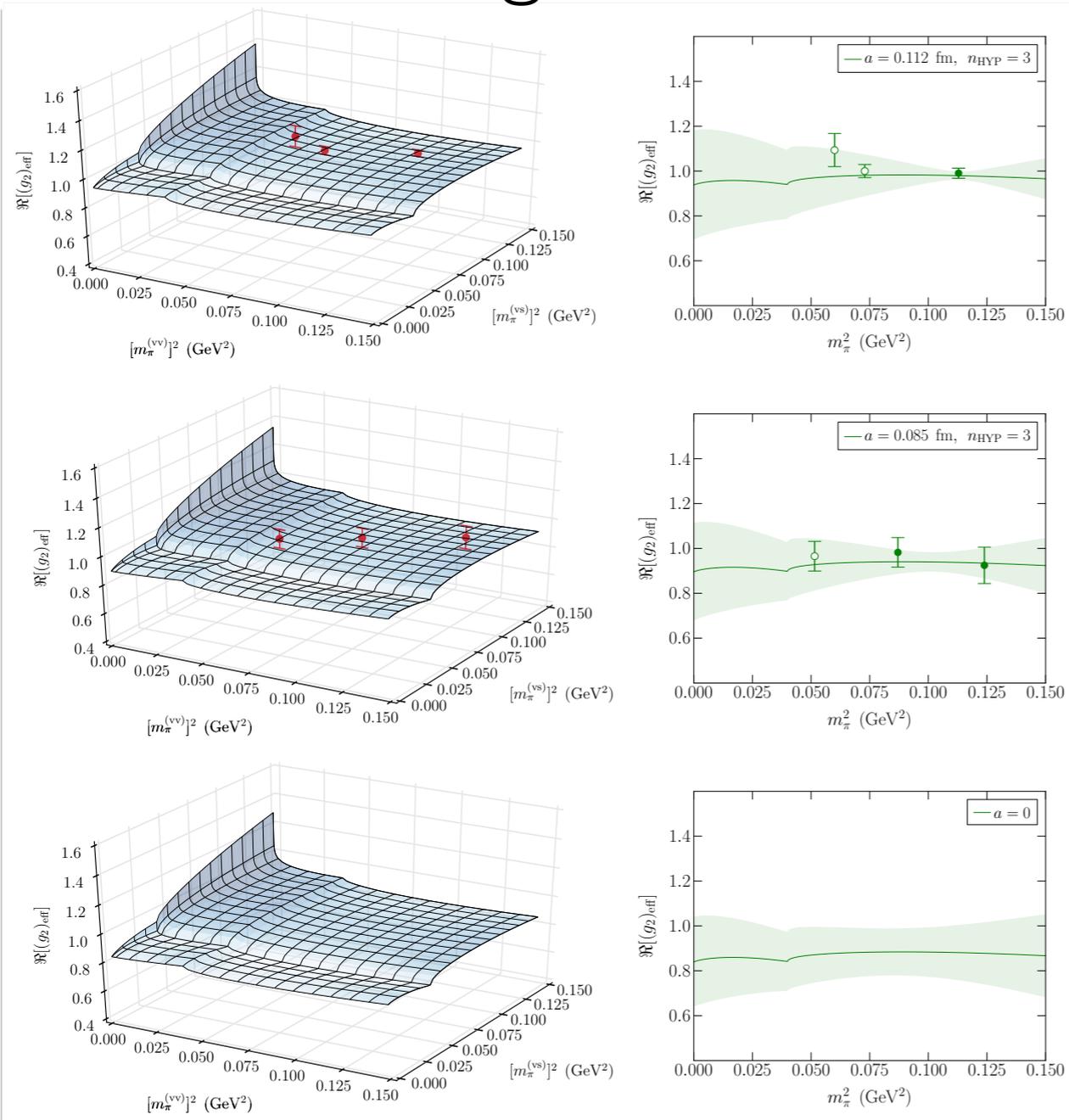


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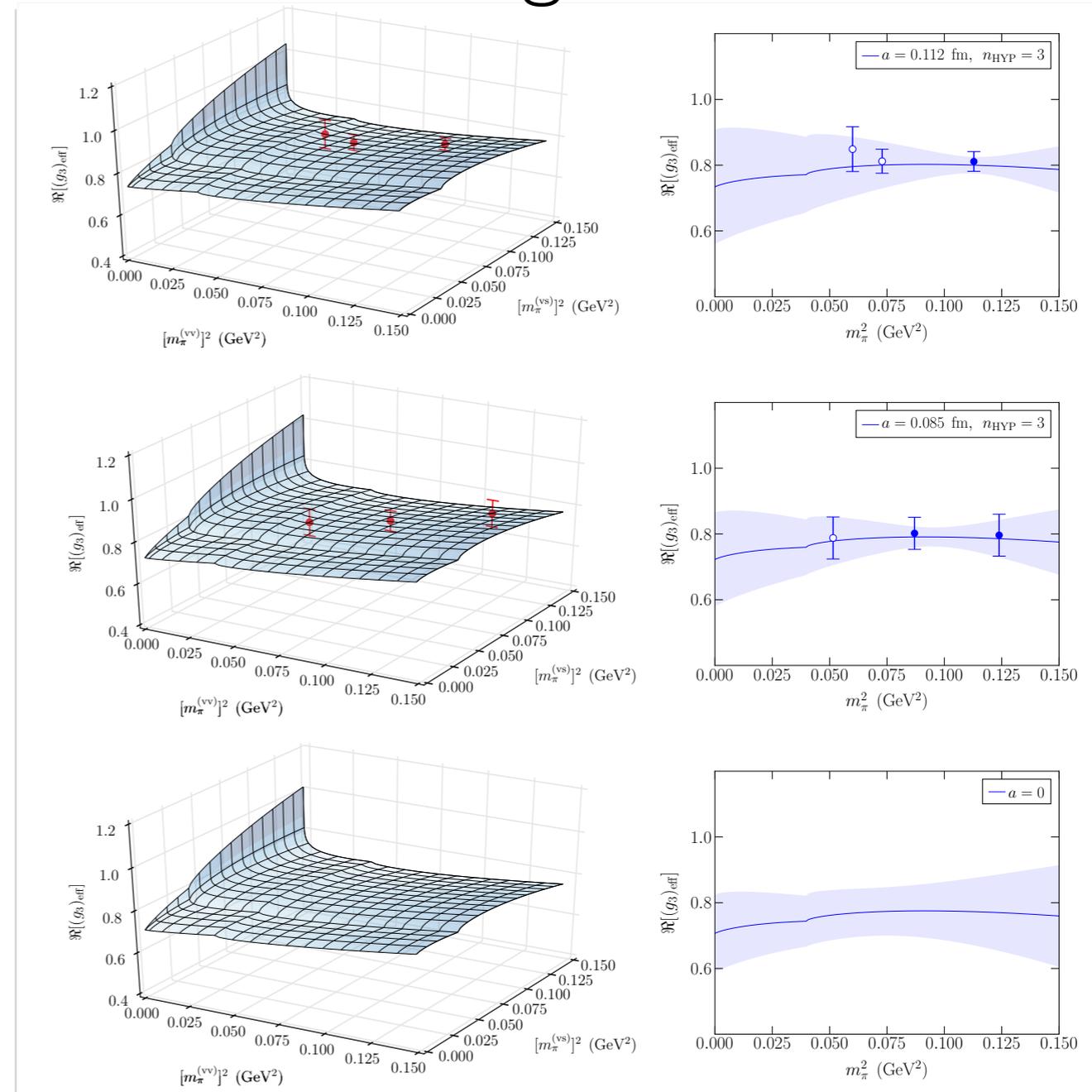


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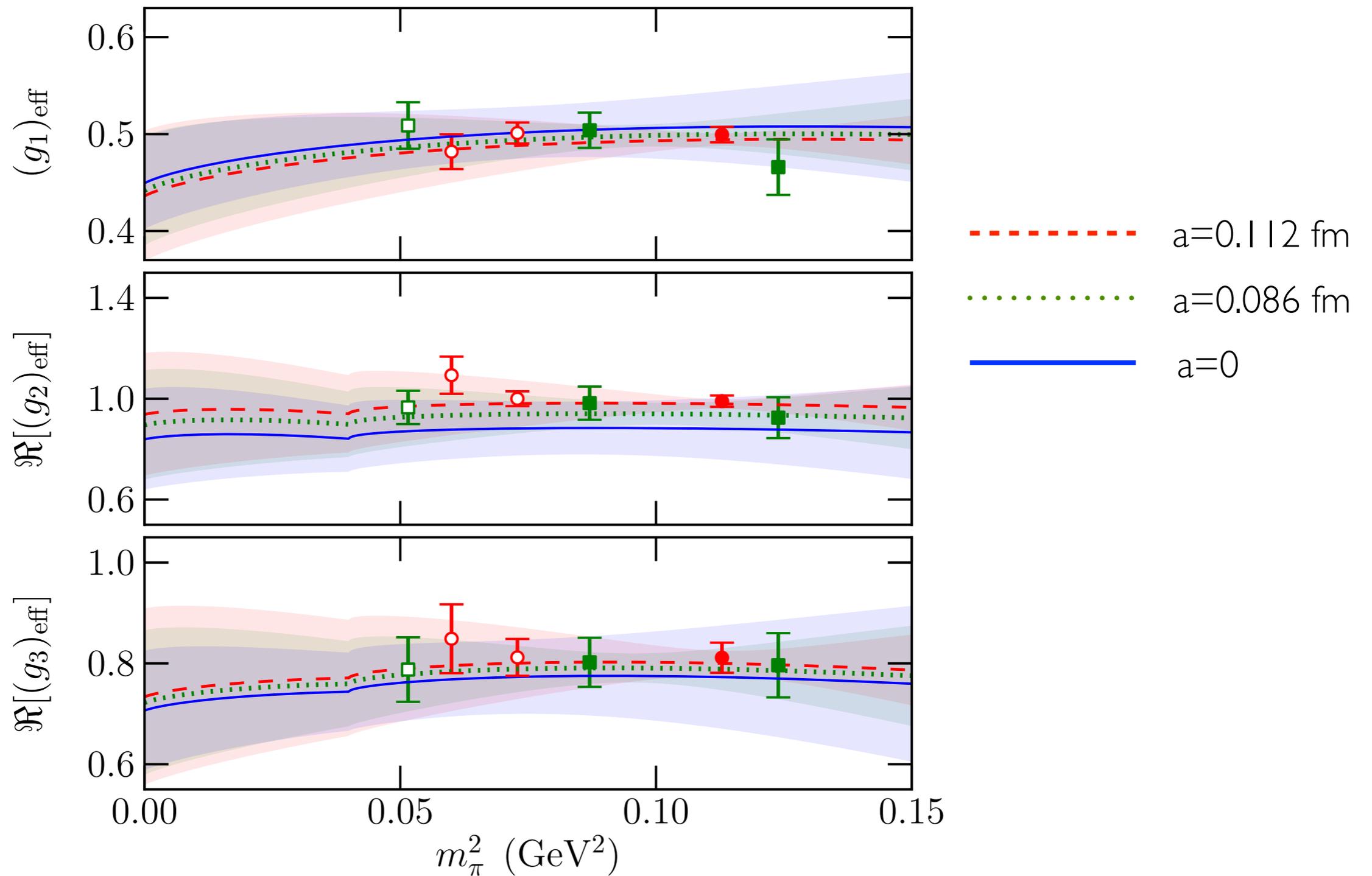
$g_2$



$g_3$



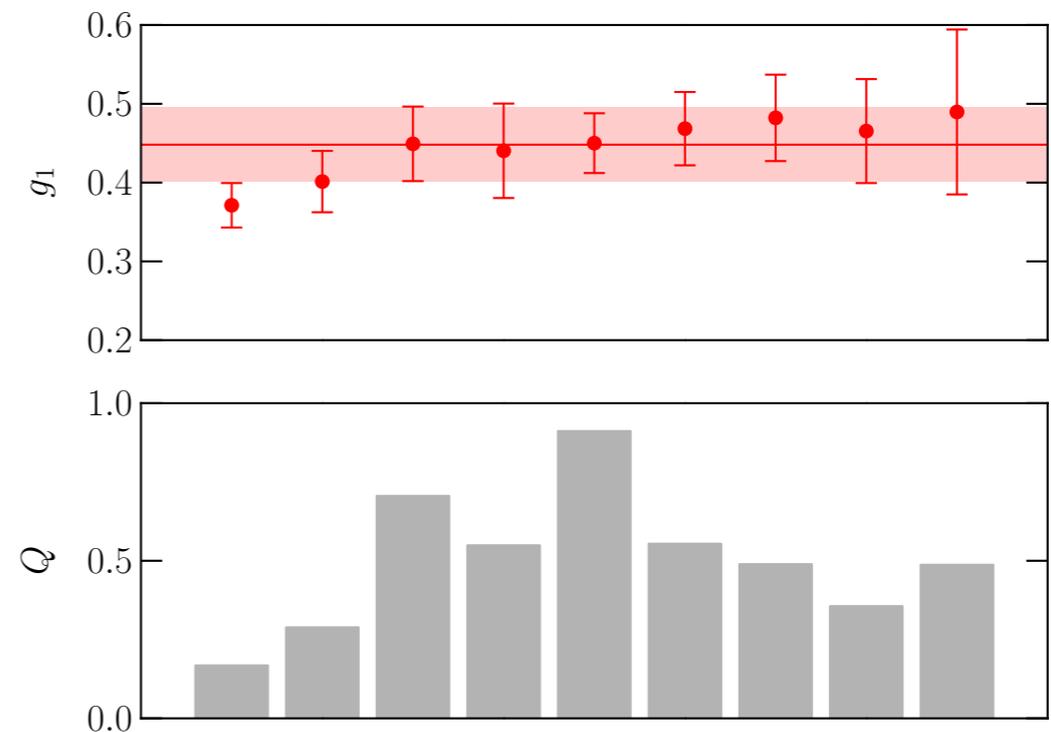
# Chiral and continuum extrapolation



# Chiral and continuum extrapolation

- Various choices of HQ actions to use in fits
- Heavy meson coupling  $g_1$

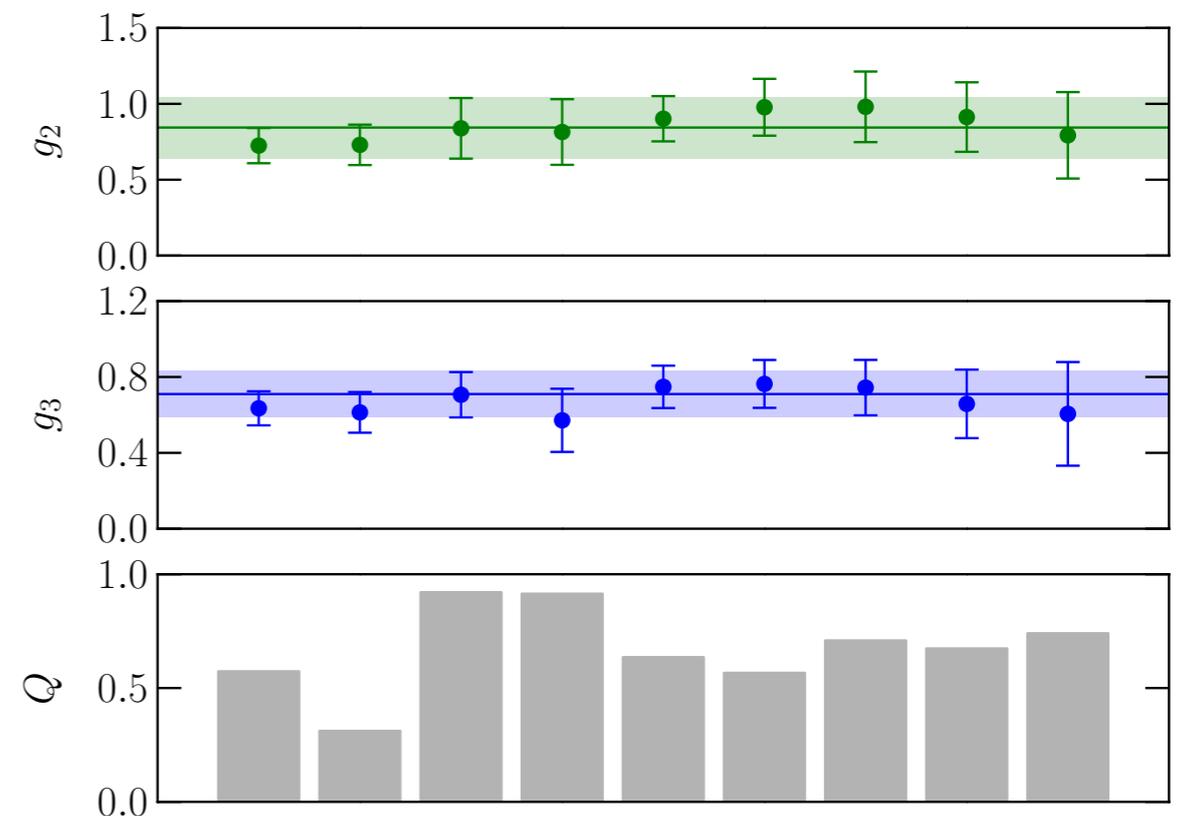
| $n_{\text{HYP}}$ | $g_1$     | d.o.f. | $\chi^2/\text{d.o.f.}$ | $Q$  |
|------------------|-----------|--------|------------------------|------|
| 1, 2, 3, 5, 10   | 0.371(28) | 30 – 8 | 1.3                    | 0.17 |
| 1, 2, 3, 5       | 0.401(39) | 24 – 7 | 1.2                    | 0.29 |
| 1, 2, 3          | 0.449(47) | 18 – 6 | 0.75                   | 0.70 |
| 1, 2             | 0.440(60) | 12 – 5 | 0.85                   | 0.54 |
| 10               | 0.450(38) | 6 – 4  | 0.09                   | 0.91 |
| 5                | 0.468(47) | 6 – 4  | 0.61                   | 0.55 |
| 3                | 0.482(55) | 6 – 4  | 0.73                   | 0.49 |
| 2                | 0.465(66) | 6 – 4  | 1.0                    | 0.36 |
| 1                | 0.49(10)  | 6 – 4  | 0.72                   | 0.49 |



# Chiral and continuum extrapolation

- Various choices of HQ actions to use in fits
- Heavy baryon couplings  $g_{2,3}$

| $n_{\text{HYP}}$ | $g_2$    | $g_3$     | d.o.f.  | $\chi^2/\text{d.o.f.}$ | $Q$  |
|------------------|----------|-----------|---------|------------------------|------|
| 1, 2, 3, 5, 10   | 0.72(12) | 0.635(90) | 58 – 16 | 0.94                   | 0.57 |
| 1, 2, 3, 5       | 0.73(13) | 0.61(11)  | 46 – 14 | 1.1                    | 0.31 |
| 1, 2, 3          | 0.84(20) | 0.71(12)  | 34 – 12 | 0.61                   | 0.92 |
| 1, 2             | 0.81(22) | 0.57(17)  | 22 – 10 | 0.50                   | 0.91 |
| 10               | 0.90(15) | 0.75(11)  | 12 – 8  | 0.64                   | 0.64 |
| 5                | 0.98(19) | 0.76(13)  | 12 – 8  | 0.74                   | 0.57 |
| 3                | 0.98(23) | 0.74(15)  | 12 – 8  | 0.54                   | 0.71 |
| 2                | 0.91(23) | 0.66(18)  | 12 – 8  | 0.51                   | 0.67 |
| 1                | 0.79(29) | 0.61(27)  | 12 – 8  | 0.42                   | 0.74 |



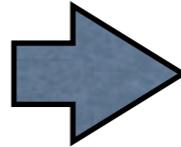
# Higher order terms

- Add higher order analytic terms in quark masses and lattice spacings

$$\begin{aligned}
 (g_i)_{\text{eff}}^{(\text{NLO}+\text{HO})}(a, m, n_{\text{HYP}}) &= (g_i)_{\text{eff}}^{(\text{NLO})}(a, m, n_{\text{HYP}}) \\
 &+ g_i \left[ c_i^{(\text{vv},\text{vv})} [m_{\pi}^{(\text{vv})}]^4 + c_i^{(\text{vs},\text{vs})} [m_{\pi}^{(\text{vs})}]^4 + c_i^{(\text{vv},\text{vs})} [m_{\pi}^{(\text{vv})}]^2 [m_{\pi}^{(\text{vs})}]^2 \right. \\
 &\quad \left. + d_{i, n_{\text{HYP}}}^{(\text{vv})} a^2 [m_{\pi}^{(\text{vv})}]^2 + d_{i, n_{\text{HYP}}}^{(\text{vs})} a^2 [m_{\pi}^{(\text{vs})}]^2 + h_{i, n_{\text{HYP}}} a^4 \right].
 \end{aligned}$$

- Refit with priors on  $c_i$ ,  $d_i$  and  $h_i$

$$\begin{aligned}
 c_i^{(\text{vv},\text{vv})} &= 0 \pm w/\Lambda_{\chi}^4, \\
 c_i^{(\text{vs},\text{vs})} &= 0 \pm w/\Lambda_{\chi}^4, \\
 c_i^{(\text{vv},\text{vs})} &= 0 \pm w/\Lambda_{\chi}^4, \\
 d_{i, n_{\text{HYP}}}^{(\text{vv})} &= 0 \pm w \Lambda_{\text{QCD}}^2/\Lambda_{\chi}^2, \\
 d_{i, n_{\text{HYP}}}^{(\text{vs})} &= 0 \pm w \Lambda_{\text{QCD}}^2/\Lambda_{\chi}^2, \\
 h_{i, n_{\text{HYP}}} &= 0 \pm w \Lambda_{\text{QCD}}^4.
 \end{aligned}$$



| $w$ | $g_1$     | $\delta\sigma(g_1)$ | $g_2$    | $\delta\sigma(g_2)$ | $g_3$    | $\delta\sigma(g_3)$ |
|-----|-----------|---------------------|----------|---------------------|----------|---------------------|
| 0   | 0.449(47) | 0                   | 0.84(20) | 0                   | 0.71(12) | 0                   |
| 1   | 0.449(47) | 0.0020              | 0.84(20) | 0.0023              | 0.71(12) | 0.0045              |
| 5   | 0.452(48) | 0.0089              | 0.84(20) | 0.014               | 0.70(12) | 0.017               |
| 10  | 0.455(50) | 0.016               | 0.84(20) | 0.024               | 0.70(12) | 0.026               |
| 50  | 0.464(72) | 0.054               | 0.82(22) | 0.099               | 0.68(15) | 0.094               |
| 100 | 0.452(94) | 0.082               | 0.78(26) | 0.17                | 0.63(21) | 0.17                |

$$\delta\sigma(g_i) = \sqrt{\sigma^2(g_i)^{(\text{NLO}+\text{HO})} - \sigma^2(g_i)^{(\text{NLO})}},$$

- $w = 10$  gives systematic uncertainty ( $w=1$  is NDA)

# Finite volume effects

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- Finite volume effects computed in HH $\chi$ PT

| $m_{\pi}^{(\text{vs})}$ (MeV) | $m_{\pi}^{(\text{vv})}$ (MeV) | $\frac{(g1)_{\text{eff}}^{(\infty)} - (g1)_{\text{eff}}^{(L)}}{(g1)_{\text{eff}}^{(\infty)}}$ | $\frac{(g2)_{\text{eff}}^{(\infty)} - (g2)_{\text{eff}}^{(L)}}{(g2)_{\text{eff}}^{(\infty)}}$ | $\frac{(g3)_{\text{eff}}^{(\infty)} - (g3)_{\text{eff}}^{(L)}}{(g3)_{\text{eff}}^{(\infty)}}$ |
|-------------------------------|-------------------------------|---|---|---|
| 294                           | 245                           | 0.0057  | 0.015   | 0.0074  |
| 304                           | 270                           | 0.0040  | 0.0070  | 0.0027  |
| 336                           | 336                           | 0.0016  | 0.00037   | -0.00079  |
| 263                           | 227                           | 0.0072  | 0.028   | 0.013   |
| 295                           | 295                           | 0.0031  | 0.00027   | -0.0012   |
| 352                           | 352                           | 0.0013  | 0.00033   | -0.00071  |

- Very small, higher order FV negligible

# Axial couplings

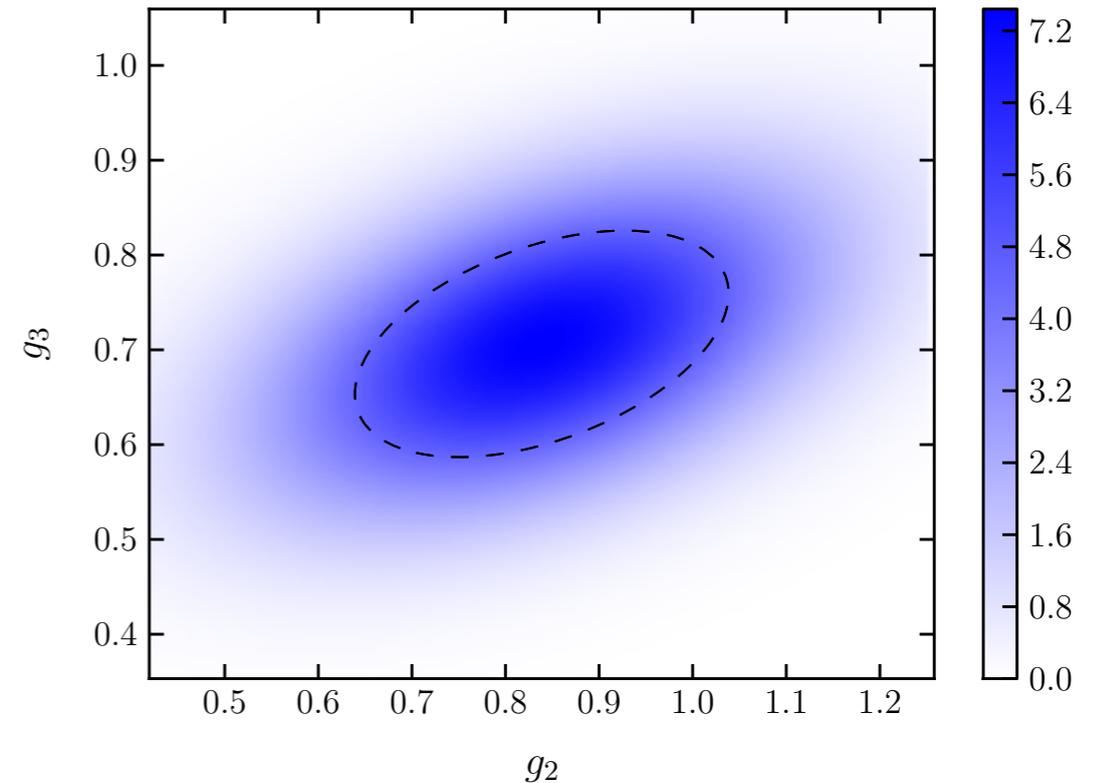
- Final extracted values

$$\begin{aligned} g_1 &= 0.449 \pm 0.047_{\text{stat}} \pm 0.019_{\text{syst}} \\ g_2 &= 0.84 \pm 0.20_{\text{stat}} \pm 0.04_{\text{syst}} \\ g_3 &= 0.71 \pm 0.12_{\text{stat}} \pm 0.04_{\text{syst}} \end{aligned}$$

- Sources of systematic errors

| Source  | $g_1$ | $g_2$ | $g_3$ |
|---|-------|-------|-------|
| NNLO terms in fits of $m_\pi$ - and $a$ -dependence | 3.6%  | 2.8%  | 3.7%  |
| Higher excited states in fits to $R_i(t)$           | 1.7%  | 2.8%  | 4.9%  |
| Unphysical value of $m_s^{(\text{sea})}$            | 1.5%  | 1.5%  | 1.5%  |
| Total   | 4.2%  | 4.3%  | 6.3%  |

- Dominated by statistical errors



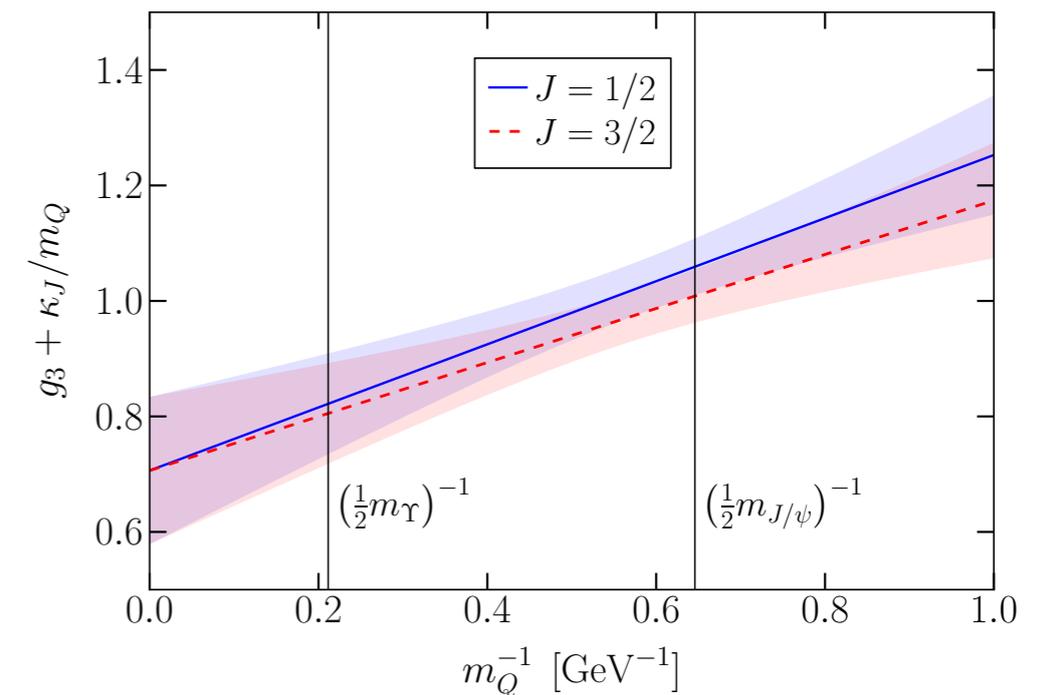
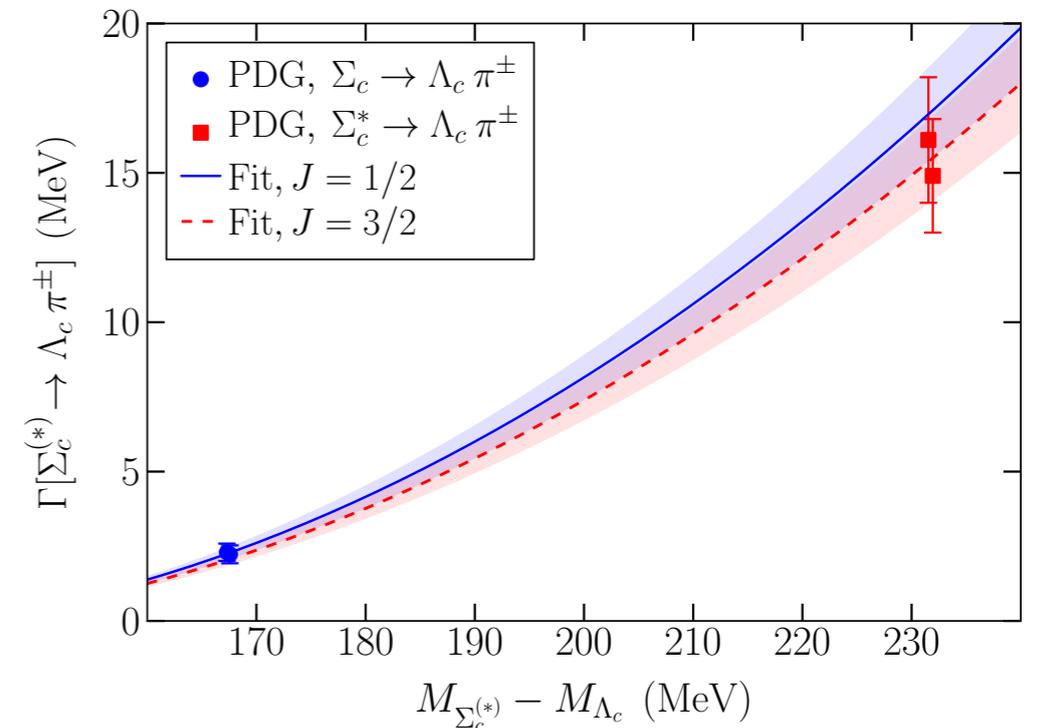
# Decay widths

- Strong decays allowed for heavy baryons

$$\Gamma[S \rightarrow T \pi] = c_f^2 \frac{1}{6\pi f_\pi^2} \left( g_3 + \frac{\kappa_J}{m_Q} \right)^2 \frac{M_T}{M_S} |\mathbf{p}_\pi|^3$$

$$c_f = \begin{cases} 1 & \text{for } \Sigma_Q^{(*)} \rightarrow \Lambda_Q \pi^\pm, \\ 1 & \text{for } \Sigma_Q^{(*)} \rightarrow \Lambda_Q \pi^0, \\ 1/\sqrt{2} & \text{for } \Xi_Q'^{(*)} \rightarrow \Xi_Q \pi^\pm, \\ 1/2 & \text{for } \Xi_Q'^{(*)} \rightarrow \Xi_Q \pi^0. \end{cases}$$

- $1/m_Q$  corrections important: determine from charm sector
- Effective coupling vs  $1/m_Q$
- Valid only at LO in  $\text{HH}\chi\text{PT}$

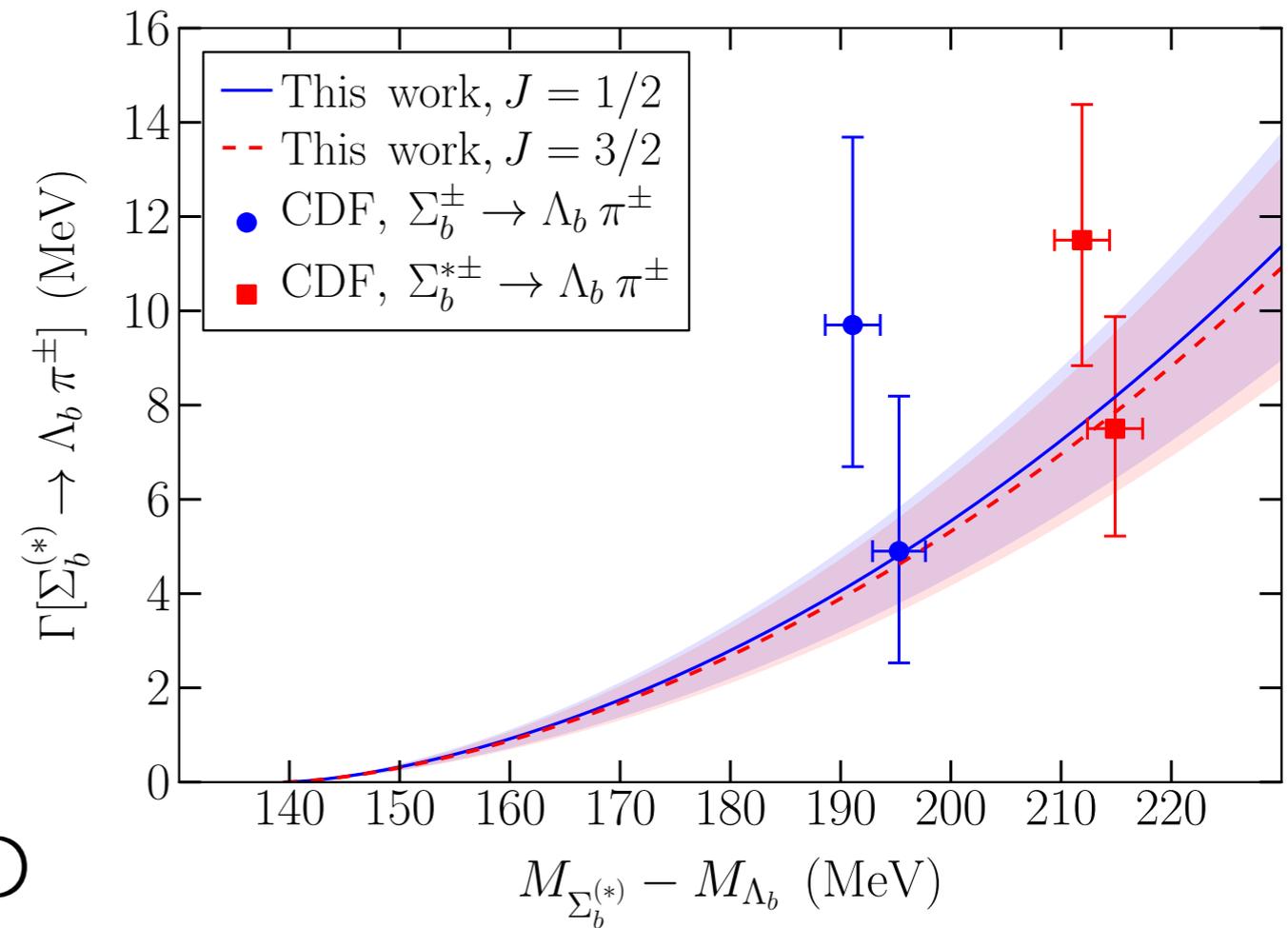


# Decay widths

- Calculate (and predict) bottom and charm baryon decay widths

| Hadron          | This work    | Experiment                        |
|-----------------|--------------|-----------------------------------|
| $\Sigma_b^+$    | 4.2(1.0)     | $9.7^{+3.8+1.2}_{-2.8-1.1}$ [13]  |
| $\Sigma_b^-$    | 4.8(1.1)     | $4.9^{+3.1}_{-2.1} \pm 1.1$ [13]  |
| $\Sigma_b^{*+}$ | 7.3(1.6)     | $11.5^{+2.7+1.0}_{-2.2-1.5}$ [13] |
| $\Sigma_b^{*-}$ | 7.8(1.8)     | $7.5^{+2.2+0.9}_{-1.8-1.4}$ [13]  |
| $\Xi'_b$        | 1.1 (CL=90%) | ...                               |
| $\Xi_b^*$       | 2.8 (CL=90%) | ...                               |
| $\Xi_c^{*+}$    | 2.44(26)     | $< 3.1$ (CL=90%) [70]             |
| $\Xi_c^{*0}$    | 2.78(29)     | $< 5.5$ (CL=90%) [71]             |

- Uses determinations of  $\Xi'_b$ ,  $\Xi_b^*$  masses from LQCD [Lewis & Woloshyn 09]



# Heavy hadron axial couplings

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- First complete calculation of axial couplings controlling all systematics

$$\begin{aligned}g_1 &= 0.449 \pm 0.047_{\text{stat}} \pm 0.019_{\text{syst}} \\g_2 &= 0.84 \pm 0.20_{\text{stat}} \pm 0.04_{\text{syst}} \\g_3 &= 0.71 \pm 0.12_{\text{stat}} \pm 0.04_{\text{syst}}\end{aligned}$$

- Considerably smaller than quark model estimates
- Pleasant consequences for convergence of  $\text{HH}\chi\text{PT}$
- Allows pre- (and post-) dictions of strong decay widths (also  $\Gamma[\Xi_c^* \rightarrow \Xi_c \gamma]$ )



[fin]

# Actions and ensembles

- Further details

| $L^3 \times T$   | $am_s^{(\text{sea})}$ | $am_{u,d}^{(\text{sea})}$ | $am_{u,d}^{(\text{val})}$ | $a$ (fm)   | $m_\pi^{(\text{ss})}$ (MeV) | $m_\pi^{(\text{vs})}$ (MeV) | $m_\pi^{(\text{vv})}$ (MeV) |
|------------------|-----------------------|---------------------------|---------------------------|------------|-----------------------------|-----------------------------|-----------------------------|
| $24^3 \times 64$ | 0.04                  | 0.005                     | 0.001                     | 0.1119(17) | 336(5)                      | 294(5)                      | 245(4)                      |
| $24^3 \times 64$ | 0.04                  | 0.005                     | 0.002                     | 0.1119(17) | 336(5)                      | 304(5)                      | 270(4)                      |
| $24^3 \times 64$ | 0.04                  | 0.005                     | 0.005                     | 0.1119(17) | 336(5)                      | 336(5)                      | 336(5)                      |
| $32^3 \times 64$ | 0.03                  | 0.004                     | 0.002                     | 0.0849(12) | 295(4)                      | 263(4)                      | 227(3)                      |
| $32^3 \times 64$ | 0.03                  | 0.004                     | 0.004                     | 0.0849(12) | 295(4)                      | 295(4)                      | 295(4)                      |
| $32^3 \times 64$ | 0.03                  | 0.006                     | 0.006                     | 0.0848(17) | 352(7)                      | 352(7)                      | 352(7)                      |

- Numbers of measurements

| $L^3 \times T$   | $am_{u,d}^{(\text{val})}$ | $t/a$         | $N_{\text{meas}}$ (approx.) |
|------------------|---------------------------|---------------|-----------------------------|
| $24^3 \times 64$ | 0.001                     | 10            | 550                         |
| $24^3 \times 64$ | 0.001                     | 9, 8, 7, 6    | 240                         |
| $24^3 \times 64$ | 0.001                     | 5             | 460                         |
| $24^3 \times 64$ | 0.001                     | 4             | 120                         |
| $24^3 \times 64$ | 0.002                     | 10            | 880                         |
| $24^3 \times 64$ | 0.002                     | 9, 8, 7, 6, 4 | 240                         |
| $24^3 \times 64$ | 0.002                     | 5             | 480                         |
| $24^3 \times 64$ | 0.005                     | 10            | 960                         |
| $24^3 \times 64$ | 0.005                     | 9, 8, 7, 6, 4 | 240                         |
| $24^3 \times 64$ | 0.005                     | 5             | 480                         |
| $32^3 \times 64$ | 0.002                     | 12            | 1200                        |
| $32^3 \times 64$ | 0.002                     | 9, 6          | 480                         |
| $32^3 \times 64$ | 0.004                     | 12            | 1200                        |
| $32^3 \times 64$ | 0.004                     | 9, 6          | 480                         |
| $32^3 \times 64$ | 0.006                     | 13            | 700                         |