Extracting B physics from lattice simulations via lattice perturbation theory

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Outline

Two applications of lattice perturbation theory (LPT)

- one-loop: f_B and f_{B_s} arXiv:1202.4914
  - motivation
  - calculating f_B
    - perturbative matching
    - automated LPT
    - simulation details
  - results
- two-loop: m_b
  - motivation
  - calculating m_b
    - perturbative matching - a mixed approach
  - results

But first: an aside on LPT

Wait, lattice perturbation theory ...?

... "**perturbation theory for lattice actions**"

Calculate:

- renormalisation parameters
- improvement coefficients
- matching to continuum QCD matrix elements

Lattice cutoff $\Rightarrow \alpha_S(\pi/a) < 1$ Lepage, arXiv:hep-lat/9607076

Calculating f_B and f_{B_s}: some motivation

f_{B_q} parameterises weak decay of B_q mesons

Direct input into Unitarity Triangle analyses of CKM matrix

Previous results from HPQCD collaboration:

1. f_{B_d}, f_{B_s}: NRQCD b and ASQTad d/s arXiv:0902.1815
   - MILC lattices with $2+1$ ASQTad sea quarks and Symanzik-improved gluons with Lüscher-Weisz coefficients
   - 5 lattice spacings
   - one-loop operator matching
   - uncertainties of $6-7\%$ in f_{B_d}, f_{B_s} and $\sim 2\%$ in f_{B_s}/f_{B_d}
2. f_{B_s}: HISQ b and s quarks arXiv:1110.4510
   - 5 lattice spacings
   - $1/M_b$ expansion up to physical b quark mass
   - uncertainties of $\sim 2\%$ in f_{B_s}

New results:

- ideally calculate f_B with HISQ b and s quarks
  - but fine lattices and light masses $\Rightarrow$ expensive
- (currently) more efficient to update NRQCD calculation
  - ASQTad $\rightarrow$ HISQ valence u/d and s quarks
  - taste-breaking discretisation errors reduced by factor of ~ 3
- calculate $f_B/f_{B_s}^{(NRQCD)} \times f_{B_s}^{(HISQ)}$
  - uncertainty in f_B and f_{B_s} largely cancels in f_{B_s}/f_B
  - result has errors $\sim 2\%$
  - require new operator matching calculation $\Rightarrow$ LPT

NRQCD action correct to $\mathcal{O}(1/M^2, v^4)$:

$$S_{\rm NRQCD} = \sum_{\mathbf{x},\tau} \psi^+(\mathbf{x},\tau)\left[\psi(\mathbf{x},\tau) - \kappa(\tau)\psi(\mathbf{x},\tau-1)\right]$$

with

$$\kappa(\tau) = \left(1 - \frac{\delta H}{2}\right)\left(1 - \frac{H_0}{2n}\right)^n U_4^\dagger \left(1 - \frac{H_0}{2n}\right)^n \left(1 - \frac{\delta H}{2}\right)$$

and

$$\delta H = -c_1 \frac{(\Delta^{(2)})^2}{8M^3} - c_4 \frac{1}{2M}\sigma\cdot\widetilde{\mathbf{B}} + c_5 \frac{\Delta^{(4)}}{24M} - c_6 \frac{(\Delta^{(2)})^2}{16nM^2}$$
$$+ c_2 \frac{ig}{8M^2}(\widetilde{\Delta}^{(\pm)}\cdot\widetilde{\mathbf{E}} - \widetilde{\mathbf{E}}\cdot\widetilde{\Delta}^{(\pm)}) - c_3 \frac{g}{8M^2}\sigma\cdot(\widetilde{\Delta}^{(\pm)}\times\widetilde{\mathbf{E}} - \widetilde{\mathbf{E}}\times\widetilde{\Delta}^{(\pm)})$$

M	c_1 and c_6	c_4	c_5
2.50	$1 + 0.95\alpha_V(2.0/a)$	$1 + 0.78\alpha_V(\pi/a)$	$1 + 0.41\alpha_V(2.0/a)$
1.72	$1 + 0.766\alpha_V(1.8/a)$	$1 + 0.691\alpha_V(\pi/a)$	$1 + 0.392\alpha_V(1.4/a)$

ASQTad action correct to $\mathcal{O}(a^2)$, strongly reduced $\mathcal{O}(\alpha_S a^2)$ errors:

$$S_{\mathrm{ASQTad}} = \sum_x \overline{\psi}(x) \left(\gamma^\mu \Delta_\mu^{\mathrm{ASQTad}} + m \right) \psi(x)$$

where

$$\Delta_\mu^{\mathrm{ASQTad}} = \Delta_\mu^F - \frac{1}{6}(\Delta_\mu)^3.$$

F indicates

$$U_\mu \to \mathcal{F}_\mu \widetilde{U}_\mu = u_0^{-1} \left[\prod_{\nu \neq \mu} \left(1 + \frac{\Delta_\nu^{(2)}}{4} \right)_{\mathrm{symm}} - \sum_{\nu \neq \mu} \frac{(\Delta_\nu)^2}{4} \right] U_\mu$$

HISQ action correct to $\mathcal{O}(a^4)$, $\mathcal{O}(\alpha_S a^2)$ with reduced taste-changing:

$$S_{\mathrm{HISQ}} = \sum_x \overline{\psi}(x)\left(\gamma^\mu \Delta_\mu^{\mathrm{HISQ}} + m\right)\psi(x)$$

where

$$\Delta_\mu^{\mathrm{HISQ}} = \Delta_\mu\left[\mathcal{F}_\mu^{\mathrm{HISQ}} U_\mu(x)\right] - \frac{1+\epsilon}{6}(\Delta_\mu)^3\left[U\mathcal{F}_\mu^{\mathrm{HISQ}} U_\mu(x)\right].$$

and

$$\mathcal{F}_\mu^{\mathrm{HISQ}} = \mathcal{F}_\mu^{\mathrm{ASQTad}} U_\mu \mathcal{F}_\mu$$

Operator matching

f_{B_q} defined through

$$\langle 0|A_\mu|B_q\rangle_{QCD} = f_{B_q} p_\mu$$

Simulations carried out with effective lattice operators

$$J_0^{(0)} = \overline{\Psi}_q \Gamma_0 \Psi_Q \qquad J_0^{(1)}(x) = -\frac{1}{2M_b}\overline{\Psi}_q \Gamma_0 \gamma \cdot \overrightarrow{\nabla}\Psi_Q$$

$$J_0^{(2)}(x) = -\frac{1}{2M_b}\overline{\Psi}_q \gamma \cdot \overleftarrow{\nabla}\gamma_0 \Gamma_0 \Psi_Q$$

- Ψ_q - four-component HISQ u/d or s quark field
- Ψ_Q - two-component NRQCD field

Matching gives

$$\langle A_0\rangle_{QCD} = (1+\alpha_s\rho_0)\,\langle J_0^{(0)}\rangle + (1+\alpha_s\rho_1)\,\langle \widetilde{J}_0^{\,(1)}\rangle + \alpha_s\rho_2\langle \widetilde{J}_0^{\,(2)}\rangle$$

where

$$\widetilde{J}_0^{\,(i)} = J_0^{(i)} - \alpha_s\zeta_{10}J_0^{(0)}$$

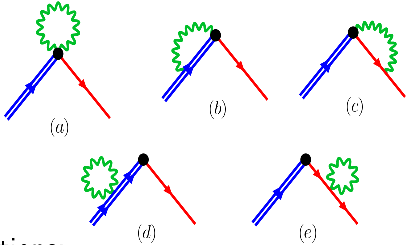
Here ρ_0, ρ_1, ρ_2 and ζ_{10} are one-loop matching coefficients

Continuum diagrams for $\langle A_0\rangle_{QCD}$:

- calculated analytically with $1/M_b$ expansion

Calculating matching coefficients

Lattice diagrams for ρ_0, ρ_1, ρ_2 and ζ_{10}:



- Independent determinations:
  1. automated lattice perturbation theory: `HiPPy` and `HPsrc`
     - `HiPPy` – `python` routines produce Feynman rules encoded as "vertex files" Hart, von Hippel, Horgan arXiv:0904.0375
     - `HPsrc` – `Fortran 90` routines reconstruct diagrams and evaluate integrals with `VEGAS`
  2. "by hand" calculation
     - `Mathematica` file handles Dirac algebra
     - `Fortran` suite extracts Feyman rules via iterated convolution
     - integrals evaluated numerically with `VEGAS`

Automated LPT: HiPPy

`HiPPy` generates Feynman rules, encoded as "vertex files"
To generate vertex files:

- Expand link variables

Lüscher and Weisz, NPB 266 (1986) 309

$$U_{\mu>0}(x) = \exp\left(gA_\mu\left(x+\frac{\hat{\mu}}{2}\right)\right) = \sum_{r=0}^{\infty}\frac{1}{r!}\left(gA_\mu\left(x+\frac{\hat{\mu}}{2}\right)\right)^r$$

with $U_{-\mu} \equiv U_\mu^\dagger(x-\hat{\mu})$

- Actions built from products of link variables - Wilson lines

$$L(x,y;U) = \sum_r \left(\frac{g^r}{r!}\right) \sum_{k_1,\mu_1,a_1} \cdots \sum_{k_r,\mu_r,a_r} \widetilde{A}_{\mu_1}^{a_1}(k_1)\cdots\widetilde{A}_{\mu_r}^{a_r}(k_r) \times V_r(k_1,\mu_1,a_1;\ldots;k_r,\mu_r,a_r)$$

where the V_r are "vertex functions"

- Vertex functions decomposed into colour structure matrix, C_r and "reduced vertex", Y_r

$$V_r(k_1, \mu_1, a_1; \ldots; k_r, \mu_r, a_r) = C_r(a_1; \ldots; a_r) Y_r(k_1, \mu_1; \ldots; k_r, \mu_r)$$

- Reduced vertices are products of exponentials

$$Y_r(k_1, \mu_1; \ldots; k_r, \mu_r) = \sum_{n=1}^{n_r} f_n \exp\left(\frac{i}{2}\left(k_1 \cdot v_1^{(n)} + \cdots + k_r \cdot v_r^{(n)}\right)\right)$$

where the f_n are amplitudes and the $v^{(n)}$ the locations of each of the r factors of the gauge potential

- Feynman rules encoded as ordered lists

$$E = (\mu_1, \cdots, \mu_r; x, y; v_1, \cdots, v_r; f)$$

For example, the product of two links, $L(0, 2x, U) = U_x(0)U_x(x)$, is

$$U_x(0)U_x(x) = \left[\sum_{r_1=0}^{\infty} \frac{1}{r_1!}\left(gA_x\left(\frac{x}{2}\right)\right)^{r_1}\right]\left[\sum_{r_2=0}^{\infty} \frac{1}{r_2!}\left(gA_x\left(\frac{3x}{2}\right)\right)^{r_2}\right]$$

$$= 1 + g\sum_{k_1} \widetilde{A}_x(k_1)e^{ik_1\cdot x/2} + g\sum_{k_2} \widetilde{A}_x(k_2)e^{i2_1\cdot 3x/2} + \dots$$

$$= 1 + g\sum_{k_1}\sum_{a_1} \widetilde{A}_x^{a_1}(k_1)T^{a_1}\left(e^{ik_1\cdot x/2} + e^{ik_1\cdot 3x/2}\right)$$

Vertex function

$$V_1(k_1, x, a_1) \equiv C_1(a_1)Y_1(k_1, x) = T^{a_1}\left(e^{ik_1\cdot x/2} + e^{ik_1\cdot 3x/2}\right)$$

Reduced vertex

$$Y_1(k_1, x) = \left(e^{ik_1\cdot x/2} + e^{ik_1\cdot 3x/2}\right)$$

Reduced vertex

$$Y_1(k_1, x) = \sum_{n=1}^{n_1=2} f_n \exp\left(\frac{i}{2}\left(k_1 \cdot v_1^{(n)}\right)\right)$$

So in this case

$$f_1 = f_2 = 1 \ ; \ v_1^{(1)} = (1/2, 0, 0, 0) \ , \ v_1^{(2)} = (3/2, 0, 0, 0)$$

We store this information as the list

$$\begin{aligned} E &= (\mu_1; x, y; v_1^{(1)}, v_1^{(2)}; f) \\ &= (x; (0,0,0,0), (2x, 0, 0, 0); (1/2, 0, 0, 0), (3/2, 0, 0, 0); (1, 1)) \end{aligned}$$

HPsrc

`HPsrc` routines build Feynman diagrams and evaluate them numerically with `VEGAS`

Vertex files read-in by appropriate module

- e.g. module `mod_vertex_qqg.F90` reads in vertex file corresponding to quark-quark-gluon vertex

Fundamental vertices defined as functions using `TaylUR` package, carrying analytic derivatives

- e.g. `vert_qqg(k_1,k_2,k_3;a_1,a_2,a_3)`

User generates Feynman diagrams by multiplying fundamental vertices as in the continuum

For example, vertex diagram contribution to ρ_0

$$\rho_0^{\rm vert} = \frac{4\pi}{(2\pi)^4} \sum_{c=1}^{8} \sum_{\mu,\nu=1}^{4} \int_{-\pi}^{\pi} \mathrm{d}^4 k \; \mathrm{vert\_nrqcd\_qqg}(\mathrm{p}, -\mathrm{k}, \mathrm{k}-\mathrm{p}; \mathrm{a}, \mathrm{c}, \mathrm{a}; \mu)$$
$$\times\, \mathrm{nrqcd\_prop}(\mathrm{k}-\mathrm{p}, -\mathrm{k}+\mathrm{p}; \mathrm{a}, \mathrm{a})$$
$$\times\, \mathrm{J}_0^{(0)}$$
$$\times\, \mathrm{hisq\_prop}(-\mathrm{k}-\mathrm{p}', \mathrm{k}+\mathrm{p}'; \mathrm{b}, \mathrm{b})$$
$$\times\, \mathrm{vert\_hisq\_qqg}(-\mathrm{k}-\mathrm{p}', \mathrm{k}, \mathrm{p}'; \mathrm{a}, \mathrm{c}, \mathrm{b}; \nu)$$
$$\times\, \mathrm{gluon\_prop}(\mathrm{k}, -\mathrm{k}; \mathrm{c}, \mathrm{c}; \mu; \nu)$$

for quarks with external momenta p, p' and colour indices a, b

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    - automated LPT
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  - calculating m_b
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But first: an aside on LPT

Simulation

Some simulation details:

- NRQCD action - see also Rachel Dowdall's talk:
  "B meson and Bottonium spectroscopy from radiatively improved NRQCD"
- HISQ action - see also Christine Davies' talk:
  "Relativistic heavy quarks on the lattice"
- Updated r_1 value from HPQCD
  $\Rightarrow$ retuned bare b quark mass, M_b
- M_b fixed via spin-averaged Υ mass
- s quark mass fixed via fictitious η_s mass

Results

We find

$$f_B = 0.191(9)\,\mathrm{GeV} \qquad \text{and} \qquad f_{B_s} = 0.227(10)\,\mathrm{GeV}$$

so

$$\frac{f_{B_s}}{f_B} = 1.188(18)$$

Agreement with previous HPQCD HISQ result

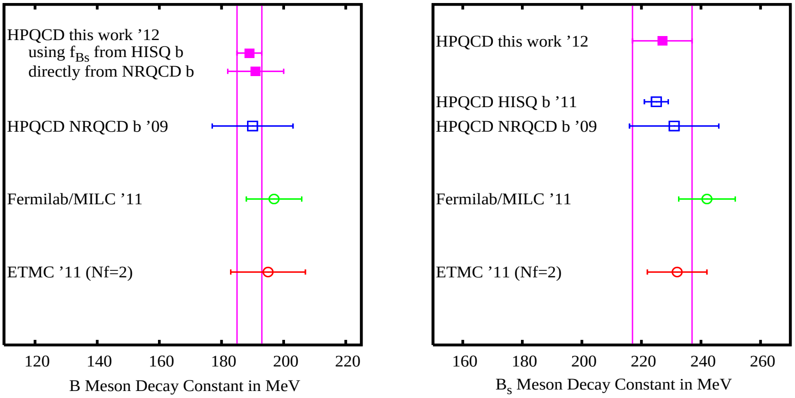
$$f_{B_s}^{(HISQ)} = 0.225(4)\,\mathrm{GeV}$$

$\Rightarrow$ non-trivial consistency check!

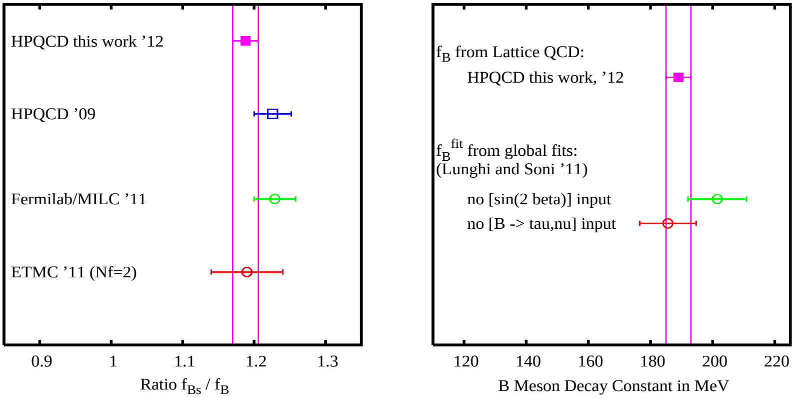
Error budget

	f_B	f_{B_s}	f_{B_s}/f_B
statistical	1.2	0.6	1.0
$\mathcal{O}(\alpha_s)$ operator matching	4.1		0.1
relativistic	1.0		0.0
r_1 scale	1.1		-
continuum extrapolation	0.9		0.9
chiral extrapolation	0.2	0.5	0.6
mass tuning	0.2	0.1	0.2
finite volume	0.1	0.3	0.36

f_B and f_{B_s} results



f_{B_s}/f_B and $f_B^{(\mathrm{fit})}$ results



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Extracting m_b

Fundamental parameter of the Standard Model

Direct input into Unitarity Triangle analyses of CKM matrix

Previous results from HPQCD collaboration:

1. NRQCD b quarks PRD 72 (2005) 094507
   - MILC lattices with $2+1$ ASQTad sea quarks and Symanzik-improved gluons with Lüscher-Weisz coefficients
   - 3 lattice spacings
   - one-loop matching dominated uncertainty of $\sim 7\%$
2. HISQ b quarks arXiv:1004.4285
   - 5 lattice spacings
   - $1/M_b$ expansion up to physical b quark mass
   - uncertainty of $\sim 0.5\%$

New results: two-loop matching reduces dominant error $\Rightarrow$ LPT

Matching condition

Calculation proceeds via pole mass:

lattice quantities ⟷ pole mass ⟷ $\overline{MS}$ mass

↓

3-loop

↓

continuum matching parameter $Z_{\rm cont}^{-1}$

$$M_b^{\rm pole} = Z_{\rm cont}(\mu, M_b^{\rm pole}) M^{\overline{MS}}(\mu).$$

Matching condition

lattice quantities ⟷ pole mass ⟷ $\overline{MS}$ mass

↓ 1-loop ↓

- via heavy quark mass renormalisation $Z_M^{\rm latt}$

$$M_b^{\rm pole} = Z_M^{\rm latt}(\mu a, M_b^{\rm latt}(a)) M_b^{\rm latt}(a).$$

$M_b^{\rm latt}(a)$ – bare lattice mass in lattice units

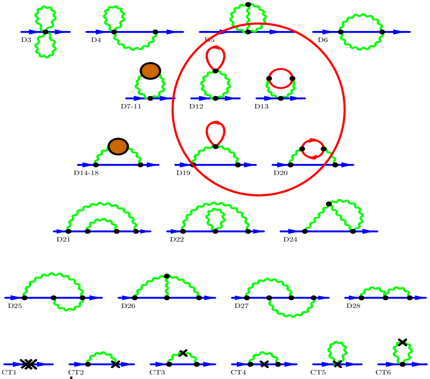
- via heavy quark energy shift E_0

$$M_\Upsilon^{\rm expt} = E^{\rm sim}(0) + 2\left(M_b^{\rm pole} - E_0\right)$$

$E^{\rm sim}(0)$ from HPQCD lattice NRQCD simulation;
$M_\Upsilon^{\rm expt} = 9.46030(26)$ from experiment

Extracting E_0

Contributions to E_0:



$\Rightarrow$ implement a mixed approach:

- quenched high-β simulations
- automated LPT

Quenched contributions

Quenched high-β simulations:

- $L^3 \times T$ lattices, $T = 3L$, $L \in [3, 10]$
- twisted boundary conditions
- 17 values of β from $\beta = 9$ to $\beta = 120$
- fix to Coulumb gauge using conjugate method
- tadpole-improved

To extract results:

1. exponential fit to

$$G(\mathbf{p}, t) = Z_\Psi \exp\left(-\left[E_0 + \frac{p^2}{2Z_M^{\mathrm{latt}} + \ldots}\right] t\right)$$

2. convert β to α_V
3. joint fit to α_V and L in $L \to \infty$ limit

with

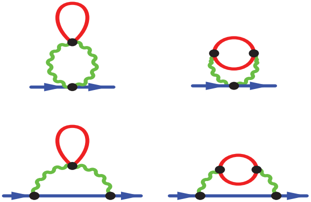
$$E_0 = E_0^{(1)}\alpha_V(3.33/a) + E_0^{(2),q}\alpha_V^2(3.33/a) + E_0^{(3),q}\alpha_V^3(3.33/a)$$

$E_0^{(1)}$ - calculated using exact mode summation

$E_0^{(2,3),q}$ - extracted with $E_0^{(1)}$ constrained to exact value

Fermionic contributions

- In this case we need to calculate



- Feynman rules: `HiPPy` – `python` routines produce vertex files
- Diagrams: `HPsrc` – `Fortran 90` routines reconstruct diagrams and evaluate integrals with `VEGAS`

Results

Extract b quark mass from

$$M_{\overline{MS}}(M_{\overline{MS}}) = \frac{1}{2} Z_{\rm cont}^{-1}(M_{\overline{MS}}, M_{\rm pole}) \left[M_{\Upsilon}^{\rm expt} - \left(E^{\rm sim}(0) - 2a^{-1} E_0 \right) \right]$$

with

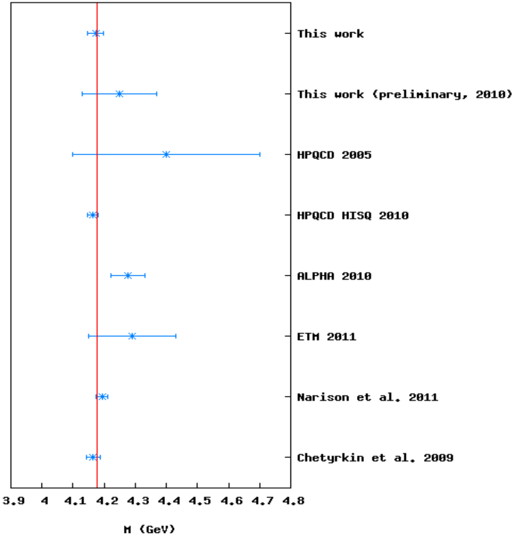
$$E_0 = E_0^{(1)} \alpha_V + \left(E_0^{(2),q} + n_f E_0^{(2),f} \right) \alpha_V^2 + E_0^{(3),q} \alpha_V^3$$

M	$E_0^{(1)}$	$E_0^{(2),q}$	$E_0^{(2),f}$	$E_0^{(3),q}$	$E^{\rm sim}(0)$ (GeV)	$M_{\overline{MS}}$ (GeV)
2.50	0.6864(5)	1.35(10)	0.2823(6)	2.2(8)	0.7397(66)	4.185(26)
1.72	0.5873(6)	1.56(11)	0.3041(3)	2.2(8)	0.96417(13)	4.154(27)

Error budget

- Statistical errors
  - $E^{\rm sim}(0)$, $M_\Upsilon^{\rm expt}$, E_0
  - ~ 3 MeV
- Perturbative errors:
  - all three-loop contributions included
  - $E_0^{(3),f}$ estimated as $\mathcal{O}(1 \times \alpha_s^3)$
  - ~ 26 MeV
- Systematic
  - lattice spacing dependence
  - best way to estimate this from two data points?
  - agree within average $\Rightarrow \sim 22$ MeV
  - fit to $F(M) = M_0 + A/a^2 M^2$

Comparison to other results



Summary

LPT important tool in extracting precise results from lattice QCD:

- HISQ/NRQCD operator matching for f_B and f_{B_s}
  $\Rightarrow$ combined results give uncertainties of $\sim 2\%$ for f_B
- extend to massive HISQ for B_c
- two-loop perturbation theory allows extraction of precise m_b from NRQCD simulations
  $\Rightarrow$ reduced uncertainties from $\sim 7\%$ to $\sim 0.6\%$ for m_b

Thank you!

Fits and correlators

- Delta function and Gaussian smearing used at both source and sink for meson correlators
- Random wall sources in operator-meson correlators
- Correlators fitted between
  - $t_{\rm min} = 2 \sim 4$ and $t_{\rm max} = 16$ on coarse ensembles
  - $t_{\rm min} = 4 \sim 8$ and $t_{\rm max} = 24$ on fine ensembles
- Bayesian multiexponential fits with $t_{\rm min}$, $t_{\rm max}$ fixed and no. exponentials increased until saturation in results

Chiral and lattice spacing fits

Fit to lattice spacing dependence as described by Rachel Dowdall, but include chiral fit.

- Fit to

$$\Phi_q = f_{B_q}\sqrt{M_{B_q}} = \Phi_0\left(1 + \delta f_q + [\mathrm{analytic}]\right)\left(1 + [\mathrm{disc.}]\right)$$

- δf_q includes chiral logs using one-loop χPT and lowest order in $1/M$
- [analytic] - powers of m_{val}/m_c and m_{sea}/m_c, with m_c scale chosen for convenience
- [disc.] - powers of $(a/r_1)^2$ with expansion coefficient functions of aM_b or am_q