B-physics calculations from Lattice QCD by ETM Collaboration

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on behalf of ETM Collaboration

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Outline

- B-physics and Lattice
- ETMC computation
  1. A novel method based on Ratios of $(h\ell)$ observables using relativistic quarks and exact knowledge of static limit for appropriate ratios ($\rightarrow m_b$, decay constants)
  2. Interpolation to b for $(h\ell)$ observables calculated in charm-like region and in the static limit ($\rightarrow$ decay constants)
- Discussion on application of Ratio method to other observables
- Outlook

[ETMC] P.D., R.Frezzotti, G.Herdoiza, V.Lubicz, C.Michael, D.Palao, G.C.Rossi, F.Sanfilippo, A.Shindler, S.Simula, C.Tarantino, M.Wagner - JHEP 2012, arXiv:1107.1441

B-physics & Lattice

- Heavy Flavour Physics is a hot topic for the lattice community
  - Non Perturbative calculations are necessary for the interpretation of ...
  - LHCb and Flavour Factories' data
  - Lattice methods aim at the calculation of decay constants, form factors, Bag-parameters
- Experimental results + Lattice calculations $\Rightarrow$

  determination of **CKM** matrix elements
- ... and (hopefully) indications for **N**ew **P**hysics

  from high precision tests of the SM

European Twisted Mass (ETM) Collaboration

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Nf=2 twisted-mass formulation

- The Mtm lattice regularization of $N_f = 2$ QCD action reads ($r = 1$)

$$S^{\rm ph}_{N_f=2} = S^{\rm YM}_L + a^4 \sum_x \bar{\psi}(x) \left[\gamma\cdot\widetilde{\nabla} - i\gamma_5\tau^3 \left(-\frac{a}{2} r\nabla^*\nabla + M_{\rm cr}(r)\right) + \mu_q\right] \psi(x)$$

- ψ is a flavour doublet, $M_{\rm cr}(r)$ is the critical mass and τ^3 acts on flavour indices

- From the "physical" basis (where the quark mass is real), the non-anomalous

$$\psi = \exp(i\pi\gamma_5\tau^3/4)\chi\,, \quad \bar{\psi} = \bar{\chi}\exp(i\pi\gamma_5\tau^3/4)$$

transformation brings the lattice action in the so-called "twisted" basis

$$S^{\rm tw}_{N_f=2} = S^{\rm YM}_L + a^4 \sum_x \bar{\chi}(x) \left[\gamma\cdot\widetilde{\nabla} - \frac{a}{2} r\nabla^*\nabla + M_{\rm cr}(r) + i\mu_q\gamma_5\tau^3\right] \chi(x)$$

- Unlike the standard Wilson regularization, in Mtm Wilson case the subtracted Wilson operator $-\frac{a}{2}r\nabla^*\nabla + M_{\rm cr}(r)$ is "chirally rotated" w.r.t. the quark mass $\Longrightarrow$ offers important advantages...

Nf=2 twisted-mass formulation

- Automatic O(a) improvement for the physical quantities
- Dirac-Wilson matrix determinant is positive

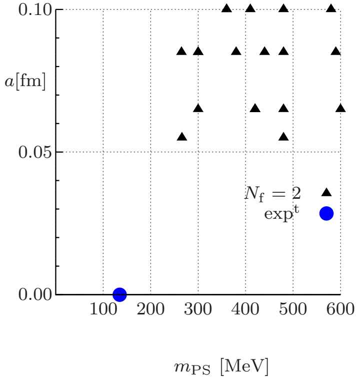
  and (lowest eigenvalue)^2 bounded from below by $\mu_q{}^2$
- Simplified (operator) renormalization ...
  - Multiplicative quark mass renormalization
  - No RC for pseudoscalar decay constant (PCAC)

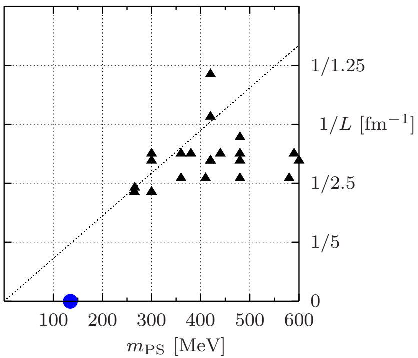
- O(a^2) breaking of parity and isospin

Frezzotti, Rossi, JHEP 2004

ETMC, Boucaud et al., Phys.Lett.B 2007

ETMC $N_f = 2$ simulations





- $a = \{0.054, \; 0.067, \; 0.085, \; 0.098\}$ fm
- $m_{ps} \in \{270, \; 600\}$ MeV
- $L \in \{1.7, \; 2.8\}$ fm , $\quad m_{ps}L \geq 3.5$

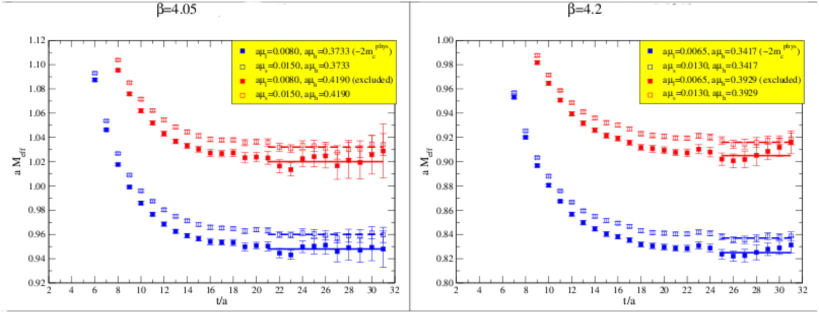
ETMC $N_f = 2$ (& heavier valence quark) simulations

β	$a\mu_\ell$	$a\mu_s$	$a\mu_h$
3.80	0.0080, 0.0110	0.0165, 0.0200 0.0250	0.2143, 0.2406, 0.2701, 0.3032 0.3403, 0.3819, 0.4287, 0.4812
3.90	0.0030, 0.0040, 0.0064, 0.0085, 0.0100	0.0150, 0.0180 0.0220	0.2049, 0.2300, 0.2582, 0.2898 0.3253, 0.3651, 0.4098, 0.4600
4.05	0.0030, 0.0060, 0.0080	0.0135, 0.0150, 0.0180	0.1663, 0.1867, 0.2096, 0.2352 0.2640, 0.2963, 0.3326, 0.3733
4.20	0.0020, 0.0065	0.0130, 0.0148 0.0180	0.1477, 0.1699, 0.1954, 0.2247 0.2584, 0.2971, 0.3417

- $\mu_h \in \{\sim m_c, \sim 2.3 m_c\}$

M_{eff} plateau quality

→ @ heavy q-mass $\gtrsim 2m_c$



Ratio method

- use relativistic quarks
- for each observable, b-mass point is reached through interpolation in $1/\mu_h$ from c-mass region to the ∞-mass (static) point.
- $c-$ mass region computation is reliable
- the b-mass point is related to its c-like counterpart by a chain of suitable, HQET-inspired ratios at successive values of μ_h
- ratios show smooth chiral and continuum limit
- Static limit value of ratios is exactly known

b-quark mass computation - 1

- observing that $\lim_{\mu_h^{\rm pole}\to\infty} \left(\frac{M_{h\ell}}{\mu_h^{\rm pole}} \right) = \text{constant}$
- construct

$$y(\bar{\mu}_h^{(n)}, \lambda; \bar{\mu}_\ell, a) \equiv \frac{M_{h\ell}(\bar{\mu}_h^{(n)}; \bar{\mu}_\ell, a)}{M_{h\ell}(\bar{\mu}_h^{(n-1)}; \bar{\mu}_\ell, a)} \cdot \frac{\bar{\mu}_h^{(n-1)}}{\bar{\mu}_h^{(n)}} \cdot \frac{\rho(\bar{\mu}_h^{(n-1)}, \mu^*)}{\rho(\bar{\mu}_h^{(n)}, \mu^*)} =$$

$$= \lambda^{-1} \frac{M_{h\ell}(\bar{\mu}_h^{(n)}; \bar{\mu}_\ell, a)}{M_{h\ell}(\bar{\mu}_h^{(n)}/\lambda; \bar{\mu}_\ell, a)} \cdot \frac{\rho(\bar{\mu}_h^{(n)}/\lambda, \mu^*)}{\rho(\bar{\mu}_h^{(n)}, \mu^*)}, \qquad n = 2, \ldots, N$$

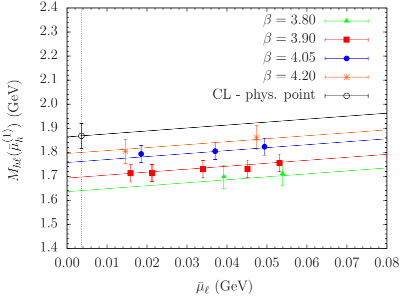
where $\frac{\bar{\mu}_h^{(n)}}{\bar{\mu}_h^{(n-1)}} = \lambda$ and $\mu_h^{\rm pole} = \rho(\bar{\mu}_h, \mu^*)\, \bar{\mu}_h(\mu^*)$ (with $\bar{\mu}_h \leftarrow \overline{\rm MS}$ scheme)
$\rho(\bar{\mu}_h, \mu^*)$ known up to N^3LO order

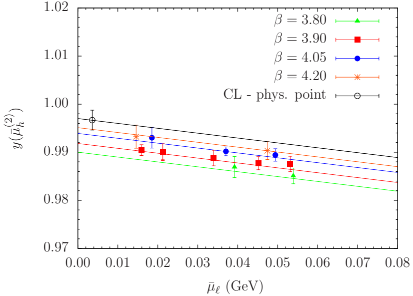
→ Static limit (*continuum*) well known

$$\lim_{\bar{\mu}_h\to\infty} y(\bar{\mu}_h, \lambda; \bar{\mu}_\ell, a = 0) = 1$$

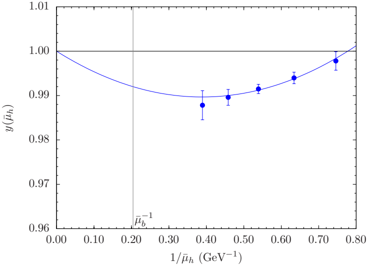
b-quark mass computation - 2

- *Triggering* input: $M_{h\ell}(\bar{\mu}_h^{(1)})$ PS meson mass (at $\bar{\mu}_h^{(1)} \sim m_c$) affected by (tolerably) small cutoff effects.
- Ratios $y(\bar{\mu}_h^{(n)}, \lambda; \bar{\mu}_\ell, a)$ have small discretisation errors





b-quark mass computation - 3



$y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2}$

(add cubic term leads to 0.5% change of the final value)

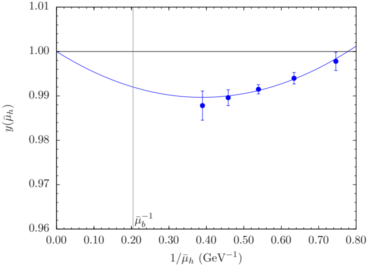
- Resolve the *master* equation

$$y(\bar{\mu}_h^{(2)})\, y(\bar{\mu}_h^{(3)}) \,\ldots\, y(\bar{\mu}_h^{(K+1)}) = \lambda^{-K} \frac{M_{hu/d}(\bar{\mu}_h^{(K+1)})}{M_{hu/d}(\bar{\mu}_h^{(1)})} \cdot \left[\frac{\rho(\bar{\mu}_h^{(1)}, \mu^*)}{\rho(\bar{\mu}_h^{(K+1)}, \mu^*)}\right]$$

- One can always adjust $(\lambda, \bar{\mu}_h^{(1)})$ such that $M_{hu/d}(\bar{\mu}_h^{(K+1)}) \equiv M_B^{expt}$ for K integer number
- our calculation: $\lambda = 1.1762$ and $\bar{\mu}_h^{(1)} = 1.14$ GeV (in $\overline{\rm MS}$, 2 GeV)

$\rightarrow$ $\bar{\mu}_b = \lambda^K \bar{\mu}_h^{(1)}$ $(K = 9)$

b-quark mass computation - 4

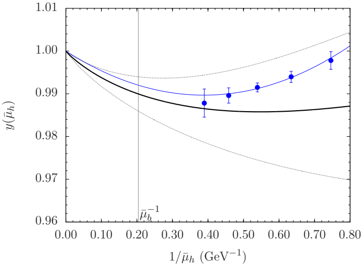


$$y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2}$$

- y deviates from its static value $\sim 1\%$ for $\bar{\mu_h} \leq m_b$.
- curvature denotes a large $1/\bar{\mu_h}^2$ contribution to ratios y.

$\rightarrow$ Is it an expected/reasonable behaviour?

b-quark mass computation - 5

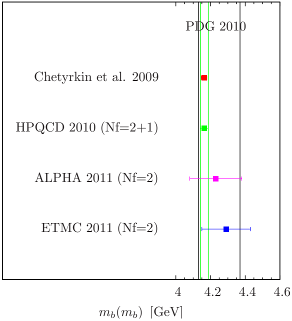


$$y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2}$$

- consider (HQET) $M_{h\ell} = \mu_h^{\rm pole} + \bar{\Lambda} - \frac{(\lambda_1+3\lambda_2)}{2}\frac{1}{\mu_h^{\rm pole}} + \mathcal{O}\left(\frac{1}{(\mu_h^{\rm pole})^2}\right)$
- and get $y = 1 - \bar{\Lambda}\frac{\lambda^{\rm pole}-1}{\mu_h^{\rm pole}} + \left(\frac{(\lambda_1+3\lambda_2)}{2}(\lambda^{\rm pole}+1) + \bar{\Lambda}^2\lambda^{\rm pole}\right)\frac{\lambda^{\rm pole}-1}{(\mu_h^{\rm pole})^2}$
  with $\lambda^{\rm pole} = \mu_h^{\rm pole}(\bar{\mu}_h)/\ \mu_h^{\rm pole}(\bar{\mu}_h/\lambda) = \lambda\,\rho(\bar{\mu}_h)/\rho(\bar{\mu}_h/\lambda)$
- use phenomenological estimates for HQET parameters, as e.g.
  $\bar{\Lambda} = 0.39(11)\ {\rm GeV}\,, \quad \lambda_1 = -0.19(10)\ {\rm GeV}^2\,, \quad \lambda_2 = 0.12(2)\ {\rm GeV}^2$
  [M. Gremm, A. Kapustin, Z. Ligeti, M.B. Wise, PhysRevLett 1996]

b-quark mass - results

- $m_b(m_b, \overline{\mathrm{MS}})|_{N_f=2} = 4.29(14)$ GeV
- compatible result for m_b (change only of $\sim 0.2\%$) *if* (*hs*)-data and M_{Bs}^{expt} used



- ETMC m_b error bugdet
  - stat. $\lesssim 0.5\%$
  - CL $\sim 1\%$
  - scale $\sim 1.5 - 2\%$
  - fit $\sim 2\%$
  - res. discr. error $\sim 1\%$

f_B and f_{Bs}

- note (HQET behaviour) $\lim_{\mu_h^{\rm pole}\to\infty} f_{h\ell(s)}\sqrt{\mu_h^{\rm pole}} = \text{constant}$
- construct

$$z(\bar\mu_h, \lambda; \bar\mu_\ell, a) \equiv \lambda^{1/2} \frac{f_{h\ell}(\bar\mu_h, \bar\mu_\ell, a)}{f_{h\ell}(\bar\mu_h/\lambda, \bar\mu_\ell, a)} \cdot \frac{C_A^{stat}(\mu_b^*, \bar\mu_h/\lambda)}{C_A^{stat}(\mu_b^*, \bar\mu_h)} \frac{[\rho(\bar\mu_h, \mu^*)]^{1/2}}{[\rho(\bar\mu_h/\lambda, \mu^*)]^{1/2}}$$

$$z_s(\bar\mu_h, \lambda; \bar\mu_\ell, \bar\mu_s, a) \equiv \lambda^{1/2} \frac{f_{hs}(\bar\mu_h, \bar\mu_\ell, \bar\mu_s, a)}{f_{hs}(\bar\mu_h/\lambda, \bar\mu_\ell, \bar\mu_s, a)} \cdot \frac{C_A^{stat}(\mu_b^*, \bar\mu_h/\lambda)}{C_A^{stat}(\mu_b^*, \bar\mu_h)} \frac{[\rho(\bar\mu_h, \mu^*)]^{1/2}}{[\rho(\bar\mu_h/\lambda, \mu^*)]^{1/2}}$$

with (HQET-QCD matching) $\Phi_{hs}(\mu_b^*) = \left[C_A^{stat}(\mu_b^*, \bar\mu_h)\right]^{-1} \cdot f_{h\ell(s)}\sqrt{M_{h\ell(s)}}|_{\bar\mu_h}$

$C_A^{stat}(\mu_b^*, \bar\mu_h)$ known up to N^{2}LO order.

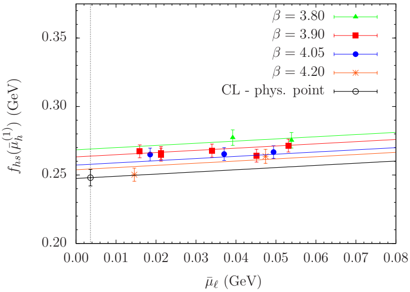
→ Static limit (*continuum*) well known

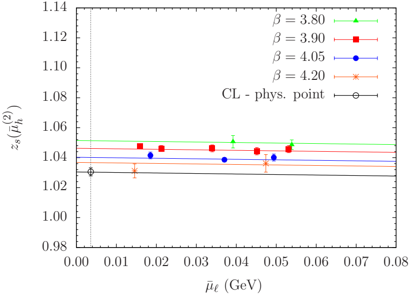
$$\lim_{\bar\mu_h\to\infty} z_{(s)}(\bar\mu_h, \lambda; \bar\mu_\ell, a=0) = 1$$

$$\lim_{\bar\mu_h\to\infty} \frac{z_s(\bar\mu_h, \lambda; \bar\mu_\ell, a=0)}{z(\bar\mu_h, \lambda; \bar\mu_\ell, a=0)} = 1$$

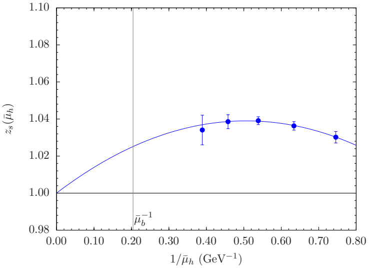
f_{Bs}

- Ratios $z_s(\bar{\mu}_h^{(n)}, \lambda; \bar{\mu}_\ell, a)$ have small discretisation effects (from 2% for the smallest to 3% for the largest heavy quark masses)
- Triggering $f_{hs}(\bar{\mu}_h^{(1)})$ pseudoscalar decay constant (at $\bar{\mu}_h^{(1)} \sim m_c$) affected by (tolerably) small cutoff effects.





f_{Bs}



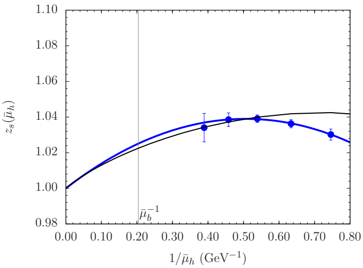
$z_s(\bar{\mu}_h) = 1 + \frac{\zeta_1}{\bar{\mu}_h} + \frac{\zeta_2}{\bar{\mu}_h^2}$

(add cubic term leads to less than 0.5% change to the final value)

- use $z_s(\bar{\mu}_h^{(2)})\, z_s(\bar{\mu}_h^{(3)}) \ \ldots \ z_s(\bar{\mu}_h^{(K+1)}) =$
  $$\lambda^{K/2} \frac{f_{hs}(\bar{\mu}_h^{(K+1)})}{f_{hs}(\bar{\mu}_h^{(1)})} \cdot \left[\frac{C_A^{stat}(\bar{\mu}_h^{(1)}, \mu^*)}{C_A^{stat}(\bar{\mu}_h^{(K+1)}, \mu^*)}\right] \left[\frac{\rho(\bar{\mu}_h^{(K+1)}, \mu^*)}{\rho(\bar{\mu}_h^{(1)}, \mu^*)}\right]^{1/2}$$
- for $\bar{\mu}_h^{(K+1)} \equiv m_b$, identify $f_{hs}(\bar{\mu}_h^{(K+1)})$ with f_{Bs}

$\rightarrow$ $f_{Bs} = 225(8)$ MeV (stat $\sim 1\%$, CL $\sim 1.5\%$, scale $\sim 1.5 - 2\%$, fit $\sim 2. - 2.5\%$)

f_{Bs}



$$z_s(\bar{\mu}_h) = 1 + \frac{\zeta_1}{\bar{\mu}_h} + \frac{\zeta_2}{\bar{\mu}_h^2}$$

(for $1/\bar{\mu}_h > 0.60$ estimated uncertainty on the black curve ~ 0.03)

- consider (HQET)

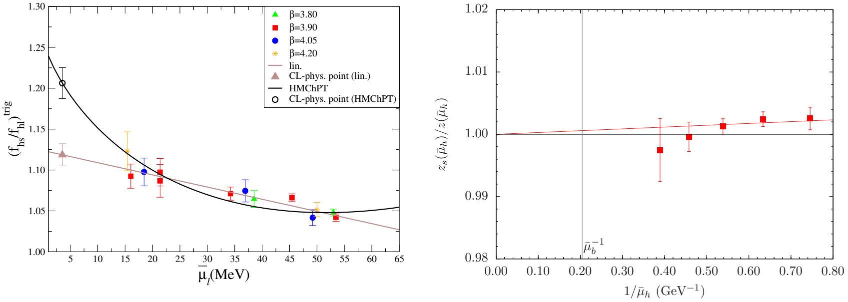
$$\Phi_{hs}(\bar{\mu}_h, \mu_b^*) = \frac{(f_{hs}\sqrt{M_{hs}})^{\rm QCD}}{C_A^{stat}(\bar{\mu}_h, \mu_b^*)} = \Phi_0(\mu_b^*)\left(1 + \frac{\Phi_1(\mu_b^*)}{\mu_h^{\rm pole}} + \frac{\Phi_2(\mu_b^*)}{(\mu_h^{\rm pole})^2}\right) + \mathcal{O}\left(\frac{1}{(\mu_h^{\rm pole})^3}\right)$$

- and get

$$y_s^{1/2}\, z_s = \frac{\Phi_{hs}(\bar{\mu}_h)}{\Phi_{hs}(\bar{\mu}_h/\lambda)} = 1 - \Phi_1 \frac{\lambda^{\rm pole} - 1}{\mu_h^{\rm pole}} - \left(\Phi_2(\lambda^{\rm pole} + 1) - \Phi_1^2 \lambda^{\rm pole}\right) \frac{\lambda^{\rm pole} - 1}{(\mu_h^{\rm pole})^2}$$

- use phenomenological values for HQET parameters
$\bar{\Lambda}_s = \bar{\Lambda} + M_{B_s} - M_B$, $\lambda_{1s} = \lambda_1$, $\lambda_{2s} = \lambda_2$, $\Phi_0 = 0.60\ {\rm GeV}^{3/2}$ and the estimates
$\Phi_1 = -0.48\ {\rm GeV}$, $\Phi_2 = 0.08\ {\rm GeV}^2$ ($\rightarrow$ values obtained from inputs at B_s and D_s.)

f_{Bs}/f_B



- $f_{hs}/f_{h\ell}$ @ triggering point ($\sim m_c$):

  HMChPT and linear fit vs. light quark mass $\rightarrow$ increase systematic uncertainty
- double ratio z_s/z shows no significant dependence on μ_l
- double ratio z_s/z *vs.* heavy quark mass: very weak dependence
- apply the master equation for z_s/z to reach the b-quark mass point ...

$\rightarrow$ $f_{Bs}/f_B = 1.18(02)(04)[05]$ (syst. error is mainly due to uncertainty in chiral extrapolation at triggering point)

Interpolation between relativistic and static data - 1

- Interpolation to b-quark mass of relativistic results $(m_c - 2.3 m_c)$ and static lattice results
- observables studied: $\Phi_{hs} = f_{hs}\sqrt{M_{hs}}$ and $\dfrac{\Phi_{hs}}{\Phi_{h\ell}} = \dfrac{f_{hs}}{f_{h\ell}}\sqrt{\dfrac{M_{hs}}{M_{h\ell}}}$
- combined continuum and chiral extrapolations at fixed heavy q-mass $\bar{\mu}_h = \{1.25, 1.50, 1.75, 2.00, 2.25\}$ MeV ...
- using various fit functions

$$\Phi_{h\ell}(a,\bar{\mu}_\ell,\bar{\mu}_h) = A_h\left[1 - \frac{3(1+3\hat{g}^2)}{4}\frac{2\,B_0\,\bar{\mu}_\ell}{(4\,\pi\,f_0)^2}\log\left(\frac{2\,B_0\,\bar{\mu}_\ell}{(4\,\pi\,f_0)^2}\right) + B_h\,\bar{\mu}_\ell + C_h\,a^2\right]$$

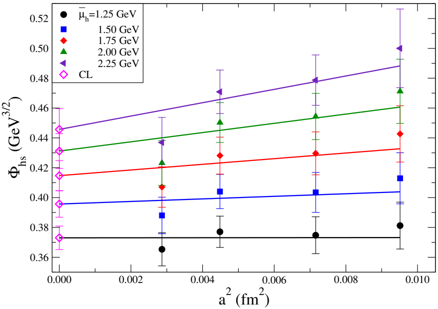
$$\Phi_{h\ell}(a,\bar{\mu}_\ell,\bar{\mu}_h) = A_h^{'}\left(1 + B_{1h}^{'}\,\bar{\mu}_\ell + (B_{2h}^{'}\,\bar{\mu}_\ell^2) + C_h^{'}\,a^2\right)$$

$$\Phi_{hs}(a,\bar{\mu}_\ell,\bar{\mu}_s,\bar{\mu}_h) = D_h\,\left(1 + E_h\,\bar{\mu}_\ell + F_h\,\bar{\mu}_s + G_h\,a^2\right)$$

- $\Phi_{hs(\ell)}(\mu_b^*) = \left[C_A^{stat}(\mu_b^*,\bar{\mu}_h)\right]^{-1}\,\Phi_{hs(\ell)}^{\rm QCD}(\bar{\mu}_h)$

Φ_{hs} – discr. errors

- $\mu_\ell \sim 50 MeV$, $\mu_s^{\rm phys}$



- reasonably <u>small</u> cutoff effects even for the heavier q-mass cases ... (partial cancelation of discr. errors between decay constant and meson mass)

- We find $\Phi_{hs}/\Phi_{h\ell}$ practically independent of latt. spacing

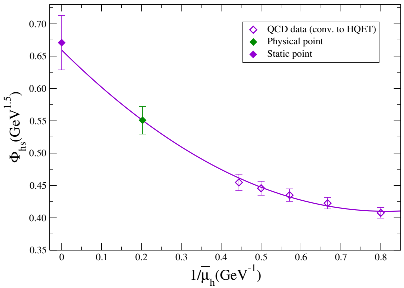
Interpolation between relativistic and static data - 2

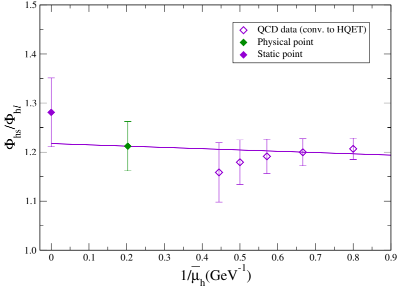
- HYP2 static action
  $S_h = a^4 \sum_x \bar{\psi}_h(x) D_0 \psi_h(x) = a^3 \sum_x \bar{\psi}_h(x)[\psi_h(x) - V^{\dagger}_{\mathrm{HYP}}(x - a\hat{0}, 0)\, \psi_h(x - a\hat{0})]$
  [M. Della Morte, A. Shindler, R. Sommer, JHEP 2005]
- computation of $\Phi^{\mathrm{stat}}_{B_q}$ at two latt. spacings ($a \sim 0.085$, 0.067 fm).
- Perturbatively calculated $Z^{\mathrm{stat}} \rightarrow$ attribute conservatively large uncertainty ($\sim$ half of the deviation from unity, 5-10%). [B. Blossier Phys.Rev.D 2012]
- $\Phi^{\mathrm{stat}}_{Bs}(\mu_b^*) = 0.67(4)\,\mathrm{GeV}^{3/2} \qquad \text{and} \qquad \Phi^{\mathrm{stat}}_{Bs}/\Phi^{\mathrm{stat}}_{B} = 1.28(7)$

$\rightarrow$ using ratio method procedure, we estimate
$\Phi^{\mathrm{stat}}_{Bs}/\Phi^{\mathrm{stat}}_{B} = 1.20(5)$ (... nice check)

Interpolation between relativistic and static data - 3

- Dependence of Φ_{hs} and $\Phi_{hs}/\Phi_{h\ell}$ on $1/\bar{\mu}_h$ and interpolation to b-quark mass value





Results from both methods

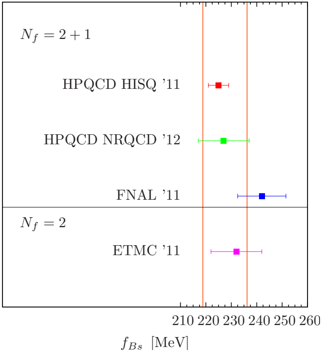
f_{Bs} [MeV]		f_{Bs}/f_B	
Ratio	Interpolation	Ratio	Interpolation
225(8)	238(10)	1.18(4)	1.19(5)
232(10)		1.19(5)	

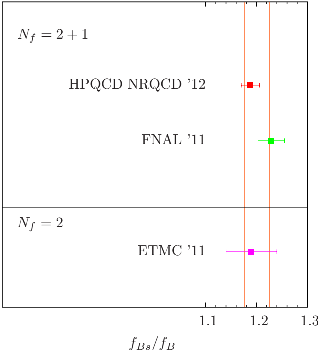
$$f_B = f_{Bs}/(f_{Bs}/f_B) = 195(12) \text{ MeV}$$

⋆ Decay constants for D & D_s mesons :

$$f_D = 212(8)\,\mathrm{MeV}\,, \qquad f_{Ds} = 248(6)\,\mathrm{MeV}, \qquad \frac{f_{Ds}}{f_D} = 1.17(5)$$

Results & Comparisons - I



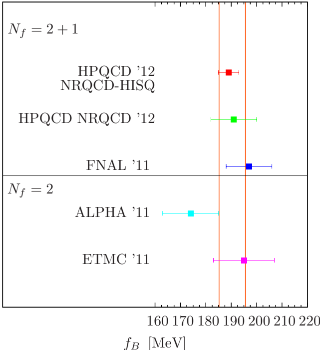


- $f_{Bs}(\mathrm{ETMC}) = 232(10)$ MeV
- $f_{Bs}/f_B(\mathrm{ETMC}) = 1.19(05)$ MeV
  - ⋆ vertical lines show average over $N_f = 2+1$ results ...

    no significant dependence on dynamical strange degree of freedom

    (within the present precision)

Results & Comparisons - II



- $f_B(\mathrm{ETMC}) = 195(12)$ MeV

Sketch of Ratio method for $\Delta B = 2$ operators

- QCD

$$
\begin{aligned}
O_1 &= [\bar{b}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha][\bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta] \\
O_2 &= [\bar{b}^\alpha (1-\gamma_5) q^\alpha][\bar{b}^\beta (1-\gamma_5) q^\beta] \\
O_3 &= [\bar{b}^\alpha (1-\gamma_5) q^\beta][\bar{b}^\beta (1-\gamma_5) q^\alpha] \\
O_4 &= [\bar{b}^\alpha (1-\gamma_5) q^\alpha][\bar{b}^\beta (1+\gamma_5) q^\beta] \\
O_5 &= [\bar{b}^\alpha (1-\gamma_5) q^\beta][\bar{b}^\beta (1+\gamma_5) q^\alpha]
\end{aligned}
$$

- HQET

$$
\begin{aligned}
\tilde{O}_1 &= [\bar{h}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha][\bar{h}^\beta \gamma_\mu (1-\gamma_5) q^\beta] \\
\tilde{O}_2 &= [\bar{h}^\alpha (1-\gamma_5) q^\alpha][\bar{h}^\beta (1-\gamma_5) q^\beta] \\
\tilde{O}_3 &= -\tilde{O}_2 - (1/2)\tilde{O}_1 \\
\tilde{O}_4 &= [\bar{h}^\alpha (1-\gamma_5) q^\alpha][\bar{h}^\beta (1+\gamma_5) q^\beta] \\
\tilde{O}_5 &= [\bar{h}^\alpha (1-\gamma_5) q^\beta][\bar{h}^\beta (1+\gamma_5) q^\alpha]
\end{aligned}
$$

⋆ Matching between QCD and HQET operators:

$$[\mathbf{W}^T_{QCD}(\mu_h, \mu)]^{-1} \langle \vec{O}(\mu) \rangle_{\mu_h} = C(\mu_h)\, [\mathbf{W}^T_{HQET}(\mu_h, \tilde{\mu})]^{-1} \langle \vec{\tilde{O}}(\tilde{\mu}) \rangle + \mathcal{O}(1/\mu_h) + \ldots$$

$[\mathbf{W}^T_{\ldots}(\mu_1, \mu_2)]^{-1}$: evolution 5x5 matrices

$C(\mu_h)$: matching matrix

(e.g. D.Becirevic, V.Gimenez, G.Martinelli, M.Papinutto, J.Reyes, JHEP 2002)

Sketch of Ratio method for $\Delta B = 2$ operators

- set
  $\vec{\Theta}(\mu_h, \mu, \tilde{\mu}) \equiv (\mathbf{W}^T_{QCD}(\mu_h, \mu) C(\mu_h) [\mathbf{W}^T_{HQET}(\mu_h, \tilde{\mu})]^{-1})^{-1} \langle \vec{O}(\mu) \rangle_{\mu_h} =$
  $\langle \vec{\tilde{O}}(\tilde{\mu}) \rangle + \mathcal{O}(1/\mu_h) + \ldots$
- $[\mathbf{W}^T_{\ldots}(\mu_1, \mu_2)]^{-1}$ and $C(\mu_h)$ are $(3 \times 3 \oplus 2 \times 2)$ block-diagonal matrices
- For B_{Bq} case, calculate ratios at successive values of $\mu_h^{(n)} = \lambda \mu_h^{(n-1)}$
  (need only 3×3 matrices):
  $Z_\Theta^{(n)} = \dfrac{\Theta_j(\mu_h^{(n)}, \mu, \tilde{\mu})}{\Theta_j(\mu_h^{(n-1)}, \mu, \tilde{\mu})}$ for $j = 1, 2, 3$
  and construct the appropriate ratio chain.
- up to LL O_1 and $\tilde{O}_1$ renormalise multipicatively; need only $j = 1$
- *computation in progress, using smeared meson fields*

Outlook - I

- Ratio method uses relativistic quarks and a well-known static limit. No static calculation needed.
- Ratio method can be used for all observables whose static limit behaviour is known from HQET.
- Interpolation method can be employed for observables whose static limit value can be computed with the necessary accuracy (... $\rightarrow$ more systematics to be kept under control).
- ETMC results for m_b, f_{Bq} and f_{Bs}/f_B are in the same ballpark with results from other collaborations.

Outlook - II

- use relativistic quarks and smeared correlators (*on going analysis*) in order to obtain better (safer) signals for heavier quark massees.
- smaller statistical errors, higher accuracy in the "$1/\mu_h$" fit; dominant systematics come from scale evaluation in computing m_b, f_{Bq} and chiral extrapolation (e.g. f_{Bs}/f_B).
- Study in progress of B-bag parameters using the ratio method and semileptonic B-decay FF.
- Repeat/extend the study to $N_f = 2 + 1 + 1$ ensembles.

Thank you for your attention !!

Thanks for discussions during the preparation of this talk to Roberto Frezzotti, Gregorio Herdoiza, Giancarlo Rossi

Back up slide - 1

$\beta = 3.90$