Status and perspectives of B physics from non-perturbative HQET with two dynamical light quarks

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for the ALPHA Collaboration

Beautiful Mesons and Baryons on the Lattice
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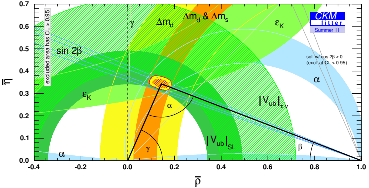
Outline

1. Motivation
2. Obstacles of HQET on the lattice
3. Computational strategy of ALPHA
4. Overview of $N_\mathrm{f} = 2$ large volume (CLS) ensembles
5. Techniques used to compute LV matrix elements
6. First $N_\mathrm{f} = 2$ results
7. Summary & outlook

Motivation

Couplings of flavor-changing *weak interactions*:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



processes with $b \to u$ transitions

- Inclusive $B \to X_u \ell \nu$

  optical theorem and heavy quark expansion

- Exclusive $B \to \pi \ell \nu$

  hadronic formfactor $f_+(q^2)$

- Leptonic $B \to \tau \nu$

  hadronic decay constant f_B

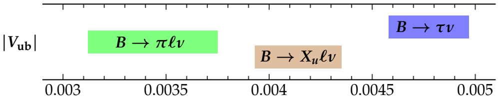
Lattice input

V_{ub} *puzzle*

$+$

$(\mathcal{B}(B \to \tau\nu), \sin(2\beta))$ discrepancy

Summer 2011: [PDG'10]



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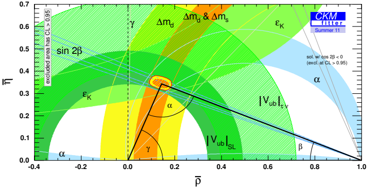

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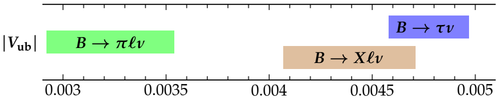
Spring 2012: [PDG'12, preliminary]



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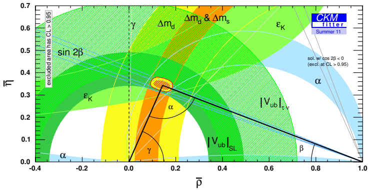

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precision??

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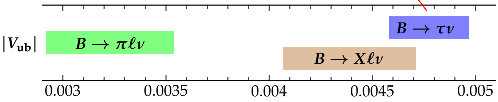

HQET ON THE LATTICE

Heavy Quark Effective Theory

Expansion in inverse heavy quark mass $1/m$ [Eichten; Isgur+Wise; Georgi]

$$\mathcal{L}_{\mathrm{HQET}} = \overline{\psi}_{\mathrm{h}} \Big[\underbrace{D_0 + \delta m}_{\text{static limit (LO)}} \underbrace{- \omega_{\mathrm{kin}}\, \mathbf{D}^2 - \omega_{\mathrm{spin}}\, \boldsymbol{\sigma}\mathbf{B}}_{\text{NLO, O(1/m)}} \Big] \psi_{\mathrm{h}} + \ldots\,, \qquad \left.\begin{matrix}\omega_{\mathrm{kin}}\\ \omega_{\mathrm{spin}}\end{matrix}\right\} \sim \frac{1}{2m}$$

operator $\mathcal{O}_{\mathrm{kin}} \equiv -\overline{\psi}_{\mathrm{h}} \mathbf{D}^2 \psi_{\mathrm{h}}$ kinetic energy from residual motion of heavy quark

operator $\mathcal{O}_{\mathrm{spin}} \equiv -\overline{\psi}_{\mathrm{h}} \boldsymbol{\sigma}\mathbf{B} \psi_{\mathrm{h}}$ chromomagnetic interaction with gluon field

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With $\mathcal{L}_{\mathrm{HQET}} = \mathcal{L}_{\mathrm{stat}} + \sum_{n\geq 1} \mathcal{L}^{(n)}$, expand integrand in functional integral repres.

$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathcal{D}[\phi]\, \mathcal{O}[\phi]\, \mathrm{e}^{-\mathcal{S}_{\mathrm{rel}} - \mathcal{S}_{\mathrm{HQET}}} , \qquad \mathcal{Z} = \int \mathcal{D}[\phi]\, \mathrm{e}^{-\mathcal{S}_{\mathrm{rel}} - \mathcal{S}_{\mathrm{HQET}}} ,$$

as a power series in $1/m$:

$$\mathrm{e}^{-\mathcal{S}_{\mathrm{HQET}}} = \exp\left\{ -a^4 \sum_x \mathcal{L}_{\mathrm{stat}}(x) \right\} \times \left\{ 1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \frac{1}{2} \left[a^4 \sum_x \mathcal{L}^{(1)}(x) \right]^2 - a^4 \sum_x \mathcal{L}^{(2)}(x) + \ldots \right\}$$

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This definition of HQET implies:

- $1/m$-terms appear as insertions of local operators only
  $\Rightarrow$ power counting: Renormalizability to each order in $1/m$
  $\Leftrightarrow$ $\exists$ continuum limit & universality (in contrast to NRQCD)
  (remark: **not** rigorously proven for *static theory to all orders in* g)
- Effective theory = (continuum) asymptotic expansion of QCD in $1/m$
- interaction with light d.o.f's still non-perturbatively (in contrast to χPT)

$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathcal{D}[\phi]\, \mathrm{e}^{-\mathcal{S}_{\mathrm{rel}} - \mathcal{S}_{\mathrm{stat}}}\, \mathcal{O} \Big\{ 1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \ldots \Big\}$$

- originally formulated by [Eichten+Hill '88-'90]: $D_0 + \delta m \to \nabla_0 + \delta m$
- again different discretisations: APE-,HYP-smeared actions
  mainly to cure bad $\frac{\text{signal}}{\text{noise}} \propto \exp[-E_{\rm stat} x_0] \sim \exp[-(c g_0^2/a) x_0]$

Explicitly: EV in HQET to subleading order

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin}\ a^4 \sum_x \langle \mathcal{O} \mathcal{O}_{\rm kin}(x) \rangle_{\rm stat} + \omega_{\rm spin}\ a^4 \sum_x \langle \mathcal{O} \mathcal{O}_{\rm spin}(x) \rangle_{\rm stat}$$
$$\equiv \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin} \langle \mathcal{O} \rangle_{\rm kin} + \omega_{\rm spin} \langle \mathcal{O} \rangle_{\rm spin}$$

with

$$\langle O \rangle_{\rm stat} = \frac{1}{\mathcal{Z}} \int_{\text{fields}} O\ \exp\left\{ -a^4 \sum_x \left[\mathcal{L}_{\rm light}(x) + \mathcal{L}_{\rm h}^{\rm stat}(x) \right] \right\}$$

Heavy Quark Effective Theory on the lattice

The Problem: power divergences

mixing of operators of different dim. in $\mathcal{L}_{\rm HQET}$ induces power divergences

- **Example:** Mass renormalization pattern at static order of HQET

  mixing of $\overline{\psi}_{\rm h} D_0 \psi_{\rm h}$ and $\overline{\psi}_{\rm h}\psi_{\rm h}$ $\rightsquigarrow$ linear divergence: $\delta m \propto a^{-1}$

$$\overline{m}_{\rm b}^{\overline{\rm MS}} = Z_{\rm pole}^{\overline{\rm MS}} \cdot m_{\rm pole}\,, \qquad m_{\rm pole} = m_{\rm b} - E_{\rm stat} - \delta m$$

$$\delta m = \frac{c(g_0)}{a} \sim {\rm e}^{+1/(2b_0 g_0^2)} \left\{ c_1 g_0^2 + c_2 g_0^4 + \ldots + O(g^{2n}) \right\}$$

- in PT: uncertainty = truncation error $\sim {\rm e}^{+1/(2b_0 g_0^2)} \cdot c_{n+1} \cdot g_0^{2n+2} \xrightarrow{g_0 \to 0} \infty$

  $\Rightarrow$ Non-perturbative $c(g_0)$ needed

  $\Rightarrow$ NP renormalization of HQET (resp. matching to QCD) required for continuum limit to exist

- power-law divergences even worse at higher orders in $1/m$:

  LO$\rightarrow$NLO: $a^{-1} \to a^{-2}$ in coeff.s of $\omega_{\rm kin}\mathcal{O}_{\rm kin}$, $\omega_{\rm spin}\mathcal{O}_{\rm spin}$ in $\mathcal{L}^{(1)}$ of $\mathcal{L}_{\rm HQET}$

Solution: NP'ly subtract power div. by exploiting finite volume

ALPHA's COMPUTATIONAL STRATEGY

General Strategy

[HeitgerSommer'01]

NP matching of QCD and HQET in small volume	$\Leftrightarrow$	relativistic b-quark feasible
+		
finite size scaling procedure	$\Leftrightarrow$	contact to large volumes

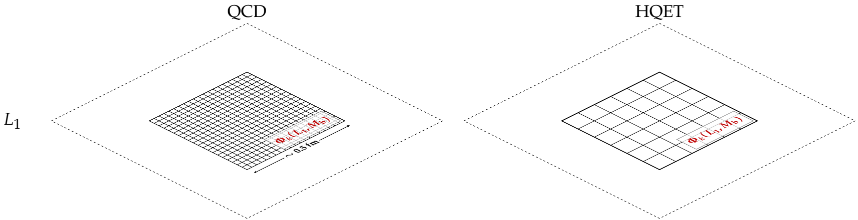
Framework:

- plaquette gauge action
- mass-degenerate doublet of non-perturbatively improved Wilson fermions
- two static quark actions (HYP discretization [HasenfratzKnechtli'01])

Ingredient: Schrödinger functional as intermediate renorm. scheme

- massless, finite volume renorm. scheme in the continuum
- Dirichlet b.c. in time $\Rightarrow$ 'IR save': $m = 0$ on the lattice
- NP definition of a running coupling $\Rightarrow$ $\bar{g}^2(\mu)$, w/ box size $L = 1/\mu$
- $N_\mathrm{f} = 2$: QCD running coupling [ALPHA'04] and mass [ALPHA'05] known

General Strategy



Step 0: define line of constant physics

'light' sector:

$\bar{g}^2(L_1/2) \equiv 2.989\,, \quad L_1 m_1 \equiv 0$ $(L_1 \approx 0.4\mathrm{fm})$ $\Rightarrow$ tuning of $(\beta, \kappa_1, L_1/a)$

QCD: $L_1/a \in \{20, 24, 32, 40\}$ $\Rightarrow a \leq 0.02\mathrm{fm}$ $\rightsquigarrow$ relativistic b-quark

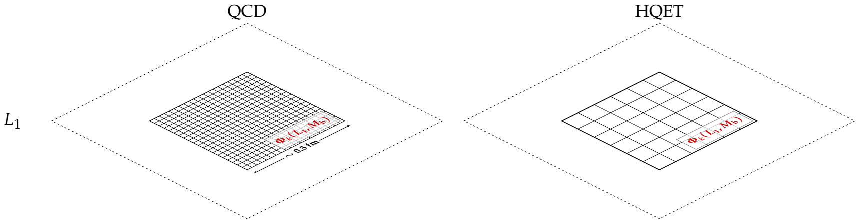
HQET: $L_1/a \in \{6, 8, 10, 12, 16\}$ coarser lattices sufficient

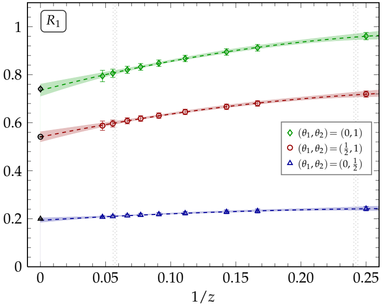
'heavy' sector in QCD: fix RGI heavy quark mass [PF,Heitger,Tantalo'11]

$$z = L_1 M = L_1 Z_M (1 + b_m a m_{q,h}) a m_{q,h} + \mathrm{O}(a^2)\,, \quad Z_M = \frac{Z(g_0) Z_A(g_0)}{Z_P(\mu, g_0)} h(L_1/2)$$

$$\in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$$

General Strategy





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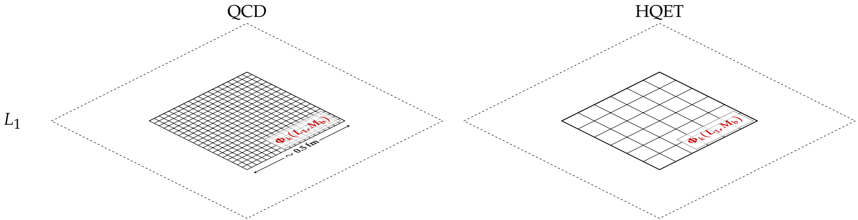


Step 1: choose proper observables for matching

$$\Phi^{\text{QCD}} = \left(L_1\Gamma_{\text{P}},\ \ln\left[-f_{\text{A}}/f_1\right],\ R_{\text{A}},\ R_1,\ \tfrac{3}{4}\ln[f_1/k_1]\right)^{\text{t}}$$

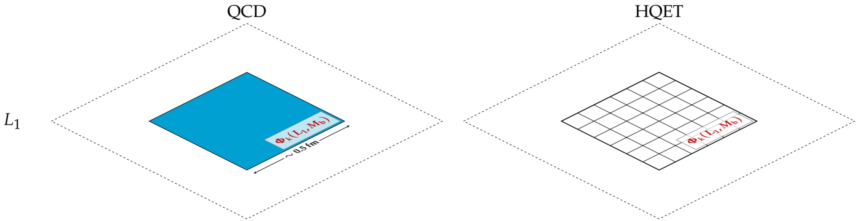
with known expansion

$$\Phi^{\text{HQET}} = \eta + \varphi \cdot \omega$$

in order to extract HQET parameters at next-to-leading order

$$\omega = \left(m_{\text{bare}},\ \ln Z_{\text{A}}^{\text{HQET}},\ c_{\text{A}}^{(1)},\ \omega_{\text{kin}},\ \omega_{\text{spin}}\right)^{\text{t}}$$

see [arXiv:1203.6516] for details

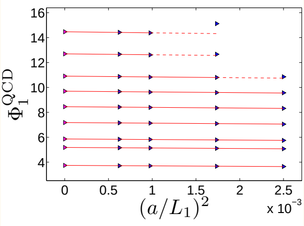


Step 2: take CL in QCD

Example: *PS eff. mass* $\Phi_1^{\mathrm{QCD}} = L_1 \Gamma_{\mathrm{P}}$

$$\Phi^{\mathrm{QCD}}(L_1, M) = \lim_{a\to 0} \Phi^{\mathrm{QCD}}(L_1, M, a)$$

for $L_1/a = 20, 24, 32, 40$



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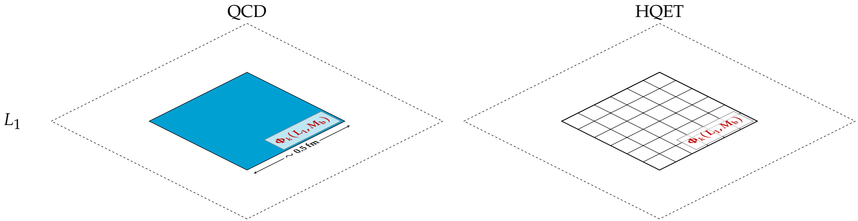

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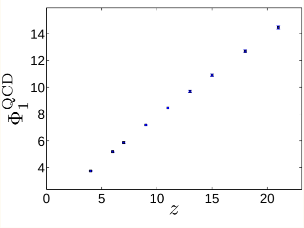

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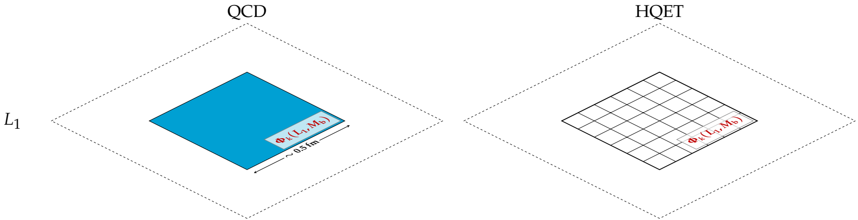

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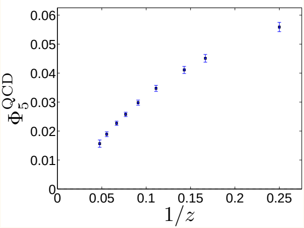

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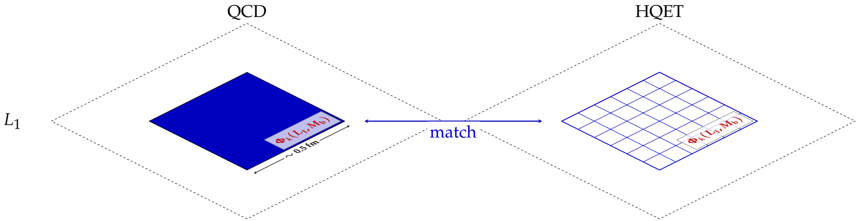


Step 3: match QCD and HQET in L_1

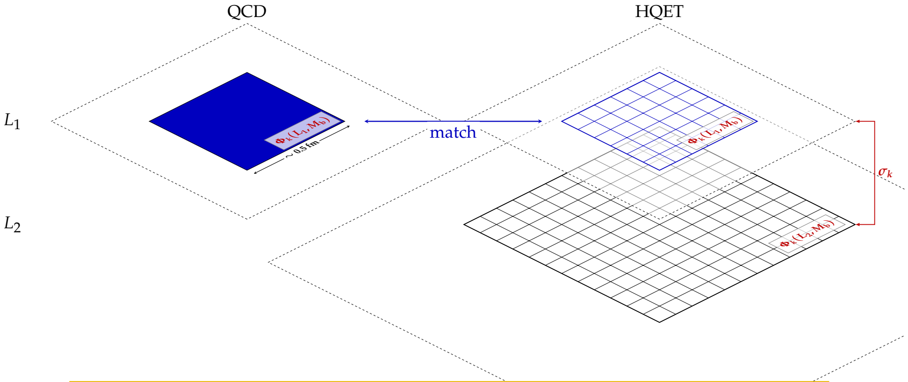
$$\Phi^{\mathrm{QCD}}(L_1, M) \equiv \Phi^{\mathrm{HQET}}(L_1, M, a) \qquad \forall z \text{ and } L_1/a \in \{6, 8, 10, 12, 16\}$$

$$\Downarrow \qquad \left(\Phi^{\mathrm{HQET}} = \eta + \varphi \cdot \omega\right)$$

$$\widetilde{\omega}(M, a) = \varphi^{-1}(L_1, a) \cdot \left(\Phi^{\mathrm{QCD}}(L_1, M) - \eta(L_1, a)\right)$$

$\widetilde{\omega}$ inherits 'mass-dependence' from yet arbitrary values of M

General Strategy



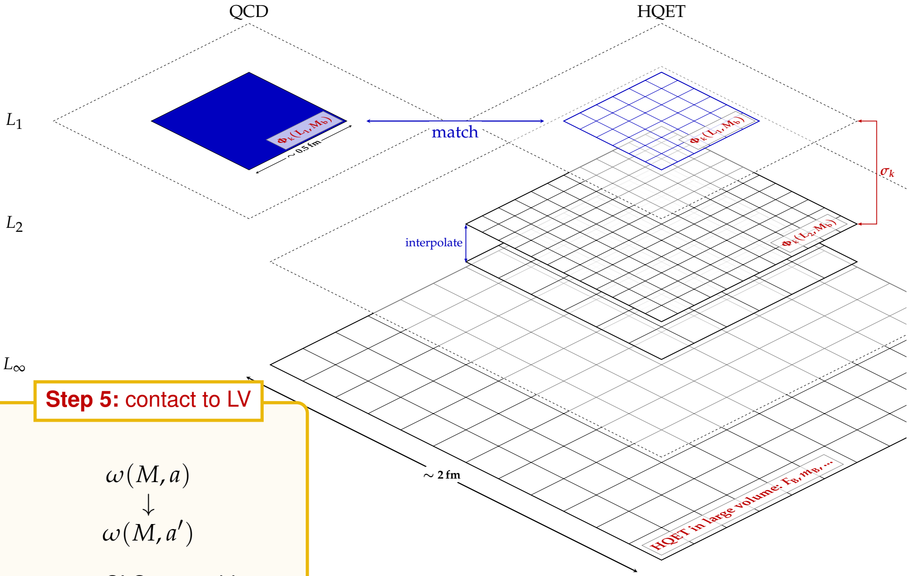
Step 4: finite size scaling

$L_1 \to L_2 = 2L_1$, while keeping bare parameters fixed

$$\begin{cases} \Phi^{\mathrm{HQET}}(L_2, M, 0) & = \lim_{a\to 0}\left[\eta(L_2,a) + \varphi(L_2,a)\cdot\tilde{\omega}(M,a)\right] \\ \omega(M,a) & = \varphi^{-1}(L_2,a)\cdot\left(\Phi^{\mathrm{HQET}}(L_2,M,0) - \eta(L_2,a)\right) \end{cases}$$

CL exists!

General Strategy



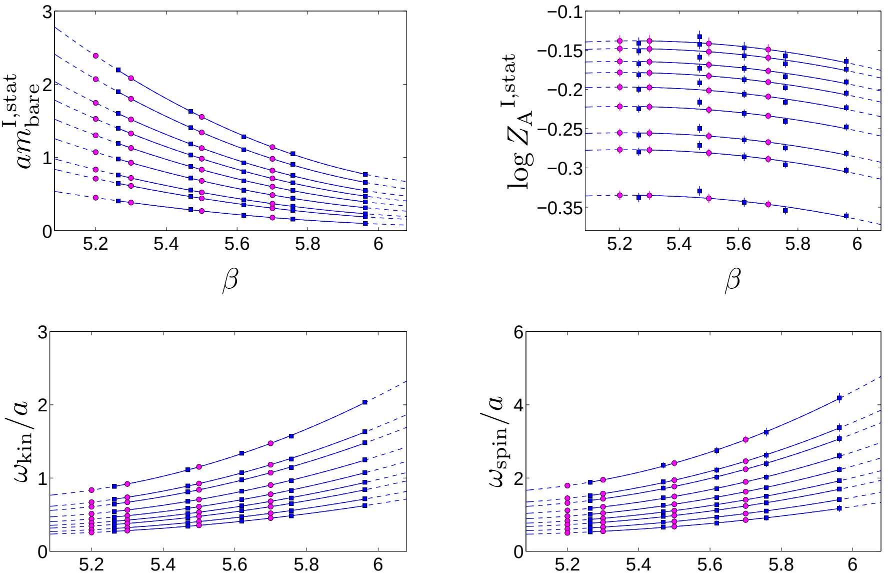
Step 5: contact to LV

$$\omega(M,a)$$
$$\downarrow$$
$$\omega(M,a')$$

to use CLS ensembles

NP'ly determined HQET parameters

[arXiv:1203.6516], $N_{\rm f} = 2$, all z



NP'ly determined HQET parameters

[arXiv:1203.6516], $N_{\mathrm{f}} = 2$, $z = 13$, HYP1, HYP2

✓ expected absorption of power divergences

Example: bare quark mass

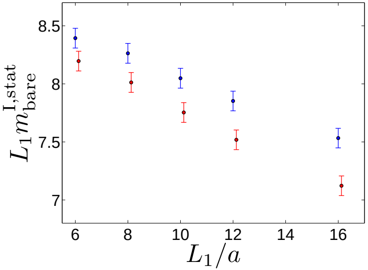
- static order:

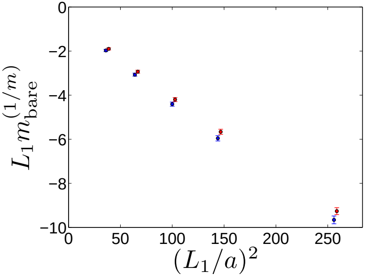
$$L_1 m_{\mathrm{bare}}^{\mathrm{stat}} \propto \frac{1}{a}$$

- $(1/m)$-correction:

$$L_1 m_{\mathrm{bare}}^{1/m} \propto \left(\frac{1}{a}\right)^2$$

✓ clear hierarchy in HQET expansion observed





NP'ly determined HQET parameters

(std. matching conditions)

parameters at β-values used in large volume simulations (HYP2)

β	LM_Q	am_{bare}	$\ln(Z_{\mathrm{A}}^{\mathrm{HQET}})$	$\frac{c_{\mathrm{A}}^{(1)}}{a}$	$\frac{\omega_{\mathrm{kin}}}{a}$	$\frac{\omega_{\mathrm{spin}}}{a}$
5.2	13	1.207(18)	−0.139(31)	−0.54(9)	0.386(7)	0.825(30)
	z_{b}	$\star$	$\star$	$\star$	$\star$	$\star$
	15	1.459(20)	−0.119(31)	−0.50(9)	0.345(7)	0.727(28)
5.3	13	0.985(17)	−0.148(32)	−0.56(10)	0.425(8)	0.899(34)
	z_{b}	$\star$	$\star$	$\star$	$\star$	$\star$
	15	1.212(18)	−0.127(32)	−0.52(10)	0.380(8)	0.791(31)
5.5	13	0.582(14)	−0.166(36)	−0.68(12)	0.533(10)	1.109(42)
	z_{b}	$\star$	$\star$	$\star$	$\star$	$\star$
	15	0.769(15)	−0.142(36)	−0.63(12)	0.476(11)	0.976(39)

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$z_{\rm b} = L_1 M_{\rm b}$ to be determined through spectrum calculation in large volume HQET

CLS
LARGE VOLUME ENSEMBLES

CLS ensembles for large volume computations

subset used in this analysis

CLS based ensembles ($T = 2L$):

$m_\pi L \gtrsim 4.0$

β	a (fm)	L/a	Lm_π	m_π (MeV)	no. of cnfg.s	separ. (MD u.)	label	code
5.2	0.075	32	4.7	380	800	8	A4 ○	DD
		32	4.0	330	200	4	A5 ○	MP3
5.3	0.065	32	4.7	440	1000	16	E5 □	DD
		48	5.0	310	500	8	F6 □	DD
		48	4.3	270	600	8	F7 □	DD
5.5	0.048	48	5.2	440	400	8	N5 ◇	DD
		64	4.2	270	700	4	O7 ◇	MP2

- full Jackknife analysis (100 bins) from small to large volume
- scale setting through f_K [ALPHA:to appear soon]

Some details about our algorithm

MP-HMC implementation [MarinkovicSchaefer'10] supersedes domain decomposed HMC [Lüscher'05]

Idea: use efficient solver from DD-HMC package and get rid off inactive links (autocorr.↘)

$\Rightarrow$ allows to reach smaller pion masses

$\Rightarrow$ drawback: increased number of parameters to optimize

- mass preconditioning [Hasenbusch'10] for arbitrary N_{pf}
- SAP-GCR with switch for
  - deflation
  - chronological inversion

  for each pseudo-fermion $1, \ldots, N_{\mathrm{pf}}$
- Multiple time scale integrator [SextonWeingarten'92]
  - 2^{nd} order integrator [OmelyanEtAl'] for pseudo-fermions
  - leapfrog for gauge field on finest integration scale

Dynamical fermion simulations

criteria for subsequent data analysis:

- FV effects small by construction

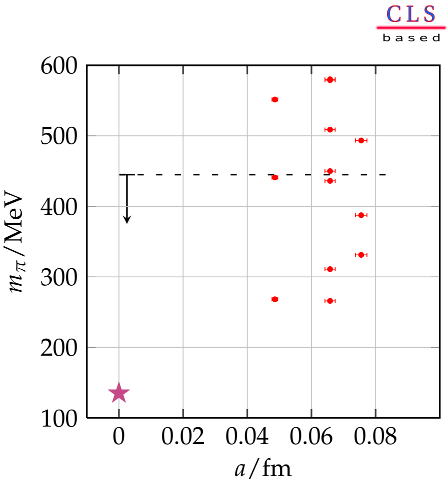
$$Lm_\pi \geq 4.0$$

- data for chiral extrapolation uses

$$(250 \lesssim m_\pi \lesssim 400 - 450)\text{MeV}$$

- lattice spacings

$$(0.048, 0.065, 0.075 < 0.1)\text{fm}$$



7 simulations fulfill our current criteria

Dynamical fermion simulations

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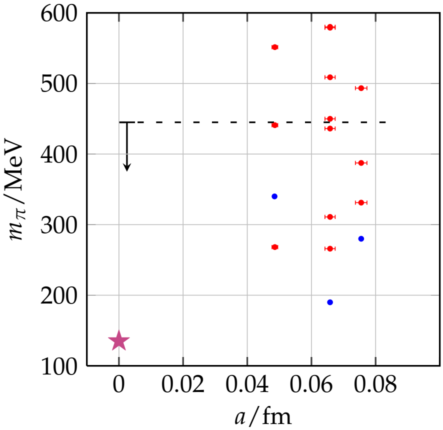
CLS based



7 simulations fulfill our current criteria

+ 3 more by end of this year

Large volume matrix elements

Large volume techniques

variance reduction through stochastic all-to-all props.

compute $N \times N$ correlator matrices

$$C_{ij}^{\mathrm{stat}}(t) = \sum_{x,\mathbf{y}} \Big\langle O_i(x_0+t,\mathbf{y})\, O_j^*(x) \Big\rangle_{\mathrm{stat}}$$

$$C_{ij}^{\mathrm{kin/spin}}(t) = \sum_{x,\mathbf{y},z} \Big\langle O_i(x_0+t,\mathbf{y})\, O_j^*(x)\, O_{\mathrm{kin/spin}}(z) \Big\rangle_{\mathrm{stat}}$$

$$C_{A^{(1)},i}^{\mathrm{stat}}(t) = \sum_{x,\mathbf{y}} \Big\langle A_0^{(1)}(x_0+t,\mathbf{y}) O_i^*(x) \Big\rangle_{\mathrm{stat}}$$

using interpolating fields

$$O_k = \overline{\psi}_{\mathrm{h}} \gamma_0 \gamma_5 \psi_{\mathrm{l}}^{(k)}, \qquad \psi_{\mathrm{h}}(x)\text{: static quark field}$$

$$O_k^* = \overline{\psi}_{\mathrm{l}}^{(k)} \gamma_0 \gamma_5 \psi_{\mathrm{h}}, \qquad \psi_{\mathrm{l}}^{(k)}(x) = \Big(1 + \kappa_{\mathrm{G}}\, a^2\, \Delta\Big)^{R_k} \psi_{\mathrm{l}}(x)$$

$N = 3$ with APE-smeared links for different levels of Gaussian smearing such that $R_k \times (a/0.3\mathrm{fm})^2 \in \{1, 4, 10\}$ kept fixed for each $C^{\rm stat}$, $C^{\rm kin/spin}$, and $C^{\rm stat}_{A^{(1)}}$, we solve the GEVP

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)\,,$$

λ_n, v_n: eigenvalue & eigenvector of $n^{\rm th}$ state

$\Rightarrow$ energies E_n and operators Q_n with largest overlap to $n^{\rm th}$ state:

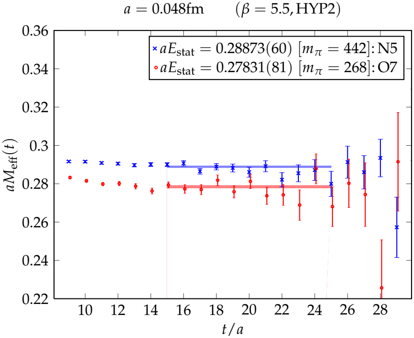
$$aE_n^{\rm eff}(t,t_0) = -\ln\left(\frac{\lambda_n(t+a,t_0)}{\lambda_n(t,t_0)}\right)$$

$$Q_n^{\rm eff}(t,t_0) = \frac{O^i(t)v_n^i(t,t_0)}{\sqrt{v_n^i(t,t_0)C_{ij}(t)v_n^j(t,t_0)}}\left(\frac{\lambda_n(t_0+a,t_0)}{\lambda_n(t_0+2a,t_0)}\right)^{t/2a}$$

Large volume techniques, results

Results for $aE_{\rm stat}$ from GEVP at finest lattice spacing

Example: static energy $aE_{\rm stat}$



with corrections (for $N = 3$)

- for energies E_X:

$$\sim e^{-t(E_4 - E_1)}$$

- and for matrix elements p^X:

$$\sim e^{-t_0(E_4 - E_1)} e^{-(t-t_0)(E_2 - E_1)}$$

$N_f = 2$ RESULTS

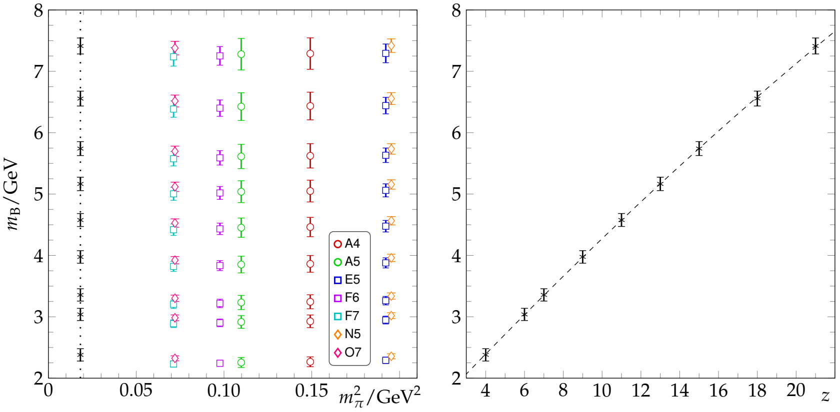
The b-quark mass

$$m_{\mathrm{B}} = m_{\mathrm{bare}} + E^{\mathrm{stat}} + \omega_{\mathrm{kin}} \cdot E^{\mathrm{kin}} + \omega_{\mathrm{spin}} \cdot E^{\mathrm{spin}} \qquad = m_{\mathrm{B}}(z, m_\pi, a)$$

- parameters $\{m_{\mathrm{bare}}, \omega_{\mathrm{kin}}, \omega_{\mathrm{spin}}\}(z,a)$ & LV energies $\{E^{\mathrm{stat}}, E^{\mathrm{kin}}, E^{\mathrm{spin}}\}(m_\pi, a)$

$$m_\mathrm{B} = m_\mathrm{bare} + E^\mathrm{stat} + \omega_\mathrm{kin} \cdot E^\mathrm{kin} + \omega_\mathrm{spin} \cdot E^\mathrm{spin} \quad = m_\mathrm{B}(z, m_\pi, a)$$

- parameters $\{m_\mathrm{bare}, \omega_\mathrm{kin}, \omega_\mathrm{spin}\}(z,a)$ & LV energies $\{E^\mathrm{stat}, E^\mathrm{kin}, E^\mathrm{spin}\}(m_\pi, a)$
- global fit: $m_\mathrm{B}\,(z, m_\pi, a) = B(z) + C \cdot m_\pi^2 + D \cdot a^2$ $\Rightarrow$ $m_\mathrm{B}(z \;, m_\pi^\mathrm{exp})$



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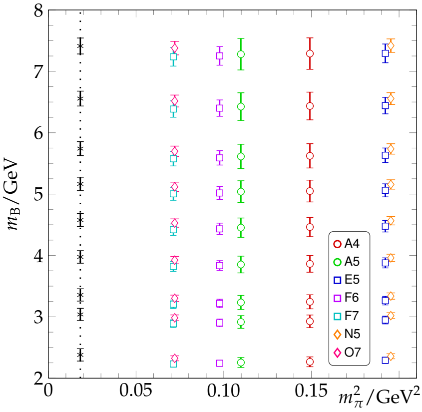
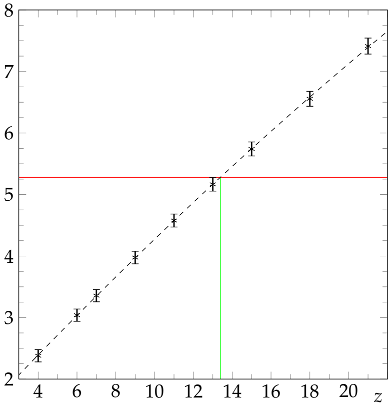
- parameters $\{m_\mathrm{bare}, \omega_\mathrm{kin}, \omega_\mathrm{spin}\}(z,a)$ & LV energies $\{E^\mathrm{stat}, E^\mathrm{kin}, E^\mathrm{spin}\}(m_\pi, a)$
- global fit: $m_\mathrm{B}(z, m_\pi, a) = B(z) + C \cdot m_\pi^2 + D \cdot a^2 \Rightarrow m_\mathrm{B}\left(z_\mathrm{b}, m_\pi^\mathrm{exp}\right) = m_\mathrm{B}^\mathrm{exp}$





The b-quark mass

we invert

$$m_{\rm B}\left(z_{\rm b}, m_\pi^{\rm exp}\right) = m_{\rm B}^{\rm exp}$$

for $m_{\rm b}(m_{\rm b})$ in MSbar scheme

$$\Downarrow$$

$$\overline{m}_{\rm b}(\overline{m}_{\rm b}) = 4.288(76)_{\rm stat}(43)_z(14)_a {\rm GeV}$$

$$\Downarrow$$

parameters at physical b-quark mass

$$\omega_i \equiv \omega_i(m_{\rm b}, a)$$

from now on

PDG: $4.19^{+0.18}_{-0.06}\,{\rm GeV}$

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$$m_{\rm B}(z_{\rm b}, m_\pi^{\rm exp}) = m_{\rm B}^{\rm exp}$$

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$$\Downarrow$$

parameters at physical b-quark mass

$$\omega_i \equiv \omega_i(m_{\rm b}, a)$$

from now on

PDG: $4.19^{+0.18}_{-0.06}\,\rm GeV$

in static approximation:

$$\overline{m}_{\rm b}(\overline{m}_{\rm b}) = 4.302(78)_{\rm stat}(43)_z(14)_a \rm GeV$$

The B-meson decay constant

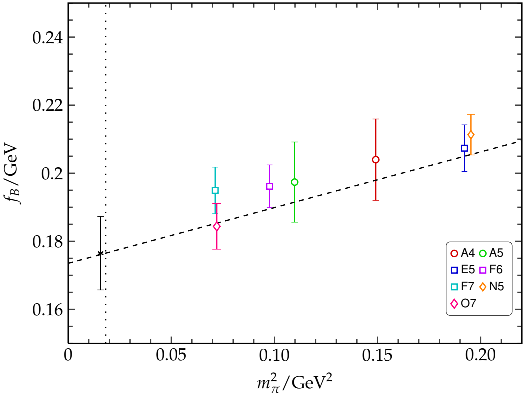
$$\ln(a^{3/2} f_{\mathrm{B}} \sqrt{m_{\mathrm{B}}/2}) = \ln(Z_{\mathrm{A}}^{\mathrm{HQET}}) + \ln(a^{3/2} p^{\mathrm{stat}}) + b_{\mathrm{A}}^{\mathrm{stat}} a m_{\mathrm{q}} + \omega_{\mathrm{kin}} p^{\mathrm{kin}} + \omega_{\mathrm{spin}} p^{\mathrm{spin}} + c_{\mathrm{A}}^{(1)} p^{\mathrm{A}^{(1)}}$$

p^X: plateau values of eff. matrix elements from GEVP analysis

The B-meson decay constant $f_{\mathrm{B}}(z)|_{z=z_{\mathrm{b}}}$

we extrapolate to physical point $f_{\mathrm{B}} \equiv \lim_{(m_\pi,a)\to(m_\pi^{\mathrm{exp}},0)} f_{\mathrm{B}}(m_\pi,a)$ using fit ansatz

$$f_{\mathrm{B}}(m_\pi,a) = b + cm_\pi^2 + da^2 \qquad \text{(LO)}$$



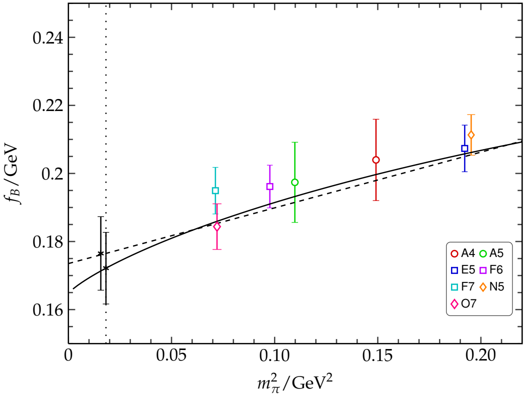
- no term in $(am_\pi)^2 \lesssim 0.02$
- $f_{\mathrm{B}} = 176(11)(5)_a$ LO

The B-meson decay constant $f_B(z)|_{z=z_b}$

we extrapolate to physical point $f_B \equiv \lim_{(m_\pi,a)\to(m_\pi^{exp},0)} f_B(m_\pi, a)$ using fit ansatz

$$f_B(m_\pi, a) = b + cm_\pi^2 + da^2 \qquad \text{(LO)}$$

$$f_B(m_\pi, a) = b'\left[1 - \frac{3}{4}\frac{1+3\hat{g}^2}{(4\pi f_\pi)^2} m_\pi^2 \ln(m_\pi^2)\right] + c' m_\pi^2 + d' a^2 \qquad \text{(HM}\chi\text{PT)}$$



- no term in $(am_\pi)^2 \lesssim 0.02$
- $f_B = 176(11)(5)_a$ LO
- $f_B = 172(11)(5)_a$ HMχPT

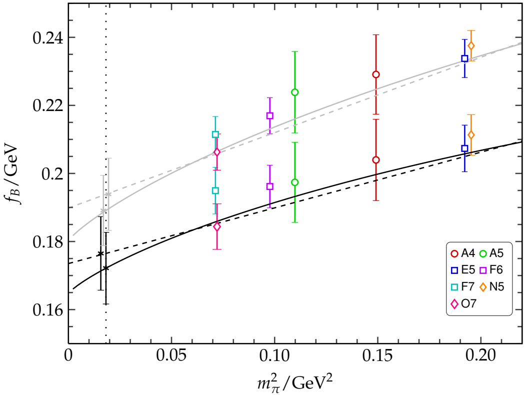
$f_\pi = f_\pi^{exp}$, $\hat{g} = 0.51(2)$

[PoS-Lat'10:BulavaETAL]

we extrapolate to physical point $f_{\mathrm{B}} \equiv \lim_{(m_\pi,a)\to(m_\pi^{\mathrm{exp}},0)} f_{\mathrm{B}}(m_\pi,a)$ using fit ansatz

$$f_{\mathrm{B}}(m_\pi,a) = b + c m_\pi^2 + d a^2 \qquad \text{(LO)}$$

$$f_{\mathrm{B}}(m_\pi,a) = b'\left[1 - \frac{3}{4}\frac{1+3\hat{g}^2}{(4\pi f_\pi)^2} m_\pi^2 \ln(m_\pi^2)\right] + c' m_\pi^2 + d' a^2 \qquad \text{(HM}\chi\text{PT)}$$



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- $f_{\mathrm{B}} = 176(11)(5)_a$ LO
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$f_\pi = f_\pi^{\mathrm{exp}}, \ \hat{g} = 0.51(2)$

[PoS-Lat'10:BulavaETAL]

- static theory

$f_{\mathrm{B}} = 194(11)(5)_a$ LO

$f_{\mathrm{B}} = 189(11)(5)_a$ HMχPT

Summary & outlook

- HQET obs. to next-to-leading order in $1/m_\mathrm{b}$ renormalized NP'ly ✓
- systematic errors included ✓
- for the first time in $N_\mathrm{f} = 2$: power divergencies canceled NP'ly and continuum limit of certain observables has been taken in large volume ✓

$$m_\mathrm{b} = 4.288(76)(43)_z(14)_a\mathrm{GeV}\,, \quad f_\mathrm{B} = 176(11)(5)_a(4)_\chi\mathrm{MeV}$$

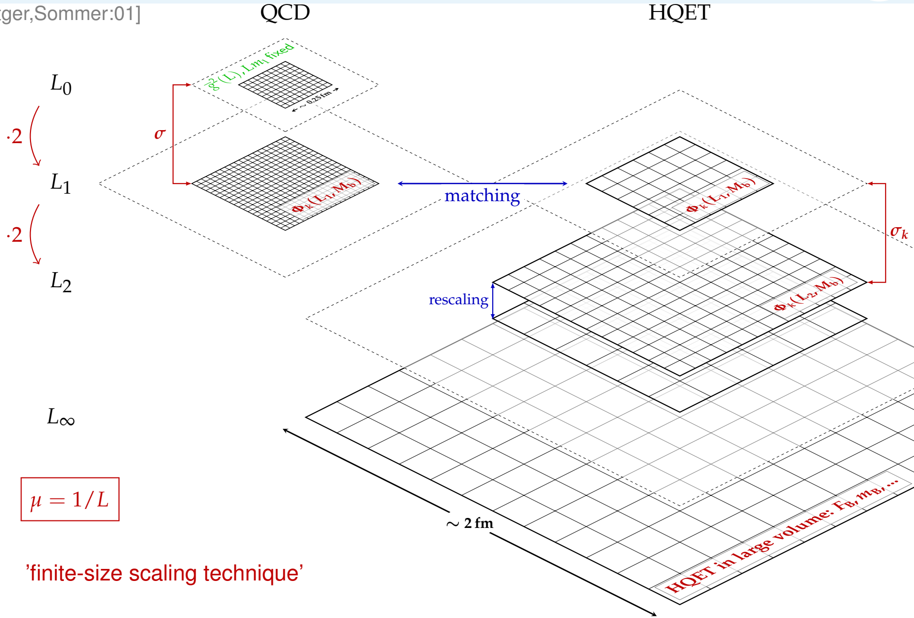
  still room to improve these results
- only truncation error $\mathrm{O}\big((\Lambda/m_\mathrm{b})^2\big)$ remains (but usually negligible)
- work in progress:
  - full analysis to be completed
  - measurements for f_{B_s}, m_{B_s}
  - $B \to \pi\ell\nu$ form factor $f_+(q^2)$
  - heavy baryons

Thanks go to: A.Athenodorou, B.Blossier, J.Bulava, M.Della Morte, M.Donnellan, N.Garron, J.Heitger, D.Hesse, G.von Hippel, M.Marinkovic, A.Ramos, S.Schaefer, H.Simma, R.Sommer, F.Virotta, . . .

BACKUP SLIDES

NP matching of HQET and QCD in a finite volume

[Heitger,Sommer:01]

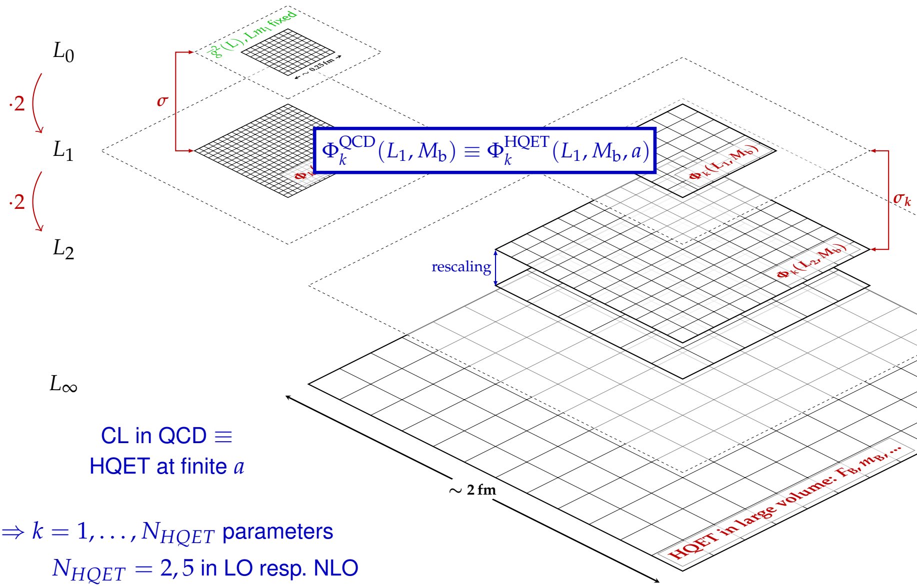


$\mu = 1/L$

'finite-size scaling technique'

NP matching of HQET and QCD in a finite volume

[Heitger,Sommer:01]



$$\Phi_k^{\mathrm{QCD}}(L_1, M_\mathrm{b}) \equiv \Phi_k^{\mathrm{HQET}}(L_1, M_\mathrm{b}, a)$$

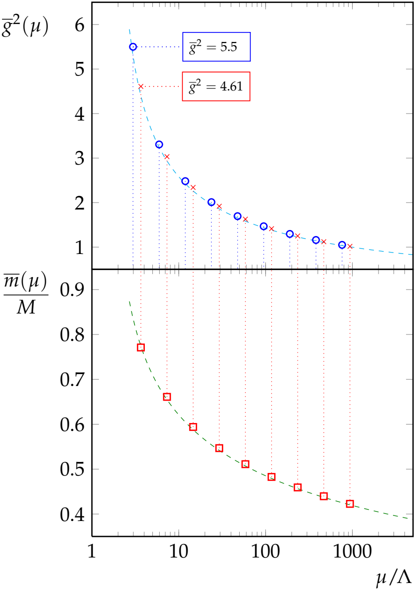
CL in QCD $\equiv$
HQET at finite a

$\Rightarrow k = 1, \ldots, N_{HQET}$ parameters

$N_{HQET} = 2, 5$ in LO resp. NLO

Scale dependence of QCD parameters

Running coupling and mass,



Renormalization group (RG) equations

1. coupling

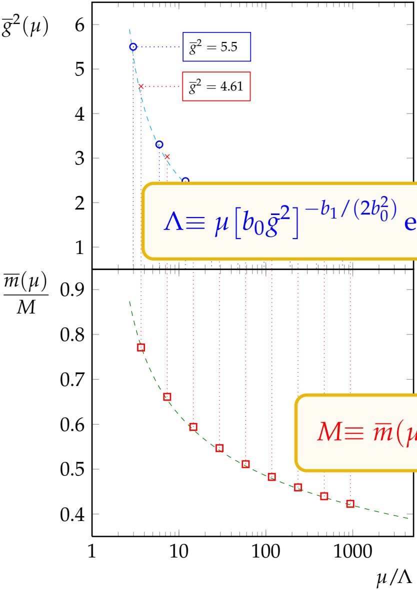
$$\mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3(b_0 + b_1\bar{g}^2 + \ldots)$$

2. mass

$$\frac{\mu}{\overline{m}} \frac{\partial \overline{m}}{\partial \mu} = \tau(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^2(d_0 + d_1\bar{g}^2 + \ldots)$$

in a massless scheme, b_0, b_1, d_0 universal

Solution leads to *exact* equations in mass-independent scheme



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1. coupling

$$\mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3(b_0 + b_1 \bar{g}^2 + \ldots)$$

$$\Lambda \equiv \mu \left[b_0 \bar{g}^2\right]^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0\bar{g}^2)} \exp\left\{-\int_0^{\bar{g}} \mathrm{d}g \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}$$

2. mass

$$\frac{\mu}{\overline{m}} \frac{\partial \overline{m}}{\partial \mu} = \tau(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^2(d_0 + d_1 \bar{g}^2 + \ldots)$$

$$M \equiv \overline{m}(\mu)\left[2b_0\bar{g}^2\right]^{-d_0/(2b_0)} \exp\left\{-\int_0^{\bar{g}} \mathrm{d}g \left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right]\right\}$$

in a massless scheme, b_0, b_1, d_0 universal

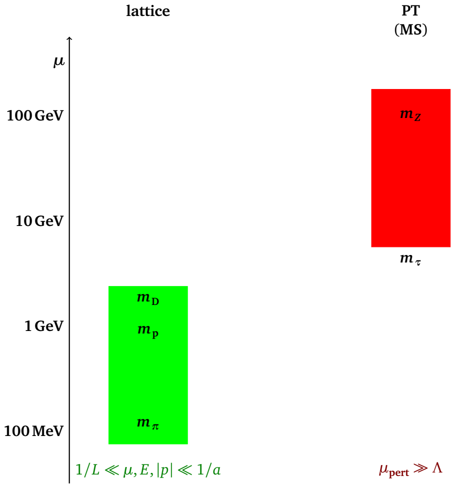
Solution leads to *exact* equations in mass-independent scheme

Generic strategy

...to connect low- & high-energy regime NP'ly

one more important ingredient:

How to connect hadronic observables from low-energies to the widely used $\overline{\text{MS}}$-scheme?



Generic strategy

. . . to connect low- & high-energy regime NP'ly

one more important ingredient:

How to connect hadronic observables from low-energies to the widely used $\overline{\mathrm{MS}}$-scheme?

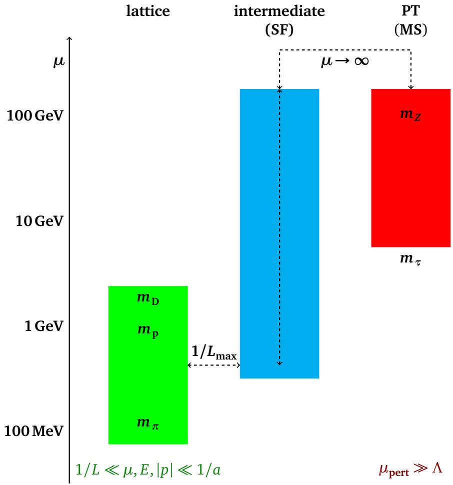
intermediate renorm. scheme

$\equiv$

Schrödinger functional

(finite volume, continuum scheme)

- RG scale evolution solved NP'ly
- thus continuum limit needs to be well controled (small cutoff effects)
- low-energy scale fixed by imposing $\bar{g}^2(L_{\mathrm{max}}) \equiv u_{\mathrm{max}}$



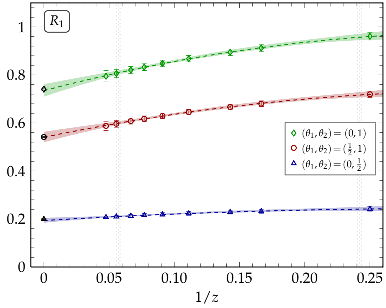
Results without any perturbative uncertainty

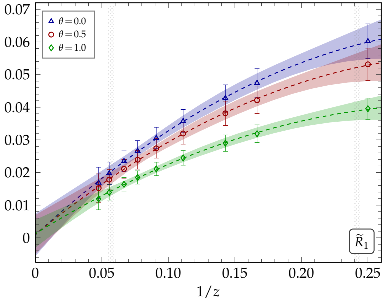
mass-dependence in the continuum, $z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$

in QCD: $f_1 = \mathcal{Z}^{-1}\langle B(L)|B(L)\rangle$ $\quad k_1 = \mathcal{Z}^{-1}\langle B^*(L)|B^*(L)\rangle$

$$R_1 = \frac{1}{4}\ln\left(\frac{f_1(\theta_1)k_1^3(\theta_1)}{f_1(\theta_2)k_1^3(\theta_2)}\right), \qquad \widetilde{R_1} = \frac{3}{4}\ln\left(\frac{f_1(\theta)}{k_1(\theta)}\right) \sim \omega_{\text{spin}}$$

their HQET expansion contains no conversion functions at LO





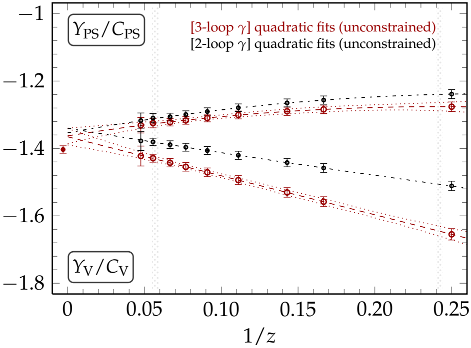
free quadratic fits in $1/z$ (static limit at $1/z = 0$)

computations in HQET & QCD absolutely independent and purely NP!

QCD-Results converted to HQET, decay constant

mass-dependence in the continuum, $z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$, $\theta \in \{0.5\}$

- impact of conversion function C_{PS} with 2- or 3-loop anomalous dimension



- barely agrees with our result at static order in HQET [DellaMorte,P.F.,Heitger'05]
- Missmatch a result of perturbative C_{PS}? 4-loops, …?
- NP matching removes this perturbative uncertainty!