$B \to V$ form factors from NRQCD

MATTHEW WINGATE
DAMTP, UNIVERSITY OF CAMBRIDGE

IN COLLABORATION WITH
ZHAOFENG LIU, STEFAN MEINEL, RON HORGAN

Motivation

- FCNC decays, rare in the SM, are being measured. Maybe BSM physics will contribute?

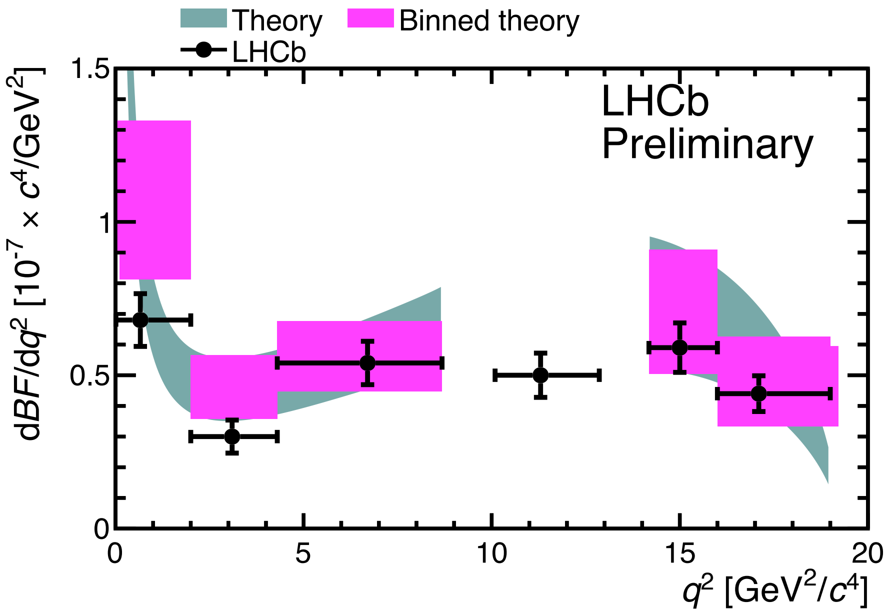
$$B \longrightarrow K^* \gamma \qquad B \longrightarrow K^* \ell^+ \ell^- \qquad B_s \longrightarrow \phi \ell^+ \ell^-$$

- Pheno strategies to fit observables to constrain Wilson coefficients in $b \to s$ effective Hamiltonian
- Light cone sum rule results, valid at large recoil, are being extrapolated to low recoil. But we can compute at low recoil.
- No unquenched lattice results for $B \to V$ form factors until now

Last month, from LHC*b*

$\mathrm{B}^0 \to K^{*0}\mu^+\mu^-$ and $\mathrm{B}_s^0 \to \phi\mu^+\mu^-$ differential branching fractions

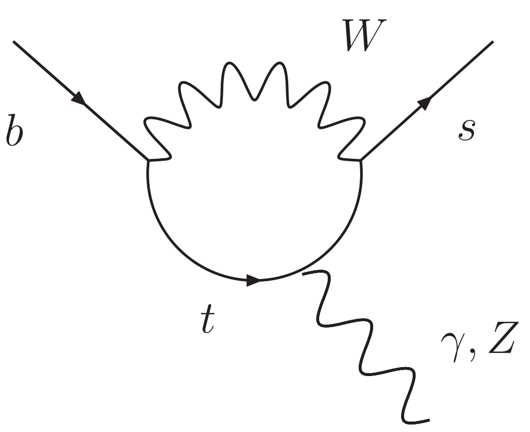
- LHCb($1.0\ \mathrm{fb}^{-1}$) : $\mathrm{B}^0 \to K^{*0}\mu^+\mu^-$: 900 ± 34 signal events



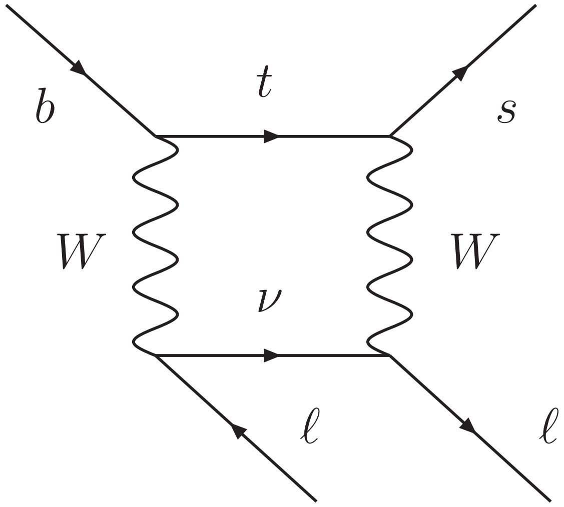
- Measurement of the $\mathrm{B}_s^0 \to \phi\mu^+\mu^-$ branching fraction reported at Moriond EW
- LHCb($1.0\ \mathrm{fb}^{-1}$) : $\mathrm{B}_s^0 \to \phi\mu^+\mu^-$: 77 ± 10 signal events
- $\mathcal{B}(\mathrm{B}_s^0 \to \phi\mu^+\mu^-) = (0.778 \pm 0.097(stat) \pm 0.061(syst) \pm 0.278(B)) \times 10^{-6}$ [preliminary]
- The most precise measurements to-date and are consistent with SM expectations [4]

Parkinson, Moriond QCD, March 2012

$b \to s$ is rare in the SM



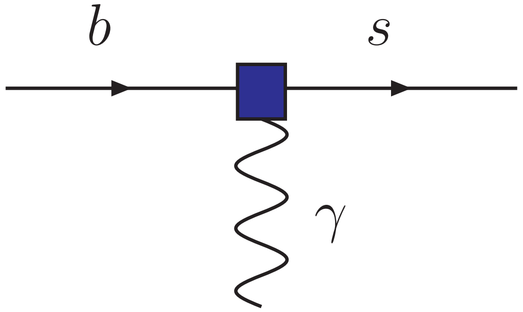
For energies ≪ m_W



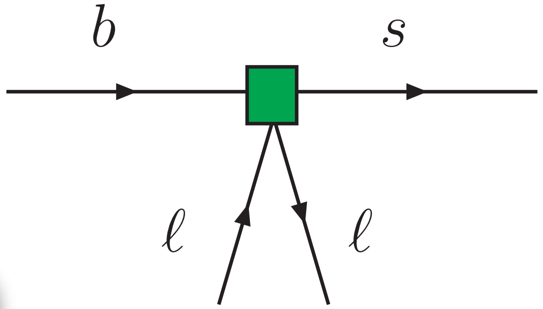
$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$

Wilson coefficients

Local operators



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$$



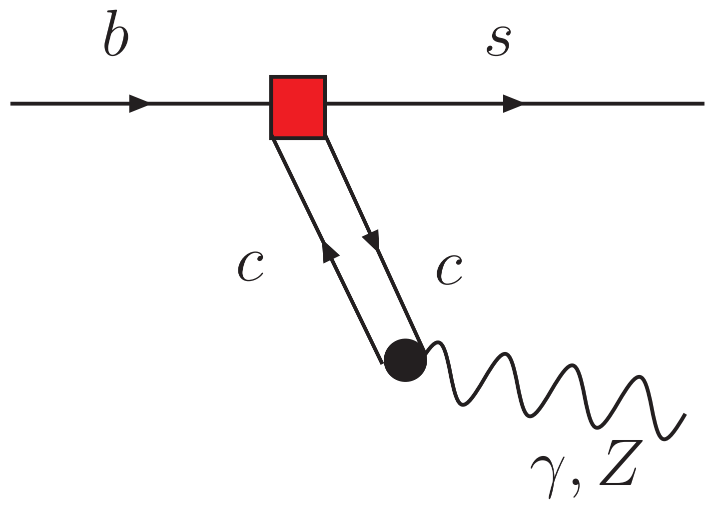
$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \, \bar{s}_i \sigma^{\mu\nu} (1+\gamma_5) b_i F_{\mu\nu}$$

$$Q_{9V} = \frac{e}{8\pi^2} (\bar{s} \, b)_{V-A} \, (\bar{\ell} \, \ell)_V$$

Theoretical issues

✤ Long distance effects

$$Q_2 = (\bar{s}\, c)_{V-A}\, (\bar{c}\, b)_{V-A}$$



Under control away from charmonium resonances

Low q^2 Large recoil	Khodjamirian, et al, PLB **402** (1997) Khodjamirian, et al, JHEP **1009** (2010)
High q^2 Low recoil	Buchalla & Isidori, NPB **525** (1998) Grinstein & Pirjol, PRD **62** (2000), PRD **70** (2004) Beylich, Buchalla, Feldmann, EPJ **C71** (2011)

✤ Narrow width approximation for K^*, φ

- ✦ Perhaps future LQCD study of decay constants across threshold will test assumptions
- ✦ Statistics willing, consistency check by comparing V_{ub} from $B \to \rho\, l\, \nu$ vs. $B \to \pi\, l\, \nu$

Lattice data

High statistics

MILC lattices (2+1 asqtad staggered)

	a(fm)	am_{sea}	Volume	$N_{conf} \times N_{src}$	am_{val}
coarse	$\sim$0.12	0.007/0.05	$20^3 \times 64$	2109×8	0.007/0.04
		0.02/0.05	$20^3 \times 64$	2052×8	0.02/0.04
fine	$\sim$0.09	0.0062/0.031	$28^3 \times 96$	1910×8	0.0062/0.031

m_π (MeV)
~300
~460
~320

Light meson momenta (units of $2\pi/L$)

- $(p_x, p_y, p_z) = (0, 0, 0)$.
- $(\tilde{q}$,0,0), (0,$\tilde{q}$,0), (0,0,$\tilde{q}$), where $\tilde{q}$=1 or 2.
- (1,1,0), (1,-1,0), (1,0,1), (1,0,-1), (0,1,1), (0,1,-1).
- (1,1,1), (1,1,-1), (1,-1,1), (1,-1,-1).

$p^2/(2\pi/L)^2$
0
1 or 4
2
3

Many Source/Sink separations (16 coarse, 22 fine)

So far, only v=0 NRQCD used (B at rest).

Leading order (HQET) current presently used.
1/m_b current matrix elements computed, analysis in progress

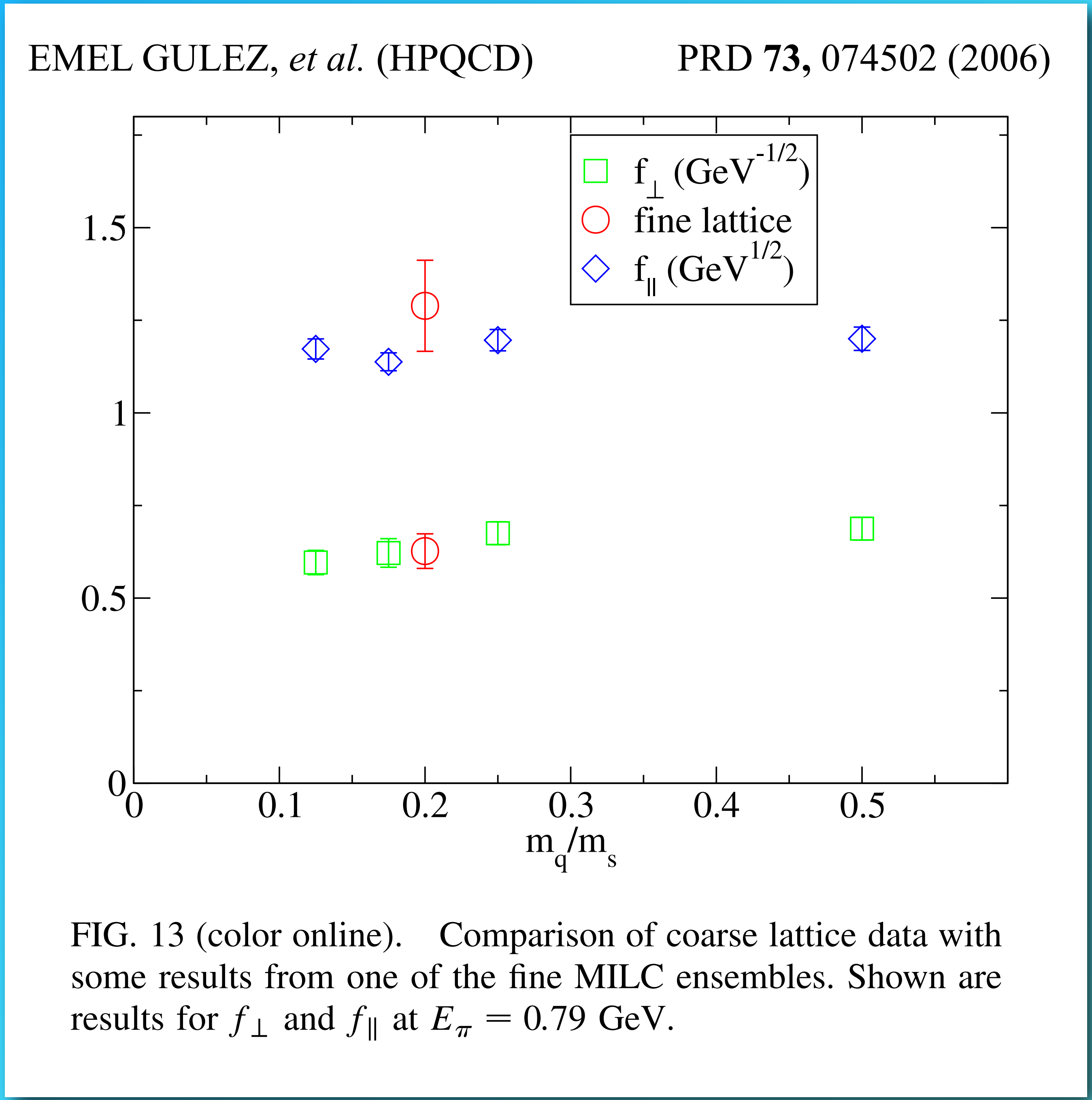
Systematic uncertainties

Combination of EFT and empirical extrapolation

- Finite volume: (2.5 fm)^3 seems large enough for B
- Discretization ($\mathscr{L}$): Improved NRQCD and AsqTad
- Discretization ($a|\mathrm{p}'|$): Moving NRQCD, but vs. statistics
- HQET currents: expand in $\Lambda_{\mathrm{QCD}}/m_b$, $|\mathrm{p}'|/m_b \Rightarrow$ work at low recoil
- Light quark mass extrapolation: χPT for pions/kaons

Systematic uncertainties

Discretization ($\mathscr{L}$): Improved NRQCD and AsqTad



EMEL GULEZ, *et al.* (HPQCD) PRD **73,** 074502 (2006)

FIG. 13 (color online). Comparison of coarse lattice data with some results from one of the fine MILC ensembles. Shown are results for $f_\perp$ and $f_\parallel$ at $E_\pi = 0.79$ GeV.

Systematic uncertainties

HQET currents: expansion in $\Lambda_{\mathrm{QCD}}/m_b$, p/m_b

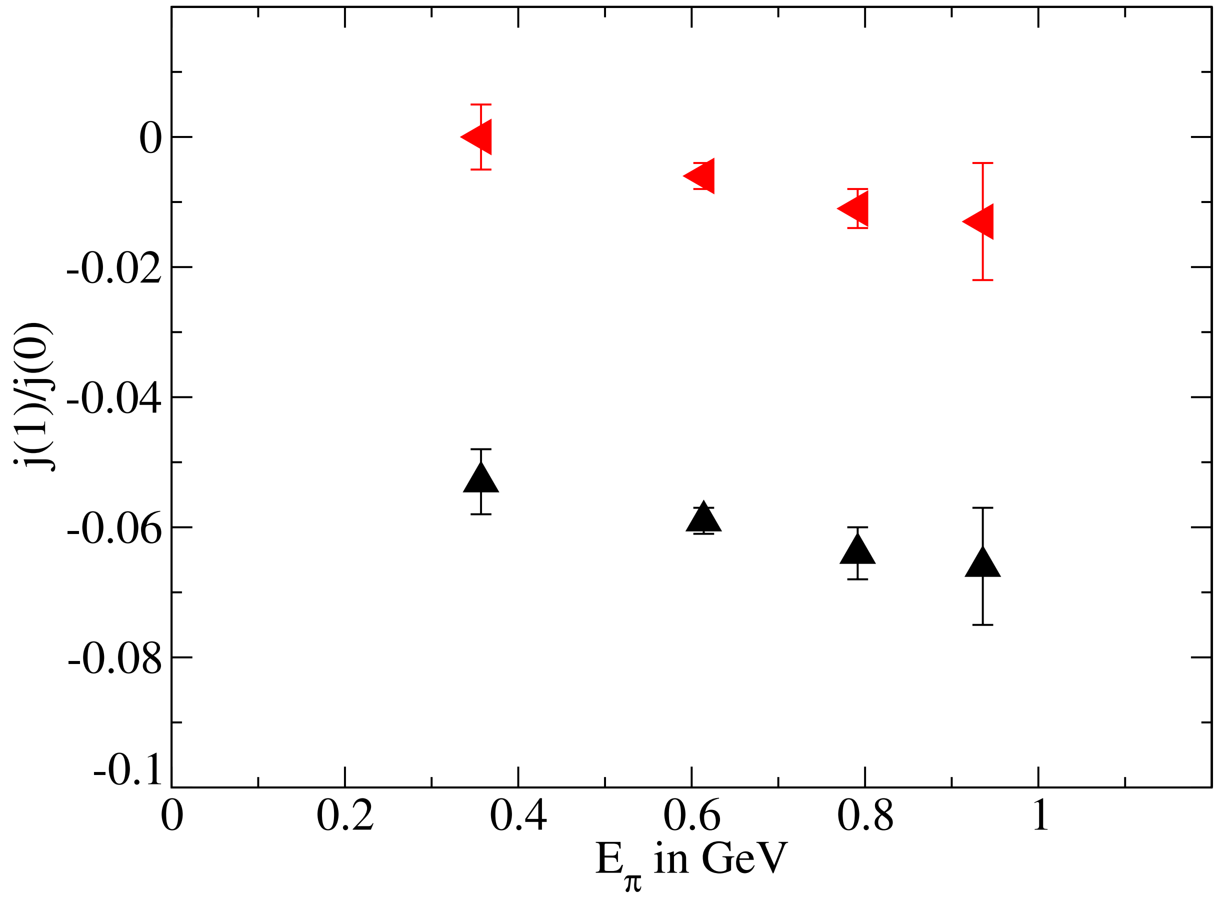


FIG. 1 (color online). The ratio $\langle J_0^{(1)}\rangle/\langle J_0^{(0)}\rangle$ for one ensemble ($u_0 am_f = u_0 am_q = 0.01$) versus the pion energy E_π. The lower points are before power law subtraction and the upper points are after power law subtraction (i.e. $\langle J_0^{(1),\mathrm{sub}}\rangle/\langle J_0^{(0)}\rangle$).

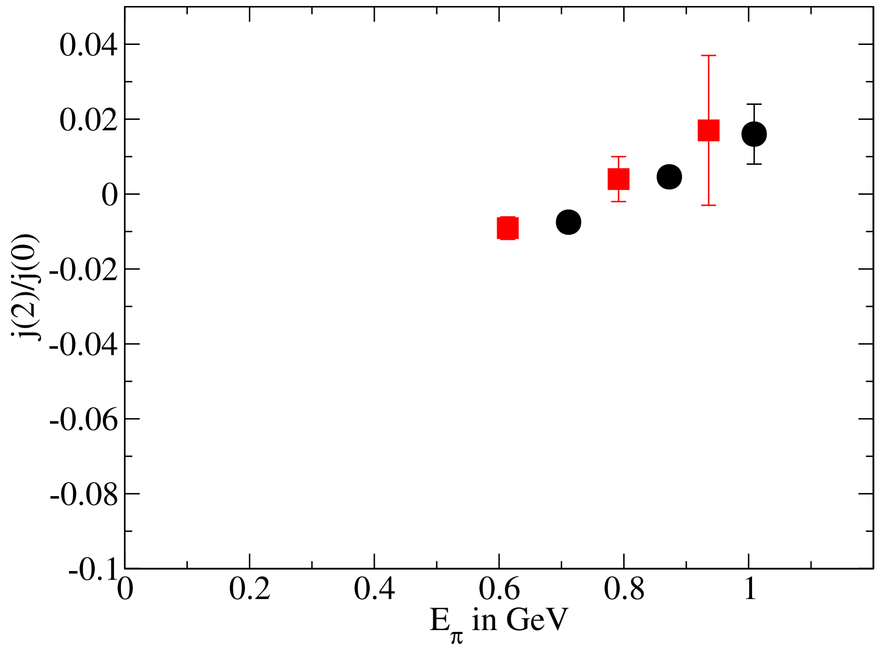


FIG. 2 (color online). The ratio $\langle J_k^{(2)}\rangle/\langle J_k^{(0)}\rangle$ for two ensembles versus the pion energy E_π. Squares are for $u_0 am_f = 0.01$ and circles for $u_0 am_f = 0.02$.

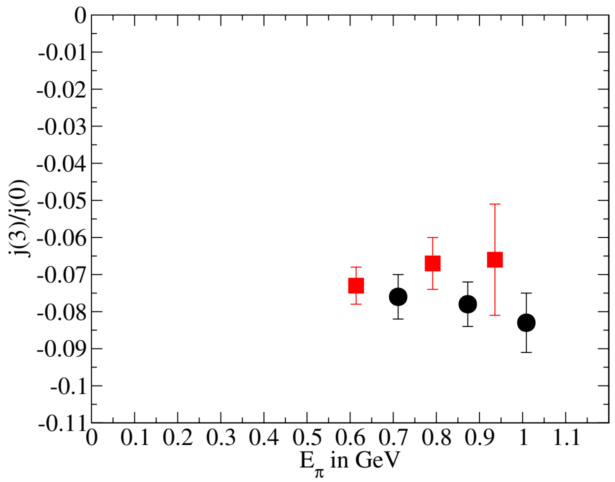


FIG. 3 (color online). Same as Fig. 2 for $\langle J_k^{(3)}\rangle/\langle J_k^{(0)}\rangle$.

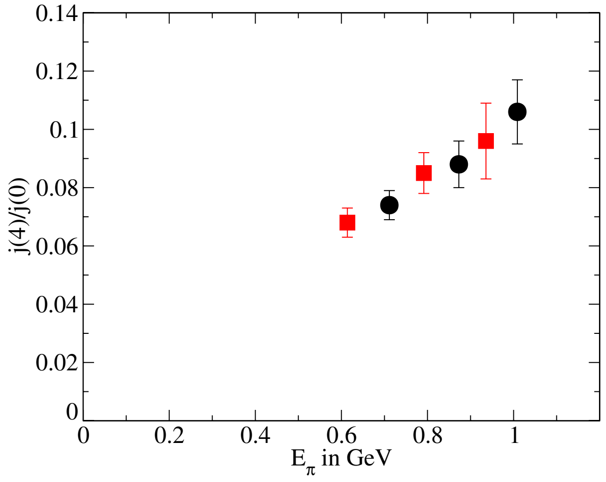
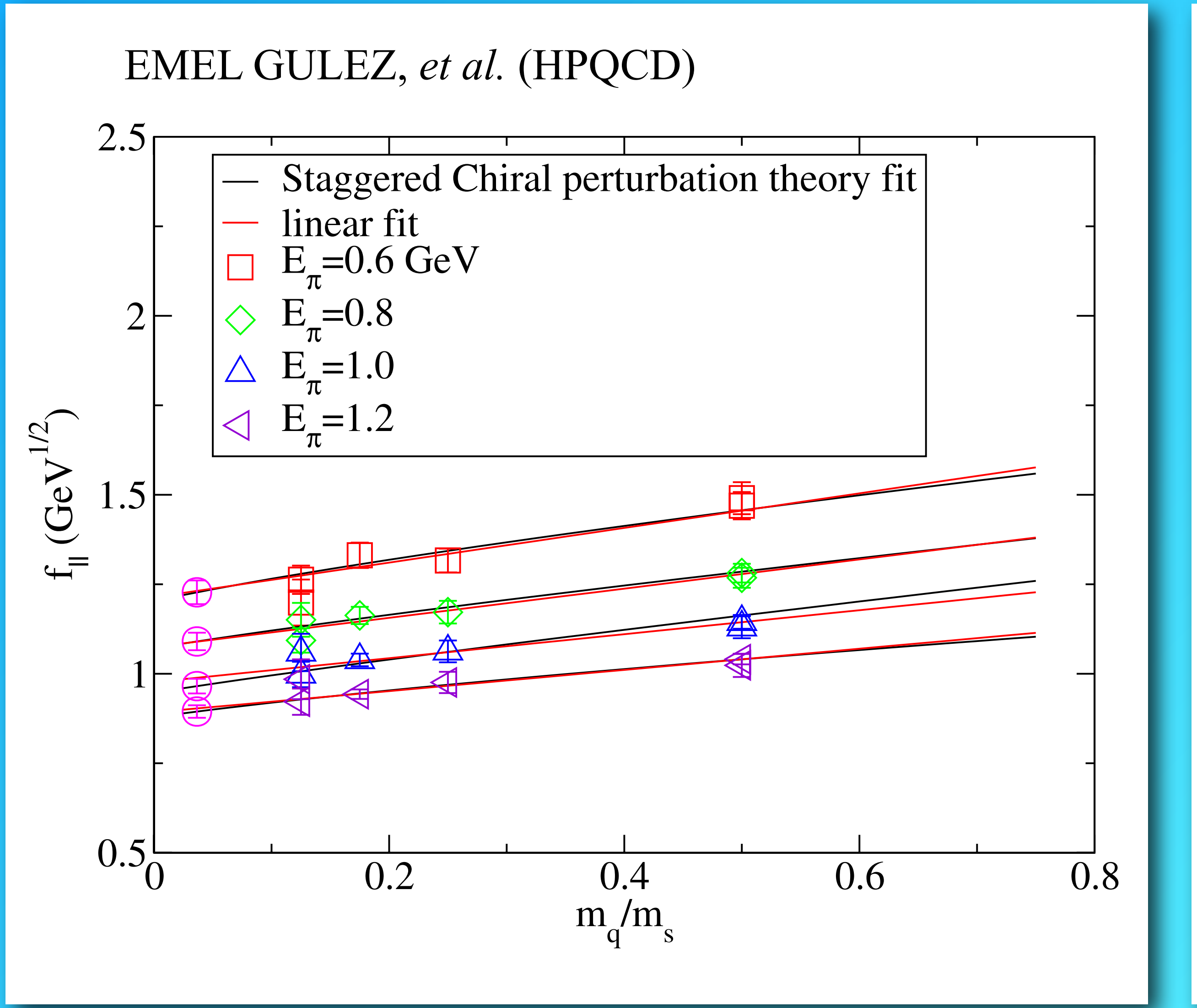


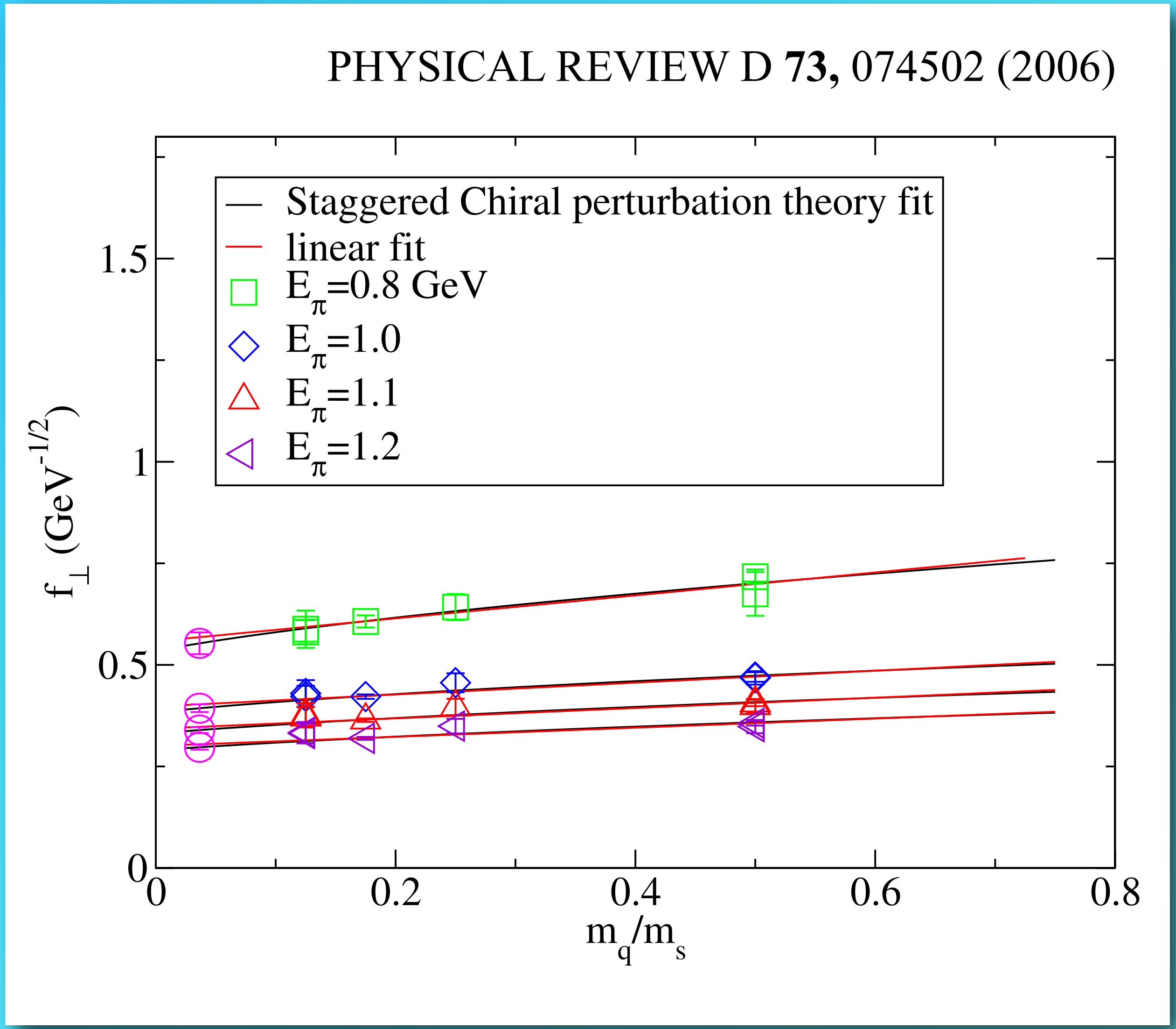
FIG. 4 (color online). Same as Fig. 2 for $\langle J_k^{(4)}\rangle/\langle J_k^{(0)}\rangle$.

Systematic uncertainties

Light quark mass extrapolation: χPT for pions/kaons

Mild mass dependence of $B \rightarrow \pi$ form factors:





Correlation functions

3-point function

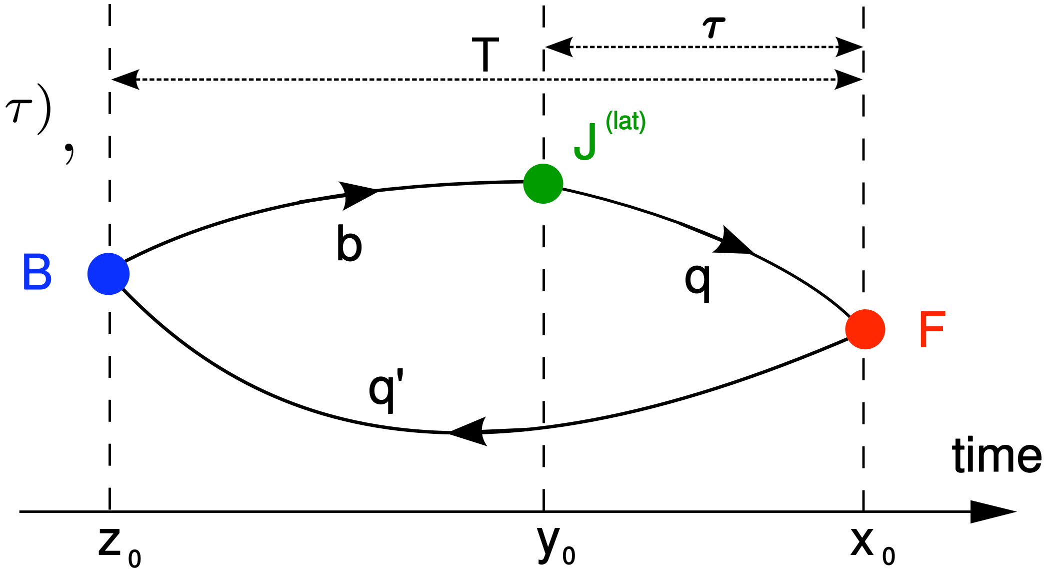
$$C_{FJB}(\mathbf{p}',\,\mathbf{p},\,x_0,\,y_0,\,z_0) = \sum_{\mathbf{y}}\sum_{\mathbf{z}}\left\langle \Phi_F(x)\,J(y)\,\Phi_B^\dagger(z)\right\rangle e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})}e^{-i\mathbf{p}\cdot(\mathbf{y}-\mathbf{z})}$$

2-point functions

$$\begin{aligned} C_{BB}(\mathbf{p},\,x_0,\,y_0) &= \sum_{\mathbf{x}}\left\langle \Phi_B(x)\,\Phi_B^\dagger(y)\right\rangle e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}, \\ C_{FF}(\mathbf{p}',\,x_0,\,y_0) &= \sum_{\mathbf{x}}\left\langle \Phi_F(x)\,\Phi_F^\dagger(y)\right\rangle e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})}. \end{aligned}$$

Large Euclidean-time behavior

$$\begin{aligned} C_{FJB}(\mathbf{p}',\,\mathbf{p},\,\tau,\,T) &\rightarrow A^{(FJB)}e^{-E_F\tau}\,e^{-E_B(T-\tau)}, \\ C_{FF}(\mathbf{p},\,\tau) &\rightarrow A^{(FF)}\,e^{-E_F\tau}, \\ C_{BB}(\mathbf{p},\,\tau) &\rightarrow A^{(BB)}\,e^{-E_B\tau}, \end{aligned}$$



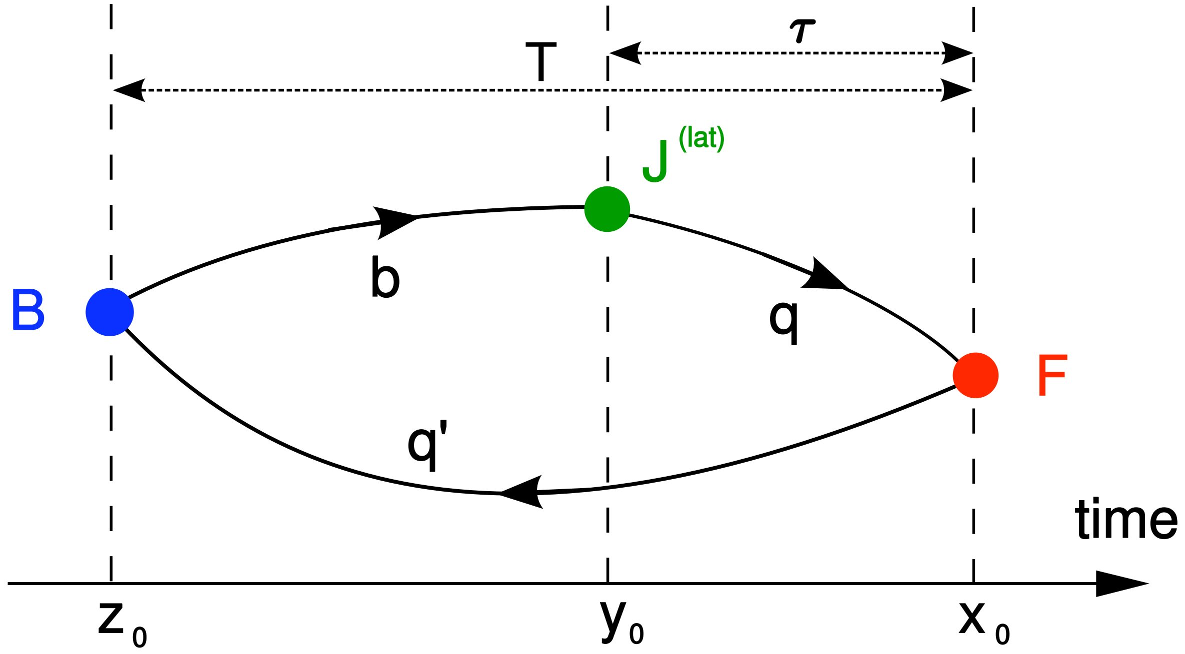
Correlation functions

Matrix element from amplitudes

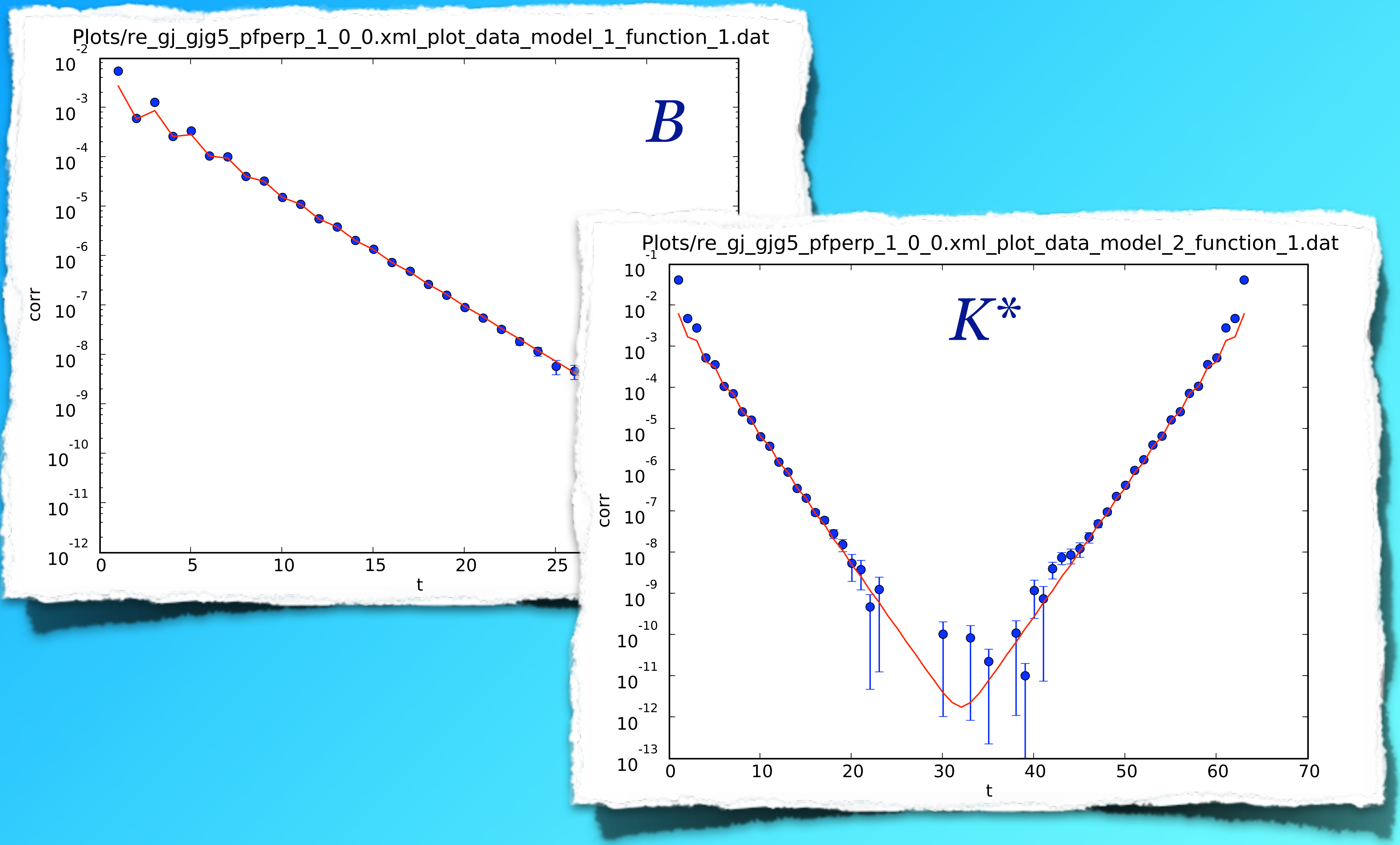
$$A^{(FJB)} = \frac{\sqrt{Z_V}}{2E_V}\frac{\sqrt{Z_B}}{2E_B}\sum_s \varepsilon_j(p',s)\,\langle V\,\big(p',\varepsilon(p',s)\big)\,|\,J\,|B(p)\rangle,$$

$$A^{(BB)} = \frac{Z_B}{2E_B},$$

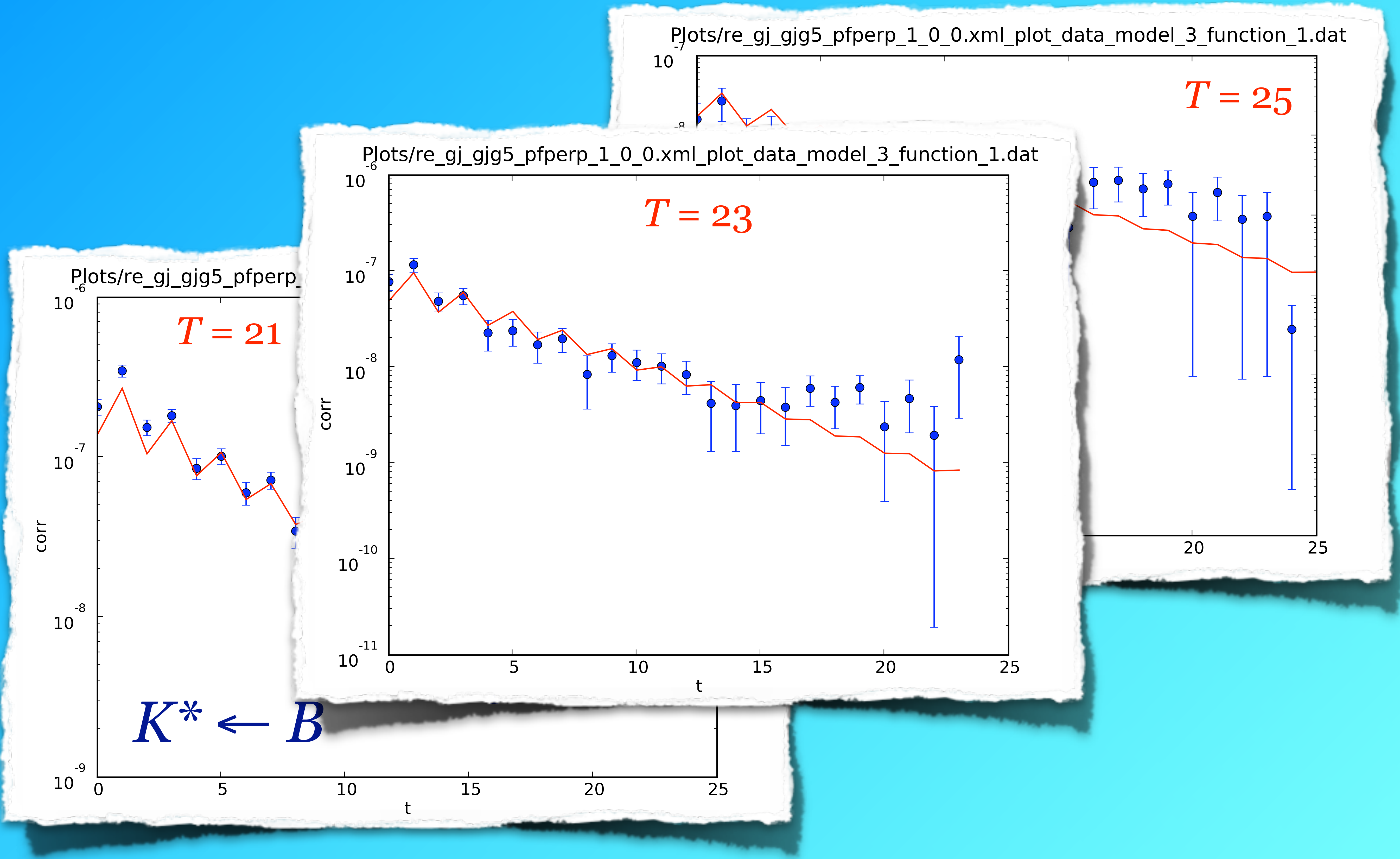
$$A^{(FF)} = \sum_s \frac{Z_V}{2E_V}\,\varepsilon_j^*(p',s)\varepsilon_j(p',s)$$



Example plots



Example plots



$K^* \leftarrow B$

5-correlator fits

Matrix element from amplitudes

$$A^{(FJB)} = \frac{\sqrt{Z_V}}{2E_V}\frac{\sqrt{Z_B}}{2E_B}\sum_s \varepsilon_j(p',s)\,\langle V\left(p',\varepsilon(p',s)\right)|\,J\,|B(p)\rangle,$$

$$A^{(BB)} = \frac{Z_B}{2E_B},$$

$$A^{(FF)} = \sum_s \frac{Z_V}{2E_V}\,\varepsilon_j^*(p',s)\varepsilon_j(p',s)$$

- ✤ One 3-point correlator whose amplitude gives matrix element
- ✤ Two 2-point correlators to divide out 2-pt amplitudes
- ✤ One 3-point correlator with precise *B* energy (*B* to *P* at |p'|=0)
- ✤ One 2-point correlator to further constrain *P* meson mass

2 analyses

- Bayesian:
    - Many-exponential fit function
    - Fit whole range of t (operator position)
    - Fit a couple values of T (source-sink separation)
- Frequentist:
    - Fewer-exponential fit functions
    - Randomly choose (plausible) t-ranges to fit
    - Fit all values of T
    - Rank “best” few fits, then use those t-ranges in bootstrap

Vector, axial-vector f.f.

$$\langle V(p',\varepsilon)|\bar{q}\hat{\gamma}^\mu b|B(p)\rangle = \frac{2\mathrm{i}V(q^2)}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p'_\rho p_\sigma$$

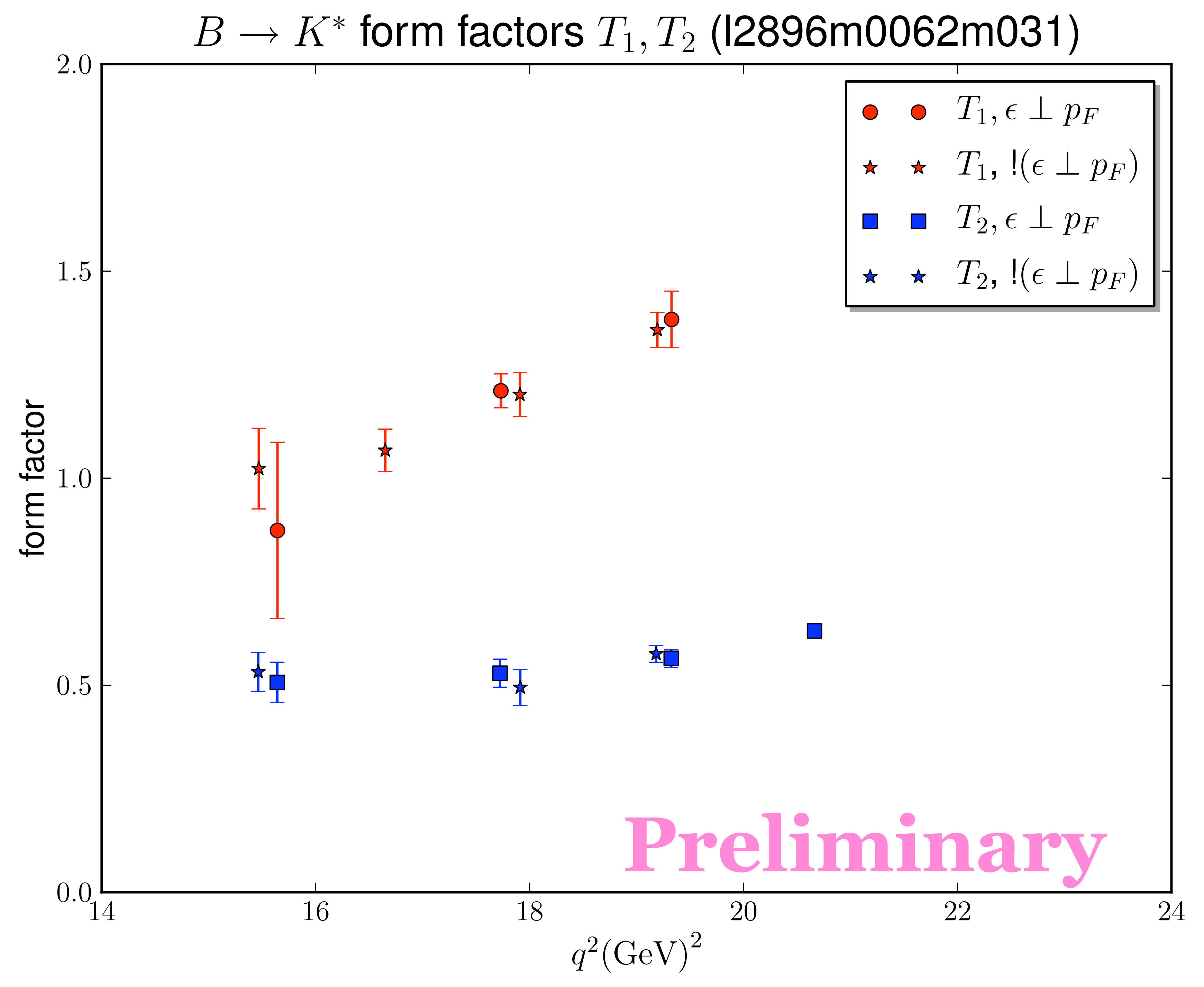
$$\begin{aligned}\langle V(p',\varepsilon)|\bar{q}\hat{\gamma}^\mu\hat{\gamma}^5 b|B(p)\rangle &= 2m_V A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu \\ &+(m_B+m_V)A_1(q^2)\left(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\right) \\ &-A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_V}\left((p+p')^\mu-\frac{m_B^2-m_V^2}{q^2}q^\mu\right)\end{aligned}$$

Tensor f.f.

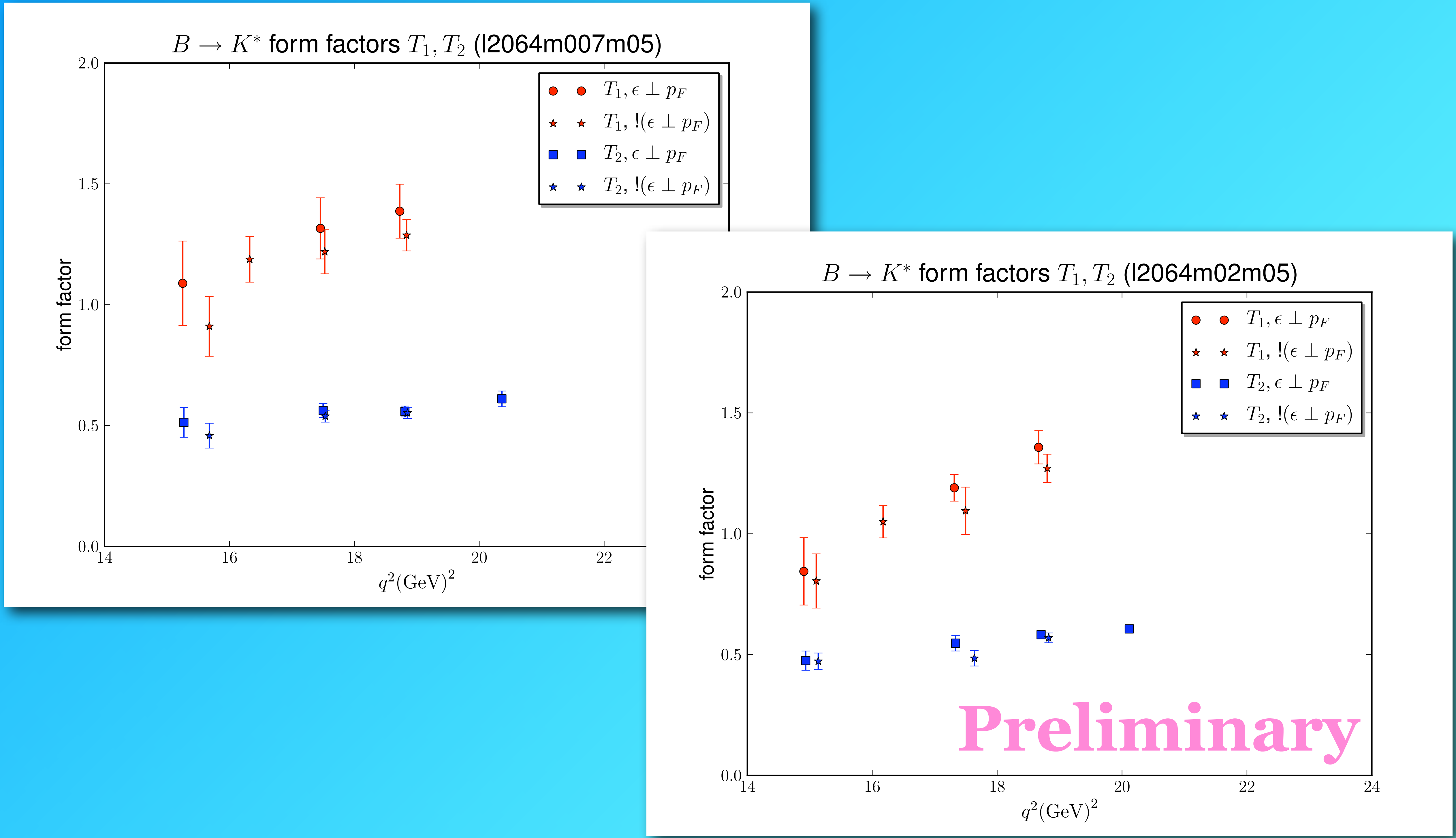
$$q^\nu \langle V(p',\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}b|B(p)\rangle = 2\mathrm{i}T_1(q^2)\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^\tau p'^\sigma$$

$$\begin{aligned} q^\nu \langle V(p',\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}\hat{\gamma}^5 b|B(p)\rangle &= \mathrm{i}T_2(q^2)[\varepsilon^*_\mu(m_B^2 - m_V^2) - (\varepsilon^*\cdot q)(p+p')_\mu] \\ &\quad +\mathrm{i}T_3(q^2)(\varepsilon^*\cdot q)\left[q_\mu - \frac{q^2}{m_B^2 - m_V^2}(p+p')_\mu\right] \end{aligned}$$

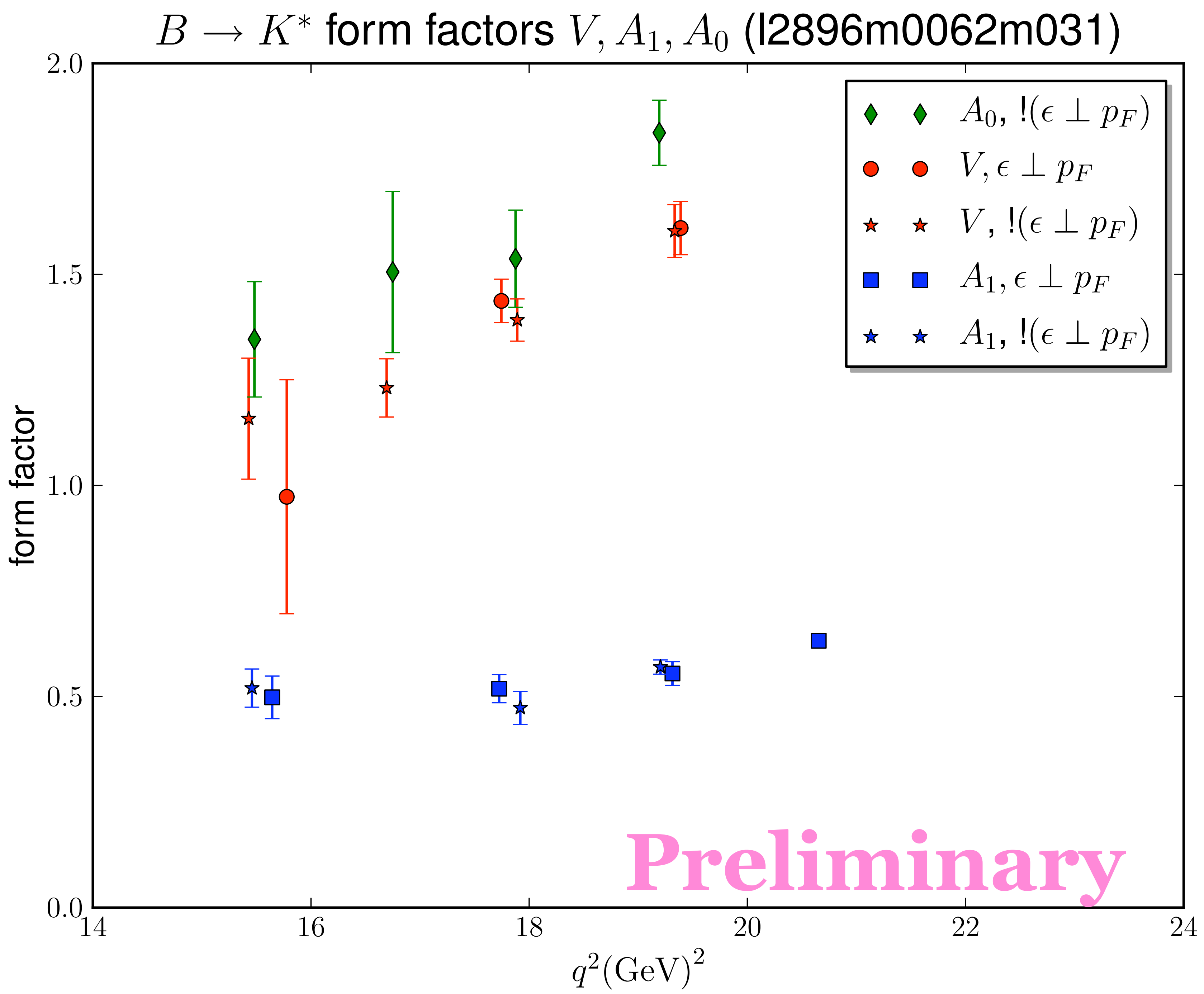
“Fine” MILC lattice



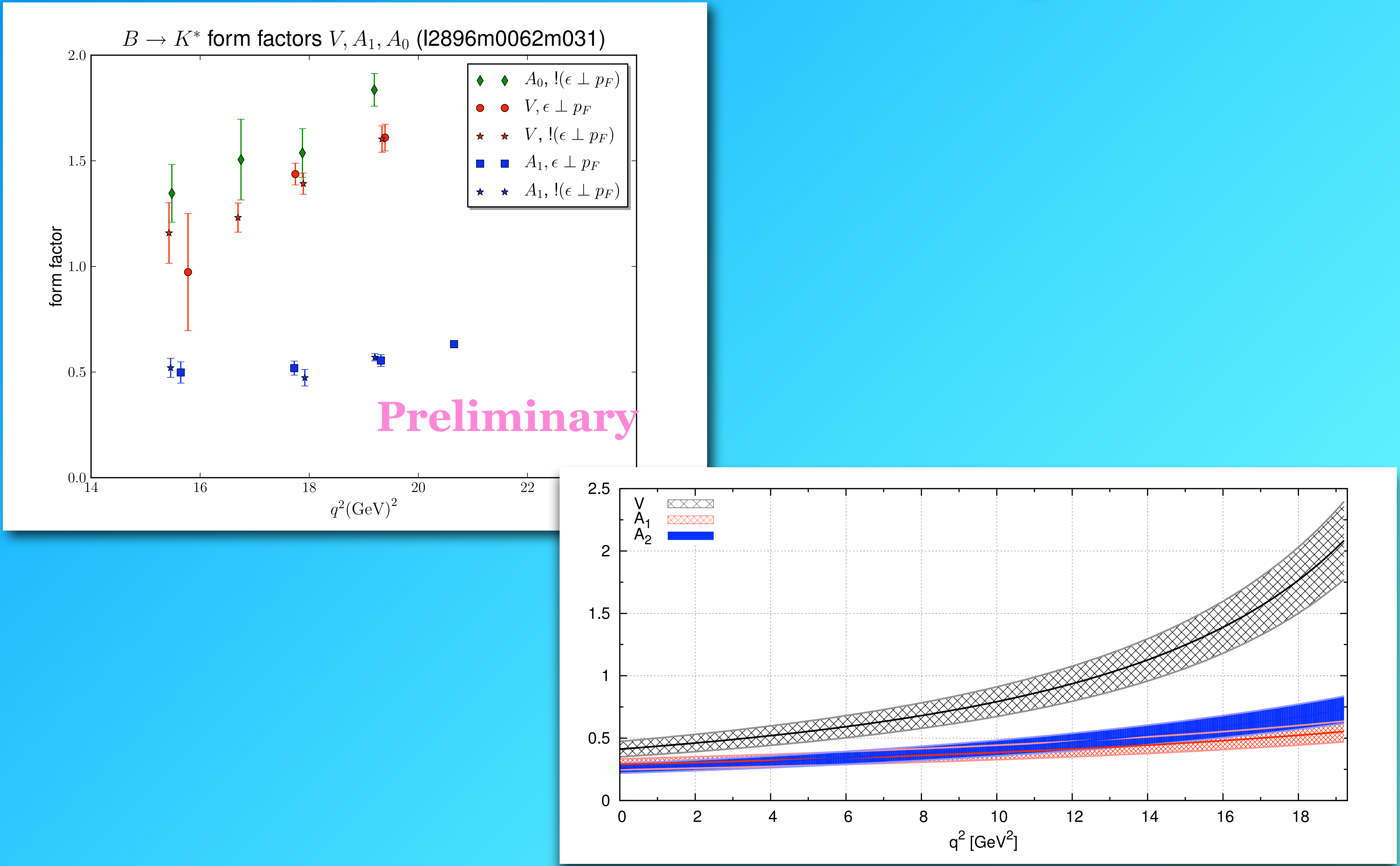
“Course” MILC lattice



“Fine” MILC lattice

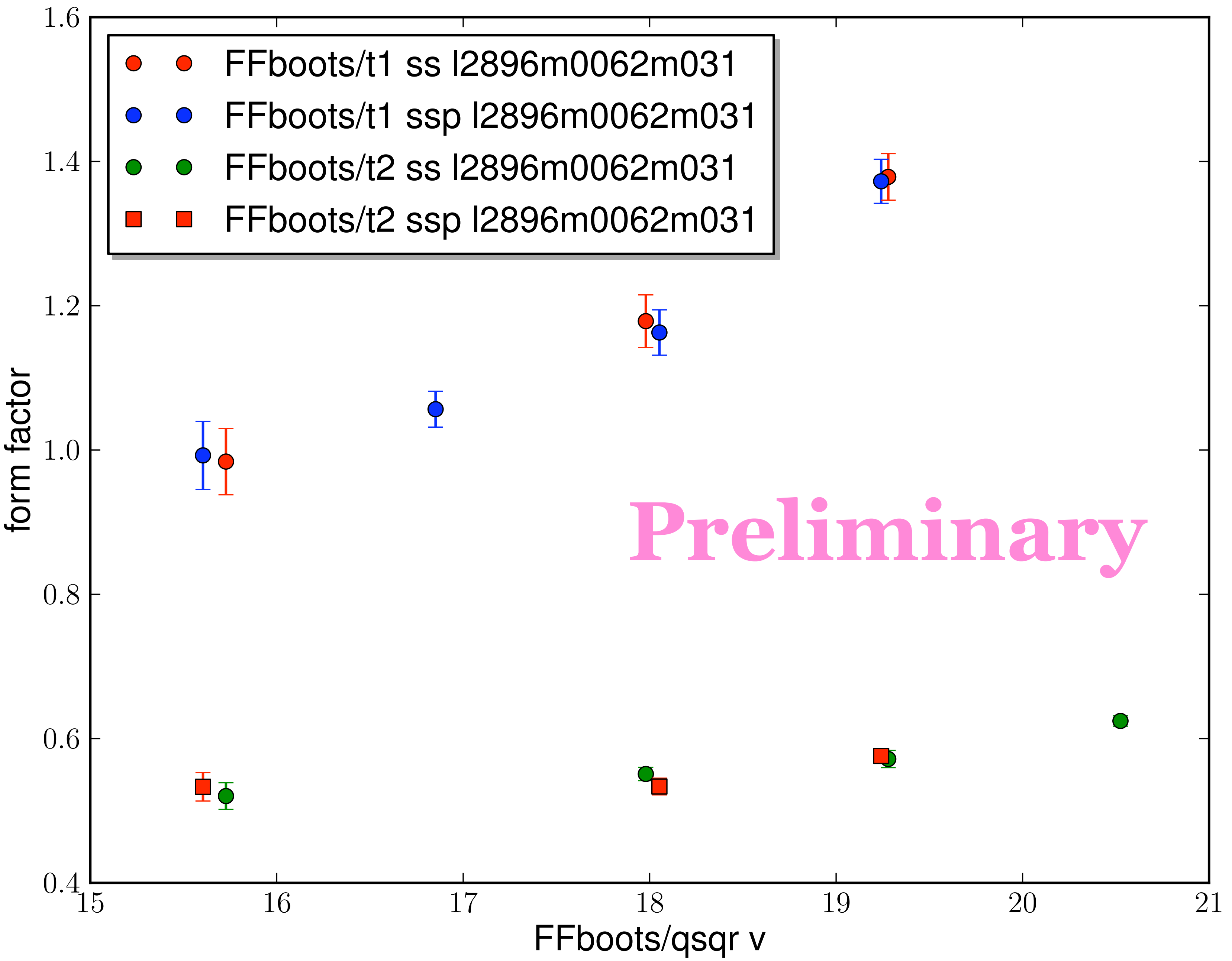


Extrapolation becomes interpolation

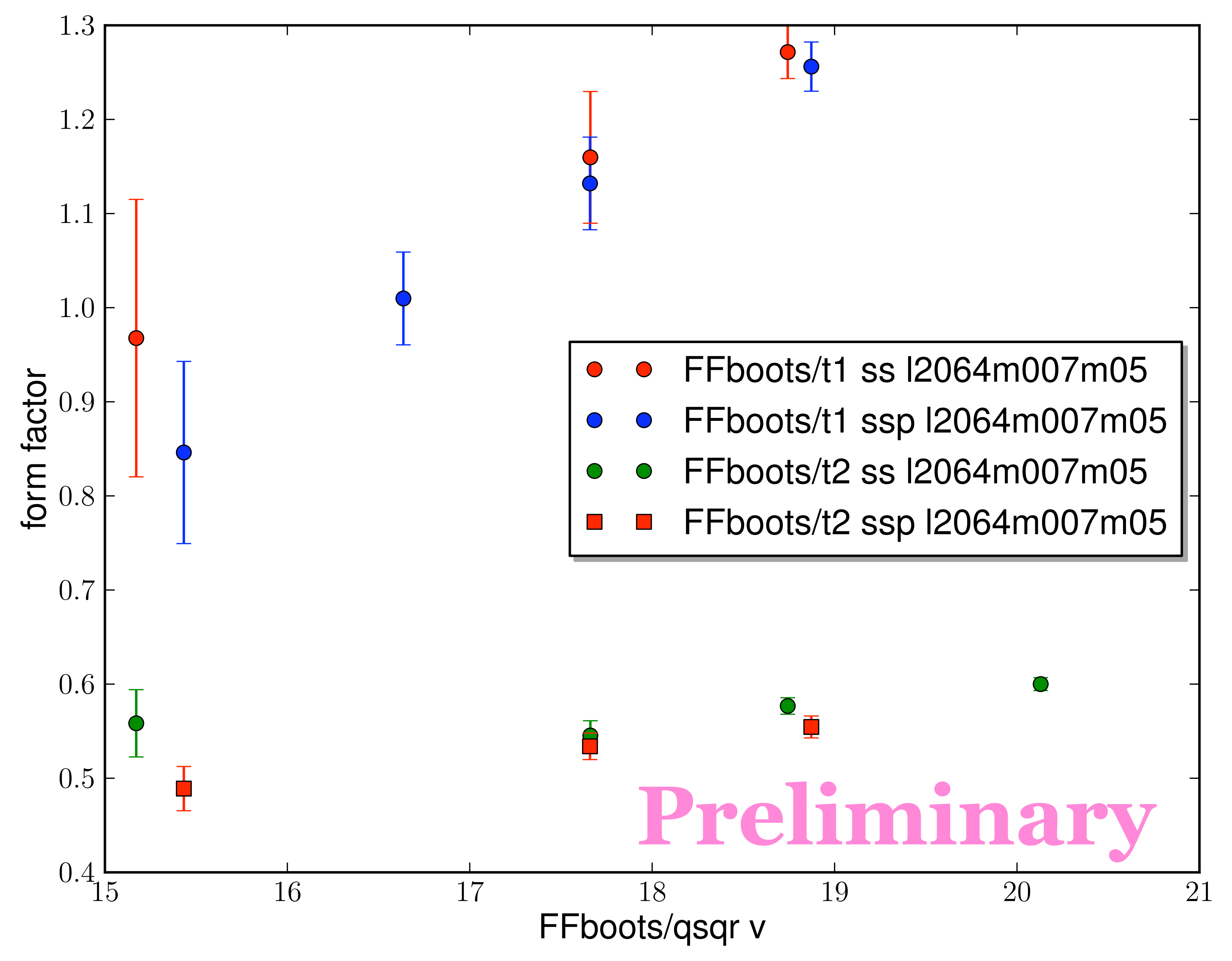


Bobeth, Hiller, van Dyk, extrapolating from Ball & Zwicky’s sum rule f.f.

B to φ; T_1, T_2; fine lattice



B to φ; T_1, T_2; coarse lattice



Form factor shape

Effective pole parameterization

Becirevic & Kaidalov, PLB **478** (2000)
Ball & Zwicky, JHEP **10** (2001), PRD **71** (2005)

$$F(q^2) = \frac{r_0}{1 - q^2/m^2_{\mathrm{res}}} + \frac{r_1}{1 - q^2/m^2_{\mathrm{fit}}} + \frac{r_2}{(1 - q^2/m^2_{\mathrm{fit}})^2}$$

- BZ set $r_0 = 0$ if m_{res} greater than 2-body threshold
- BZ find $r_2 \approx 0$ for A_1 and T_2

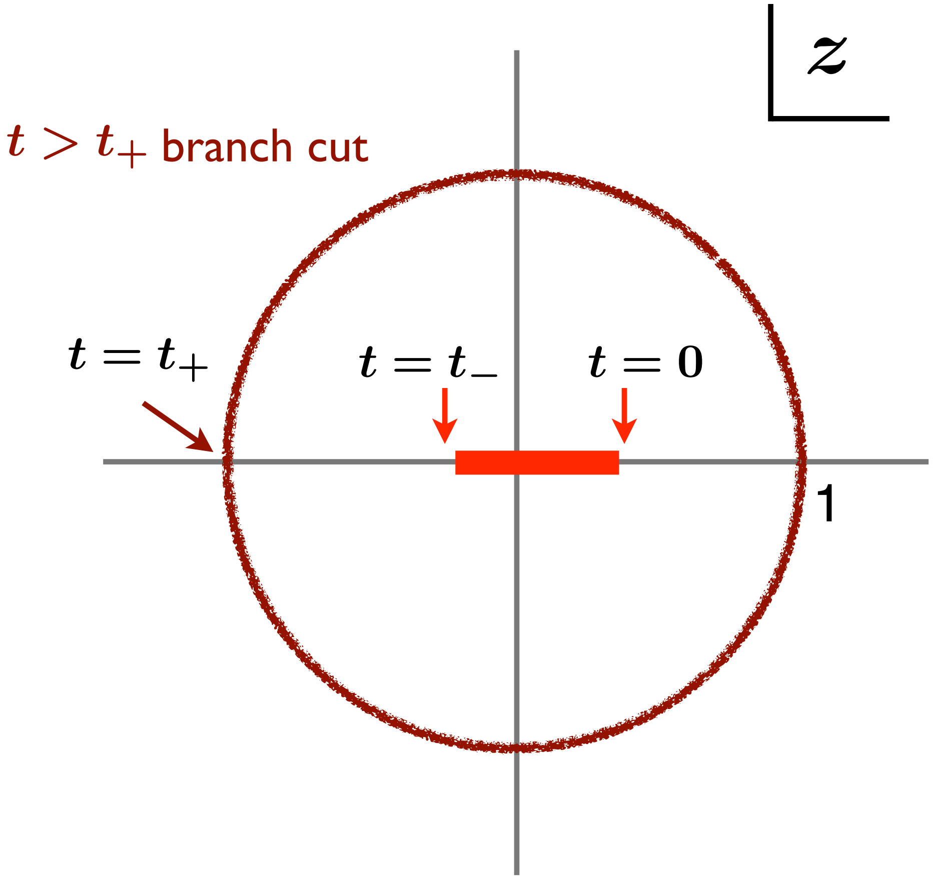
Form factor shape

Series (z) expansion

$$t = q^2 \qquad t_\pm = (m_B \pm m_F)^2$$

Choose, *e.g.* $t_0 = 12\ \mathrm{GeV}^2$

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$



Simplified series expansion

$$F(t) = \frac{1}{1 - t/m_{\mathrm{res}}^2} \sum_n a_n z^n$$

Bourrely, Caprini, Lellouch PRD **79** (2009) following Okubo; Bourrely, Machet, de Rafael; Boyd, Grinstein, Lebed; Boyd & Savage; Arneson *et al.*; ...

Continuum-chiral-kinematic fits

FNAL/MILC, HPQCD

$$F(t) = \frac{1}{1 - t/m_{\rm res}^2}[1 + b_1(aE_F)^2 + \ldots]\sum_n a_n d_n z^n$$

discretization errors

$$d_n = 1 + c_{n1}\frac{m_P^2}{(4\pi f)^2} + \ldots$$

quark mass dependence

Very preliminary fits not yet public

Summary

- Nice experimental data on rare *B* to *V* semileptonic decays
- Life is difficult but let's see what we can do
- At least improve what has been done in the narrow width approximation (the φ is somewhat narrow)
- Unquenched LQCD results at large q^2 complement what has been done with light cone sum rules