1. Unitary Fermi Gas and the ϵ-Expansion ..... Dr M Wingate

Consider a gas of nonrelativistic spin-$\frac{1}{2}$ fermions which have a short-range attractive interaction. This is a good description of fermionic atoms cooled in laser traps, as well as a good toy model of neutron matter. For a *dilute* gas, the scattering between fermions occurs at low energy, so that only S-wave ($\ell = 0$) scattering is relevant. We may then represent the interaction by a δ-function potential with a tunable coefficient c_0, which we can tune to match to the physical scattering length. That is, the system is described by the following Langrangian:

$$\mathcal{L} \;=\; \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi_\sigma \;+\; c_0\, \psi_\uparrow{}^\dagger \psi_\downarrow{}^\dagger\, \psi_\downarrow\, \psi_\uparrow \,. \tag{1}$$

We are especially interested in the special case when the physical scattering length a is tuned to infinity. (In atomic physics experiments, an external magnetic field is tuned to a Feshbach resonance in the particular atomic system.) The first part of your essay should discuss the motivation for studying this system and outline the theoretical set-up [1]. One specific calculation the essay should show is how to match the bare coupling c_0 to physical values of the scattering length [2].

Next, consider the role of dimensionality. Nussinov and Nussinov showed that this system is much simpler in 2 and 4 (spatial) dimensions than in 3 dimensions [3]. Based on these observations, Nishida and Son developed an ϵ-expansion [4,5]. Rederive the ϵ-expansion, filling in some of the details they take for granted.

Relevant Courses

Essential: Statistical Field Theory
Useful: Quantum Field Theory, Advanced Quantum Field Theory

References

[1] E.g. see the review by S. Giorgini, L. P. Pitaevskii, and S. Stringari, arXiv:0706.3360, sections I-V.
[2] E.g. see the lecture notes by D. B. Kaplan, arXiv:nucl-th/0510023, sections 4.2-4.5.
[3] Z. Nussinov and S. Nussinov, arXiv:cond-mat/0410597.
[4] Y. Nishida and D. T. Son, Phys. Rev. Lett. **97**, 050403 (2006).
[5] Y. Nishida and D. T. Son, Phys. Rev. A **75**, 063617, (2007).