1. Spontaneous Symmetry Breaking and Effective Field Theory .....

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The goal of this essay is to discuss the low energy dynamics of theories whose ground state spontaneously breaks a global symmetry of the underlying theory. Arguably the most important example of this phenomenon is given by Quantum Chromodynamics. The QCD Lagrangian with N_f flavours of massless quarks is invariant under $SU(N_f)_L \times SU(N_f)_R$ transformations. However, the vacuum is invariant only under transformations belonging to the diagonal subgroup $SU(N_f)_V$: $\langle \bar{q}q \rangle \neq 0$.

In such cases, one or more Nambu-Goldstone bosons occur in the spectrum of physical states. In QCD, the pion is a pseudo-Nambu-Goldstone boson. Its dynamics are described by the the so-called Chiral Lagrangian, constructed using only symmetry principles and a derivative expansion.

A successful essay will cover the following topics to varying degrees of depth

- Why spontaneously broken (global) symmetries lead to Nambu-Goldstone excitations in the physical spectrum
- The principles behind constructing a low energy effective Lagrangian which has these bosons as its degrees-of-freedom
- Demonstrate these principles for the example of a classical spin model or a quantum anti-ferromagnet
- Discuss these steps if the global symmetry is only an approximate symmetry due to a small external symmetry-breaking source, e.g. an external field (in QCD, this effect is due to nonzero quark masses)
- Discuss effects of finite volume

Everything in this list should be mentioned at least briefly. Some of the topics should be presented in considerable detail, while others may be brief but pedagogical sketches.

Relevant Courses

Essential: Quantum Field Theory
Useful: Statistical Field Theory

References

Ref. [1,2] discuss these topics in detail and in generality. Ref. [3] contains pedagogical reviews of the principles of effective theories in general (not just

for theories with spontaneous symmetry breaking). Ref. [4] discusses finite volume effects in an $O(n)$ model. More references may be provided upon consultation.

[1] S. Weinberg, *The Quantum Theory of Fields*, Vol. 2, Chap. 19, (Cambridge Univ. Press, 1996).

[2] C.G. Callan, S. Coleman, J. Wess, and B. Zumino, Phys. Rev 177, 2247 (1969).

[3] D.B. Kaplan, "Five Lectures of Effective Field Theory," arXiv:nucl-th/0510023.

[4] P. Hasenfratz and H. Leutwyler, Nucl. Phys. B343, 241 (1990).